Black hole formation at the correspondence point

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We study the process of bound state formation in a D-brane collision. We consider two mechanisms for bound state formation. The first, operative at weak coupling in the worldvolume gauge theory, is pair creation of W-bosons. The second, operative at strong coupling, corresponds to formation of a large black hole in the dual supergravity. These two processes agree qualitatively at intermediate coupling, in accord with the correspondence principle of Horowitz and Polchinski. We show that the size of the bound state and time scale for formation of a bound state agree at the correspondence point. The time scale involves matching a parametric resonance in the gauge theory to a quasinormal mode in supergravity.

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I. INTRODUCTION AND SUMMARY

Understanding black hole microstates from a D-brane or fundamental string perspective is a long-standing theme in string theory. The original observation that vibrating strings qualitatively resemble a black hole [1,2] was followed by a quantitative worldvolume derivation of black hole entropy for certain Bogomol'nyi-Prasad-Sommerfeld states [3]. This relationship eventually became a fundamental aspect of the holographic duality between gauge and gravity degrees of freedom [4]. According to this duality, microstates of a black hole are in one-to-one correspondence with microstates of a strongly coupled gauge theory. This duality also applies to time-dependent processes such as black hole formation and evaporation, leading to the viewpoint that these processes should be unitary, contrary to [5].

To gain insight into black hole formation, and a better understanding of the microstructure of the resulting black hole, in this paper we study the process of bound state formation from two perspectives: perturbative gauge theory and supergravity. In perturbative gauge theory a D-brane bound state can be formed through a process of open string creation. In supergravity we will see that open string creation is not possible, and one instead forms a bound state through the gravitational or closed-string process of black hole formation.

The perturbative gauge theory and supergravity calculations of bound state formation do not have an overlapping range of validity. But we will show that they agree qualitatively at an intermediate value of the coupling, in accord with the correspondence principle introduced by Horowitz and Polchinski [6]. This suggests that there is a smooth transition between the process of open string creation at weak coupling and black hole formation at strong coupling.

As a first test of these ideas, in Sec. II we study bound state formation in D0-brane collisions and show that the sizes of the bound states match at the correspondence point. In Sec. III we extend this analysis to general Dp-branes.

Next we consider the time development of the bound states after they have formed. In Sec. IV we show that the weakly coupled gauge theory has a parametric resonance which exponentially amplifies the number of open strings present, and we identify the time scale for the production of additional open strings at weak coupling. In the gravitational description, a perturbed black hole approaches equilibrium on a time scale determined by the quasinormal frequencies. In Sec. V we compare these two time scales and show that they agree at the correspondence point.

In Sec. VI we compare properties of the bound state as initially formed to equilibrium properties of the black hole, and show that at the correspondence point the bound state is created in a state of near-equilibrium. In Sec. VII we study a different initial configuration, in which a bound state is formed by collapse of a spherical shell of D0-branes, and show that the picture of a smooth transition between open string production and black hole formation continues to hold. We conclude in Sec. VIII.

The present work is related to several studies in the literature. In gauge–gravity duality, a black hole on the gravity side is dual to a thermal state of the gauge theory, where all $\mathcal{O}(N^2)$ degrees of freedom are excited [7,8]. There have been many studies of 0-brane black hole microstates from matrix quantum mechanics, along with their associated thermalization process. Some previous studies of 0-brane black holes from matrix quantum mechanics include [9–15]. Also see [16,17] for studies of black hole

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formation from the gravity perspective, and [18–23] for studies from the gauge theory perspective. In particular parametric resonance has been discussed in relation to thermalization in the closely related work [21]. Open string production has been studied as a mechanism for trapping moduli at enhanced symmetry points in [24], while open string production in relativistic D-brane collisions has been studied in [25].

II. BOUND STATE FORMATION IN 0-BRANE COLLISIONS

Consider colliding two clusters of 0-branes as shown in Fig. 1. We would like to understand whether a bound state is formed during the collision. Two mechanisms for bound state formation have been discussed in the literature:

(1) In a perturbative description of D-brane dynamics, open strings can be produced and lead to formation of a bound state. This occurs for impact parameters $b \lesssim \sqrt{v\alpha'}$ [26]. This can be understood as the condition for violating the adiabatic approximation. For a review of the calculation see the Appendix.

(2) At large N and strong coupling the D-brane system has a dual gravitational description [27]. In this description, according to the hoop conjecture of Thorne [28–30], a black hole should form if the two D-brane clusters are contained within their own Schwarzschild radius.

Our goal is to understand in what regimes these two mechanisms for bound state formation are operative, and whether they are connected in any way.

It will be convenient to work in terms of a radial coordinate U with units of energy, $U = r/\alpha'$. Here r is the distance between the clusters, $r = \sqrt{b^2 + v^2 t^2}$. The 't Hooft coupling of the M(atrix) quantum mechanics is $\lambda = g_{YM}^2 N$, which in string and M-theory units can be expressed as

$$\lambda = g_s N/\ell_s^3 = R^3 N/\ell_{11}^6. \quad (1)$$

Here g_s is the string coupling, ℓ_s is the string length, R is the radius of the M-theory circle, and ℓ_{11} is the M-theory Planck length. The mass of a single D0-brane is

$$m_0 = \frac{1}{g_s \ell_s} = \frac{1}{R}. \quad (2)$$

FIG. 1. Colliding stacks of 0-branes with relative velocity v and impact parameter b.

A. Perturbative string production

We work in the center of mass frame, with momenta

$$p_1 = \frac{N_1}{R} v_1, \qquad p_2 = \frac{N_2}{R} v_2, \qquad p_1 + p_2 = 0. \quad (3)$$

We consider a fixed total energy E, which determines the asymptotic relative velocity v:

$$\frac{1}{2}\frac{N_1}{R} v_1^2 + \frac{1}{2}\frac{N_2}{R} v_2^2 = E,$$
$$\Rightarrow v = v_1 - v_2 \sim \left(\frac{NER}{N_1 N_2}\right)^{1/2} = \left(\frac{\lambda E l_s^4}{N_1 N_2}\right)^{1/2}. \quad (4)$$

In terms of the U coordinate, the asymptotic relative velocity is

$$\dot{U} = \left(\frac{\lambda E}{N_1 N_2}\right)^{1/2}. \quad (5)$$

As reviewed in the Appendix, open string production sets in when

$$U \sim \sqrt{\dot{U}} = \left(\frac{\lambda E}{N_1 N_2}\right)^{1/4}. \quad (6)$$

Note that the radius at which open strings are produced depends on how we split the total D-brane charge. The radius is minimized when $N_1 = N_2 = N/2$, which gives the minimum radius for open string production as

$$U_0 \sim \left(\frac{\lambda E}{N^2}\right)^{1/4}. \quad (7)$$

This is the case which is interesting for matching to supergravity.

There are some checks we should perform to make sure this perturbative result is valid. As discussed in [31], the effective action has a double expansion in λ/U^3 and $\dot{U}^2/U^4$. The expansion in powers of λ/U^3 is the Yang-Mills loop expansion, which is valid provided $U_0 > \lambda^{1/3}$. From (7) this requires

$$E > N^2 \lambda^{1/3}. \quad (8)$$

At the critical point where the loop expansion breaks down, $U_0 \sim \lambda^{1/3}$, the inequality (8) is saturated.

The expansion in powers of $\dot{U}^2/U^4$ is the derivative expansion, which is valid when $\dot{U}^2 < U^4$. Note that the derivative expansion breaks down at the point where open strings are produced. Up to this point, i.e. for $U > \sqrt{\dot{U}}$, one can trust the two-derivative terms in the effective action, which means the asymptotic velocity (4) is a good approximation to the actual velocity.[1] So the only condition for the validity of the perturbative description of open string production is (8).

[1] As we will see, this is not the case in the supergravity regime.

B. Bound state formation in gravity

At large N the M(atrix) quantum mechanics has a dual gravitational description at strong coupling, meaning for $U < \lambda^{1/3}$. So let us imagine the 0-brane clusters approach to within this distance, and study whether a bound state can form.

At first, one might think a bound state could form via open string production. As noted in [27], the metric factors cancel out of the Nambu-Goto action, and even in the supergravity regime the mass of an open string connecting the two clusters of D-branes is $m_W \sim U$. The adiabatic approximation breaks down, and these open strings should be produced, if $\dot{U}/U^2 > 1$. However this velocity cannot be attained in the regime where supergravity is valid, since it violates the causality bound [32,33]. This can be seen in the probe approximation, where the Dirac-Born-Infeld (DBI) action for a probe is (see, for example, [9])

$$S = \frac{1}{g_{\rm YM}^2}\int dt \frac{U^7}{\lambda}\left(1 - \sqrt{1 - \frac{\lambda \dot{U}^2}{U^7}}\right). \tag{9}$$

Thus causality bounds the velocity of the probe,

$$\frac{\lambda \dot{U}^2}{U^7} < 1. \tag{10}$$

Rather remarkably, the probe has to slow down significantly as $U \to 0$. In any case, in the supergravity regime we have $\frac{\dot{U}^2}{U^4} < \frac{U^3}{\lambda}$, and since $\frac{U^3}{\lambda} < 1$ at strong coupling, open strings can never be produced.

This means black hole formation is the only way to form a bound state in the supergravity regime. Since open string production is ruled out, we reach the sensible conclusion that the formation of a horizon is a purely gravitational closed-string process. The hoop conjecture states that a black hole will form if the energy E is contained within its own Schwarzschild radius. For a ten-dimensional black hole with N units of 0-brane charge, the Schwarzschild radius is

$$U_0 = \left(\frac{\lambda^2 E}{N^2}\right)^{1/7}. \tag{11}$$

This 10-D supergravity description is only valid if the curvature and string coupling are small at the horizon, which requires

$$\lambda^{1/3} N^{-4/21} < U_0 < \lambda^{1/3}. \tag{12}$$

For smaller U_0 one must lift to M-theory; for larger U_0 the M(atrix) quantum mechanics is weakly coupled. At the outer radius where the supergravity approximation breaks down, $U_0 \sim \lambda^{1/3}$, Eq. (11) tells us that $E \sim N^2 \lambda^{1/3}$.

C. Correspondence point

We have found that open string production is only possible at weak coupling, while black hole formation can only occur within the bubble where supergravity is valid. One could ask if the two phenomena are smoothly connected. Is there a correspondence point where both descriptions are valid?

From the perturbative point of view, the transition happens when the condition (8) is saturated, $E = N^2\lambda^{1/3}$. In this case open strings are produced, but at a radius $U_0 \sim \lambda^{1/3}$ where the system is just becoming strongly coupled.

From the supergravity point of view, the transition happens when the energy of the black hole is $E = N^2\lambda^{1/3}$, corresponding to a Schwarzschild radius $U_0 \sim \lambda^{1/3}$. In this case the black hole fills the entire region where supergravity is valid.

This suggests that open string production and black hole formation are indeed continuously connected. Since the transition between the two descriptions happens when the curvature at the horizon is of order string scale,

$$\alpha' R \sim (\lambda/U^3)^{-1/2} \sim 1, \tag{13}$$

this is an example of the correspondence principle of Horowitz and Polchinski [6]. Note that for a given black hole energy, one can view the condition of being at the correspondence point, $E = N^2\lambda^{1/3}$, as fixing the total 0-brane charge,

$$N = \left(\frac{E^3 \ell_s^3}{g_s}\right)^{1/7}. \tag{14}$$

III. Dp-BRANE COLLISIONS

In this section we generalize our 0-brane results and consider Dp-branes wrapped on a p-torus of volume V_p. We first record some general formulas then analyze particular cases.

The Yang-Mills coupling is $g_{\rm YM}^2 = g_s/\ell_s^{3-p}$ and the 't Hooft coupling is $\lambda = g_{\rm YM}^2 N$. In terms of $U = r/\alpha'$, the effective dimensionless 't Hooft coupling is

$$\lambda_{\rm eff} = \frac{\lambda}{U^{3-p}}. \tag{15}$$

The Yang-Mills theory is weakly coupled when $\lambda_{\rm eff} < 1$. It has a dual gravitational description at large N when $\lambda_{\rm eff} > 1$ [27].

Imagine colliding two stacks of wrapped Dp-branes at weak coupling, with a fixed energy density ϵ as measured in the Yang-Mills theory. The mass of a wrapped p-brane is $V_p/g_s\ell_s^{p+1}$, so in the center of mass frame the relative velocity is

$$\dot{U} = \left(\frac{\lambda\epsilon}{N_1 N_2}\right)^{1/2}. \tag{16}$$

Open string production sets in when

$$U \sim \sqrt{\dot{U}} \sim \left(\frac{\lambda\epsilon}{N_1 N_2}\right)^{1/4}. \tag{17}$$

The radius at which open strings are produced depends on how we divide the total D-brane charge. The radius is minimized by setting $N_1 = N_2 = N/2$, which gives the minimum radius for open string production as

$$U_0 \sim \left(\frac{\lambda\epsilon}{N^2}\right)^{1/4}. \tag{18}$$

This is the case which is interesting for comparison to supergravity.

Just as for 0-branes, open string production is not possible in the supergravity regime. The DBI action for a probe brane is

$$S = \frac{1}{g_{\rm YM}^2}\int d^{p+1}x \frac{U^{7-p}}{\lambda}\left(1 - \sqrt{1 - \frac{\lambda\dot{U}^2}{U^{7-p}}}\right). \tag{19}$$

Thus the causality bound is $\dot{U}^2/U^4 < U^{3-p}/\lambda = 1/\lambda_{\rm eff}$ [32], which rules out open string production (at least in the probe approximation). Instead we have the process of black hole formation, with a horizon radius $U_0 = (g_{\rm YM}^4\epsilon)^{1/(7-p)}$ [27].

Further analysis depends on the dimension of the branes.

$p = 0, 1, 2$.—For $p < 3$ the Yang-Mills theory is weakly coupled when $U > \lambda^{1/(3-p)}$ and has a dual gravitational description when $U < \lambda^{1/(3-p)}$. Thus open string production is possible at large distances, while black hole formation is possible at small distances. The correspondence point, where the two descriptions match on to each other, occurs when

$$\epsilon = N^2\lambda^{\frac{1+p}{3-p}} \qquad U_0 = \lambda^{1/(3-p)}.$$

At this energy density open string production occurs just as the Yang-Mills theory is becoming strongly coupled. From the supergravity perspective, the resulting black brane fills the entire region in which supergravity is valid.

$p = 3$.—In this case the Yang-Mills theory is conformal and dual to $\mathrm{AdS}_5 \times S^5$ [4]. The 't Hooft coupling is dimensionless. For $\lambda \lesssim 1$ open string production is possible, while for $\lambda \gtrsim 1$ black holes can form. The two descriptions match on to each other at the correspondence point $\lambda = 1$. Note that, unlike other values of p, the correspondence point is independent of the energy density ϵ.

As a test of this idea, note that the radius at which open strings form is

$$U_0 = (\lambda\epsilon/N^2)^{1/4} \tag{20}$$

while for $p = 3$ the horizon radius is

$$U_0 = (g_{\rm YM}^4\epsilon)^{1/4}. \tag{21}$$

These two expressions for U_0 agree when $\lambda = 1$. This suggests that the process of open string production for $\lambda \lesssim 1$ smoothly matches on to black hole formation for $\lambda \gtrsim 1$.

$p = 4, 5, 6$.—For $p > 3$ the Yang-Mills theory is strongly coupled in the UV and has a dual supergravity description (modulo some subtleties described in [27]). In the IR the Yang-Mills theory is weakly coupled. Black hole production is possible in the supergravity regime, where $U > \lambda^{1/(3-p)}$, while open string production is possible for $U < \lambda^{1/(3-p)}$. The correspondence point where the two descriptions match is at

$$\epsilon = N^2\lambda^{\frac{1+p}{3-p}} \tag{22}$$

$$U_0 = \lambda^{1/(3-p)}. \tag{23}$$

IV. PARAMETRIC RESONANCE IN PERTURBATIVE SYM

In this section we study the evolution of a bound state formed at weak coupling by open string creation. We show that the number of open strings increases exponentially with time due to a parametric resonance in the gauge theory. For simplicity we consider 0-brane collisions; the generalization to Dp-branes is straightforward and will be mentioned in Sec. V B.

Suppose a cluster of N_1 incoming 0-branes collides with a stack of N_2 coincident 0-branes at rest. We assume weak coupling but do not require large N. In the collision suppose n open strings are produced. These open strings produce a linear confining potential, so the system will begin to oscillate. The conserved total energy is

$$E = \frac{1}{2}mv^2 + n\tau x. \tag{24}$$

Here we are adopting a nonrelativistic description, appropriate to the form of the D0-brane quantum mechanics, while m is the mass of the incoming 0-branes, v is their velocity, n is the number of open strings created, $\tau = 1/2\pi\alpha'$ is the fundamental string tension, and x is the length of the open strings. The period of oscillation is

$$\Delta t = 4\left(\frac{m}{2}\right)^{1/2}\int_0^{E/n\tau}\frac{dx}{\sqrt{E - n\tau x}} \sim \frac{\sqrt{mE}}{n\tau}. \tag{25}$$

So up to numerical factors, the frequency of oscillation is

$$\Omega = \frac{n\tau}{\sqrt{mE}} \tag{26}$$

while the amplitude of oscillation (the maximum value of x) is

$$L = \frac{E}{n\tau}. \tag{27}$$

We introduce this as a classical M(atrix) background by setting $X^i = X^i_{\rm cl} + x^i$ where

$$X^1_{\rm cl} = \begin{pmatrix} L\sin\Omega t \mathbb{1}_{N_1} & 0 \\ 0 & 0\end{pmatrix} \qquad X^2_{\rm cl} = \cdots = X^9_{\rm cl} = 0. \tag{28}$$

We have decomposed the $N \times N$ matrix into blocks; $\mathbb{1}_{N_1}$ is the $N_1 \times N_1$ unit matrix. Expanding to quadratic order in the fluctuations, the M(atrix) Lagrangian[2]

$$\mathcal{L}_{\rm YM} = \frac{1}{2g_{\rm YM}^2}\,\mathrm{Tr}\Big(\dot{X}^i\dot{X}^i + \frac{1}{2}[X^i,X^j][X^i,X^j]\Big) \tag{29}$$

reduces to

$$\mathcal{L}_{\rm YM} = \frac{1}{2g_{\rm YM}^2}\,\mathrm{Tr}(\dot{x}^1\dot{x}^1) + \frac{1}{2g_{\rm YM}^2}\sum_{i=2}^{9}\mathrm{Tr}(\dot{x}^i\dot{x}^i + [x^i,X^1_{\rm cl}][x^i,X^1_{\rm cl}]). \tag{30}$$

Note that the potential for x^1 vanishes. We also have the Gauss constraint associated with setting $A_0 = 0$, namely

$$\sum_i [X^i,\dot{X}^i] = 0. \tag{31}$$

To quadratic order this reduces to $[X^1_{\rm cl},\dot{x}^1] = [\dot{X}^1_{\rm cl},x^1]$ which only constrains x^1. The simplest solution is to set $x^1 = 0$.

To study the remaining degrees of freedom we decompose

$$x^i = \begin{pmatrix} a^i & b^{i\dagger} \\ b^i & c^i \end{pmatrix}, \tag{32}$$

where a^i is an $N_1 \times N_1$ matrix, b^i is an $N_1 \times N_2$ rectangular matrix and c^i is an $N_2 \times N_2$ matrix. We will often suppress the index $i = 2,\ldots,9$. To quadratic order the a and c entries have trivial dynamics, since $[x^i, X^1_{\rm cl}]$ does not involve a and c. On the other hand, the equation of motion for b is

$$\ddot{b} + L^2\sin^2(\Omega t)b = 0. \tag{33}$$

Defining $s = \Omega t$ this reduces to Mathieu's equation,

$$\frac{d^2b}{ds^2} + (a - 2q\cos 2s)b = 0 \tag{34}$$

with the particular values $a = 2q = L^2/2\Omega^2$. Mathieu's equation admits Floquet solutions

$$b(t) = e^{i\gamma\Omega t}P(\Omega t), \tag{35}$$

where $P(\cdot)$ is a periodic function with period π. As a function of a and q there are intervals where γ has a negative imaginary part and the solution grows exponentially. These intervals correspond to band gaps in the Bloch interpretation of Mathieu's equation. The imaginary part of γ is plotted as a function of $a = 2q$ in Fig. 2. There are clearly many intervals where the solution is unstable, with a typical exponent $|\mathrm{Im}\gamma| \sim 0.25$.

This instability corresponds to an exponential growth in the number of open strings present. Note that in our case[3]

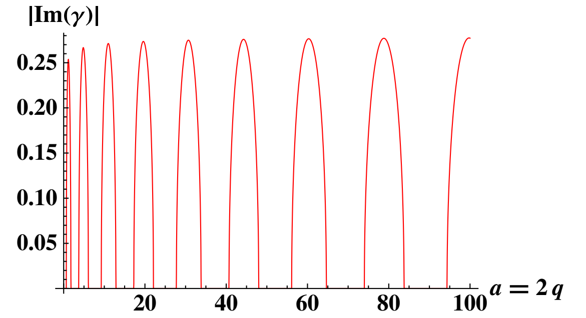

FIG. 2 (color online). The imaginary part of the Mathieu characteristic exponent as a function of $a = 2q$.

$$a = 2q \sim mE^3/n^4. \tag{36}$$

After the initial collision the energy E in the oscillating background will decrease as the system begins to thermalize, while the number n of open strings gets larger. So we expect the value of a to decrease with time. This means the system will scan across the different instability bands available to it.

To summarize, we have found that the oscillating background resulting from a 0-brane collision is unstable. The $16N_1N_2$ real degrees of freedom contained in b^i for $i = 2,\ldots,9$ behave as parametrically driven oscillators. Their amplitude grows exponentially, on a time scale

$$t_{\rm YM} \sim 1/\Omega \sim \sqrt{mE}/n\tau. \tag{37}$$

Here m is the mass of the N_1 incoming 0-branes, E is the total energy of the system, n is the number of open strings present in the off-diagonal block b and τ is the fundamental string tension.

V. COMPARISON OF TIME SCALES

We compare the time scale associated with parametric resonance to the quasinormal modes of a black hole. We consider parametric resonance for D0-branes in Sec. V A, generalize to Dp-branes in Sec. V B, and compare to quasinormal modes in Sec. V C.

A. 0-brane parametric resonance

As we saw in Sec. IV, the time scale for parametric resonance is determined by the period of oscillation. In a 0-brane collision this is given by

$$t_{\rm YM} \sim 1/\Omega \sim \sqrt{mE}/n\tau. \tag{38}$$

For N_1 incoming D0-branes the mass is $m = N_1/R$, where $R = g_s l_s$ is the radius of the M-theory circle. Also E is the total energy of the system, n is the number of open strings and $\tau \sim 1/l_s^2$ is string tension. We consider the case $N_1 \sim N_2 \sim N$, with N large to compare to supergravity. Then the off-diagonal block b contains $\mathcal{O}(N^2)$ elements, so as shown in the Appendix $\mathcal{O}(N^2)$ open strings are created by parametric resonance.

[2]We are setting $2\pi\alpha' = 1$ and $A_0 = 0$.
[3]Restoring units, we would have $L^2 \to L^2\tau^2$ in (33) and $a = 2q \sim mE^3/n^4\tau^2$ in (36).

Using $R = g_s \ell_s$, $\tau \sim 1/\ell_s^2$, $n \sim N^2$ and $g_s \sim g_{YM}^2 \ell_s^3$ we obtain

$$t_{YM} \sim \sqrt{\frac{NE}{R}} \frac{1}{n\tau} \sim \frac{\sqrt{E}}{\lambda^{1/2} N}. \tag{39}$$

At the correspondence point

$$E \sim N^2 \lambda^{1/3} \tag{40}$$

which means

$$t_{YM} \sim \lambda^{-1/3}. \tag{41}$$

At the correspondence point the time scale for parametric resonance is independent of N and is set by the 't Hooft scale. As we will see in Sec. V C, the same holds true for the quasinormal frequencies of a black hole at the correspondence point.

B. p-brane parametric resonance

It is straightforward to extend this result to Dp-branes. First, the mass of a single D0-brane in the previous section is replaced by the mass of Dp-brane wrapped on a volume V_p. So we should replace

$$1/R \rightarrow V_p / g_s l_s^{p+1}. \tag{42}$$

The energy of the incoming Dp-branes is related to the energy density ϵ by

$$E = \epsilon V_p. \tag{43}$$

The tension of the strings is the same, $\tau \sim 1/\ell_s^2$. So for Dp-branes, in place of (38), the oscillation time scale is

$$t_{YM} \sim \frac{\sqrt{mE}}{n\tau} \rightarrow V_p \sqrt{\frac{N\epsilon}{g_s l_s^{p+1}}} \frac{1}{n\tau}. \tag{44}$$

The number of open strings n is modified. As shown in the Appendix, for $N_1 \sim N_2$ and $p \neq 3$, the number density of open strings at the correspondence point is set by the 't Hooft scale. Thus

$$n \sim N^2 V_p \lambda^{\frac{p}{3-p}}. \tag{45}$$

Using this together with $g_s N = g_{YM}^2 N \ell_s^{3-p} = \lambda \ell_s^{3-p}$ we obtain

$$t_{YM} \sim V_p \sqrt{\frac{N\epsilon}{g_s \ell_s^{p+1}}} \frac{1}{n\tau} \sim \frac{\lambda^{-\frac{p}{3-p}} \sqrt{\epsilon}}{\lambda^{1/2} N}. \tag{46}$$

From (22) the energy density at the correspondence point is

$$\epsilon \sim N^2 \lambda^{\frac{1+p}{3-p}} \tag{47}$$

so the time scale is

$$t_{YM} \sim \lambda^{-\frac{1}{3-p}}. \tag{48}$$

Just as for 0-branes, the time scale for parametric resonance is independent of N and set by the 't Hooft scale.

3-branes are a special case since the 't Hooft coupling is dimensionless. The correspondence point is defined by $\lambda \sim 1$. As shown in the Appendix, for $N_1 \sim N_2$ the number of open strings at the correspondence point is

$$n \sim N^2 V_3 U_0^3, \tag{49}$$

where U_0 is the horizon radius of the black brane. The energy density at the correspondence point is $\epsilon \sim N^2 U_0^4$, so the parametric resonance time scale is

$$t_{YM} \sim V_p \sqrt{\frac{N\epsilon}{g_s \ell_s^{p+1}}} \frac{1}{n\tau} \sim \frac{1}{U_0}. \tag{50}$$

Thus for D3-branes the parametric resonance time scale is $1/U_0$, which also happens to be the inverse temperature of the black brane.

C. Comparison to quasinormal modes

Quasinormal modes for nonextremal Dp-branes were studied in [34,35] following earlier work on AdS-Schwarzschild black holes [36]. The basic idea is to solve the scalar wave equation in the near-horizon geometry of N coincident nonextremal Dp-branes, with a Dirichlet boundary condition at infinity and purely ingoing waves at the future horizon. This gives rise to a discrete set of complex quasinormal frequencies, whose imaginary parts govern the decay of scalar perturbations of the black hole. It was found that the quasinormal frequencies are proportional to the temperature, with a coefficient of proportionality that was found numerically in [34].

Recall that the temperature, energy density and entropy density of these black branes are related to their horizon radius U_0 by [27,34]

$$T \sim \frac{1}{\sqrt{\lambda}} U_0^{(5-p)/2}, \qquad \epsilon \sim \frac{N^2}{\lambda^2} U_0^{7-p},$$
$$s \sim \frac{N^2}{\lambda^{3/2}} U_0^{(9-p)/2}.$$

Assuming $p \neq 3$, at the correspondence point we have $U_0 \sim \lambda^{1/(3-p)}$ so that

$$T \sim \lambda^{\frac{1}{3-p}}, \qquad \epsilon \sim N^2 \lambda^{\frac{p+1}{3-p}}, \qquad s \sim N^2 \lambda^{\frac{p}{3-p}}.$$

These quantities all obey the expected large-N counting, and since the 't Hooft coupling λ has units of $(\text{energy})^{3-p}$, these results could have been guessed on dimensional grounds. In the special case $p = 3$ the 't Hooft coupling is dimensionless and the correspondence point is defined by $\lambda = 1$. At the correspondence point the horizon radius U_0 remains arbitrary, with

$$T = U_0, \qquad \epsilon = N^2 U_0^4, \qquad s = N^2 U_0^3.$$

Again these results could have been guessed on dimensional grounds.

As we saw in Secs. VA and VB the time scale for parametric resonance is

$$t_{\rm YM} \sim \begin{cases} \lambda^{-1/(3-p)} & \text{for } p \neq 3 \\ 1/U_0 & \text{for } p = 3. \end{cases} \tag{51}$$

For all p this matches the inverse temperature of the black brane, $t_{\rm YM} \sim 1/T$. Thus at the correspondence point the time scale for parametric resonance matches the time scale for the decay of quasinormal excitations of the black brane.

VI. COMPARISON TO EQUILIBRIUM PROPERTIES

It is interesting to compare the properties of the bound state as initially formed to the equilibrium properties of the black hole. This will show us that, at the correspondence point, very little additional evolution is required to reach equilibrium—perhaps just a few e-foldings of parametric resonance will suffice.

First, in a 0-brane collision, note that the total number of open strings produced is $\sim N_1 N_2$. With equal charges $N_1 = N_2 = N/2$ the number of open strings is $\mathcal{O}(N^2)$. At the correspondence point these strings have a mass $\sim\lambda^{1/3}$, so the total energy and entropy in open strings is

$$E \sim N^2 \lambda^{1/3}, \qquad S \sim N^2.$$

This matches the equilibrium energy and entropy of the black hole, suggesting that black hole formation at the correspondence point is a simple one-step procedure, in which the open strings that are formed in the initial collision essentially account for the equilibrium properties of the black hole. The analogous result for p-branes is that the number of open strings at the correspondence point is, for $p \neq 3$,

$$n \sim N^2 V_p \lambda^{\frac{p}{3-p}}, \tag{52}$$

where we have used (A2) and the fact that $U \sim \lambda^{\frac{1}{3-p}}$. Since the open strings have a mass $\sim U$, this corresponds to a total energy and entropy in open strings

$$E \sim N^2 V_p \lambda^{\frac{p+1}{3-p}}, \qquad S \sim N^2 V_p \lambda^{\frac{p}{3-p}}$$

which again matches the equilibrium energy and entropy of the black brane. This again suggests that the black hole is essentially fully formed in the initial collision, with very little additional evolution required to reach equilibrium.[4]

Another quantity we can compare at the correspondence point is the size of the bound state. At weak coupling, after n open strings have been formed, the amplitude of oscillation of the resulting bound state is, from (27),

$$L = \frac{E}{n\tau}. \tag{53}$$

[4]When $p = 3$ the matching is $n \sim N^2 V_3 U_0^3$, $E \sim N^2 V_3 U_0^4$, $S \sim N^2 V_3 U_0^3$.

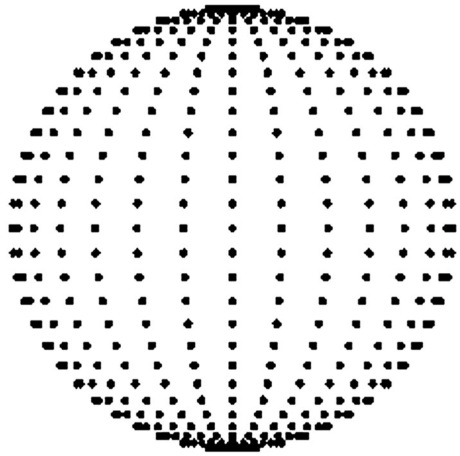

FIG. 3. A collapsing shell of 0-branes. Initially the 0-branes are spread uniformly over an S^8 with velocities toward the center.

At the correspondence point for general p we have

$$E \sim N^2 V_p U_0^{p+1} \tag{54}$$

while the initial number of open strings created is

$$n \sim N^2 V_p U_0^p. \tag{55}$$

Thus the initial amplitude of oscillation as measured in the U coordinate is

$$L/\ell_s^2 = E/n \sim U_0. \tag{56}$$

In other words, the initial oscillation amplitude matches the equilibrium horizon radius of the black brane. Again this suggests that after the initial collision, only a small amount of additional evolution is required to reach equilibrium.

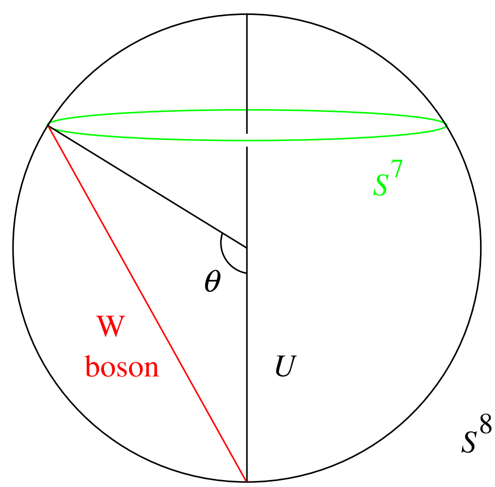

FIG. 4 (color online). The 0-branes are spread over an S^8 of radius U. The green S^7 has radius $U \sin\theta$ and the red W boson has length $2U \sin\theta/2$.

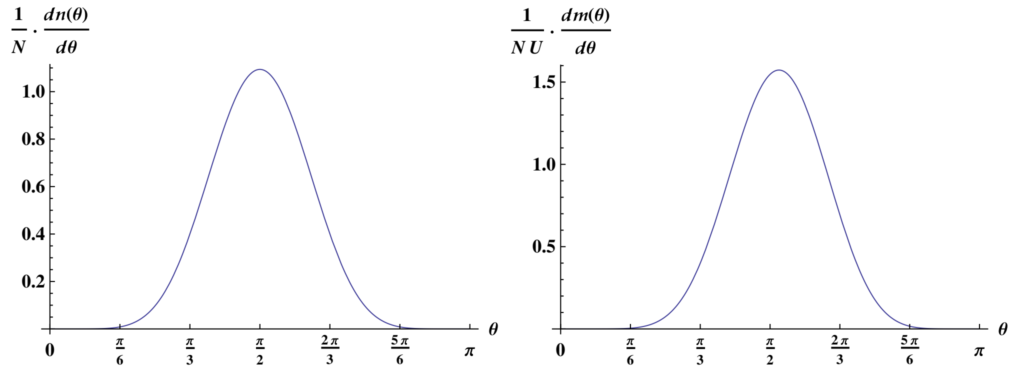

FIG. 5 (color online). On the left, the W-boson number density $\frac{1}{N}\frac{dn}{d\theta}$. On the right, the W-boson mass density $\frac{1}{NU}\frac{dm}{d\theta}$.

VII. SHELL COLLAPSE

So far we have studied bound state formation in a collision between two clusters of D-branes, in the geometry shown in Fig. 1. Here we study a different initial configuration, in which N D0-branes are uniformly distributed over a collapsing spherical shell as in Fig. 3. We will see that the correspondence principle applies and a similar outcome is obtained in this case.

We consider an initial configuration in which the 0-branes are uniformly spread over an S^8 of radius U in nine spatial dimensions. The 0-branes are localized but uniformly distributed over the sphere, with velocities directed toward the center. Intuitively we argue as follows. Since the total volume of the sphere scales as U^8, each 0-brane occupies a volume $\sim U^8/N$, and the distance between nearest-neighbor 0-branes scales as $U/N^{1/8}$. This means virtual open strings connecting nearest-neighbor 0-branes are quite light, with a mass $\sim U/N^{1/8}$ that goes to zero at large N. However the typical open string is much heavier, with a mass $\sim U$ that is independent of N. We expect these typical open strings to dominate the bound-state formation process, and therefore expect to have a well-defined correspondence point at large N.

To argue this in more detail, it is useful to consider a 0-brane located at the south pole and study the number of virtual open strings as a function of the angle θ to the other 0-brane. See Fig. 4. The number of distinct open strings dn in the interval $(\theta, \theta + d\theta)$ is

$$dn = \frac{N}{\frac{32\pi^4 U^8}{105}} \times \frac{\pi^4}{3}(U\sin\theta)^7 \times U d\theta. \tag{57}$$

The first factor $N/(\frac{32\pi^4 U^8}{105})$ is the number density of 0-branes on the S^8, the second factor $\frac{\pi^4}{3}(U\sin\theta)^7$ is the volume of an S^7 located at an angle θ from the south pole. Thus the number density of open strings is

$$\frac{dn}{d\theta} = \frac{35}{32} N \sin^7\theta. \tag{58}$$

We can also find the mass density of open strings $\frac{dm}{d\theta}$. Since an open string subtending an angle θ has a mass $2U\sin\theta/2$, this is given by

$$\frac{dm}{d\theta} = \frac{dn}{d\theta}\cdot 2U\sin\frac{\theta}{2} = \frac{35}{16} N U \sin^7\theta \sin\frac{\theta}{2}. \tag{59}$$

The W-boson number density $\frac{1}{N}\frac{dn}{d\theta}$ and mass density $\frac{1}{NU}\frac{dm}{d\theta}$ are plotted in Fig. 5.

As can be seen in the figure, there are light open strings at large N. However the number of these strings is tiny, since $\frac{dn}{d\theta} \sim \theta^7$ at small angles.[5] Most of the W-bosons are concentrated around $\theta = \pi/2$. Therefore a spherical shell is basically the same as having W-bosons distributed in the interval $\theta_0 < \theta < \pi - \theta_0$, where θ_0 is determined by the fraction of 0-branes pairs we neglect. For example, if we neglect $\frac{dn}{d\theta} \leq 10^{-7}N$, then $\theta_0 \sim 0.1$. Since the masses of the W-bosons near $\theta = \pi/2$ are all $O(U)$, we can simply approximate the entire W-boson spectrum by taking $m_W \sim U$.

We now consider what happens when we give the shell of 0-branes some velocity toward the origin. The analysis is almost identical to the colliding clusters considered in Sec. II. Given N D0-branes with total energy E, the asymptotic relative velocity is

$$E \sim \text{mass} \times v^2 \sim \frac{N}{R} v^2 \Rightarrow v \sim \left(\frac{ER}{N}\right)^{1/2} = \left(\frac{E\lambda l_s^4}{N^2}\right)^{1/2}. \tag{60}$$

In terms of the U coordinate, this becomes

$$\dot{U} = \left(\frac{E\lambda}{N^2}\right)^{1/2}. \tag{61}$$

This matches the result in Sec. II for $N_1 = N_2 \sim N$. Since the W-boson masses are concentrated around $m_W \sim U$, open string production again sets in when

[5] This is due to the fact that the 0-branes are spread on an S^8. The distribution would be less sharply peaked in lower dimensions, with $\frac{dn}{d\theta} \sim \theta^{d-1}$ on an S^d.

$$U \sim \sqrt{\dot{U}} \sim \left(\frac{E\lambda}{N^2}\right)^{1/4}. \qquad (62)$$

At the correspondence point, where the effective gauge coupling becomes order one, we have

$$U \sim \lambda^{1/3} \qquad (63)$$

and therefore

$$E \sim N^2 \lambda^{1/3}. \qquad (64)$$

Just as in Sec. II, this matches the radius and energy of a black hole at the correspondence point.

VIII. CONCLUSIONS

In this paper we studied D-brane collisions. We argued that the process of open string creation, which leads to formation of a D-brane bound state at weak coupling, smoothly matches on to a process at strong coupling, namely black hole formation in the dual supergravity. The transition happens at an intermediate value of the coupling, given by the correspondence principle of Horowitz and Polchinski. The size of the bound state, the time scale for approaching equilibrium, and the thermodynamic properties of the bound state all agree between the two descriptions. The latter agreement happens quickly, which suggests that the bound state is formed by the initial collision in a near-equilibrium configuration.

We considered two types of initial configurations, namely colliding clusters of wrapped Dp-branes and a collapsing shell of D0-branes. The main difference between the two configurations was that the shell had a tail of light open strings which we argued could be neglected. In fact, this distinction between the two configurations is somewhat artificial, since with somewhat more generic initial conditions the 0-branes which make up the clusters could have some small random relative velocities. One would then expect a bit of open string production within the clusters, which would put the two examples on much the same footing.

In the examples we studied the powers of N were fixed by large-N counting, so at the correspondence point there was essentially only a single length scale in the problem, namely the 't Hooft scale (for $p \neq 3$) or the horizon radius (when $p = 3$). In a sense this guaranteed the matching between perturbative gauge theory and gravity results, just on dimensional grounds. To explore this further it would be interesting to study multicharged black holes, or to deform the background in a way which introduces another length scale, and ask whether there is still a simple transition between perturbative worldvolume dynamics and black hole formation.

A step in this direction would be to consider 0-brane collisions but with $N_1 \neq N_2$. In this case, as we saw in Sec. VI, the matching between perturbative gauge and gravity results must be more complicated, because the energy and entropy in open strings that are created in the initial collision do not match the equilibrium energy and entropy of the black hole. This means further dynamical evolution is required before the bound state reaches equilibrium. It would be interesting to study this, perhaps by going beyond the linearized approximation made when studying parametric resonance in Sec. IV. There are several related interesting examples to consider, for example a situation in which several concentric layers of shells are collapsing.

Another direction would be to use the present results to better understand the microstructure of black holes. The picture that emerges, that a black hole is a thermal bound state of D-branes and open strings, is reminiscent of the fuzzball proposal [37]. However the real question, relevant for understanding firewalls [38] or the energetic curtains of [39], is whether this thermal state could be a dual description of the interior geometry of the black hole.

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APPENDIX: STRING PRODUCTION IN A D-BRANE COLLISION

We review the process of open string production in a D-brane collision, following [26,40].

Consider colliding two 0-branes with relative velocity v and impact parameter b. Setting $2\pi\alpha' = 1$, the virtual open strings connecting the two 0-branes have an energy or frequency $\omega = \sqrt{v^2 t^2 + b^2}$. As long as this frequency is changing adiabatically open strings will not be produced. The adiabatic approximation breaks down when $\dot{\omega}/\omega^2 \gtrsim 1$. The peak value of this quantity is $\dot{\omega}/\omega^2 \sim v/b^2$ when $vt \sim b$, so (restoring units) open strings are produced for $b \lesssim \sqrt{v\alpha'}$. In terms of the radial coordinate $U = r/\alpha'$, where r is the distance between 0-branes, the energy of an open string is $m_W = U/2\pi$. So the adiabatic approximation breaks down and open strings are produced when $\dot{U}/U^2 \sim 1$.[6]

Now consider colliding two p-branes wrapped on a torus of volume V_p, with relative velocity v and impact parameter b in the transverse dimensions. Consider a virtual open string that connects the two p-branes and has momentum k

[6]In principle we should distinguish between the asymptotic relative velocity $\dot{U} = v/\alpha'$ and the actual time-dependent value $\dot{U} = \frac{v}{\alpha'} \frac{vt}{\sqrt{b^2+v^2t^2}}$. But at $vt \sim b$ this distinction can be ignored.

along the p-brane worldvolumes. Setting $2\pi\alpha' = 1$, this virtual open string has an energy or frequency

$$\omega = \sqrt{k^2 + v^2 t^2 + b^2}.$$

If $k = 0$ then the condition for open string production is just what it was for 0-branes, $b \lesssim \sqrt{v}$. Having nonzero k increases ω and suppresses open string production. Effectively there is a cutoff, that open strings are produced up to a maximum momentum $k \sim b \sim \sqrt{v}$. Restoring units, the maximum momentum is $k \sim \sqrt{v/\alpha'} = \dot{U}^{1/2}$. This cutoff corresponds to a number density of open strings on the p-brane worldvolume,

$$\frac{\#\text{open strings}}{\text{volume}} \sim \dot{U}^{p/2}.$$

Again these open strings are produced when $\dot{U}/U^2 \sim 1$.

If we collide two stacks of Dp-branes with charges N_1 and N_2 respectively, it is easy to estimate the total number of open strings that are produced. At weak coupling the individual brane collisions are independent events. So for 0-branes the total number of open strings produced is

$$n \sim N_1 N_2$$

while for p-branes the total number of open strings produced is

$$n \sim N_1 N_2 V_p \dot{U}^{p/2} \tag{A1}$$

or equivalently, in terms of the radius at which open string production takes place

$$n \sim N_1 N_2 V_p U^p. \tag{A2}$$

There is, however, an important consistency check on this result: we need to make sure the incoming D-branes have enough kinetic energy to produce this number of open strings. Equivalently, we need to make sure that the back-reaction of open string production on the velocities of the D-branes is under control. Given the number of open strings (A2), the energy in open strings is

$$E_{\text{string}} = nU = N_1 N_2 V_p \left(\frac{\lambda\epsilon}{N_1 N_2}\right)^{\frac{p+1}{4}},$$

where we have used (17). On the other hand the kinetic energy of the incoming branes is

$$E = \epsilon V_p.$$

Thus the ratio

$$\frac{E_{\text{string}}}{E} = \lambda \left(\frac{\lambda\epsilon}{N_1 N_2}\right)^{\frac{p-3}{4}} \tag{A3}$$

and the consistency condition $E_{\text{string}}/E < 1$ is equivalent to

$$\lambda U^{p-3} < 1.$$

This is nothing but the condition $\lambda_{\text{eff}} < 1$. Thus at weak coupling energy conservation does not limit the number of open strings that are produced and the simple estimate (A2) can be trusted.

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