Covariant action and equations of motion for the eleven dimensional multiple M0-brane system

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We study the properties of the covariant supersymmetric and κ-symmetric action for a system of N nearly coincident M0-branes (mM0 system) in flat 11-dimensional (11D) superspace and obtain supersymmetric equations for this dynamical system. Although a single M0-brane is the massless 11D superparticle, center-of-energy motion of the mM0 system is characterized by a non-negative constant mass M constructed from the matrix fields describing the relative motion of mM0 constituents. We show that a bosonic solution of the mM0 equations can be supersymmetric iff this effective mass vanishes, $M^2 = 0$, and that all the supersymmetric bosonic solutions preserve just one half of the 11D supersymmetry.

I. INTRODUCTION

In [1] it was motivated that an approximate description of the system of nearly coincident Dirichlet p-branes (Dp-branes) is provided by maximal $d = p + 1$ dimensional supersymmetric Yang-Mills (SYM) theory with the gauge group $U(N)$, which can be obtained by the dimensional reduction of D = 10 U(N) SYM theory. This includes $D − p − 1$ Hermitian matrices of scalar fields, the diagonal elements of which describe the positions of different Dp-branes, while the off-diagonal elements account for the strings stretched between different Dp-branes.

As a single Dp-brane was known to be described by a sum of supersymmetric Dirac-Born-Infeld action, providing a nonlinear generalization of the U(1) Yang-Mills action, and a Wess-Zumino term (see [2] and Refs. therein), it was natural to search for a nonlinear generalization of the non-Abelian SYM action, providing a more complete nonlinear description of the system of nearly coincident Dp-branes. For the bosonic limit of multiple nearly coincident Dp-branes (mDp system), the most popular description is provided by the Myers "dielectric brane" action [3]. This was obtained by a chain of T-duality transformations from the 10D symmetric trace non-Abelian Born-Infeld action, proposed by Tseytlin [4] for purely bosonic limit of the system of multiple space-time filling D9-branes (mD9 system). Both the actions of [3,4] resisted the attempts to construct their supersymmetric generalizations for many years; in addition, the Myers action does not possess the Lorentz symmetry.

The supersymmetric and Lorentz covariant description of the mDp system was reached in [5–7] in the frame of the so-called "boundary fermion approach." However, this description is provided at the "minus one quantization level," which means that to reach the description of the mDp system similar to the description of Dp-branes in e.g. [2], one has to perform quantization of the dynamical system. This task is nontrivial and has not been solved in

a complete form,[1] which motivated further attempts to obtain a possibly approximate but Lorentz covariant and supersymmetric description of the mDp system going beyond the SYM approximation (see e.g. [8]). Only for the case of the mD0 system does a nonlinear, supersymmetric and Lorentz-invariant candidate for the wanted mD0 action exist [9].

As Dp-branes with $p = 0$, 2, 4 can be obtained by a dimensional reduction of the 11-dimensional M0-, M2- and M5-branes, it is natural to expect that the corresponding mDp system can be obtained from the respective mMp system. However, for the case of mM5, even the question of what should be a counterpart of the very low-energy approximate SYM description is still obscure (see e.g. [10] for related study and references). For the case of the mM2-brane, such a problem was unsolved for many years, but relatively recently the $d = 3$ $\mathcal{N} = 8$ supersymmetric BLG model [11] based on a three-algebra (see [12] and Refs. therein) instead of Lie algebra, and then a more conventional ABJM model [13] (with $SU(N) \times SU(N)$ gauge symmetry and only $\mathcal{N} = 6$ manifest supersymmetries) were found and accepted for this role.

As far as the multiple M0-brane (mM0) system is considered, a purely bosonic candidate was constructed in [14] as the 11D generalization of the Myers "dielectric D0-brane" action. On the other hand, an approximate but supersymmetric and Lorentz covariant equations of motion

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[1]For the bosonic limit, i.e. when the boundary fermions are the only non-vanishing fermions, it was shown in [6] that replacing these by suitable Dirac matrices, replacing the Poisson brackets by (anti)commutators and replacing the integration over the boundary fermions by symmetric trace of the product of matrices one obtains a result which agrees with that of [3]. The supersymmetrization of the action of [6] was constructed in [7] with the use of Bernstein-Leites integration over the boundary fermion coordinates. It is invariant under κ-symmetry with parameters dependent on the boundary fermions, which suggests it should be a matrix after quantization of the boundary fermion sector. Such a quantization of supersymmetric action of [7] has not been developed yet.

for mM0-system in flat target 11D superspace were obtained in [15] in the frame of superembedding approach (see [16,17] as well as [18,19] and Refs. therein). The generalization of these equations for the case of mM0-system in curved 11D supergravity superspace, which describes the generalization of the M(atrix) theory [20] (see also earlier [21]) for the case of its interaction with arbitrary supergravity background, were presented and studied in [22]. In [23] it was shown that in the case of 11D pp-wave background these equations reproduce (in an approximation) the so-called BMN matrix model proposed for this background by Berenstein, Maldacena and Nastase in [24]. This result, confirming that the equations of [22] describe the Matrix theory interacting with supergravity background, also have shown that, due to the superspace origin of these equations, their applications for a certain, even purely bosonic supergravity background are sufficiently complicated: it requires the lifting of the bosonic supersymmetric solution of the supergravity equations till the complete superfield solution of the 11D superspace supergravity constraints [25]. This made desirable to find an action which reproduces the Matrix model equations of [22] or their generalizations.

For the case of mM0 system in flat target superspace such an action was proposed in [26], where it was shown that it possesses local $\mathcal{N} = 16$ 1d supersymmetry. The main aim of this paper is to derive and to study equations of motion of the mM0 system described by that action. We will study the properties of the supersymmetric solutions of these equations, show that their center-of-energy sector is similar to the solution of the equation for a single M0-brane, and also present two examples of nonsupersymmetric solutions with different properties of the center-of-energy motion. It was noticed in [26] that all the supersymmetric solutions of the mM0 equations are characterized by vanishing of the effective mass of the center-of-energy motion. Here we not only reproduce this result proving that $M^2 = 0$ appears as the BPS equation, but also show that all the supersymmetric solutions of mM0 equations preserve just $1/2$ of the 11D supersymmetry, so that all the supersymmetric mM0 BPS states are $1/2$ BPS.

The paper is organized as follows. In Secs. II and III, we review the spinor moving frame formulation of single M0-brane, this is to say of 11D massless superparticle. We describe there the moving frame and 11D spinor moving frame variables (Secs. II C and II D and also II E and II F), discuss the M0-brane equations of motion (Sec. III) obtained from the spinor moving frame action (of Sec. II A) and show (in Sec. III A) that supersymmetric solution of these equations preserve just $1/2$ (16 of 32) of the 11D supersymmetries i.e. describe $\frac{1}{2}$ BPS states. We also discuss there the irreducible κ-symmetry of the spinor moving frame formulation of superparticle (Sec. II B), stress its identification with the local worldline supersymmetry[2] and

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describe (in Sec. III) the composite 1d $\mathcal{N} = 16$ supergravity multiplet corresponding to it. This supergravity induced by embedding of the M0 worldline into the target 11D superspace allows to make local the originally global supersymmetry of, say, 1d $\mathcal{N} = 16$ supersymmetric Yang-Mills (SYM) theory living on the worldline and plays an important role in the mM0 action of [26], which is the subject of our investigation here. This action (described in Sec. IV) is given by the sum of the 1d $\mathcal{N} = 16$ SYM action coupled to the induced 1d $\mathcal{N} = 16$ supergravity and of the same spinor moving frame action functional as we have used to describe the single M0, which now describes the center-of-energy motion of the mM0 system. In Sec. V we vary this mM0 action and obtain the set of covariant and supersymmetric equations of motion for mM0 system. The properties of the solutions of these equations are studied in Secs. VI, VII, and VIII. Particularly, Sec. VII is devoted to supersymmetric solutions of mM0 equations. We show there that all these are characterized by vanishing center-of-energy effective mass, $M^2 = 0$. In Sec. VIII we present two examples of nonsupersymmetric solutions with $M^2 \neq 0$. We conclude and discuss our results in Sec. IX. Appendixes A and C contain the complete lists of the equations of motion for single M0-brane and for the mM0 systems, respectively. Appendix B collects the properties of the moving frame and spinor moving frame variables.

II. SINGLE M0-BRANE IN SPINOR MOVING FRAME FORMULATION

In this section we review the spinor moving frame formulation of single M0-brane, this is to say 11D massless superparticle, developed in [28], and summarize the properties of spinor moving frame variables used in this formulation as well as in the description of multiple M0-brane (mM0) system.

A. Twistorlike spinor moving frame action and its irreducible κ symmetry

The spinor moving frame action of the M0-brane reads (see [28] and also [29–34])

$$S_{M0} = \int_{W^1} \rho^\# \hat{E}^= = \int_{W^1} \rho^\# u_a^= E^a(\hat{Z}) \qquad (2.1)$$

$$= 1/16 \int_{W^1} \rho^\# (v_q^- \Gamma_a v_q^-) \hat{E}^a. \qquad (2.2)$$

In the first line of this equation, (2.1), $\rho^\#(\tau)$ is a Lagrange multiplier,

$$\hat{E}^a := E^a(\hat{Z}) = d\hat{Z}^M(\tau) E_M^a(\hat{Z}) =: d\tau \hat{E}_\tau^a(Z) \quad (2.3)$$

is the pull-back of the bosonic supervielbein of the 11D target superspace $(a = 0, 1, \ldots, 10)$, $E^a = E^a(Z) = dZ^M E_M^a(Z)$, to the worldline W^1 parametrized by proper

time τ. In the case of flat target superspace the supervielbein can be chosen in the form[3,4]

$$E^a = \Pi^a = dx^a - id\theta\Gamma^a\theta, \qquad E^\alpha = d\theta^\alpha. \tag{2.4}$$

Finally, $\hat{E}^= = \hat{E}^a u_a^=$ and $u_a^= = u_a^=(\tau)$ is a lightlike 10D vector, $u^{=a}u_a^= = 0$.

One can write the action (2.1) in a probably more conventional from, extracting $d\tau$ measure from the pullback of the supervielbein 1-form (see (2.3))

$$S_{M0} = \int_{W^1} d\tau\rho^\# \hat{E}_\tau^= = \int_{W^1} d\tau\rho^\# \partial_\tau \hat{Z}^M(\tau) E_M^a(\hat{Z}(\tau)) u_a^=(\tau). \tag{2.5}$$

We however, prefer to hide $d\tau$ inside of differential form, define the Lagrangian 1-form by $\mathcal{L}_1 = d\tau\mathcal{L}_\tau$, and write our actions as integral of this 1–form over the worldline, $\int_{W^1} \mathcal{L}_1$, rather than as an integral over $d\tau$ of a density, $\int d\tau\mathcal{L}_\tau$.

If we were stoping at this stage, one can easily observe that the action (2.1) can be obtained from the first order form of 11D version of the Brink-Schwarz action,

$$S_{BS} = \int_{W^1} \left(p_a\hat{E}^a - \frac{e}{2} p_a p^a d\tau \right), \tag{2.6}$$

by solving the constraints $p_a p^a = 0$ (equations of motion for Lagrange multiplier $e(\tau)$) and substituting them back to the action. Furthermore, one might wonder why the solution $p_a = \rho^\# u_a^=$ is written with a multiplier $\rho^\#(\tau)$ instead of just stating that it has the form of $S = \int_{W^1} p_a\hat{E}^a$ with p_a constrained by $p_a p^a = 0$. We will answer that question a bit later, just announcing now that $\rho^\#$ is a kind of Stückelberg variable allowing to introduce an $SO(1,1)$ gauge symmetry; although looking artificial at this stage, this symmetry allows to clarify the group theoretical meaning of $u_a^=$ and also of the set of 16 constrained spinors appearing in the second representation of S_{M0}, Eq. (2.2).

The lightlike vector $u_a^=$ can be considered as a composite of (any of) the 16 spinors $v_q^{-\alpha}$ provided these are constrained by

$$v_q^{-\alpha}(\Gamma^a)_{\alpha\beta}v_p^{-\beta} = \delta_{qp}u_a^= \tag{2.7a}$$

$$2v_q^{-\alpha}v_q^{-\beta} = u_a^=\tilde{\Gamma}^{a\alpha\beta}. \tag{2.7b}$$

Notice that the traces of both equations give $16u_a^= = v_q^{-\alpha}(\Gamma^a)_{\alpha\beta}v_q^{-\beta}$ which can be read off (2.2) and (2.1). The set of spinors $v_q^{-\alpha}$ constrained by (2.7) are called spinor moving frame variables (hence the name 'spinor moving frame' for the formulation of superparticle mechanics based on the action (2.1) and (2.2)). Before discussing their origin and nature (in Sec. II C) [29,30,33,35,36], in the next Sec. II B we would like to try to convince the reader in the usefulness of these 'square roots' of the lightlike vector $u_a^=$.

B. Irreducible κ symmetry of the spinor moving frame action

The action (2.1) and (2.2) is invariant under the following local fermionic κ symmetry transformations:

$$\delta_\kappa\hat{x}^a = -i\hat{\theta}\Gamma^a\delta_\kappa\hat{\theta}, \qquad \delta_\kappa\hat{\theta}^\alpha = \epsilon^{+q}(\tau)v_q^{-\alpha},$$
$$\delta_\kappa\rho^\# = 0, \qquad \delta_\kappa u_a^= = 0 \Leftarrow \delta_\kappa v_q^{-\alpha} = 0. \tag{2.8}$$

This symmetry is irreducible in the sense of that each of 16 fermionic parameters[5] $\epsilon^{+q}(\tau)$ acts efficiently on the variables of the theory and can be used to remove some components of fermionic field $\hat{\theta}^\alpha(\tau)$ thus reducing the number of the degrees of freedom in it to 16 (while $\alpha = 1, \ldots, 32$).

In contrast, the κ symmetry of the original Brink-Schwarz superparticle action (2.6) [38],

$$\delta_\kappa\hat{x}^a = -i\hat{\theta}\Gamma^a\delta_\kappa\hat{\theta}, \qquad \delta_\kappa\hat{\theta}^\alpha = p_a\tilde{\Gamma}^{a\alpha\beta}\kappa_\beta(\tau),$$
$$\delta_\kappa e = -4i\kappa_\beta d\hat{\theta}^\beta, \tag{2.9}$$

is infinitely reducible. It is parametrized by the 32-component fermionic spinor function $\kappa_\beta(\tau)$, which however is not acting efficiently on the variable of the theory.[6]

The irreducible κ symmetry of the spinor moving frame formulation (2.8) can be obtained from the infinitely

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[3]The action (2.1) and (2.2) makes sense when supervielbein $E^a = dZ^M E_M^a(Z)$ obeys the 11D superspace supergravity constraints [25]. In this paper we will restrict ourselves by the case of flat target superspace, described by Eqs. (2.4).

[4]We use the (real) matrices $\Gamma_{\alpha\beta}^a = \Gamma_{\beta\alpha}^a = \Gamma_\alpha^{a\gamma}C_{\gamma\beta}$ and $\tilde{\Gamma}_a^{\alpha\beta} = \tilde{\Gamma}_a^{\beta\alpha} = C^{\alpha\gamma}\Gamma_\gamma^{a\beta}$ constructed as a product of 11D Dirac matrices $\Gamma_\beta^{a\gamma}$ (obeying $\Gamma^a\Gamma^b + \Gamma^b\Gamma^a = 2\eta^{ab}I_{32\times32}$) with, respectively, the 11D charge conjugation matrix $C_{\gamma\beta} = -C_{\beta\gamma}$ and its inverse $C^{\alpha\beta} = -C^{\beta\alpha}$. Both $\Gamma_\beta^{a\gamma}$ and $C_{\beta\gamma}$ are pure imaginary in our mostly minus notation $\eta^{ab} = \text{diag}(1, -1, \ldots, -1)$.

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[5]The κ symmetry was discovered in [37,38] and was shown to coincide with the local worldline supersymmetry in [27]. Our notation $\epsilon^{+q}(\tau)$ for the (irreducible) κ-symmetry parameter is an implicit reference to this later result which will be useful in the discussion below.

[6]Roughly speaking, due to the constraint $p_a p^a = 0$, κ_α and $\kappa_\alpha + p_a\tilde{\Gamma}_{\alpha\beta}^a\kappa^{(1)\beta}(\tau)$ produce the same κ variation of the Brink-Schwarz superparticle variables. One says that the above transformation has a null vector $\kappa^{(1)\beta}(\tau)$ and, hence, the symmetry is reducible. But this is not the end of story. One easily observes that $\kappa^{(1)\beta}(\tau)$ and $\kappa^{(1)\beta}(\tau) + p_a\tilde{\Gamma}^{a\alpha\beta}\kappa_\beta^{(2)}(\tau)$, with an arbitrary $\kappa_\beta^{(2)}(\tau)$, makes the same change of the parameter κ_α. This implies that there is a null-vector for null-vector and that the κ-symmetry possesses at least the second rank of reducibility. Furthermore, one sees that this process of finding higher null vectors can be continued up to infinity (next stages are completely equivalent to the first two ones) so that one speaks about infinite reducibility of the κ symmetry of the Brink-Schwarz superparticle. The number of the fermionic degrees of freedom which can be removed by κ symmetry is then calculated as an infinite sum $32 - 32 + 32 - 32 + \cdots = 32 \cdot (1 - 1 + 1 - 1 + \cdots) = 32 \cdot \lim_{q\to1}(1 - q + q^2 - \cdots) = 32 \cdot \lim_{q\to1}\frac{1}{1+q} = 16$.

reducible (2.9) by substituting for p_a the solution $p_a = \rho^\# u_a^=$ of the constraint $p_a p^a = 0$; furthermore, using (2.7), we find

$$\epsilon^{+q} = 2\rho^\# v_q^{-\alpha} \kappa_\alpha. \tag{2.10}$$

Let us stress that this relation, as well as the transformation rules of the irreducible κ symmetry (2.8), necessarily involves the constrained spinors $v_q^{-\alpha}$. Thus the covariant irreducible form of the κ symmetry is a characteristic property of the spinor moving frame and similar (twistor-like) formulations of the superparticle mechanics.[7]

The importance of the κ symmetry is related to the fact that it reflects a part of target space supersymmetry which is preserved by ground state of the brane under consideration [39–41] thus insuring that it is a BPS state. Its irreducible form, reached in the frame of spinor moving frame formulation, is useful not only for clarifying its nature as worldline supersymmetry [27], but also for finding the corresponding induced supergravity multiplet which is necessary for constructing the mM0 action. To address this issue we need to comment on some properties of moving frame and spinor moving frame variables.

C. Moving frame and spinor moving frame

To clarify the origin and nature of the set of spinors $v_q^{-\alpha}$ which provide the square root of the lightlike vector $u_a^=$ in the sense of Eqs. (2.7), and which have been used to present the κ symmetry in the irreducible form (2.8), it is useful to complete the lightlike vector $u_a^=$ till the moving frame matrix,

$$U_b^{(a)} = \left(\frac{u_b^= + u_b^\#}{2}, u_b^i, \frac{u_b^\# - u_b^=}{2} \right) \in SO(1, 10) \tag{2.11}$$

($i = 1, \ldots, 9$). The statement that this matrix is an element of the $SO(1, 10)$, having been made in (2.11), is tantamount to saying that

$$U^T \eta U = I, \qquad \eta^{ab} = \text{diag}(+1, -1, \ldots, -1), \tag{2.12}$$

which in its turn implies that the moving frame vectors obey the following set of constraints [42]:

$$u_a^= u^{a=} = 0, \qquad u_a^= u^{ai} = 0, \qquad u_a^= u^{a\#} = 2, \tag{2.13}$$

$$u_a^\# u^{a\#} = 0, \qquad u_a^\# u^{ai} = 0, \tag{2.14}$$

$$u_a^i u^{aj} = -\delta^{ij}. \tag{2.15}$$

The 11D spinor moving frame variables (appropriate for our case) can be defined as 16×32 blocks of the Spin(1, 10) valued matrix,

$$V_{(\beta)}{}^\alpha = \begin{pmatrix} v_q^{+\alpha} \\ v_q^{-\alpha} \end{pmatrix} \in \text{Spin}(1, 10), \tag{2.16}$$

double covering the moving frame matrix (2.11). This statement implies that the similarity transformations with the matrix V leave the 11D charge conjugation matrix invariant and, when applied to the 11D Dirac matrices, produce the same effect as 11D Lorentz rotation with matrix U,

$$VCV^T = C, \tag{2.17}$$

$$V\Gamma_b V^T = U_b^{(a)} \Gamma_{(a)}, \tag{2.18}$$

$$V^T \tilde{\Gamma}^{(a)} V = \tilde{\Gamma}^b u_b^{(a)}. \tag{2.19}$$

The two seemingly mysterious constraints (2.7) appear as a 16×16 block of the second of these relations, (2.17), and as a component $V^T \tilde{\Gamma}^= V = \tilde{\Gamma}^b u_b^=$ of the third one, (2.19) (with an appropriate representation of the 11D Gamma matrices). The other blocks/components of these constraints involve the second set of constrained spinors,

$$v_q^+ \Gamma_a v_p^+ = u_a^\# \delta_{qp}, \qquad v_q^- \Gamma_a v_p^+ = -u_a^i \gamma_{qp}^i, \tag{2.20}$$

$$2 v_q^{+\alpha} v_q^{+\beta} = \tilde{\Gamma}^{a\alpha\beta} u_a^\#, \qquad 2 v_q^{-(\alpha} v_q^{+\beta)} = -\tilde{\Gamma}^{a\alpha\beta} u_a^i. \tag{2.21}$$

Here γ_{qp}^i are the 9d Dirac matrices; they are real, symmetric, $\gamma_{qp}^i = \gamma_{pq}^i$, and obey the Clifford algebra

$$\gamma^i \gamma^j + \gamma^j \gamma^i = 2\delta^{ij} I_{16\times16}, \tag{2.22}$$

as well as the following identities

$$\gamma_{q(p_1}^i \gamma_{p_2 p_3)}^i = \delta_{q(p_1} \delta_{p_2 p_3)}, \tag{2.23}$$

$$\gamma_{q(q'}^{ij} \gamma_{p')p}^i + \gamma_{p(q'}^{ij} \gamma_{p')q}^i = \gamma_{q'p'}^j \delta_{qp} - \delta_{q'p'} \gamma_{qp}^j. \tag{2.24}$$

Thus $v_q^{-\alpha}$ and $v_q^{+\alpha}$ can be identified as square roots of the lightlike vectors $u_a^=$ and $u_a^\#$, respectively, while to construct u_a^i one needs both these sets of constrained spinors.

The first constraint, Eq. (2.17), implies that the inverse spinor moving frame matrix

$$V_\alpha^{(\beta)} = (v_{\alpha q}{}^+, v_{\alpha q}{}^-) \in \text{Spin}(1, 10),$$

$$V_{(\beta)}{}^\gamma V_\gamma^{(\alpha)} = \delta_{(\beta)}{}^{(\alpha)} = \begin{pmatrix} \delta_{qp} & 0 \\ 0 & \delta_{qp} \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} v_q^{-\alpha} v_{\alpha p}{}^+ = \delta_{qp} = v_q^{+\alpha} v_{\alpha p}{}^-, \\ v_q^{-\alpha} v_{\alpha p}{}^- = 0 = v_q^{+\alpha} v_{\alpha p}{}^+, \end{cases} \tag{2.25}$$

can be constructed from $v_q^{\mp\alpha}$,

$$v_{\alpha q}{}^- = iC_{\alpha\beta} v_q^{-\beta}, \qquad v_{\alpha q}{}^+ = -iC_{\alpha\beta} v_q^{+\beta}. \tag{2.26}$$

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[7]Notice that in D = 3, 4 and 6 dimensions the counterpart of $v_q^{-\alpha}$ can be chosen to be unconstrained spinors; see references in e.g. [28,29,31].

D. Cartan forms, differentiation and variation of the (spinor) moving frame variables

To vary the action and to clarify the structure of the equations of motion one needs to vary and to differentiate the moving frame and spinor moving frame variables. As these are constrained, at the first glance this problem might look complicated, but, actually, this is not the case. The clear group theoretical structure beyond the moving frame and spinor moving frame variables makes their differential calculus and variational problem extremely simple.

Referring again for the details to [15,28], let us just state that the derivatives of the moving frame and spinor moving frame variables can be expressed in terms of the $so(1, 10)$-valued Cartan forms $\Omega^{(a)(b)} = U^{(a)c}dU_c^{(b)}$ the set of which can be split onto the covariant Cartan forms

$$\Omega^{=i} = u^{=a}du_a^i, \qquad \Omega^{\#i} = u^{\#a}du_a^i, \qquad (2.27)$$

providing the basis for the coset $\frac{SO(1,10)}{SO(1,1)\times SO(9)}$, and the forms

$$\Omega^{(0)} = \frac{1}{4}u^{=a}du_a^\#, \qquad (2.28)$$

$$\Omega^{ij} = u^{ia}du_a^j, \qquad (2.29)$$

which have the properties of the $SO(1, 1)$ and $SO(9)$ connection respectively. These can be used to define the $SO(1, 1) \times SO(9)$ covariant derivative D. The covariant derivative of the moving frame vectors is expressed in terms of the covariant Cartan forms (2.27)

$$Du_b^= := du_b^= + 2\Omega^{(0)}u_b^= = u_b^i\Omega^{=i}, \qquad (2.30)$$

$$Du_b^\# := du_b^\# - 2\Omega^{(0)}u_b^\# = u_b^i\Omega^{\#i}, \qquad (2.31)$$

$$Du_b^i := du_b^i - \Omega^{ij}u_b^j = \frac{1}{2}u_b^\#\Omega^{=i} + \frac{1}{2}u_b^=\Omega^{\#i}. \qquad (2.32)$$

The same is true for the spinor moving frame variables,

$$Dv_q^{-\alpha} := dv_q^{-\alpha} + \Omega^{(0)}v_q^{-\alpha} - \frac{1}{4}\Omega^{ij}\gamma_{qp}^{ij}v_p^{-\alpha}$$
$$= -\frac{1}{2}\Omega^{=i}v_p^{+\alpha}\gamma_{pq}^i, \qquad (2.33)$$

$$Dv_q^{+\alpha} := dv_q^{+\alpha} - \Omega^{(0)}v_q^{+\alpha} - \frac{1}{4}\Omega^{ij}\gamma_{qp}^{ij}v_p^{+\alpha}$$
$$= -\frac{1}{2}\Omega^{\#i}v_p^{-\alpha}\gamma_{pq}^i. \qquad (2.34)$$

The variation of moving frame and spinor moving frame variables can be obtained from the above expression for derivatives by a formal contraction with variation symbol, $i_\delta d = \delta$ (this is to say, by taking the Lie derivatives). The independent variations are then described by i_δ contraction of the Cartan forms, $i_\delta\Omega^{(a)(b)}$. Furthermore, $i_\delta\Omega^{(0)}$ and $i_\delta\Omega^{ij}$ are the parameters of the $SO(1, 1)$ and $SO(9)$ transformations, which are manifest gauge symmetries of the

model. Then the essential variation of the moving frame and spinor moving frame variables, this is to say, variations which produce (better to say, which may produce) nontrivial equations of motion, are expressed in terms of $i_\delta\Omega^{=i}$ and $i_\delta\Omega^{\#i}$,

$$\delta u_b^= = u_b^i i_\delta\Omega^{=i}, \qquad \delta u_b^\# = u_b^i i_\delta\Omega^{\#i}, \qquad (2.35)$$

$$\delta u_b^i = \frac{1}{2}u_b^\# i_\delta\Omega^{=i} + \frac{1}{2}u_b^= i_\delta\Omega^{\#i}. \qquad (2.36)$$

$$\delta v_q^{-\alpha} = -\frac{1}{2}i_\delta\Omega^{=i}v_p^{+\alpha}\gamma_{pq}^i, \qquad (2.37)$$

$$\delta v_q^{+\alpha} = -\frac{1}{2}i_\delta\Omega^{\#i}v_p^{-\alpha}\gamma_{pq}^i. \qquad (2.38)$$

E. K_9 gauge symmetry of the spinor moving frame action of the M0-brane

A simple application of the above formulae begins by observing that the parameter $i_\delta\Omega^{\#i}$ does not enter the variation of neither $u_a^=$ nor $v_q^{-\alpha}$. However, the M0-brane (2.1) and (2.2) involves only these (spinor) moving frame variables. Hence the transformation of the spinor moving frame corresponding to τ dependent parameters $k^{\#i} = i_\delta\Omega^{\#i}$ are gauge symmetries of this M0 action. These so-called K_9 symmetry transformations

$$\delta u_b^= = 0, \quad \delta u_b^\# = u_b^i k^{\#i}, \quad \delta u_b^i = \frac{1}{2}u_b^= k^{\#i}, \quad (2.39)$$

$$\delta v_q^{-\alpha} = 0, \qquad \delta v_q^{+\alpha} = -\frac{1}{2}k^{\#i}v_p^{-\alpha}\gamma_{pq}^i \qquad (2.40)$$

should be taken into account when calculating the number of M0 degrees of freedom.

Quite interesting remnant of this K9 symmetry survives in the multiple M0 case and will be essential to understand the structure of mM0 equations of motion.

F. Derivatives and variations of the Cartan forms

One can easily check that the covariant Cartan forms are covariantly constant,

$$D\Omega^{=i} = 0, \qquad D\Omega^{\#i} = 0, \qquad (2.41)$$

where the covariant derivatives include the induced connection (2.28) and (2.29).[8] The curvatures of these connections are

$$F^{(0)} := d\Omega^{(0)} = \frac{1}{4}\Omega^{=i} \wedge \Omega^{\#i}, \qquad (2.42)$$

$$G^{ij} := d\Omega^{ij} + \Omega^{ik} \wedge \Omega^{kj} = -\Omega^{=[i} \wedge \Omega^{\#j]}, \qquad (2.43)$$

can be calculated, e.g., from the integrability conditions of Eqs. (2.30), (2.31), and (2.32),

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[8]$D\Omega^{=i} := d\Omega^{=i} + 2\Omega^{=i} \wedge \Omega^{(0)} + \Omega^{=j} \wedge \Omega^{ji}$, see (2.30), (2.31), and (2.32).

$$DDu_a^{\#} = -2F^{(0)}u_a^{\#}, \qquad DDu_a^i = u_a^j G^{ji}. \qquad (2.44)$$

As in the case of moving frame variables (see Sec. II D), the variations of the Cartan forms can be obtained from the above expressions using the Lie derivative formula. Omitting the transformations of manifest gauge symmetries $SO(1,1)$ and $SO(9)$ (parametrized by $i_\delta\Omega^{(0)}$ and $i_\delta\Omega^{ij}$), we present the essential variations:

$$\delta\Omega^{\#i} = Di_\delta\Omega^{\#i}, \qquad \delta\Omega^{=i} = Di_\delta\Omega^{=i}, \quad (2.45)$$

$$\delta\Omega^{ij} = -\Omega^{=[i}i_\delta\Omega^{\#j]} - \Omega^{\#[i}i_\delta\Omega^{=j]}, \quad (2.46)$$

$$\delta\Omega^{(0)} = \frac{1}{4}\Omega^{=i}i_\delta\Omega^{\#i} - \frac{1}{4}\Omega^{\#i}i_\delta\Omega^{=i}. \qquad (2.47)$$

These equations will be useful to vary the multiple M0-brane action in Sec. V. For deriving the equations of motion of single M0-brane it is sufficient to use Eqs. (2.35), (2.37), (2.30), (2.31), (2.32), (2.33), and (2.34).

III. EQUATIONS OF MOTION OF A SINGLE M0 BRANE AND INDUCED $\mathcal{N} = 16$ SUPERGRAVITY ON THE WORLDLINE W^1

In this section we summarize the equation of motion for a single M0-brane obtained from the spinor moving frame action [28] and discuss the 1d $\mathcal{N} = 16$ supergravity multiplet induced by embedding of the worldline in target 11D superspace. This induced supergravity will be used to write the mM0 action. We also show here that a supersymmetric solutions of the equations of motion of single M0-brane preserve just one half of the 11D supersymmetry. This result, seemingly new although not unexpected, is necessary to make similar conclusion about supersymmetric solutions of the mM0 equations.

A. Equations of motion for spinor moving frame variables

The moving frame matrix $U_a^{(b)}$ (2.11) provides a 'bridge' between the 11D Lorentz group and its $SO(9) \otimes SO(1,1)$ subgroup in the sense that it carries one index $(_a)$ of $SO(1,10)$ and one index $(^{(b)})$ transformed by a matrix from $SO(9) \otimes SO(1,1)$ subgroup of $SO(1,10)$. Contracting the pull-back of the bosonic supervielbein form $\hat{E}^b$ we arrive at

$$\hat{E}^{(a)} = \hat{E}^b U_b^{(a)} = (\hat{E}^=, \hat{E}^{\#}, \hat{E}^i) \qquad (3.1)$$

which is split covariantly in three types of one forms. These are inert under $SO(1,10)$ but carry the nontrivial $SO(9)$ vector index (in the case of $\hat{E}^i$) or $SO(1,1)$ weights (in the cases of $\hat{E}^=$ and $\hat{E}^{\#}$). The corresponding decomposition of the vector representation of $SO(1,10)$ with respect to its $SO(9) \otimes SO(1,1)$ subgroup,

$$\mathbf{11} \mapsto \mathbf{1}_{-2} + \mathbf{1}_{+2} + \mathbf{9}_0,$$

is even better illustrated by the equation $\hat{E}^{(a)}U_{(a)}{}^b = \hat{E}^b$ which, in more detail, reads

$$\hat{E}^a = \frac{1}{2}\hat{E}^= u^{a\#} + \frac{1}{2}\hat{E}^{\#}u^{a=} - \hat{E}^i u^{ai}. \qquad (3.2)$$

Thus the moving frame vectors help to split the pull-back of the supervielbein in a Lorentz covariant manner. The $SO(9)$ singlet one form with $SO(1,1)$ weight -2, $\hat{E}^= = \hat{E}^b u_b^=$ enters the action (2.1) multiplied by the weight $+2$ worldline scalar field $\rho^{\#}(\tau)$. This clearly has the meaning of the Lagrange multiplier: its variation results in vanishing of $\hat{E}^=$,

$$\hat{E}^= := \hat{E}^a u_a^= = 0. \qquad (3.3)$$

Now, the variation of $\hat{E}^=$ contains two different contributions, $\delta\hat{E}^= = \delta\hat{E}^a u_a^= + \hat{E}^a \delta u_a^=$. The first comes from the variation of the pull-back of the bosonic supervielbein form which in our case of flat target superspace can be easily calculated with the result

$$\delta\hat{E}^a = -id\hat{\theta}\Gamma^a\delta\hat{\theta} + d(\delta\hat{x}^a - i\delta\hat{\theta}\Gamma^a\hat{\theta}). \qquad (3.4)$$

The second term contains the variation of the lightlike vector $u_a^=$ which can be written as in Eq. (2.35), $\delta u_a^= = u_a^i i_\delta\Omega^{=i}$ with an arbitrary $i_\delta\Omega^{=i}$. The corresponding variation of the action (2.1) reads $\delta_u S_{M0} = \int_{W^1} \rho^{\#}\delta u_a^= \hat{E}^a = \int_{W^1} \rho^{\#}u_a^i \hat{E}^a i_\delta\Omega^{=i}$ and produce the equation of motion

$$\hat{E}^i := \hat{E}^a u_a^i = 0. \qquad (3.5)$$

Using Eq. (3.2) one can collect Eqs. (3.3) and (3.5) in

$$\hat{E}^a := \frac{1}{2}\hat{E}^{\#}u^{a=}. \qquad (3.6)$$

This equation shows that the M0-brane worldline W^1 is a lightlike line in target (super)space, as it should be for the massless superparticle.

B. Induced supergravity on the worldline of single M0-brane

Furthermore (3.6) suggests to consider $\hat{E}^{\#}$ as einbein on the worldline W^1; this composite einbein is induced by embedding of W^1 into the target superspace. The transformation of $\hat{E}^{\#}$ under the irreducible κ symmetry (2.8) is given by $\delta_\kappa\hat{E}^{\#} = -2i\hat{E}^{+q}\epsilon^{+q}$. In the light of the identification of κ symmetry with local worldline supersymmetry [27], this equation suggests to consider the covariant $\mathbf{16}_+$ projection, $\hat{E}^{+q} = \hat{E}^\alpha v_\alpha^{+q}$, of the pull-back of the fermionic 1-form E^α as induced "gravitino" companion of the induced 1d "graviton" $\hat{E}^{\#}$. Indeed under the κ symmetry (2.8), this set of forms shows the typical transformation rules of (1d $\mathcal{N} = 16$) supergravity multiplet,

$$\delta_\kappa\hat{E}^{+q} = D\epsilon^{+q}(\tau), \qquad \delta_\kappa\hat{E}^{\#} = -2i\hat{E}^{+q}\epsilon^{+q}. \quad (3.7)$$

Here $D = d\tau D_\tau$ is the $SO(1, 1) \times SO(9)$ covariant derivative which we will specify below. The connections in this covariant derivative are defined in terms of moving frame variables and, hence, are inert under the κ symmetry; in this sense the induced 1d $\mathcal{N} = 16$ supergravity multiplet is described essentially by 1 bosonic and 16 fermionic 1-forms $\hat{E}^\#$ and $\hat{E}^{+q}$. Our action for the mM0 system, which we present in the next section, will contain the coupling of this induced 1d supergravity to the matter describing the relative motion of the mM0 constituents.

C. Dynamical equations of single M0-brane

The other, $\mathbf{16}_-$ projection $\hat{E}^{-q} = \hat{E}^\alpha v_\alpha^{-q}$ of the pull-back of fermionic supervielbein form to W^1 vanishes on the mass shell,

$$\hat{E}^{-q} := \hat{E}^\alpha v_\alpha^{-q} = 0. \quad (3.8)$$

Indeed, varying the coordinate functions in the action (2.1) we arrive at equation $\frac{\delta S_{M0}}{\delta \hat{Z}^M} = 0$ which reads

$$\partial_\tau(\rho^\# u_a^= E_M^a(\hat{Z})) = 0. \quad (3.9)$$

In our case of flat target superspace $E_M^a(\hat{Z}) = \delta_M^a - i\delta_M^\alpha(\Gamma^a \hat{\theta})_\alpha$ and one can easily split (3.9) into the bosonic vector and fermionic spinor equations (which we prefer to write with the use of $d = d\tau \partial_\tau$)

$$d(\rho^\# u_a^=) = 0, \quad (3.10)$$

$$\rho^\# u_a^= (\Gamma^a \partial_\tau \hat{\theta})_\alpha = 0. \quad (3.11)$$

Using (2.7b) and assuming $\rho^\# \neq 0$ we find that (3.11) is equivalent to Eq. (3.8). This implies that the $d\hat{\theta}^\alpha$ can be expressed through the induced gravitino,

$$\hat{E}^\alpha = d\hat{\theta}^\alpha = \hat{E}^{+q} v_q^{-\alpha}. \quad (3.12)$$

Let us come back to the equation for the bosonic coordinate functions, (3.10) (or equivalently, $\partial_\tau(\rho^\# u_a^=) = 0$). Using (2.30) we can write this in the form $0 = D\rho^\# u_a^= + \rho^\# u_a^i \Omega^{=i}$. Here and below we use the covariant derivatives defined in (2.30), (2.31), and (2.32). Contracting that equation with $u^{a\#}$ gives us

$$D\rho^\# = 0, \quad (3.13)$$

while the nontrivial part of the bosonic equations of motion of a single M0-brane, which can be read off from the coefficient for u_a^i, states that the covariant Cartan form $\Omega^{=i}$ vanishes,

$$\Omega^{=i} = 0. \quad (3.14)$$

Coming back to Eq. (2.30), we see that Eq. (3.14) can be expressed by stating that the covariant derivative of the lightlike vector $u_a^=$ vanishes,

$$Du_a^= = 0, \quad (3.15)$$

or, equivalently, by

$$Dv_q^{-\alpha} = 0. \quad (3.16)$$

On the other hand, using

$$D = d\tau D_\tau = \hat{E}^\# D_\#, \quad (3.17)$$

we can write Eq. (3.15) in the form $D_\# u_a^= = 0$, and, as far as (3.6) implies $u_a^= = 2\hat{E}_\#^a$, in the following more standard form

$$D_\# \hat{E}_\#^a = 0, \quad (3.18)$$

or, in more detail,

$$D_\# D_\# \hat{x}^a = iD_\#(D_\# \hat{\theta} \Gamma^a \hat{\theta}). \quad (3.19)$$

Two more observations will be useful below. The first is that Eq. (3.13), $0 = D\rho^\# = d\rho^\# - 2\rho^\# \Omega^{(0)}$, can be solved with respect to the induced $SO(1, 1)$ connection,

$$\Omega^{(0)} = \frac{d\rho^\#}{2\rho^\#}. \quad (3.20)$$

Notice that this is in agreement with the statement that one can always gauge away any 1d connection: using the local $SO(1,1)$ symmetry to fix the gauge $\rho^\# = \text{const}$ we arrive at $\Omega^{(0)} = 0$.

The second comment concerns the supersymmetric pure bosonic solutions of the above equations of motion.

D. All supersymmetric solutions of the M0 equations describe 1/2 BPS states

As far as the fermionic coordinate function $\hat{\theta}^\alpha$ is transformed by both spacetime supersymmetry and by the worldline supersymmetry (κ symmetry) (4.24), $\delta \hat{\theta}^\alpha = -\varepsilon^\alpha + \epsilon^{+q}(\tau) v_q^{-\alpha}(\tau)$, the purely bosonic solutions of the M0 equations, having

$$\hat{\theta}^\alpha = 0, \quad (3.21)$$

may preserve a part of target space supersymmetry. This is characterized by parameter

$$\varepsilon^\alpha = \epsilon^{+q}(\tau) v_q^{-\alpha}(\tau). \quad (3.22)$$

The left hand side of this equation contains a constant fermionic spinor $d\varepsilon^\alpha = 0$, so that $d(\epsilon^{+q} v_q^{-\alpha}) = D\epsilon^{+q} v_q^{-\alpha} + \epsilon^{+q} Dv_q^{-\alpha} = 0$. Furthermore, taking into account that the equations of motion for the bosonic coordinate function, Eq. (3.19), implies (3.16), one finds that the consistency of (3.22) is the covariant constancy of the κ symmetry parameter $\epsilon^{+q}(\tau)$,

$$D\epsilon^{+q} = 0. \quad (3.23)$$

In 1d system the connection can be gauged away so that this condition can be reduced to the existence of a constant $SO(9)$ spinor ϵ^q. For instance gauging away the $SO(9)$ connection and using Eq. (3.20), we can present (3.23) in

the form $d(\epsilon^{+q}/\sqrt{\rho^{\#}}) = 0$ and solve it by $\epsilon^{+q} = \sqrt{\rho^{\#}}\epsilon^q$ with $d\epsilon^q = 0$.

This implies that any purely bosonic solution of the M0 equations preserves exactly 1/2 of the spacetime supersymmetry.

IV. COVARIANT ACTION FOR MULTIPLE M0-BRANE SYSTEM

In this section we obtain the action for mM0 system, first presented in [26]. We start with 1d $\mathcal{N} = 16$ $SU(N)$ SYM action, make it supersymmetry local by coupling to 1d $\mathcal{N} = 16$ supergravity, and add to such a locally supersymmetric functional the counterpart of a single M0 action for the center-of-energy variables which induces the above supergravity multiplet on the center-of-energy worldline. The local supersymmetry of the induced supergravity is produced by a generalization of the κ symmetry transformations of a single M0-brane acting on the center-of-energy variables.

A. Variables describing the mM0 system

Let us introduce the dynamical variables describing the system of multiple M0-branes, which we abbreviate as mM0. Its dimensional reduction is expected to produce the system of N nearly coincident D0-branes (mD0 system) and at very low energy this later is described by the action of 1d $\mathcal{N} = 16$ supersymmetric Yang-Mills theory (SYM) with the gauge group $U(N)$, which is given by dimensional reduction of the 10D $\mathcal{N} = 1$ U(N) SYM down to d = 1. Now, the set of fields of the U(N) SYM can be split onto the non-Abelian SU(N) SYM and Abelian U(1) SYM multiplets. Roughly speaking, this later describes the center-of-energy motion of the mD0 system while the former corresponds to the relative motion of the constituents of the mD0 system. Then it is natural to assume that the relative motion of the mM0 constituents is also described by the fields of $SU(N)$ SYM multiplet.

Now let us turn to the center-of-energy motion. We begin by noticing that the $U(1)$ SYM fields can be seen in the single D0-brane action (see [2] and Refs. therein) after fixing the gauge with respect to κ symmetry and reparametrization symmetry. Originally the action of a single D0-brane is written in terms of 10 bosonic and 32 fermionic coordinate functions, worldline fields corresponding to the coordinates of type IIA $D = 10$ superspace. The above gauge fixing reduces the number of fermionic fields to 16 and the number of bosonic coordinate functions to 9. These are the same as the number of physical fields in 1d reduction of the 10D SYM theory. This also contains the time component of the gauge field which can be gauged away by the U(1) gauge symmetry transformation and do not carry degrees of freedom. The U(1) SYM multiplet describing the center-of-energy motion of the mD0 system can be obtained by fixing the

gauge with respect to κ symmetry and reparametrization symmetry on the coordinate functions, the same as in the case of single D0-brane.

In light of the above discussion, it is natural to describe the center-of-energy motion of the mM0 system by the 11 bosonic and 32 fermionic coordinate functions, the same as used to describe the motion of single M0-brane, and to assume that the wanted mM0 action possesses κ symmetry and reparametrization symmetry, like the single M0-brane action does.

To resume, following [15,22,26] we will describe the center-of-energy motion of N nearly coincident M0-branes (mM0 system) by the 11 commuting and 32 anti-commuting coordinate functions

$$\hat{Z}^M(\tau) = (\hat{x}^\mu(\tau), \hat{\theta}^\alpha(\tau)),$$
$$\mu = 0, 1, \ldots, 10; \quad \alpha = 1, 2, \ldots, 32 \quad (4.1)$$

(the same as used to describe single M0-brane), and the relative motion of the mM0 constituents by the fields of the $SU(N)$ SYM supermultiplet. These are the bosonic and fermionic hermitian traceless $N \times N$ matrices fields

$$\mathbb{X}^i(\tau) \text{ and } \Psi_q(\tau) \quad (i = 1, \ldots, 9, q = 1, \ldots, 16) \quad (4.2)$$

depending on a (center of energy) proper time variable τ. The bosonic $\mathbb{X}^i(\tau)$ carries the index $i = 1, \ldots, 9$ of the vector representation of $SO(9)$, while the fermionic Ψ_q transforms as a spinor under $SO(9)$, $q = 1, \ldots, 16$.

B. First order form of the 1d $\mathcal{N} = 16$ SYM Lagrangian as a starting point to build mM0 action

The standard 1d $\mathcal{N} = 16$ SYM Lagrangian (obtained by dimensional reduction of 10D SYM) can be written in the following first order form

$$d\tau L_{\text{SYM}} = \text{tr}(-\mathbb{P}^i \nabla_\tau \mathbb{X}^i + 4i \Psi_q \nabla_\tau \Psi_q) + d\tau \mathcal{H} \quad (4.3)$$

where the Hamiltonian

$$\mathcal{H} = \frac{1}{2} \text{tr}(\mathbb{P}^i \mathbb{P}^i) + \mathcal{V}(\mathbb{X}) - 2 \text{tr}(\mathbb{X}^i \Psi \gamma^i \Psi) \quad (4.4)$$

contains the positively definite scalar potential

$$\mathcal{V} = -\frac{1}{64} \text{tr}[\mathbb{X}^i, \mathbb{X}^j]^2 \equiv +\frac{1}{64} \text{tr}[\mathbb{X}^i, \mathbb{X}^j] \cdot [\mathbb{X}^i, \mathbb{X}^j]^\dagger. \quad (4.5)$$

Eqs. (4.3) involve the auxiliary 'momentum' fields, the nanoplet of traceless $N \times N$ matrices $\mathbb{P}^i$, and also the gauge field $\mathbb{A}_\tau(\tau)$ which enters the covariant derivatives $\nabla = d\tau \nabla_\tau$ of the above bosonic and fermionic fields,

$$\nabla \mathbb{X}^i = d\mathbb{X}^i + [\mathbb{A}, \mathbb{X}^i], \quad \nabla \Psi_q = d\Psi_q + [\mathbb{A}, \Psi_q]. \quad (4.6)$$

The action with the above Lagrangian is invariant under the following d = 1 $\mathcal{N} = 16$ supersymmetry transformations with constant fermionic parameter ε^q

$$\delta_\varepsilon \mathbb{X}^i = 4i\varepsilon^q(\gamma^i \Psi)_q, \quad \delta_\varepsilon \mathbb{P}^i = [\varepsilon^q(\gamma^i \Psi)_q, \mathbb{X}^j], \quad (4.7)$$

$$\delta_\varepsilon \Psi_q = \frac{1}{2} \varepsilon^p \gamma^i_{pq} \mathbb{P}^i - \frac{i}{16} \epsilon^p \gamma^{ij}_{pq} [\mathbb{X}^i, \mathbb{X}^j], \qquad (4.8)$$

$$\delta_\varepsilon \mathbb{A} = -d\tau \varepsilon^q \Psi_q. \qquad (4.9)$$

The mM0 action should describe the coupling of the above SYM theory to the center-of-energy variables (4.1). As we have discussed above, such an action should possess the reparametrization symmetry and a 16 parametric local fermionic symmetry, a counterpart of the irreducible κ symmetry (2.8) of the single M0 action. It is natural also to think on this fermionic gauge symmetry as on the local version of the above rigid d $= 1$ $\mathcal{N} = 16$ supersymmetry of the SYM action, Eqs. (4.7), (4.8), and (4.9).

C. Induced supergravity on the center-of-energy worldline

The natural way to make a supersymmetry local is to couple it to supergravity multiplet. As a by-product such a coupling should guaranty the reparametrization (general coordinate) invariance. Now is the time to recall about the induced supergravity multiplet on the worldline of the single M0-brane constructed in Sec. III. Similarly, we can associate a moving frame (2.11) and spinor moving frame (2.16) to the center-of-energy motion of the mM0 system and use these together with center of energy coordinate functions (4.1) to build the composite d $= 1$ $\mathcal{N} = 16$ supergravity multiplet including the 1d "graviton" and "gravitino,"

$$\hat{E}^\# = \hat{E}^a u^\#_a = (d\hat{x}^a - i d\hat{\theta} \Gamma^a \hat{\theta}) u^\#_a, \qquad (4.10)$$

$$\hat{E}^{+q} = \hat{E}^\alpha v^{+q}_\alpha = d\hat{\theta}^\alpha v^{+q}_\alpha, \qquad (4.11)$$

transforming under the local supersymmetry as in (3.7),

$$\delta_\epsilon \hat{E}^{+q} = D\epsilon^{+q}(\tau), \qquad \delta_\epsilon \hat{E}^\# = -2i\hat{E}^{+q} \epsilon^{+q}. \qquad (4.12)$$

Notice that the use of such a composite supergravity induced by embedding of the center of energy worldline into the flat target 11D superspace implies that the local supersymmetry parameter carries the weight $+1$ of the $SO(1, 1)$ group transformations defined on the moving frame variables. This implies the necessity to adjust the $SO(1, 1)$ weight also to the fields describing the relative motion of the mM0 constituents. Following [15,22,26] we define the $SO(1, 1)$ weight of the bosonic and fermionic fields to be -2 and -3, respectively, so that in a more explicit notation (and using the conventions where the upper $^-$ index indicates the same -1 weight as the lower $+$ one)[9]

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[9]Such a choice of weight of the basic matrix fields is preferable for the description in the frame of superembedding approach, like developed in [15,22]. Once using the density $\rho^\# = \rho^{++}$ which enters the spinor moving frame action for single M0, we can easily change the weights of the fields multiplying them by corresponding power of $\rho^\#$. However, we find it more convenient to work with the "weighted" fields (4.13) and (4.14).

$$\mathbb{X}^i = \mathbb{X}^i_\# := \mathbb{X}^i_{++}, \qquad i = 1, \ldots, 9, \qquad (4.13)$$

$$\Psi_q = \Psi_{\#+q} := \Psi_{+++q} = \Psi_{\#q}^{\ -}, \qquad q = 1, \ldots, 16. \qquad (4.14)$$

As in the case of single M0-brane, we expect the $SO(1, 1)$ as well as $SO(9)$ transformation to be a gauge symmetry of our action. This implies the use of a covariant derivative with $SO(1, 1)$ and $SO(9)$ connection. As in the case of single M0-brane, we define these connections to be constructed from the moving frame variables

$$\Omega^{(0)} = \frac{1}{4} u^{=a} du^\#_a, \qquad \Omega^{ij} = u^{ia} du^j_a \qquad (4.15)$$

[see Eqs. (2.28) and (2.29)], which are now associated to the center-of-energy motion of the mM0 system. The covariant derivatives of the su(N)-valued matrix fields (4.13) are defined by

$$D\mathbb{X}^i := d\mathbb{X}^i + 2\Omega^{(0)}\mathbb{X}^i - \Omega^{ij}\mathbb{X}^j + [\mathbb{A}, \mathbb{X}^i], \qquad (4.16)$$

$$D\Psi_q := d\Psi_q + 3\Omega^{(0)}\Psi_q - \frac{1}{4}\Omega^{ij}\gamma^{ij}_{qp}\Psi_p + [\mathbb{A}, \Psi_q]. \qquad (4.17)$$

They also involve the $SU(N)$ connection $\mathbb{A} = d\tau \mathbb{A}_\tau(\tau)$ on the center of energy worldline W^1. The anti-Hermitian traceless $N \times N$ matrix gauge field $\mathbb{A}_\tau(\tau)$ is an independent variable of our model. Let us stress, however, that, as any 1d gauge field, it can be gauged away and thus does not carry any degree of freedom.

The covariant derivative of the supersymmetry parameter in (4.12) reads

$$D\epsilon^{+q} = d\epsilon^{+q} - \Omega^{(0)}\epsilon^{+q} + \frac{1}{4}\Omega^{ij}\epsilon^{+p}\gamma^{ij}_{pq}, \qquad (4.18)$$

so that the induced connection (4.15) are also the members of the composite d $= 1$ $\mathcal{N} = 16$ supergravity multiplet.

D. A way towards mM0 action

Now we are ready to present the action for the system of N nearly coincident M0-branes (mM0 system) which was proposed in [26]. It can be considered as a result of "gauging" of rigid d $= 1$ $\mathcal{N} = 16$ supersymmetry (4.7), (4.8), and (4.9) of the $SU(N)$ SYM action with the Lagrangian (4.3) achieved by coupling it to a composite d $= 1$ $\mathcal{N} = 16$ supergravity (4.10), (4.11), and (4.15) induced by embedding of the center-of-energy worldline of the mM0 system into the target 11D superspace.

The natural first step on this way is to make the Lagrangian (4.3) covariant by coupling it to a 1d gravity. This can be reached by just replacing $d\tau$ in the right hand side of (4.3) by the 1-form $\hat{E}^\#$ of (4.10). Then, to provide also the $SO(1, 1)$ and $SO(9)$ gauge symmetries, which play the role of Lorentz and R-symmetries in our induced 1d

$\mathcal{N} = 16$ supergravity, we should replace the Yang-Mills covariant derivatives in (4.6) by the $SO(1, 1) \times SO(9)$ covariant derivatives defined in (4.16) and (4.17), and to multiply the Lagrangian 1-form thus obtained by $(\rho^\#)^3$. The next stage is suggested by the fact that setting N = 1 in the action for the system of N nearly coincident M0-brane one should arrive a single M0-brane action. As the SU(N) SYM Lagrangian, and all the matrix fields involved in it, vanish when N = 1, this implies the necessity just to add the single M0 action to the integral of the above described Lagrangian form. Then the coupling to induced gravitino can be restored from the requirement of local supersymmetry invariance of the mM0 action.

E. mM0 action

In such a way we arrive at the mM0 action proposed in [26]. It reads

$$S_{mM0} = \int_{W^1} \rho^\# \hat{E}^= + \int_{W^1} (\rho^\#)^3 (\text{tr}(-\mathbb{P}^i D\mathbb{X}^i + 4i\Psi_q D\Psi_q)$$
$$+ \hat{E}^\# \mathcal{H}) + \int_{W^1} (\rho^\#)^3 \hat{E}^{+q} \text{tr}\Big(4i(\gamma^i \Psi)_q \mathbb{P}^i$$
$$+ \frac{1}{2}(\gamma^{ij}\Psi)_q [\mathbb{X}^i, \mathbb{X}^j]\Big), \qquad (4.19)$$

where $\mathcal{H}$ is the relative motion Hamiltonian (cf. (4.4))

$$\mathcal{H} := \mathcal{H}_{\#\#\#\#}(\mathbb{X}, \mathbb{P}, \Psi)$$
$$= \frac{1}{2} \text{tr}(\mathbb{P}^i \mathbb{P}^i) + \mathcal{V}(\mathbb{X}) - 2 \text{tr}(\mathbb{X}^i \Psi \gamma^i \Psi) \qquad (4.20)$$

including the scalar potential (cf. (4.5))

$$\mathcal{V} := \mathcal{V}_{\#\#\#\#}(\mathbb{X}) = -\frac{1}{64} \text{tr}[\mathbb{X}^i, \mathbb{X}^j]^2 \qquad (4.21)$$

$$= +\frac{1}{64} \text{tr}[\mathbb{X}^i, \mathbb{X}^j] \cdot [\mathbb{X}^i, \mathbb{X}^j]^\dagger, \qquad (4.22)$$

and the Yukawa-type coupling $\text{tr}(\mathbb{X}^i \Psi \gamma^i \Psi)$.

The covariant derivatives D are defined in (4.16) and (4.17). Their connection are build from the (spinor) moving frame variables, Eq. (4.15), which are related to the center-of-energy motion of the mM0 system. These are also used to construct the composite graviton and gravitino 1-forms $\hat{E}^\#$ and $\hat{E}^{+q}$, Eqs. (4.10) and (4.11). The 1-form $\hat{E}^=$ is the same as in the case of single M0-brane

$$\hat{E}^= = \hat{E}^a u_a^=. \qquad (4.23)$$

For the completeness of this section, let us recall that in these equations $\hat{E}^a$ is the pull-back of the bosonic super-vielbein to the center-of-energy worldline W^1, Eq. (2.3) and (2.4), $\hat{E}^\alpha = d\hat{\theta}^\alpha(\tau)$, $u_a^=$ and $u_a^\#$ are lightlike moving frame vectors (2.11), (2.13), and (2.14), and v_α^{+q} is an element of spinor moving frame (2.16).

Although the first term in (4.19) coincides with the single M0-brane action (2.1), now the Lagrange multiplier

$\rho^\#$ and spinor moving frame variables are also present in the second and third terms. This results in that their equations of motion differ from (3.6), and, as we discuss in the next section, generically, the center-of-energy motion of the mM0 system is not lightlike.

F. Local supersymmetry of the mM0 action

The action (4.19) is invariant under the transformation of the 16 parametric local worldline supersymmetry

$$\delta_\epsilon \hat{\theta}^\alpha = \epsilon^{+q}(\tau) v_q^{-\alpha}, \qquad (4.24)$$

$$\delta_\epsilon \hat{x}^a = -i\hat{\theta}\Gamma^a \delta_\epsilon \hat{\theta} + \frac{1}{2} u^{a\#} i_\epsilon \hat{E}^=, \qquad (4.25)$$

$$\delta_\epsilon \rho^\# = 0, \qquad (4.26)$$

$$\delta_\epsilon v_q^{\pm\alpha} = 0 \Rightarrow \delta_\epsilon u_a^= = \delta_\epsilon u_a^\# = \delta_\epsilon u_a^i = 0, \qquad (4.27)$$

$$\delta_\epsilon \mathbb{X}^i = 4i\epsilon^+ \gamma^i \Psi, \qquad \delta_\epsilon \mathbb{P}^i = [(\epsilon^+ \gamma^{ij}\Psi), \mathbb{X}^j], \qquad (4.28)$$

$$\delta_\epsilon \Psi_q = \frac{1}{2}(\epsilon^+ \gamma^i)_q \mathbb{P}^i - \frac{i}{16}(\epsilon^+ \gamma^{ij})_q [\mathbb{X}^i, \mathbb{X}^j], \qquad (4.29)$$

$$\delta_\epsilon \mathbb{A} = -\hat{E}^\# \epsilon^{+q}\Psi_q + \hat{E}^+ \gamma^i \epsilon^+ \mathbb{X}^i, \qquad (4.30)$$

where

$$i_\epsilon \hat{E}^= = 6(\rho^\#)^2 \text{tr}\Big(i\mathbb{P}^i \epsilon^+ \gamma^i \Psi - \frac{1}{8}\epsilon^+ \gamma^{ij}\Psi[\mathbb{X}^i, \mathbb{X}^j]\Big). \qquad (4.31)$$

The local supersymmetry transformations of the fields describing relative motion of mM0 constituents, (4.28) and (4.29) coincide with the SYM supersymmetry (4.7) and (4.8) modulo the fact that now the fermionic parameter is an arbitrary function of the center-of-energy proper time, $\epsilon^{+q} = \epsilon^{+q}(\tau)$. The local supersymmetry transformation of the 1d $SU(N)$ gauge field (4.30) differs from the SYM transformation by additional term involving the composite gravitino.

The transformations of the center-of-energy variables Eqs. (4.24), (4.25), (4.26), and (4.27) describe a deformation of the irreducible κ symmetry (2.8) of the free massless superparticle. Actually, the deformation touches the transformation rule (4.25) for the the bosonic coordinate function, $\delta_\epsilon \hat{x}^a$ only. The Lagrange multiplier $\rho^\#$ and the (spinor) moving frame variables are invariant under the supersymmetry, like they are under the κ symmetry of a single superparticle.

V. mM0 EQUATIONS OF MOTION

In this section we derive and study the equations of motion for the multiple M0-brane system which follow from the action (4.19).

A. Equations of the relative motion

Varying the action with respect to the momentum matrix field $\mathbb{P}^i$ gives us the equation

$$D\mathbb{X}^i = \hat{E}^\# \mathbb{P}^i + 4i\hat{E}^{+q}(\gamma^i \Psi)_q \qquad (5.1)$$

which allows to identify $\mathbb{P}^i$, modulo fermionic contribution, with the covariant time derivative of $\mathbb{X}^i$,

$$\mathbb{P}^i = D_\# \mathbb{X}^i - 4i\hat{E}_\#^+ \gamma^i \Psi. \qquad (5.2)$$

Here

$$D_\# = \frac{1}{\hat{E}_\tau^\#} D_\tau, \qquad \hat{E}_\#^{+q} = \frac{1}{\hat{E}_\tau^\#} \hat{E}_\tau^{+q}, \qquad (5.3)$$

are covariant derivative and the induced 1d gravitino field corresponding to the induced einbein on the worldvolume, $\hat{E}^\# = \hat{E}^a u_a^\# =: d\tau \hat{E}_\tau^\#$, in the sense of that

$$D = \hat{E}^\# D_\#, \qquad \hat{E}^{+q} = \hat{E}^\# \hat{E}_\#^{+q}. \qquad (5.4)$$

The variation with respect to the worldline gauge field $\mathbb{A} = d\tau \mathbb{A}_\tau$ gives

$$[\mathbb{P}^i, \mathbb{X}^i] = 4i\{\Psi_q, \Psi_q\} \qquad (5.5)$$

and the variation with respect to $\mathbb{X}^i$ results in

$$D\mathbb{P}^i = -\frac{1}{16} \hat{E}^\#[[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] + 2\hat{E}^\# \Psi \gamma^i \Psi \\ + \hat{E}^{+q} \gamma_{qp}^{ij}[\Psi_p, \mathbb{X}^j]. \qquad (5.6)$$

Using (5.1) we can easily present this equation in the form

$$D_\# D_\# \mathbb{X}^i = -\frac{1}{16}[[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] + 2\Psi \gamma^i \Psi \\ + 4iD_\#(\hat{E}_\#^+ \gamma^i \Psi_p) + \hat{E}_\#^+ \gamma^{ij}[\Psi, \mathbb{X}^j]. \qquad (5.7)$$

Finally, the variation with respect to the traceless matrix fermionic field Ψ_q produces

$$D\Psi = \frac{i}{4} \hat{E}^\#[\mathbb{X}^i, (\gamma^i \Psi)] + \frac{1}{2} \hat{E}^+ \gamma^i \mathbb{P}^i - \frac{i}{16} \hat{E}^+ \gamma^{ij}[\mathbb{X}^i, \mathbb{X}^j]. \qquad (5.8)$$

B. A convenient gauge fixing

To simplify the above equations, let us use the fact that 1-dimensional connection can always be gauged away and fix the gauge where the composed $SO(9)$ connection (2.29) and also the $SU(N)$ gauge field vanish

$$\Omega^{ij} = d\tau \Omega_\tau^{ij} = 0, \qquad (5.9)$$

$$\mathbb{A} = d\tau \mathbb{A}_\tau = 0. \qquad (5.10)$$

This breaks the local $SO(9)$ and $SU(N)$, but the symmetry under the rigid $SO(9) \otimes SU(N)$ transformations remains.

As far as the $SO(1,1)$ gauge symmetry is concerned, we would not like to fix it but rather use a part $\frac{1}{2} u^{a\#} \frac{\delta S_{mM0}}{\delta \hat{x}^a} = 0$

of the equations of motion for the center-of-energy coordinate functions $\hat{x}^a$ (discussed below in full),

$$D\rho^\# = 0, \qquad (5.11)$$

to find the explicit form of the induced $SO(1,1)$ connection (2.28), $\Omega^{(0)} := \frac{1}{4} u^{a=} du_a^\#$. Indeed, as far as $D\rho^\# = d\rho^\# - 2\rho^\# \Omega^{(0)}$, Eq. (5.11) implies

$$\Omega^{(0)} = \frac{d\rho^\#}{2\rho^\#}. \qquad (5.12)$$

In the gauge (5.9) and (5.10) the set of bosonic gauge symmetries is reduced to the Abelian $SO(1,1)$, τ-reparametrization and b symmetry (which we describe below in Sec. V E), and the covariant derivatives simplify to

$$D\mathbb{X}^i = (\rho^\#)^{-1} d(\rho^\# \mathbb{X}^i), \qquad D\mathbb{P}^i = (\rho^\#)^{-2} d((\rho^\#)^2 \mathbb{P}^i),$$

$$D\Psi_q = (\rho^\#)^{-3/2} d((\rho^\#)^{3/2} \Psi_q). \qquad (5.13)$$

As a result, Eqs. (5.7) and (5.8) can be written in the following (probably more transparent) form:

$$\partial_\tau \tilde{\Psi} = \frac{i}{4} e[\tilde{\mathbb{X}}^i, (\gamma^i \tilde{\Psi})] + \frac{1}{2\sqrt{\rho^\#}} \hat{E}_\tau^+ \gamma^i \tilde{\mathbb{P}}^i \\ - \frac{i}{16\sqrt{\rho^\#}} \hat{E}_\tau^+ \gamma^{ij}[\tilde{\mathbb{X}}^i, \tilde{\mathbb{X}}^j], \qquad (5.14)$$

$$\partial_\tau \left(\frac{1}{e} \partial_\tau \tilde{\mathbb{X}}^i \right) = -\frac{e}{16}[[\tilde{\mathbb{X}}^i, \tilde{\mathbb{X}}^j], \tilde{\mathbb{X}}^j] + 2e\tilde{\Psi}\gamma^i \tilde{\Psi} \\ + 4i\partial_\tau \left(\frac{\hat{E}_\tau^+ \gamma^i \tilde{\Psi}}{e\sqrt{\rho^\#}} \right) + \frac{1}{\sqrt{\rho^\#}} \hat{E}_\tau^+ \gamma^{ij}[\tilde{\Psi}, \tilde{\mathbb{X}}^j]. \qquad (5.15)$$

Writing Eqs. (5.14) and (5.15) we used the redefined fields

$$\tilde{\mathbb{X}}^i = \rho^\# \mathbb{X}^i, \qquad \tilde{\Psi}_q = (\rho^\#)^{3/2} \Psi_q, \qquad (5.16)$$

$$\tilde{\mathbb{P}}^i = (\rho^\#)^2 \mathbb{P}^i = \frac{1}{e} \left(\partial_\tau \tilde{\mathbb{X}}^i - \frac{4i}{\sqrt{\rho^\#}} \hat{E}_\tau^+ \gamma^i \tilde{\Psi} \right), \qquad (5.17)$$

which are inert under the $SO(1,1)$, and

$$e(\tau) = \hat{E}_\tau^\# / \rho^\# \qquad (5.18)$$

which has the properties of the einbein of the Brink-Schwarz superparticle action (2.6).

C. Bypass technical comment on derivation of the equations for the center-of-energy coordinate functions

This is the place to present some comments on the convenient way to derive equations of motion for the center-of-energy variables (which were actually also used when working with single M0 in Sec. III). A reader not interested in technical details may omit this subsection.

To find the manifestly covariant and supersymmetric invariant linear combinations of the equations of motion for the bosonic and fermionic coordinate functions, $\frac{\delta S_{mM0}}{\delta \hat{x}^a} = 0$ and $\frac{\delta S_{mM0}}{\delta \hat{\theta}^\alpha} = 0$, we introduce the covariant basis $i_\delta \hat{E}^A$ in the space of variation such that

$$\delta_{\hat{Z}^M} S_{mM0} = \int_{W^1} \left(\delta \hat{x}^a \frac{\delta S_{mM0}}{\delta \hat{x}^a} + \delta \hat{\theta}^\alpha \frac{\delta S_{mM0}}{\delta \hat{\theta}^\alpha} \right)$$
$$= \int_{W^1} \left(i_\delta \hat{E}^a \frac{\delta S_{mM0}}{i_\delta \hat{E}^a} + i_\delta \hat{E}^\alpha \frac{\delta S_{mM0}}{i_\delta \hat{E}^\alpha} \right). \quad (5.19)$$

In the generic case of curved superspace $i_\delta \hat{E}^A = \delta \hat{Z}^M E_M^A(\hat{Z})$; in our case of flat target superspace this implies

$$i_\delta \hat{E}^a = \delta \hat{x}^a - i \delta \hat{\theta} \Gamma^a \hat{\theta}, \qquad i_\delta \hat{E}^\alpha = \delta \hat{\theta}^\alpha. \quad (5.20)$$

Furthermore, it is convenient to use the moving frame variables to split covariantly the set of bosonic equations $\frac{\delta S_{mM0}}{i_\delta \hat{E}^a} = 0$ into

$$\frac{\delta S_{mM0}}{i_\delta \hat{E}^=} = \frac{1}{2} u^{a\#} \frac{\delta S_{mM0}}{i_\delta \hat{E}^a}, \qquad \frac{\delta S_{mM0}}{i_\delta \hat{E}^\#} = \frac{1}{2} u^{a=} \frac{\delta S_{mM0}}{i_\delta \hat{E}^a},$$
$$\frac{\delta S_{mM0}}{i_\delta \hat{E}^i} = -u^{ai} \frac{\delta S_{mM0}}{i_\delta \hat{E}^a}, \quad (5.21)$$

and the set of fermionic equations, $\frac{\delta S_{mM0}}{i_\delta \hat{E}^\alpha} = \frac{\delta S_{mM0}}{\delta \hat{\theta}^\alpha}$, into

$$\frac{\delta S_{mM0}}{i_\delta \hat{E}^{-q}} = v_q^{+\alpha} \frac{\delta S_{mM0}}{\delta \hat{\theta}^\alpha}, \qquad \frac{\delta S_{mM0}}{i_\delta \hat{E}^{+q}} = v_q^{-\alpha} \frac{\delta S_{mM0}}{\delta \hat{\theta}^\alpha}. \quad (5.22)$$

To resume,

$$\delta_{\hat{Z}^M} S_{mM0} = \int_{W^1} (\delta \hat{x}^a - i \delta \hat{\theta} \Gamma^a \hat{\theta}) \left(u_a^= \frac{\delta S_{mM0}}{i_\delta \hat{E}^=} \right.$$
$$\left. + u_a^\# \frac{\delta S_{mM0}}{i_\delta \hat{E}^\#} + u_a^i \frac{\delta S_{mM0}}{i_\delta \hat{E}^i} \right)$$
$$+ \int_{W^1} \delta \hat{\theta}^\alpha \left(v_\alpha^{-q} \frac{\delta S_{mM0}}{i_\delta \hat{E}^{-q}} + v_\alpha^{+q} \frac{\delta S_{mM0}}{i_\delta \hat{E}^{+q}} \right). \quad (5.23)$$

D. Equations for the center-of-energy coordinate functions

As we have already stated, the bosonic equation $\frac{\delta S_{mM0}}{i_\delta \hat{E}^=} := \frac{1}{2} u^{a\#} \frac{\delta S_{mM0}}{\delta \hat{x}^a} = 0$ results in Eq. (5.11) which is equivalent to (5.12). This observation is useful to extract consequences of the next equation, $\frac{\delta S_{mM0}}{i_\delta \hat{E}^\#} = 0$, which reads

$$D((\rho^\#)^3 \mathcal{H}) = 0. \quad (5.24)$$

Using (5.11) one can write Eq. (5.24) in the form of

$$d((\rho^\#)^4 \mathcal{H}) = 0, \quad (5.25)$$

or, equivalently, $(\rho^\#)^4 \mathcal{H} = $ const. Due to the structure of $\mathcal{H}$, Eq. (4.20), this constant is non-negative. Furthermore, as it has been shown in [26] (see also Sec. VIII C), it can be

identified (up to numerical multiplier) with the mass parameter M^2 characterizing the center-of-energy motion,

$$M^2 = 4(\rho^\#)^4 \mathcal{H} = \text{const} \geq 0. \quad (5.26)$$

The remaining projection of the equation for the bosonic center-of-energy coordinate functions, $\frac{\delta S_{mM0}}{i_\delta \hat{E}^i} := -\frac{1}{2} u^{ai} \frac{\delta S_{mM0}}{\delta \hat{x}^a} = 0$, gives us the relation between covariant $\frac{SO(1,9)}{SO(1,1) \times SO(9)}$ Cartan forms (2.27),

$$\Omega^{=i} = -(\rho^\#)^2 \mathcal{H} \Omega^{\#i} = -\frac{M^2}{4(\rho^\#)^2} \Omega^{\#i}. \quad (5.27)$$

The nontrivial part of the fermionic equation of the center-of-energy motion, $\frac{\delta S_{mM0}}{i_\delta \hat{E}^{-q}} := v_q^{-\alpha} \frac{\delta S_{mM0}}{i_\delta \hat{E}^\alpha} = 0$, reads

$$\hat{E}^{-q} = -\frac{1}{2} \Omega^{\#i} \gamma_{qp}^i v_{\#p}^-, \quad (5.28)$$

where

$$v_{\#q}^- := (\rho^\#)^2 \text{tr}\left((\gamma^j \Psi)_q \mathbb{P}^j - \frac{i}{8} (\gamma^{jk} \Psi)_q [\mathbb{X}^j, \mathbb{X}^k] \right). \quad (5.29)$$

E. Noether identities for gauge symmetries. First look

Actually one can show that Eq. (5.24) is satisfied identically when other equations are taken into account. (To be precise, Eqs. (5.1), (5.5), (5.6), (5.8), and (5.11) have to be used). This is the Noether identity for the "tangent space" copy of the reparametrization symmetry (sometimes called "*b symmetry*") with the parameter function $i_\delta \hat{E}^\#$. Similarly, one can find the Noether identity reflecting the existence of the $\mathcal{N} = 16$ 1d gauge supersymmetry (4.24), (4.25), (4.26), (4.27), (4.28), (4.29), (4.30), and (4.31) with the basic parameter $\epsilon^{+q} = i_\delta \hat{E}^{+q}$. It states the dependence of the one half of the fermionic equations, namely $\frac{\delta S_{mM0}}{i_\delta \hat{E}^{+q}} := v_q^{-\alpha} \frac{\delta S_{mM0}}{i_\delta \hat{E}^\alpha} = 0$, which reads

$$D v_{\#q}^- = \rho^2 \hat{E}^{+q} \mathcal{H} \quad (5.30)$$

or $D v_{\#q}^- = \rho^{\#2} \hat{E}^{+q} \mathcal{H}_{\#\#\#\#}$ in a more complete notation.

F. Equations which follow from the auxiliary field variations and simplification of the above equations

Variation with respect to the Lagrange multiplier $\rho^\#$, $\frac{\delta S_{mM0}}{\delta \rho^\#} = 0$, express the projection $\hat{E}^= := \hat{E}^a u_a^=$ of the pull-back $\hat{E}^a$ of the bosonic supervielbein to the center-of-energy worldline through the relative motion variables,

$$\hat{E}^= := \hat{E}^a u_a^= = -3(\rho^\#)^2 \mathcal{L}_{\#\#\#}$$
$$= 3(\rho^\#)^2 \text{tr}\left(\frac{1}{2} \mathbb{P}^i D \mathbb{X}^i + \frac{1}{64} \hat{E}^\# [\mathbb{X}^i, \mathbb{X}^j]^2 \right.$$
$$\left. - \frac{1}{4} (E^+ \gamma^{ij} \Psi) [\mathbb{X}^i, \mathbb{X}^j] \right). \quad (5.31)$$

The $\hat{E}^i := \hat{E}^a u_a^i$ projection of this pull-back is expressed by equations appearing as a result of variation with respect to the spinor moving frame variables. According to Eqs. (2.35), (2.36), (2.37), and (2.38), those should appear as coefficients for $i_\delta \Omega^{=i}$ and $i_\delta \Omega^{\#i}$ in the variation of the action. Equation $\frac{\delta S_{mM0}}{i_\delta \Omega^{=i}} = 0$ reads

$$\hat{E}^i := \hat{E}^a u_a^i = -(\rho^\#)^{-1} \Omega^{\#j} (J^{ij} + \delta^{ij} J), \qquad (5.32)$$

where we have introduced the notation

$$J^{ij} := (\rho^\#)^3 \text{tr}(\mathbb{P}^{[i} \mathbb{X}^{j]} - i\Psi \gamma^{ij} \Psi), \qquad (5.33)$$

$$J := \frac{(\rho^\#)^3}{2} \text{tr}(\mathbb{P}^i \mathbb{X}^i). \qquad (5.34)$$

The $(\rho^\#)^3$ multipliers are introduced to make J^{ij} and J inert under the $SO(1, 1)$ transformations.

In this notation, equation $\frac{\delta S_{mM0}}{i_\delta \Omega^{\#i}} = 0$ reads

$$(\rho^\#)^3 \mathcal{H} \hat{E}^i = -\Omega^{=j}(J^{ij} - \delta^{ij} J) - 2i(\rho^\#) \hat{E}^{-q}(\gamma^i \nu_\#^-)_q. \qquad (5.35)$$

Using (5.27), (5.32), (5.26), and (5.28), one can rewrite Eq. (5.35) as equation for $\Omega^{\#i}$,

$$\Omega^{\#j}(M^2 J^{ij} - 2i(\rho^\#)^3 \nu_\#^- \gamma^{ij} \nu_\#^-) = 0. \qquad (5.36)$$

Actually, as we are going to show in the next Sec., V G, taking into account the remnant of the K_9 gauge symmetry of single M0-brane [see (2.39) and (2.40)], which is present in the mM0 action, one can present the above equation in the form of

$$\Omega^{\#i} = 0, \qquad (5.37)$$

or, in terms of component, $\Omega_\tau^{\#j} = 0$. Due to (5.27) Eq. (5.37) implies

$$\Omega^{=i} = 0 \qquad (5.38)$$

and (5.32) acquires the same form as in the case of single M0-brane,

$$\hat{E}^i := \hat{E}^a u_a^i = 0. \qquad (5.39)$$

Furthermore, the fermionic equation of motion (5.28) also becomes homogeneous, of the same form as the equation for single M0-brane,

$$\hat{E}^{-q} = 0. \qquad (5.40)$$

Eqs. (5.37) and (5.38) also imply that all the moving frame and spinor moving frame variables are covariantly constant,

$$Du_a^\# = 0, \qquad Du_a^= = 0, \qquad Du_a^i = 0, \qquad (5.41)$$

$$Dv_q^{+\alpha} = 0, \qquad Dv_q^{-\alpha} = 0. \qquad (5.42)$$

Notice that in the case of single M0-brane such a form of equations for moving frame variables can be reached after

gauge fixing the K_9 gauge symmetry with parameter $i_\delta \Omega^{\#i}$. In the mM0 case only a part (remnant) of K_9 symmetry is present so that a part of variations $i_\delta \Omega^{\#i}$ produce nontrivial equations which, together with the above mentioned remnant of K_9 symmetry, results in Eqs. (5.41) and (5.42).

G. Noether identity, remnant of the K_9 gauge symmetry and the final form of the $\Omega^{\#i}$ equation

In this section we present the remnant of K_9 gauge symmetry leaving invariant the mM0 action and show that, modulo this gauge symmetry, Eq. (5.36) is equivalent to (5.37).

Let us write Eq. (5.36) as

$$\lambda^{ij} \Omega_\tau^{\#j} = 0, \qquad (5.43)$$

where

$$\lambda^{ij} = M^2 J^{ij} - 2i(\rho^\#)^3 \nu_\#^- \gamma^{ij} \nu_\#^-. \qquad (5.44)$$

As this 9×9 matrix is antisymmetric, it has rank 8 or lower, $\text{rank}(\lambda^{ij}) \leq 8$. In other words, it has at least one null vector, this is to say a vector V^i which obeys[10]

$$\exists V^i, i = 1, \ldots, 9: \qquad \lambda^{ij} V^j = 0. \qquad (5.45)$$

Actually, the matrix λ^{ij} is constructed from the dynamical variables of our model in such a way (according to Eqs. (5.44) and (5.33)) that the number of its null vectors depends on the configuration of the fields describing the relative motion of the mM0 constituents. However, as one null vector always exists, it is sufficient to consider a configuration with $\text{rank}(\lambda^{ij}) = 8$, and λ^{ij} having just one null vector, at some neighborhood $\Delta\tau$ of a proper-time moment τ; the generalization for a more complicated configuration or neighborhood is straightforward.

Then, on one hand, the solution of Eq. (5.43) in the neighborhood $\Delta\tau$ is given by $\Omega_\tau^{\#i} \propto V^i$, or, equivalently,

$$\Omega_\tau^{\#i} = f V^i, \qquad (5.46)$$

where $f = f(\tau)$ is an arbitrary function of the center-of-energy proper time τ. [For configurations/neighborhoods with several null vectors V_r^i, $r = 1, \ldots, (9 - \text{rank}\lambda)$ the solution will be $\Omega_\tau^{\#i} = f^r V_r^i$ with arbitrary functions $f^r = f^r(\tau)$].

On the other hand, the existence of null vector, Eq. (5.45), implies that a part of Eqs. (5.43) is satisfied identically

$$\Omega_\tau^{\#i} \lambda^{ij} V^j \equiv 0, \qquad (5.47)$$

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[10]This should not be confused with lightlike vectors which can exist in the space with indefinite metric. In particular, our 11D moving frame vectors $u_a^=$ and $u_a^\#$ are lightlike. To exclude any confusion, in this paper we never use the name "null vectors" for the lightlike vectors.

when some other equations are taken into account. This is the Noether identity reflecting the existence of the gauge symmetry with the basic variation[11]

$$i_\delta \Omega^{\#i} = \alpha V^i \qquad (5.48)$$

with an arbitrary function $\alpha = \alpha(\tau)$. This is clearly a remnant of the K_9 gauge symmetry (2.39) of the action (2.1) for single M0-brane.

The generic variation of the Cartan 1-form $\Omega^{\#i}$ can be expressed as in Eq. (2.45), which in our 1d case can also be written as

$$\delta \Omega_\tau^{\#i} = D_\tau i_\delta \Omega^{\#i}. \qquad (5.49)$$

Applying (5.49) to the variation of the solution (5.46) of Eq. (5.43) under (5.48), we find that

$$\delta f(\tau) = \partial_\tau \alpha(\tau). \qquad (5.50)$$

Hence, one can use the local symmetry (5.48) to set $f = 0$ and, thus, to gauge away (to trivialize) the solution (5.46) of Eq. (5.43).

This proves that the gauge fixing version of Eq. (5.43) is given by Eq. (5.37), $\Omega^{\#i} = 0$.

In Sec. VIII we give more detailed discussion of the above local symmetry and its Noether identities reproducing independently the above conclusion for the purely bosonic case.

VI. GROUND STATE SOLUTION OF THE RELATIVE MOTION EQUATIONS

The natural first step in studying the above obtained mM0 equations is to address the sector of

$$\Psi_q = 0. \qquad (6.1)$$

As far as the fermionic equations of motion have the same form (5.40) as for the single M0-brane, $\hat{E}_q^- = 0$, the only possible fermionic contribution to the relative motion equations might come from the induced gravitino $\hat{E}^{+q} = d\hat{\theta}^\alpha v_\alpha^{+q}$. However, with (6.1), the fermionic equation of the relative motion (5.8) results in

$$\hat{E}^+ \gamma^i \mathbb{P}^i - \frac{i}{8} \hat{E}^+ \gamma^{ij} [\mathbb{X}^i, \mathbb{X}^j] = 0. \qquad (6.2)$$

As it will be clear after our discussion below, for $M^2 > 0$ this equation has only trivial solution $\hat{E}^{+q} = 0$, while for $M^2 = 0$ the 1d gravitino $\hat{E}^{+q}$ remains arbitrary.

A. Ground state of the relative motion

It is easy to see that a particular configuration of the bosonic fields for which Eq. (6.2) is satisfied is

$$\mathbb{P}^i = 0, \qquad [\mathbb{X}^i, \mathbb{X}^j] = 0. \qquad (6.3)$$

Then the fermionic 1-form $\hat{E}^{+q}$ remains arbitrary (and pure gauge) as it is in the case of a single M0-brane.

Together with (6.1), Eqs. (6.3) describe the ground state of the relative motion. For it the relative motion Hamiltonian (4.20) and the center-of-energy effective mass vanish,

$$M^2 = 0 \qquad (6.4)$$

so that the center-of-energy motion is lightlike. Moreover, when Eqs. (6.1) and (6.3) hold, all the equations of the center-of-energy motion coincide with the equations for single M0-brane.

The ground state of the mM0 system is thus described by Eqs. (6.1) and (6.3) and by a (pure bosonic) ground state solution of the single M0 equations. This preserves all 16 worldline supersymmetries, which corresponds (as we have discussed in Sec. II) to the preservation of 16 of 32 spacetime supersymmetries.

B. Solutions with $M^2 = 0$ have relative motion in the ground state sector

Curiously enough, being in the ground state of the relative motion is the only possibility for the mM0 system to have the lightlike center-of-energy motion characterized by zero effective mass

$$M^2 = 0 \Leftrightarrow \mathcal{H} = \frac{1}{2} \mathrm{tr}(\mathbb{P}^i \mathbb{P}^i) - \frac{1}{64} \mathrm{tr}[\mathbb{X}^i, \mathbb{X}^j]^2 = 0. \quad (6.5)$$

Indeed, the pure bosonic relative motion Hamiltonian $\mathcal{H}$ is given by the sum of two terms both of which are traces of squares of hermitian operators ($[[\mathbb{X}^i, \mathbb{X}^j]^\dagger = [\mathbb{X}^j, \mathbb{X}^i] = -[\mathbb{X}^i, \mathbb{X}^j]$); hence, the sum vanishes, $\mathcal{H} = 0$, iff both equations in (6.3) hold,[12] $\mathbb{P}^i = D_\# \mathbb{X}^i = 0$ and $[\mathbb{X}^i, \mathbb{X}^j] = 0$.[13]

Thus any nontrivial configuration of the relative motion, with either $\mathbb{P}^i \neq 0$ or/and $[\mathbb{X}^i, \mathbb{X}^j] \neq 0$, creates a nonzero effective mass of the center-of-energy motion, $M^2 = 0$.

VII. SUPERSYMMETRIC SOLUTIONS OF mM0 EQUATIONS

In this section we will show that supersymmetric solutions of the mM0 equations have vanishing effective

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[11]See Sec. VIII for more details on these Noether identity and gauge symmetry in the purely bosonic case. Here let us just recall that Eq. (5.36) appears as an essential part of the coefficient for $i_\delta \Omega^{\#i}$ in the variation of the mM0 action.

[12]We do not discuss here the possible nilpotent contributions, like the possibility to solve the equation $a^2 = 0$ for a real bosonic $a(\tau)$ by $a = \beta_{\alpha_1 \ldots \alpha_{17}} \hat{\theta}^{\alpha_1} \ldots \hat{\theta}^{\alpha_{17}}$ with 17 center-of-energy fermions $\hat{\theta}^\alpha(\tau)$ contracted with some fermionic $\beta_{\alpha_1 \ldots \alpha_{17}} = \beta_{[\alpha_1 \ldots \alpha_{17}]}$.

[13]This is true for finite size matrices. In the $N \mapsto \infty$ limit (mM0 condensate) one can consider a "noncommutative plane" solution with $[\mathbb{X}^i, \mathbb{X}^j] = i\Theta^{ij}$ and c number valued $\Theta^{ij} = -\Theta^{ji}$, see for instance, [43]. In the case of finite N this solution cannot be used as far as the right hand side is assumed to be proportional to the unity matrix, $I_{N \times N}$ while the trace of the commutator vanishes.

center-of-energy mass, $M^2 = 0$, and that they can preserve only 1/2 of the target space supersymmetry. The first statement, which is tantamount to saying that mM0 BPS states are massless, can be found in [26], while the second, which is tantamount to saying that mM0 BPS states are 1/2 BPS, is a new result of this paper.

A. Supersymmetric solutions of the mM0 equations have $M^2 = 0$

From Eq. (4.29) one concludes that a solution of the mM0 equations with vanishing relative motion fermionic fields, Eq. (6.1), can be supersymmetric if

$$(\epsilon^+ \gamma^i)_q \mathbb{P}^i - \frac{i}{8}(\epsilon^+ \gamma^{ij})_q [\mathbb{X}^i, \mathbb{X}^j] = 0. \qquad (7.1)$$

All the 16 worldline supersymmetries (1/2 of the target space supersymmetries) can be preserved iff this equation is satisfied for arbitrary ϵ^{+p}. This implies

$$\gamma^i_{qp} \mathbb{P}^i - \frac{i}{8}\gamma^{ij}_{qp}[\mathbb{X}^i, \mathbb{X}^j] = 0 \qquad (7.2)$$

the only solution of which is given by the ground state of the relative motion, Eq. (6.3).

Thus all the bosonic solutions of mM0 equations preserving 16 supersymmetries have the trivial relative motion sector described by Eq. (6.3) which is characterized by the lightlike center-of-energy motion, $M^2 = 0$.

This suggests that $M^2 = 0$, is the BPS condition, i.e. the necessary condition for the 1/2 supersymmetry preservation. As we are going to show, this is indeed the case, and, moreover

$$M^2 = 0 \qquad (7.3)$$

is the BPS equation for preservation of any part of the target space supersymmetry.

Indeed, on one hand, tracing Eq. (7.1) with $\gamma^j \mathbb{P}^j$ and using the properties of "tr" we find

$$\epsilon^{+q} \text{tr}(\mathbb{P}^i \mathbb{P}^i) = \frac{i}{8}(\epsilon^+ \gamma^{jk} \gamma^i)_q \text{tr}(\mathbb{P}^i [\mathbb{X}^j, \mathbb{X}^k]).$$

On the other hand, tracing (7.1) with $\frac{i}{8}\gamma^{jk}[\mathbb{X}^j, \mathbb{X}^k]$ and using the Jacobi identities $[\mathbb{X}^{[i}[\mathbb{X}^j, \mathbb{X}^{k]}]] \equiv 0$ we find

$$\frac{i}{8}(\epsilon^+ \gamma^i \gamma^{jk})_q \text{tr}(\mathbb{P}^i [\mathbb{X}^j, \mathbb{X}^k]) = \frac{1}{32}(\epsilon^{+q} \text{tr}([\mathbb{X}^j, \mathbb{X}^k]^2).$$

Taking the sum of these two equations and using (5.5) (with fermionic fields set to zero) we find $\epsilon^{+q} \mathcal{H} = 0$ which, using (5.26), can be written as $\epsilon^{+q} M^2 = 0$,

$$\epsilon^{+q} M^2 = 0 \Leftarrow \epsilon^{+q} \mathcal{H} = 0. \qquad (7.4)$$

For $M^2 \neq 0$ this implies $\epsilon^{+q} = 0$, so that the supersymmetry is broken. Thus all the supersymmetric solutions of mM0 equation are characterized by $M^2 = 0$.

This fact is very important: it means that the existence of our action does not imply the existence of a new type of

supersymmetric solutions of the 11D SUGRA equations.[14] A BPS solution is in correspondence with the ground state of the brane or of the multiple brane system; the ground state of mM0 system is characterized by the vanishing effective mass and with the center-of-energy motion characteristic for the single M0-brane. Thus a supersymmetric solution of 11D SUGRA equations corresponding to single M-wave also describe the mM0 (multiple M-wave) ground state.

B. All BPS states of mM0 system are 1/2 BPS

As we have shown, a solution of mM0 equations can preserve some part of the 16 worldline supersymmetries (and some part ($\leq 1/2$) of the target space supersymmetry) if and only if $M^2 = 0$. Now, in the light of the observation in Sec. VI B, $M^2 = 0$ implies that the relative motion of the mM0 constituents is in its ground state, Eq. (6.3). This has two consequences. Firstly, as the ground state trivially solves the Killing spinor equation (7.1), it preserves all the supersymmetries allowed by the center-of-energy motion. Secondly, when the relative motion sector is in its ground state, the center-of-energy sector of supersymmetric solution is described by the same equations as the motion of single M0-brane (massless 11D superparticle). Now, as we have shown in Sec. III D, the supersymmetric solutions of these M0 equations preserve just 1/2 of the target space supersymmetry.

This proves that all the supersymmetric solutions of the equations of motion of the mM0 system preserve just one half of 32 target space supersymmetries. In other words, all the mM0 BPS states are 1/2 BPS.

VIII. ON SOLUTIONS OF mM0 EQUATIONS WITH $M^2 > 0$

When $M^2 \neq 0$, Eq. (6.2) has only trivial solutions. (The proof of this fact follows the stages of Sec. VII A). This means that (6.1) results in

$$\hat{E}^{+q} = 0, \qquad (8.1)$$

so that, when $M^2 > 0$, a configuration with vanishing relative motion fermion is purely bosonic.

A. Purely bosonic equations in the case of $M^2 > 0$

The complete list of nontrivial pure bosonic equations for mM0 system with nonvanishing center-of-energy mass, $M^2 > 0$, reads

$$D\rho^\# = 0 \Leftrightarrow \Omega^{(0)} = \frac{d\rho^\#}{2\rho^\#}, \qquad (8.2)$$

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[14]Although this statement can be done about the solutions preserving 1/2 of the 11D supersymmetry, as it will be clear in a moment, it is universal as far as a supersymmetric solution of mM0 equations can preserve only 1/2 of the tangent space supersymmetry.

$$D_\# D_\# \mathbb{X}^i = -\frac{1}{16}[[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j], \tag{8.3}$$

$$[D_\# \mathbb{X}^i, \mathbb{X}^i] = 0, \tag{8.4}$$

$$\hat{E}^= := d\hat{x}^a u_a^= = 3\hat{E}^\#\left((\rho^\#)^2 \text{tr}(D_\# \mathbb{X}^i)^2 - \frac{M^2}{4(\rho^\#)^2}\right), \tag{8.5}$$

$$\hat{E}^i := d\hat{x}^a u_a^i = 0, \tag{8.6}$$

$$\Omega^{\#i} = 0, \tag{8.7}$$

$$\Omega^{=i} = 0, \tag{8.8}$$

where $\hat{E}^\# = d\hat{x}^a u_a^\#$ and the center-of-energy mass M is defined by Eq. (5.26), $M^2 = 4(\rho^\#)^4 \mathcal{H}$, with the relative motion Hamiltonian

$$\mathcal{H} = \text{tr}\left(\frac{1}{2}(D_\# \mathbb{X}^i)^2 - \frac{1}{64}[\mathbb{X}^i, \mathbb{X}^j]^2\right). \tag{8.9}$$

Notice that (as we have discussed in the general case) the currents

$$J^{ij} = (\rho^\#)^3 \text{tr} D_\# \mathbb{X}^{[i}\mathbb{X}^{j]}, \qquad J = \frac{(\rho^\#)^3}{2} \text{tr} D_\# \mathbb{X}^i \mathbb{X}^i \tag{8.10}$$

disappear from the final form of equations when one takes into account the presence of the remnants of the K_9 symmetry. As far as this statement is very important in the analysis of the mM0 equations, we are going to give more detail on this symmetry and gauge fixing now.

But before let us make an observation that the current J^{ij} is covariantly constant on the mass shell (i.e. when the above equations of motion are taken into account),

$$DJ^{ij} = 0. \tag{8.11}$$

In contrast, in the generic purely bosonic configuration the scalar current is not a constant, $DJ = dJ \neq 0$.

B. Remnant of K_9 symmetry in the bosonic limit of the mM0 action and $\Omega^{\#i}$ equations

The variation of the bosonic limit of the mM0 action (4.19) can be written in the form

$$\delta S_{mM0}^{\text{bosonic}} = \int_{W^1} \mathcal{E}_u^{=i} i_\delta \Omega^{\#i} + \int_{W^1} \mathcal{E}_u^{\#i} i_\delta \Omega^{=i} - \int_{W^1} \mathcal{E}_{\hat{x}}^i i_\delta \hat{E}^i + \cdots. \tag{8.12}$$

where

$$\mathcal{E}_u^{=i} = M^2 \hat{E}^i / 4\rho^\# + \Omega^{=j}(J^{ij} - \delta^{ij}J),$$
$$\mathcal{E}_u^{\#i} = \rho^\# \hat{E}^i + \Omega^{\#j}(J^{ij} + \delta^{ij}J),$$
$$\mathcal{E}_{\hat{x}}^i = \rho^\# \Omega^{=i} + M^2 \Omega^{\#i} / 4\rho^\#, \tag{8.13}$$

with J^{ij} and J defined in (8.10), and dots denote the terms involving the other basic variations ($\delta\rho^\#$, $i_\delta \hat{E}^=$ etc.). Furthermore, one can rearrange the terms in (8.12) in the following way:

$$\delta S_{mM0}^{\text{bosonic}} = \int_{W^1} \mathcal{E}_u^{\#i}\left(i_\delta \Omega^{=i} - \frac{M^2}{4(\rho^\#)^2} i_\delta \Omega^{\#i}\right)$$
$$- \int_{W^1} \mathcal{E}_{\hat{x}}^i\left(i_\delta \hat{E}^i + \frac{1}{\rho^\#}(J^{ij} + \delta^{ij}J)i_\delta \Omega^{\#j}\right)$$
$$+ \frac{M^2}{2(\rho^\#)^2}\int_{W^1} d\tau \Omega_\tau^{\#i} J^{ij} i_\delta \Omega^{\#j} + \cdots, \tag{8.14}$$

In this form it is transparent that the equations of motion corresponding to the $i_\delta \Omega^{\#j}$ variation can be written in the form

$$\Omega_\tau^{\#i} J^{ij} = 0, \tag{8.15}$$

which is the bosonic limit of Eq. (5.36). As we have already discussed in the general case, Eq. (8.15) always has a nontrivial solution as far as the antisymmetric 9×9 matrix $J^{ij} = -J^{ji}$ always has at least one null vector, a nonzero vector V^i such that $V^i J^{ij} = 0$.

Each null vector generates a nontrivial solution of (8.15), but also a gauge symmetry of the mM0 action. Indeed, as one can easily see from (8.14), the transformations with τ-dependent parameter $i_\delta \Omega^{\#j}$ obeying

$$J^{ij} i_\delta \Omega^{\#j} = 0, \tag{8.16}$$

completed by

$$i_\delta \Omega^{=i} = \frac{M^2}{4(\rho^\#)^2} i_\delta \Omega^{\#i},$$
$$i_\delta \hat{E}^i = -(J^{ij} + J\delta^{ij})i_\delta \Omega^{\#j}/\rho^\#, \tag{8.17}$$

leave the action invariant, $\delta S_{mM0}^{\text{bosonic}} = 0$, and, thus define the gauge symmetries of the mM0 action. The transformations of $\Omega_\tau^{\#i}$ under this gauge symmetry are $\delta\Omega_\tau^{\#i} = D_\tau i_\delta \Omega^{\#i}$ (5.49). As far as in purely bosonic limit $DJ^{ij} = 0$ on the mass shell [see Eq. (8.11)],

$$J^{ij} D_\tau i_\delta \Omega^{\#j} = 0 \tag{8.18}$$

is also obeyed. Furthermore, in 1d case all the connection can be gauged away so that the transformation rules of the nontrivial solution of Eq. (8.15) can be summarized as follows

$$\delta\Omega_\tau^{\#i} = \partial_\tau i_\delta \Omega^{\#i}, \qquad \begin{cases} \Omega_\tau^{\#i} J^{ij} = 0 \\ J^{ij} i_\delta \Omega^{\#j} = 0, \\ \partial_\tau J^{ij} = 0. \end{cases} \tag{8.19}$$

This form makes transparent that any nontrivial solution of Eq. (8.15) can be gauged away using local symmetry (8.18) and (8.17). Thus, modulo the gauge symmetry, Eq. (8.15) is equivalent to Eq. (8.7), $\Omega^{\#i} = 0$.

C. Center-of-energy velocity and momentum for $M^2 \neq 0$

Let us notice one property of the center-of-energy motion of our M0 system, which on first glance might look strange, and try to convince the reader that it is rather a natural manifestation of the influence of relative motion on the center-of-energy dynamics.

Using Eqs. (8.5) and (8.6) we can easily calculate center-of-energy velocity of the bosonic limit of our mM0 system,

$$\dot{\hat{x}}^a := \partial_\tau \hat{x}^a = \frac{1}{2}\hat{E}_\tau^= u^{\#a} + \frac{1}{2}\hat{E}_\tau^\# u^{=a} - \hat{E}_\tau^i u^{ia}$$

$$= \frac{1}{2}\hat{E}_\tau^\#\left(u^{=a} + 3u^{\#a}\left((\rho^\#)^2 \text{tr}(D_\#\mathbb{X}^i)^2 - \frac{M^2}{4(\rho^\#)^2}\right)\right). \quad (8.20)$$

On the other hand, the canonical momentum conjugate to the center-of-energy coordinate function $\dot{\hat{x}}^a$ is[15]

$$p_a = \frac{\partial \mathcal{L}_\tau^{mM0}}{\partial \dot{x}^a} = \rho^\#\left(u_a^= + u_a^\# \frac{M^2}{4(\rho^\#)^2}\right). \quad (8.21)$$

This equation justifies our identification of the constant M^2 as a square of the effective mass of the mM0 system as it gives

$$p^a p_a = M^2. \quad (8.22)$$

Thus, generically, the center-of-energy velocity and its momentum are oriented in different directions of 11D spacetime,

$$\dot{\hat{x}}_a \propto (p_a - \mathcal{A}_a), \quad (8.23)$$

$$\mathcal{A}_a = u_a^\#\left(\frac{M^2}{\rho^\#} - 3(\rho^\#)^3 \text{tr}(D_\#\mathbb{X}^i)^2\right). \quad (8.24)$$

Equation (8.23) might look strange if one expects the center-of-energy motion to be similar to the motion of a free particle. However, this relation is characteristic for a charged particle moving in a background Maxwell field (see e.g. [44]). In our case the counterpart (8.24) of the electromagnetic potential $\mathcal{A}_a$ is constructed in terms of the relative motion variables. It vanishes when the relative motion is in its ground state.

Thus the seemingly unusual effect of that the mM0 center-of-energy velocity and momentum are not parallel one to another is just one of the manifestations of the mutual influence of the center-of-energy and the relative motion in mM0 system. The relative motion variables, when they are not in ground state, generate a counterpart of the 11D background vector potential for the center-of-energy motion.

D. An example of nonsupersymmetric solutions

Let us fix the gauge (5.9) and (5.10), $\Omega^{ij} = 0 = \mathbb{A}$, use the $SO(1, 1)$ gauge symmetry to set $\rho^\# = 1$ and the reparametrization symmetry to fix $\hat{E}_\tau^\# = 1$,[16]

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[15] $\mathcal{L}_\tau^{mM0}$ is the Lagrangian of the mM0 action (4.19), $S_{mM0} = \int d\tau \mathcal{L}_\tau^{mM0}$.

[16] Actually, to be precise, there exists an obstruction to fix such a gauge by τ reparametrization [45]. The best one can do is to fix $\partial_\tau \hat{E}_\tau^\# = 0$, while the constant value remains indefinite. This is especially important for path integral quantization, where the integration over this constant value (modulus) should be included in the definition of the path integral measure. As here we do not need in this level of precision, we allow ourselves to simplify the formulas by just setting this indefinite constant to unity.

$$\Omega_\tau^{ij} = 0 = \mathbb{A}_\tau, \qquad \hat{E}_\tau^\# = 1 = \rho^\#. \quad (8.25)$$

Then

$$D_\# = \partial_\tau \quad (8.26)$$

and Eqs. (8.3) simplify to

$$\ddot{\mathbb{X}}^i = -\frac{1}{16}[[\mathbb{X}^i, \mathbb{X}^j]\mathbb{X}^j], \quad (8.27)$$

$$[\dot{\mathbb{X}}^i, \mathbb{X}^i] = 0. \quad (8.28)$$

These very well known equations describe the 1d reduction of the 10D $SU(N)$ Yang-Mills gauge theory.

A very simple solution of Eqs. (8.27) and (8.28) is provided by

$$\mathbb{X}^i(\tau) = (A^i\tau + B^i)\mathbb{Y}, \quad (8.29)$$

where $\mathbb{Y}$ is a constant traceless $N \times N$ matrix, A^i and B^i are constant $SO(9)$ vectors, and τ is the proper time of the mM0 center of energy. The center-of-energy effective mass is defined by the trace of $\mathbb{Y}^2$ and by the length of vector $\vec{A} = \{A^i\}$,

$$M^2 = 4\mathcal{H} = 2\vec{A}^2 \text{tr}\mathbb{Y}^2, \qquad \vec{A}^2 := A^i A^i. \quad (8.30)$$

Actually, by choosing the initial point of the proper time, $\tau \mapsto \tau - a$, we can always make the constant SO(9) vectors A^i and B^i orthogonal,

$$\vec{A}\,\vec{B} := A^i B^i = 0. \quad (8.31)$$

Then the "currents" (8.10) read

$$J^{ij} = A^{[i}B^{j]}\text{tr}\mathbb{Y}^2 = \frac{A^{[i}B^{j]}}{2\vec{A}^2}M^2, \qquad J = \frac{\tau}{4}M^2. \quad (8.32)$$

Now the equations for the center-of-energy coordinate functions (5.31) and (8.6) and the gauge fixing condition $\hat{E}_\tau^\# = 1$ imply

$$\dot{\hat{x}}^a u_a^= = 3M^2/4, \quad (8.33)$$

$$\dot{\hat{x}}^a u_a^i = 0, \quad (8.34)$$

$$\dot{\hat{x}}^a u_a^\# = 1. \quad (8.35)$$

With our gauge fixing, Eqs. (5.41), which follow from (8.7) and (8.8), imply that moving frame vectors are constant

$$\dot{u}_a^\# = 0, \qquad \dot{u}_a^= = 0, \qquad \dot{u}_a^i = 0. \quad (8.36)$$

Thus (8.33), (8.34), and (8.35) is a simple system of linear differential equations

$$\dot{\hat{x}}^= = 3M^2/4, \quad (8.37)$$

$$\dot{\hat{x}}^i = 0, \quad (8.38)$$

$$\dot{\hat{x}}^{\#} = 1, \tag{8.39}$$

for the variables

$$\hat{x}^{=} = \hat{x}^{a} u_{a}^{=}, \qquad \hat{x}^{\#} = \hat{x}^{a} u_{a}^{\#}, \qquad \hat{x}^{i} = \hat{x}^{a} u_{a}^{i}. \tag{8.40}$$

This system can be easily solved for the 'comoving frame' coordinate functions (8.40). The solution in an arbitrary frame

$$\hat{x}^{\mu}(\tau) = \hat{x}^{\mu}(0) + \frac{\tau}{2}\left(u^{=\mu} + \frac{3M^{2}}{4}u^{\#\mu}\right) \tag{8.41}$$

describe a timelike motion of the center of energy characterized by a nonvanishing effective mass (8.30). The velocity of this motion,

$$\dot{x}^{\mu} = \frac{1}{2}\left(u^{=\mu} + \frac{3M^{2}}{4}u^{\mu\#}\right) \tag{8.42}$$

is not parallel to the canonical momentum [see (8.21)]

$$p_{\mu} = u_{\mu}^{=} + \frac{M^{2}}{4}u_{\mu}^{\#}. \tag{8.43}$$

As was discussed in general in Sec. VIII C, this is due to the influence of the relative motion of the mM0 constituents on the center-of-energy motion and can be considered as an effect of the induction by the relative motion dynamics of a counterpart of the Maxwell background field interacting with the center-of-energy coordinate functions. In the case under consideration this induced Maxwell field is constant, $\mathcal{A}_{\mu} = -u_{\mu}^{\#}M^{2}/2$.

E. Another nonsupersymmetric formal solution

In the case of the system of two M0-branes, the 2×2 matrix field $\mathbb{X}^{i}$ can be decomposed on Pauli matrices, $\mathbb{X}^{i} = f_{J}^{i}(\tau)\sigma^{J}$,

$$\sigma^{I}\sigma^{J} = \delta^{IJ}I_{2\times2} + i\epsilon^{IJK}\sigma^{K}, \qquad I, J, K = 1, 2, 3. \tag{8.44}$$

The simplest ansatz which solves the Gauss constraint (8.28) is $f_{J}^{i}(\tau) = \delta_{J}^{i}f(\tau)$ so that

$$\mathbb{X}^{i}(\tau) = f(\tau)\delta_{J}^{i}\sigma^{J}, \qquad i = 1, \ldots, 9;$$
$$I, J, K = 1, 2, 3. \tag{8.45}$$

Eq. (8.27) then implies that this function should obey

$$\ddot{f} + \frac{1}{2}f^{3} = 0. \tag{8.46}$$

The simplest solution of this equation is given by $f(\tau) = \frac{2i}{\tau}$ which is complex and thus breaks the condition that $\mathbb{X}^{i}$ is a hermitian matrix. Actually one can consider this solution,

$$\mathbb{X}^{i}(\tau) = \frac{2i}{\tau}\delta_{J}^{i}\sigma^{J}, \qquad J = 1, 2, 3. \tag{8.47}$$

as an analog of instanton as far as the Wick rotation $\tau \mapsto i\tau$ restores the hermiticity properties.

Ignoring for a moment the problem with hermiticity we can calculate the Hamiltonian and find that it is equal to zero. Thus (8.47) is a solution with vanishing center-of-energy mass, $M^{2} = 0$.

A configuration (8.45) with nonzero effective center-of-energy mass can be obtained by observing that (8.46) has a more general solution given by the so-called Jackobi elliptic function [46]. These functions obey

$$\dot{f}^{2} = -f^{4}/4 + C \tag{8.48}$$

with an arbitrary constant C. The above discussed particular solution (8.47) of (8.46) solves (8.48) with $C = 0$ which suggests the relation of C with M^{2}. Indeed, a straightforward calculation shows that $C = M^{2}/12$ so that a solution of the mM0 equations of relative motion is given by 2×2 matrices (8.45) with the function $f(\tau)$ obeying

$$\dot{f}^{2} = \frac{M^{2} - 3f^{4}}{12}. \tag{8.49}$$

The set of equations for the center-of-energy motion includes (8.38) and (8.39) and

$$\dot{\hat{x}}^{=} = 3M^{2}/4 - 9(f(\tau))^{4}/2. \tag{8.50}$$

This equations can be solved numerically, but its detailed study goes beyond the scope of this paper.

IX. CONCLUSIONS AND DISCUSSION

In this paper we obtain and study the equations of motion of multiple M0-brane (multiple M-wave or shortly mM0) system. In particular, we have shown that all the supersymmetric solutions of mM0 equations preserve just one half of the 11D supersymmetry and are characterized by a trivial relative motion sector. This implies that all the mM0 BPS states are 1/2 BPS and have the same properties as BPS states of single M0-brane. In the light of the possibility to describe the BPS states by the solution of the supergravity field equations this implies that our results do not suggest existence of new exotic solutions of 11D supergravity: the mM0 BPS states are described by the same type supergravity solutions as the single M-wave (see [47] for discussion on this solution).

Our mM0 equations follow from the covariant supersymmetric and κ-symmetric mM0 action proposed in [26] and we have also studied the gauge symmetries of these action. In particular we have found that this mM0 action is invariant under an interesting reminiscent of the so-called K_{9} gauge symmetry characteristic for the spinor moving frame formulation of 11D massless superparticle (which is to say of a single M0-brane). The accounting of this symmetry is necessary to find the final form of the bosonic equations of motion for the center-of-energy coordinate functions. This allows to check that the center-of-energy dynamics does not suffer indefiniteness, as might seem when looking on the original form of the center-of-energy

equations which includes some number of arbitrary functions of proper time: just the above mentioned reminiscent of the K_9 symmetry allows to gauge away all these arbitrary functions.

Our equations for the system of N M0-branes are split on the equations for center-of-energy coordinate functions and moving frame variables, which are of the same type as the fields describing a single M0-brane, and the relative motion equations involving the bosonic and fermionic traceless $N \times N$ matrix fields $\mathbb{X}^i$ and Ψ_q (as well as auxiliary matrix fields: momentum $\mathbb{P}^i$ and 1d $SU(N)$ gauge potential $\mathbb{A}$). The center-of-energy variables also enter the relative motion equations. There exists also the backreaction—the influence of the relative motion on the motion of the center of energy. This is characteristic for the purely bosonic Myers actions [3] and their generalizations [14], but was not catched by the superembedding approach to mM0 system developed in [15,22] because it was based on the standard superembedding approach equation for the center-of-energy variables. How to change this center-of-energy superembedding equation to account for backreaction of the relative motion on the center-of-energy dynamics is one of the interesting problems for future.

The most important effect of the backreaction of the relative motion (noticed already in [26]) is that, in distinction to the case of a single M0-brane, the generic center-of-energy motion of mM0 system is characterized by a nonvanishing effective mass M constructed from the matrix field describing the relative motion. Its square is expressed by $M^2 = 4(\rho^\#)^4 \mathcal{H}$ in terms of relative motion Hamiltonian $\mathcal{H}$ and the Lagrange multiplier $\rho^\#$ (which can be gauged to a constant). Both $\rho^\#$ and $\mathcal{H}$ are covariantly constant on mass shell (i.e. when equations of motion are taken into account) and this guaranties that M^2 is constant. The fact that this constant in non-negative can be easily seen from the explicit expression for the relative motion hamiltonian $\mathcal{H}$.

Another backreaction effect consists in that, when the relative motion is not in its ground state, the center-of-energy velocity and the canonical momentum conjugate to the center-of-energy coordinate function are oriented in different directions of the 11D spacetime. This can be treated as an effect of interaction of the center-of-energy coordinate degrees of freedom with the counterpart of Maxwell background field induced by the relative motion.

All the backreaction effects disappear when $M^2 = 0$. In the purely bosonic case, it is easy to see (Sec. VI B) that, when $M^2 = 0$, the relative motion is in its ground state described by constant commuting $\mathbb{X}^i$ matrices, Eqs. (6.3). Moreover, we have found that $M^2 = 0$ is the BPS conditions for supersymmetric purely bosonic solutions of the mM0 equations (Sec. VII A). This implies that all the supersymmetric bosonic solutions of the mM0 equations preserve just 1/2 of the target space supersymmetry (16 of 32), which implies that all the BPS states of mM0 system

are 1/2 BPS. The proof uses, among the others, the fact that all the BPS states of a single M0-brane are 1/2 BPS, which we have demonstrated in the introductory Sec. II devoted to spinor moving frame formulation of a single M0-brane (11D massless superparticle) model.

Furthermore, we have shown that all the supersymmetric solutions of mM0 equations have the relative motion sector in its ground state. For this the relative motion Hamiltonian vanishes $\mathcal{H} = 0$, and, hence, the effective mass of the center-of-energy motion of the mM0 system is equal to zero, $M^2 = 0$. Then the center-of-energy momentum is lightlike and parallel to the center-of-energy velocity. Moreover, all the equations of the center-of-energy motion acquire the same form as equations for single M0-brane, so that all the supersymmetric solutions of the mM0 equations are characterized by a solution of single M0-brane equations, describing the lightlike movement of the center of energy of these supersymmetric mM0 configuration plus the nanoplet of constant commuting traceless $N \times N$ matrices $\mathbb{X}^i$ (where N is the number of constituents of the mM0 system). These latter moduli of the mM0 system are the same as in 1d $SU(N)$ SYM theory.

One of the most important problems for future study is the search for generalization of our mM0 action for the mM0 system in an arbitrary 11D supergravity background. Such a search does not promise to be simple (see [48] for relevant studies of related bosonic models) so that different approximations seems to be welcome. Probably a good starting point is to search for the generalization to the case of curved superspace with constant fluxes, such as $\mathrm{AdS}_{4(7)} \times S^{7(4)}$ and pp-wave superspaces.[17]

Another important problem is to understand whether it is possible to generalize our mM0 action for the case of multiple M2-brane (mM2) system. Both these problems are under investigation now.

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[17]The purely bosonic Myers-type action for the mM0 system in a bosonic pp-wave background was proposed in [49]. The supersymmetric and Lorentz covariant equations for mM0 system in pp-wave superspace has been deduced in [23] from the superembedding approach description of [22]. This has been developed for an arbitrary 11D supergravity superspace, but the dynamics of the center of energy in it has been governed by the the same superembedding equation as describing single M0-brane. As a result, center-of-energy dynamics was considered to be not influenced by the relative motion, and plays a role of background for this. The present study of mM0 system in flat superspace shows that such an influence does exist, so that the center-of-energy superembedding equation has to be modified by the terms involving the fields describing relative motion of the mM0 constituents. Probably such a modified superembedding description of mM0 in pp-wave superspace, or the generalization of the flat superspace action of this paper to this case, may catch additional nonlinear terms in the mM0 equations, which are not present in [23]. The advantage of the action principle is that this distinguishes between three form potential A_3 of 11D supergravity and its dual A_6, so that its development for curved superspace might describe the interaction with A_6, similar to the one presented in the purely bosonic action of [49].

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APPENDIX A: EQUATIONS OF MOTION FOR A SINGLE M0-BRANE

In this appendix we collect the equations of motion for the single M0-brane obtained from the spinor moving frame action (2.1) and (2.2). They read

$$\hat{E}^{=} := \hat{E}^a u_a^{=} = 0, \qquad (A1)$$

$$\hat{E}^i := \hat{E}^a u_a^i = 0, \qquad (A2)$$

$$D\rho^{\#} = 0 \Leftrightarrow \Omega^{(0)} = \frac{d\rho^{\#}}{2\rho^{\#}}, \qquad (A3)$$

$$\Omega^{=i} = 0 \Leftrightarrow Du_a^{=} = 0 \Leftrightarrow Dv_q^{-\alpha} = 0, \qquad (A4)$$

$$\hat{E}^{-q} := \hat{E}^{\alpha} v_{\alpha}^{-q} = 0. \qquad (A5)$$

These equations are formulated in terms of pull-backs of bosonic and fermionic supervielbein forms of flat 11D superspace to the mM0 worldline W^1

$$\hat{E}^a = d\hat{x}^a - id\hat{\theta}\Gamma^a\hat{\theta}, \qquad a = 0, 1, \ldots, 10, \qquad (A6)$$

$$E^{\alpha} = d\hat{\theta}^{\alpha} \qquad \alpha = 1, \ldots, 32, \qquad (A7)$$

which are constructed from the coordinate functions $\hat{x}^a(\tau)$, $\hat{\theta}^{\alpha}(\tau)$ of the proper time τ, and of the moving frame and spinor moving frame variables $u_b^{=}$, u_b^i, v_{α}^{-q}. The properties of these latter as well as of the Cartan forms $\Omega^{=i}$, $\Omega^{(0)}$ and covariant derivatives D are collected in the next Appendix B.

In (A6) and in the main text we have used the real symmetric 32×32 11D Γ-matrices $\Gamma^a_{\alpha\beta} = (\gamma^a C)_{\alpha\beta}$ which, together with $\tilde{\Gamma}_a^{\alpha\beta} = (C\gamma_a)^{\alpha\beta}$, obey $\Gamma^{(a}\tilde{\Gamma}^{b)} = \eta^{ab}\mathbb{I}_{32\times32}$.

APPENDIX B: MOVING FRAME AND SPINOR MOVING FRAME VARIABLES

Moving frame and spinor moving frame variables are defined as blocks of, respectively, $SO(1, 10)$ and Spin(1, 10) valued matrices,

$$U_b^{(a)} = \left(\frac{u_b^{=} + u_b^{\#}}{2}, u_b^i, \frac{u_b^{\#} - u_b^{=}}{2}\right) \in SO(1, 10) \qquad (B1)$$

$(i = 1, \ldots, 9)$ and

$$V_{(\beta)}{}^{\alpha} = \begin{pmatrix} v_q^{+\alpha} \\ v_q^{-\alpha} \end{pmatrix} \in \text{Spin}(1, 10). \qquad (B2)$$

We also use

$$V_{\alpha}^{(\beta)} = (v_{\alpha q}{}^+, v_{\alpha q}{}^-) \in \text{Spin}(1, 10), \qquad (B3)$$

with

$$v_{\alpha q}{}^- = iC_{\alpha\beta}v_q^{-\beta}, \qquad v_{\alpha q}{}^+ = -iC_{\alpha\beta}v_q^{+\beta} \qquad (B4)$$

obeying

$$V_{(\beta)}{}^{\gamma}V_{\gamma}^{(\alpha)} = \delta_{(\beta)}{}^{(\alpha)} = \begin{pmatrix} \delta_{qp} & 0 \\ 0 & \delta_{qp} \end{pmatrix}$$
$$\Leftrightarrow \begin{cases} v_q^{-\alpha}v_{\alpha p}{}^+ = \delta_{qp} = v_q^{+\alpha}v_{\alpha p}{}^-, \\ v_q^{-\alpha}v_{\alpha p}{}^- = 0 = v_q^{+\alpha}v_{\alpha p}{}^+. \end{cases} \qquad (B5)$$

The algebraic properties of moving frame and spinor moving frame variables are summarized as

$$u_a^{=}u^{a=} = 0, \qquad u_a^{=}u^{ai} = 0, \qquad u_a^{=}u^{a\#} = 2, \qquad (B6)$$

$$u_a^{\#}u^{a\#} = 0, \qquad u_a^{\#}u^{ai} = 0, \qquad (B7)$$

$$u_a^i u^{aj} = -\delta^{ij}. \qquad (B8)$$

$$v_q^-\Gamma_a v_p^- = u_a^{=}\delta_{qp}, \qquad v_q^+\Gamma_a v_p^+ = u_a^{\#}\delta_{qp},$$
$$v_q^-\Gamma_a v_p^+ = -u_a^i\gamma^i_{qp}, \qquad (B9)$$

$$2v_q^{-\alpha}v_q^{-\beta} = \tilde{\Gamma}^{a\alpha\beta}u_a^{=}, \qquad 2v_q^{+\alpha}v_q^{+\beta} = \tilde{\Gamma}^{a\alpha\beta}u_a^{\#},$$
$$2v_q^{-(\alpha}v_q^{+\beta)} = -\tilde{\Gamma}^{a\alpha\beta}u_a^i. \qquad (B10)$$

In (B9) and (B10) we have used real symmetric 16×16 9d Dirac matrices $\gamma^i_{qp} = \gamma^i_{pq}$ which obey Clifford algebra

$$\gamma^i\gamma^j + \gamma^j\gamma^i = 2\delta^{ij}I_{16\times16}, \qquad (B11)$$

and

$$\gamma^i_{q(p_1}\gamma^i_{p_2p_3)} = \delta_{q(p_1}\delta_{p_2p_3)}, \qquad (B12)$$

$$\gamma^{ij}_{q(q'}\gamma^i_{p')p} + \gamma^{ij}_{p(q'}\gamma^i_{p')q} = \gamma^j_{q'p'}\delta_{qp} - \delta_{q'p'}\gamma^j_{qp}. \qquad (B13)$$

Derivatives of the moving frame and spinor moving frame variables are expressed in terms of covariant $\frac{SO(1,10)}{SO(1,1)\times SO(9)}$ Cartan forms

$$\Omega^{=i} = u^{=a}du_a^i, \qquad \Omega^{\#i} = u^{\#a}du_a^i, \qquad (B14)$$

and induced $SO(1, 1) \times SO(9)$ connection

$$\Omega^{(0)} = \frac{1}{4}u^{=a}du_a^{\#}, \qquad (B15)$$

$$\Omega^{ij} = u^{ia}du_a^j. \qquad (B16)$$

It is convenient to use these latter to define covariant derivative. Then

$$Du_b^= := du_b^= + 2\Omega^{(0)}u_b^= = u_b^i\Omega^{=i}, \tag{B17}$$

$$Du_b^\# := du_b^\# - 2\Omega^{(0)}u_b^\# = u_b^i\Omega^{\#i}, \tag{B18}$$

$$Du_b^i := du_b^i - \Omega^{ij}u_b^j = \frac{1}{2}u_b^\#\Omega^{=i} + \frac{1}{2}u_b^=\Omega^{\#i}. \tag{B19}$$

$$Dv_q^{-\alpha} := dv_q^{-\alpha} + \Omega^{(0)}v_q^{-\alpha} - \frac{1}{4}\Omega^{ij}\gamma_{qp}^{ij}v_p^{-\alpha}$$
$$= -\frac{1}{2}\Omega^{=i}v_p^{+\alpha}\gamma_{pq}^i, \tag{B20}$$

$$Dv_q^{+\alpha} := dv_q^{+\alpha} - \Omega^{(0)}v_q^{+\alpha} - \frac{1}{4}\Omega^{ij}\gamma_{qp}^{ij}v_p^{+\alpha}$$
$$= -\frac{1}{2}\Omega^{\#i}v_p^{-\alpha}\gamma_{pq}^i. \tag{B21}$$

The Cartan forms obey

$$D\Omega^{=i} = 0, \qquad D\Omega^{\#i} = 0, \tag{B22}$$

$$F^{(0)} := d\Omega^{(0)} = \frac{1}{4}\Omega^{=i} \wedge \Omega^{\#i}, \tag{B23}$$

$$G^{ij} := d\Omega^{ij} + \Omega^{ik} \wedge \Omega^{kj} = -\Omega^{=[i} \wedge \Omega^{\#j]}. \tag{B24}$$

Notice that, e.g.

$$DDu_a^\# = -2F^{(0)}u_a^\#, \qquad DDu_a^i = u_a^j G^{ji}. \tag{B25}$$

The essential variations of moving frame and spinor moving frame variables can be written as

$$\delta u_b^= = u_b^i i_\delta\Omega^{=i}, \qquad \delta u_b^\# = u_b^i i_\delta\Omega^{\#i}, \tag{B26}$$

$$\delta u_b^i = \frac{1}{2}u_b^\# i_\delta\Omega^{=i} + \frac{1}{2}u_b^= i_\delta\Omega^{\#i}. \tag{B27}$$

$$\delta v_q^{-\alpha} = -\frac{1}{2}i_\delta\Omega^{=i}v_p^{+\alpha}\gamma_{pq}^i, \tag{B28}$$

$$\delta v_q^{+\alpha} = -\frac{1}{2}i_\delta\Omega^{\#i}v_p^{-\alpha}\gamma_{pq}^i, \tag{B29}$$

where $i_\delta\Omega^{=i}$ and $i_\delta\Omega^{\#i}$ are independent variations.

The essential variations of the Cartan forms read

$$\delta\Omega^{\#i} = Di_\delta\Omega^{\#i}, \qquad \delta\Omega^{=i} = Di_\delta\Omega^{=i}, \tag{B30}$$

$$\delta\Omega^{ij} = \Omega^{=[i}i_\delta\Omega^{\#j]} - \Omega^{\#[i}i_\delta\Omega^{=j]}, \tag{B31}$$

$$\delta\Omega^{(0)} = \frac{1}{4}\Omega^{=i}i_\delta\Omega^{\#i} - \frac{1}{4}\Omega^{\#i}i_\delta\Omega^{=i}. \tag{B32}$$

APPENDIX C: mM0 EQUATIONS OF MOTION

The mM0 system, which is to say an interacting system of N nearly coincident M0-branes, is described in terms of center-of-energy variables, which similar to the variables of a single M0-brane described in Appendix A, and the traceless $N \times N$ matrices $\mathbb{X}^i$ ($i = 1, \ldots, 9$), Ψ_q ($q = 1, \ldots, 16$). Our action includes also the auxiliary $N \times N$ matrix fields: momentum $\mathbb{P}^i$ and the 1d SU(N) gauge field $\mathbb{A}_\tau$ ($\mathbb{A} = d\tau\mathbb{A}_\tau$).

The complete list of equations of motion for the mM0 system splits naturally on the equations for the relative motion variables,

$$D\mathbb{X}^i = \hat{E}^\#\mathbb{P}^i + 4i\hat{E}^{+q}(\gamma^i\Psi)_q,$$
$$[\mathbb{P}^i, \mathbb{X}^i] = 4i\{\Psi_q, \Psi_q\},$$
$$D\mathbb{P}^i = -\frac{1}{16}\hat{E}^\#[[\mathbb{X}^i, \mathbb{X}^j]\mathbb{X}^j] + 2\hat{E}^\#\Psi\gamma^i\Psi$$
$$+ \hat{E}^{+q}\gamma_{qp}^{ij}[\Psi_p, \mathbb{X}^j],$$
$$D\Psi = \frac{i}{4}\hat{E}^\#[\mathbb{X}^i, (\gamma^i\Psi)] + \frac{1}{2}\hat{E}^+\gamma^i\mathbb{P}^i - \frac{i}{16}\hat{E}^+\gamma^{ij}[\mathbb{X}^i, \mathbb{X}^j] \tag{C1}$$

and the center-of-energy equations which can be considered as a deformation of the system of equations for single M0-brane. After fixing the gauge under a reminiscent of the K_9 symmetry, these equation read

$$\hat{E}^= := \hat{E}^a u_a^=$$
$$= 3(\rho^\#)^2\text{tr}\left(\frac{1}{2}\mathbb{P}^i D\mathbb{X}^i + \frac{1}{64}\hat{E}^\#[\mathbb{X}^i, \mathbb{X}^j]^2\right.$$
$$\left. -\frac{1}{4}(E^+\gamma^{ij}\Psi)[\mathbb{X}^i, \mathbb{X}^j]\right), \tag{C2}$$

$$\hat{E}^i := \hat{E}^a u_a^i = 0, \tag{C3}$$

$$\hat{E}^{-q} := \hat{E}^\alpha v_\alpha^{-q} = 0, \tag{C4}$$

$$\left.\begin{matrix}\Omega^{=i} = 0 \\ \Omega^{\#i} = 0\end{matrix}\right\} \Leftrightarrow \begin{cases} Du_a^= = 0, & Du_a^\# = 0, \\ Du_a^i = 0, \\ Dv_q^{-\alpha} = 0, & Dv_q^{+\alpha} = 0, \end{cases} \tag{C5}$$

$$D\rho^\# = 0 \Leftrightarrow \Omega^{(0)} = \frac{d\rho^\#}{2\rho^\#}. \tag{C6}$$

As a consequence of the above equation the effective mass M of the mM0 center-of-energy motion,

$$M^2 = 4(\rho^\#)^4\mathcal{H}, \tag{C7}$$

is a constant

$$dM^2 = 0. \tag{C8}$$

Eq. (C7) expresses M^2 in terms of Lagrange multiplier $\rho^\#$ and the relative motion Hamiltonian (4.4)

$$\mathcal{H} = \frac{1}{2}\text{tr}(\mathbb{P}^i\mathbb{P}^i) - \frac{1}{64}\text{tr}[\mathbb{X}^i, \mathbb{X}^j]^2 - 2\text{tr}(\mathbb{X}^i\Psi\gamma^i\Psi). \tag{C9}$$

The definition and properties of the covariant derivatives of the spinor moving frame variables and of the Cartan forms, described in the main text, are collected in Appendix B

If we fix the gauge where the composed SO(9) connection and also the SU(N) gauge field vanish,

$$\Omega^{ij} = d\tau \Omega_\tau^{ij} = 0, \qquad \mathbb{A} = d\tau \mathbb{A}_\tau = 0, \tag{C10}$$

the equations of relative motion and Eq. (C2) simplify to

$$\partial_\tau \tilde{\Psi} = \frac{i}{4} e[\tilde{\mathbb{X}}^i, (\gamma^i \tilde{\Psi})] + \frac{1}{2\sqrt{\rho^\#}} \hat{E}_\tau^+ \gamma^i \tilde{\mathbb{P}}^i - \frac{i}{16\sqrt{\rho^\#}} \hat{E}_\tau^+ \gamma^{ij}[\tilde{\mathbb{X}}^i, \tilde{\mathbb{X}}^j],$$

$$\partial_\tau \left(\frac{1}{e} \partial_\tau \tilde{\mathbb{X}}^i \right) = -\frac{e}{16}[[\tilde{\mathbb{X}}^i, \tilde{\mathbb{X}}^j]\tilde{\mathbb{X}}^j] + 2e\tilde{\Psi}\gamma^i\tilde{\Psi} + 4i\partial_\tau \left(\frac{\hat{E}_\tau^+ \gamma^i \tilde{\Psi}}{e\sqrt{\rho^\#}} \right) + \frac{1}{\sqrt{\rho^\#}} \hat{E}_\tau^+ \gamma^{ij}[\tilde{\Psi}, \tilde{\mathbb{X}}^j],$$

$$\partial_\tau \tilde{\mathbb{X}}^i = e\tilde{\mathbb{P}}^i + \frac{4i}{\sqrt{\rho^\#}}(\hat{E}_\tau^+ \gamma^i \tilde{\Psi}), \qquad [\tilde{\mathbb{P}}^i, \tilde{\mathbb{X}}^i] = 4i\{\tilde{\Psi}_q, \tilde{\Psi}_q\},$$

$$\rho^\# \hat{E}_\tau^= = 3 \,\mathrm{tr}\left(\frac{1}{2} \tilde{\mathbb{P}}^i \partial_\tau \tilde{\mathbb{X}}^i + \frac{1}{64} e[\tilde{\mathbb{X}}^i, \tilde{\mathbb{X}}^j]^2 - \frac{1}{4\sqrt{\rho^\#}}(\hat{E}_\tau^+ \gamma^{ij}\tilde{\Psi})[\tilde{\mathbb{X}}^i, \tilde{\mathbb{X}}^j] \right). \tag{C11}$$

These equations are written in terms of redefined fields,

$$\tilde{\mathbb{X}}^i = \rho^\# \mathbb{X}^i, \qquad \tilde{\Psi}_q = (\rho^\#)^{3/2}\Psi_q, \qquad \tilde{\mathbb{P}}^i = (\rho^\#)^2 \mathbb{P}^i = \frac{1}{e}\left(\partial_\tau \tilde{\mathbb{X}}^i - \frac{4i}{\sqrt{\rho^\#}} \hat{E}_\tau^+ \gamma^i \tilde{\Psi} \right), \tag{C12}$$

and

$$e(\tau) = \hat{E}_\tau^\# / \rho^\#. \tag{C13}$$

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