Lorentz-violating regulator gauge fields as the origin of dynamical flavor oscillations

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We show how a mass mixing matrix can be generated dynamically, for two massless fermion flavors coupled to a Lorentz invariance violating (LIV) gauge field. The LIV features play the role of a regulator for the gap equations, and the nonanalytic dependence of the dynamical masses, as functions of the gauge coupling, allows us to consider the limit where the LIV gauge field eventually decouples from the fermions. Lorentz invariance is then recovered, to describe the oscillation between two free fermion flavors, and we check that the finite dynamical masses are the only effects of the original LIV theory. We also discuss briefly a connection of our results with the case of Majorana neutrinos in both, the standard model, where only left-handed (active) neutrinos are considered, and extensions thereof, with sterile right-handed neutrinos.

I. INTRODUCTION AND MOTIVATION

The generation of quark, lepton and vector boson masses, as described in the standard model due to their coupling with the Higgs boson (spontaneous symmetry breaking), seems to have been confirmed by the latest experimental results at the Large Hadron Collider [1], with the discovery of a Higgs-like (scalar) particle. However, the origin of neutrino masses is still not well established, although the seesaw mechanism seems the most elegant and simple for such a purpose [2]. Seesaw mechanisms involve necessarily Majorana mass type fermions, and heavy right-handed neutrino states, without standard model interactions (sterile), whose exchange explains the smallness of the active neutrino (left-handed) species of the standard model. Such sterile neutrinos have not yet been discovered in Nature [3].

The possibility, therefore, of generating neutrino masses dynamically without the involvement of heavy right-handed states is still at play. In this article we take some preliminary steps in this direction and envisage scenarios in which flavor oscillations can arise dynamically, from the flavor-mixing interaction of two massless bare fermions with an Abelian gauge field, which has a Lorentz-Invariance-Violating (LIV) propagator. Lorentz symmetry violation is achieved by higher order space derivatives, which are suppressed by a large mass scale M. This mass scale allows the dynamical generation of fermion masses, as was shown in [4] with the Schwinger-Dyson approach. Another role of this mass scale is to lead to a finite gap equation, and therefore to regulate the model. Further studies using a similar model were done in [5] to generate a fermion mass hierarchy.

Moreover, LIV U(1) gauge models of the form suggested in [4] have been shown [6] to arise in the low-energy limit of some consistent quantum gravity theories, for instance when the U(1) gauge theory is embedded in a stringy space-time foam model, with the foamy structures being provided by (pointlike) D-brane space-time defects ("D-particles"). In such microscopic models, the gauge field was one of the physical excitations on brane world universes interacting with the D-particles. It was observed in [6] that the LIV Lagrangian of [4] can be obtained from a Born-Infeld-type Lagrangian of the U(1) gauge field in the D-particle background, upon an expansion in derivatives. Lorentz violation arises locally in such models as a result of the recoil of the D-particle defects during their interaction with open strings representing the U(1) excitations. Other works involving quantization of higher-order-derivative extensions of quantum electrodynamics can be found in [7].

An important point is the following structure of the dynamical fermion mass [4]

$$m_{\text{dyn}} \simeq M \exp(-a/e^2), \tag{1}$$

where a is a positive constant and e is the gauge coupling. Such a nonanalytical form is well known in the studies of magnetic catalysis [8], and it can be derived from a nonperturbative approach only, as the Schwinger-Dyson derivation of a gap equation, used in [4] and here. From the expression (1), one can see that it is possible to take the simultaneous limits

$$M \to \infty \quad \text{and} \quad e \to 0 \tag{2}$$

in such a way that the dynamical mass (1) remains finite, corresponding to a *physical* fermion mass. This procedure is consistent in the string-embedding case of [6], where Lorentz symmetry is recovered in the limit of vanishing density of D-particles. In that model, the LIV scale can diverge in the case of vanishing D-particle density and zero fluctuations of the recoil velocity (evaluated over a stochastic population of D-particles), where also the

coupling can go to zero [9], in such a way that the dynamically generated fermion mass remains finite. This is a physical case in which the vector U(1) fields appear as LIV regulators, implying dynamical mass for fermions.

In the present article we shall consider the regularization (2) in a more generic sense. We note at this point that the role of Lorentz symmetry breaking as an UV regulator of a quantum field theory has been considered in [10], but from a rather different perspective than ours. Our aim here is to discuss the dynamically generated mass for fermions and/ or the induced oscillations among fermion species, using the coupling of the fermions with such a *LIV regulator gauge field.*

Oscillations of massless neutrinos were already studied in [11], where neutrinos are considered open systems, interacting with an environment. Such oscillations have also been studied in [12], in the framework of LIV models, involving nonvanishing vacuum expectation values for vectors and tensors. Other constraints and consequences of these LIV models are given in [13]. Whilst these studies have been questioned by phenomenological constraints [14], our present model, based on higher order space derivatives, is not excluded. We note that dynamical generation of flavor oscillations was also studied in the context of Lifshitz theories [15], and a detailed analysis of this mechanism was done in [16], for two Lifshitz fermions coupled by a renormalizable four-fermion interaction.

In the limit (2), the nonphysical gauge field decouples from the theory, and hence the gauge dependence of the dynamical mass is avoided (although this problem can be understood perturbatively in the framework of the pinch technique [17], as explained in [6]). We stress here an essential feature of the mechanism described in the present article. Although LIV operators are suppressed by a large mass scale, so that the corresponding effect is negligible at the classical level, quantum corrections completely change this picture, and lead to finite effects. In our present study, the finite effect is the dynamical generation of fermion masses, which is present even after setting the LIV-suppressing mass scale M to infinity. Note that the order of the steps followed is important: quantization is done for finite mass M and coupling e, after which the simultaneous limits (2) are taken.

The structure of the article is the following: The next section II introduces the model and derives the corresponding gap equations which must be satisfied by the dynamical masses. We consider the corresponding constraints and calculate the dynamical masses in the relevant cases in Sec. III. In subsection III F we discuss the "Lorentz-invariant limit" (2), in which the LIV gauge field decouples from fermions, and we demonstrate that relativistic dispersion relations for fermions are indeed recovered. The extension of the Dirac fermion case to chiral Majorana fermions, as appropriate for neutrinos either in the standard model or in seesaw-type extensions thereof, involving

sterile neutrinos, is discussed in Sec. IV. Finally conclusions and outlook are presented in Sec. V. Technical aspects of our work are given in two Appendices.

II. DYNAMICAL FERMION MASS MATRIX

A. The field theory model

The LIV model we consider is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} \left(1 - \frac{\Delta}{M^2}\right) F^{\mu\nu} + \bar{\Psi}(i\slashed{\partial} - \tau\slashed{A})\Psi, \quad (3)$$

where $F_{\mu\nu}$ is the Abelian field strength for the gauge field A^μ and $\Delta = -\partial_i \partial^i$ is the Laplacian (the metric used throughout this work is diag$(1, -1, -1, -1)$). The mass scale M suppresses the LIV derivative operator Δ, and can be thought of as the Plank mass, which eventually will be set to infinity. Ψ is a massless fermion doublet

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad (4)$$

and the flavor mixing matrix τ features the gauge couplings (e_1, e_2, ϵ) as

$$\tau = \begin{pmatrix} e_1 & -i\epsilon \\ i\epsilon & e_2 \end{pmatrix} = \frac{e_1 + e_2}{2} \mathbf{1} + \frac{e_1 - e_2}{2} \sigma_3 + \epsilon\sigma_2, \quad (5)$$

where σ_i are the usual Pauli matrices and $\mathbf{1}$ is the 2×2 identity matrix. The fermions ψ_1 and ψ_2 in (3) are Dirac, but the structure of the gap equations that will be derived below remains the same in the case of Majorana fermions, hence the corresponding dynamical masses are independent of the nature of fermions. As already noted in the previous section, the Lagrangian (3) can be derived from a stringy space-time foam model, as shown in [6]. We mention in passing that such a space-time foam model was already used to study decoherence in flavor oscillations, both in flat space-time and in a Friedman-Robertson-Walker metric [18].

The gauge field bare propagator is

$$D_{\mu\nu} = -\frac{i}{1 + \vec{p}^2/M^2} \left(\frac{\eta_{\mu\nu}}{\omega^2 - \vec{p}^2} + \zeta \frac{p_\mu p_\nu}{(\omega^2 - \vec{p}^2)^2}\right), \quad (6)$$

where ζ is a gauge fixing parameter, which appears in the final expression for the dynamical masses, but does not play a role in the simultaneous limits

$$M \to \infty \quad \text{and} \quad e_1, e_2, \epsilon \to 0, \quad (7)$$

that leave the dynamical masses finite, as we discuss further on.

We note that the flavor mixing interaction $\bar{\Psi}\tau\slashed{A}\Psi$ can be at the origin of a gauge boson mass, which is dynamically generated, as fermion masses. This alternative to the Higgs mechanism is explained in [19], and was extended to a LIV model in [20]. In the present article, we disregard the possibility to generate a gauge boson mass dynamically,

since, as we shall demonstrate below, the flavor mixing coupling ϵ vanishes *necessarily* for consistency of the model in the case there is dynamical generation of *fermion oscillations*.

The bare fermion propagator is $S = i\not{p}/p^2$, where $p_\mu = (\omega, \vec{p})$, and we assume the dynamical generation of the fermion mass matrix

$$\mathbf{M} = \begin{pmatrix} m_1 & \mu \\ \mu & m_2 \end{pmatrix} = \frac{m_1 + m_2}{2}\mathbf{1} + \frac{m_1 - m_2}{2}\sigma_3 + \mu\sigma_1,$$

(8)

with eigenvalues

$$\lambda_{m\pm} = \frac{m_1 + m_2}{2} \pm \frac{\sqrt{(m_1 - m_2)^2 + 4\mu^2}}{2}.$$

(9)

Because the mass matrix contains in general nondiagonal elements, the flavor eigenstates $|\psi_i\rangle$, $i = 1, 2$ are not the same as the mass eigenstates $|\psi_\pm\rangle$ and there is mixing and oscillations, provided the energy eigenvalues $E_\pm = \sqrt{p^2 + \lambda_\pm^2}$ are different.[1] As usual, the flavor eigenstates are connected to the mass (energy) eigenstates by a unitary transformation, parametrized by a mixing angle θ:

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

(10)

and if at time $t = 0$ one has a flavor $\psi_1(t = 0)$ then the probability of obtaining (under Hamiltonian evolution) the other flavor $\psi_2(t)$ at $t > 0$ is nontrivial and given by

$$\mathcal{P}_{12}(t) = \sin^2 2\theta \sin^2\left(\frac{E_+ - E_-}{2}t\right)$$

(11)

and the survival probability $\mathcal{P}_{11} = 1 - \mathcal{P}_{12}$. These constitute the *flavor oscillations*. We stress that, as becomes evident from (11), nontrivial mixing, $\theta \neq 0$, is not sufficient for oscillatory behavior among flavors, one needs necessarily different energy levels $E_+ \neq E_-$ as well. In what follows we shall identify cases where mixing and/or oscillations are generated dynamically, as a result of the coupling of the fermions with the LIV gauge bosons.

If we neglect other quantum corrections, the dressed fermion propagator G, obtained by solving the equation

$$G(\not{p} - \mathbf{M}) = i\mathbf{1}$$

(12)

is then

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[1]We shall check in subsection III F that the relativistic dispersion relations for the fermions are indeed obtained in the Lorentz invariant limit (7), after (finite) dynamical mass generation.

$$G = i\frac{p^2 + \not{p}(m_1 + m_2) + m_1 m_2 - \mu^2}{(p^2 - m_1^2)(p^2 - m_2^2) - 2\mu^2(p^2 + m_1 m_2) + \mu^4}$$
$$\times\left[\left(\not{p} - \frac{m_1 + m_2}{2}\right)\mathbf{1} + \frac{m_1 - m_2}{2}\sigma_3 + \mu\sigma_1\right].$$

(13)

We must check in what follows that the dynamical masses m_1, m_2, μ assumed here can indeed be generated by quantum corrections, which are obtained using the Schwinger-Dyson approach.

B. Schwinger-Dyson gap equations

The self-consistent Schwinger-Dyson equation for the fermion propagator has the usual structure [21], and is not modified by the LIV term in the Lagrangian (3). If we neglect corrections to the wave functions, the vertices and the gauge propagator, the Schwinger-Dyson equation reads for our model

$$G^{-1} - S^{-1} = \int_p D_{\mu\nu}\tau\gamma^\mu G\tau\gamma^\nu.$$

(14)

The previous loop integral is finite as a consequence of the LIV term $\vec{p}^2/M^2$ in the denominator of the gauge propagator (6). We show in Appendix A that the equation (14) leads to the following four gap equations, which must be satisfied by the three masses m_1, m_2, μ,

$$\frac{m_1}{4 + \zeta} = (e_1^2 m_1 + \epsilon^2 m_2)I_1 + (\mu^2 - m_1 m_2)(e_1^2 m_2 + \epsilon^2 m_1)I_2,$$
$$\frac{m_2}{4 + \zeta} = (e_2^2 m_2 + \epsilon^2 m_1)I_1 + (\mu^2 - m_1 m_2)(e_2^2 m_1 + \epsilon^2 m_2)I_2,$$
$$\frac{\mu}{4 + \zeta} = \mu(e_1 e_2 - \epsilon^2)[I_1 - (\mu^2 - m_1 m_2)I_2],$$
$$0 = \epsilon(e_1 m_1 + e_2 m_2)I_1 + \epsilon(\mu^2 - m_1 m_2)(e_1 m_2 + e_2 m_1)I_2,$$

(15)

where

$$I_1 = \frac{J(A_+^2) - J(A_-^2)}{A_+^2 - A_-^2},$$
$$I_2 = \frac{1}{A_+^2 - A_-^2}\left[\frac{J(A_+^2)}{A_+^2} - \frac{J(A_-^2)}{A_-^2}\right],$$

(16)

and

$$J(A_\pm^2) = \frac{1}{4\pi^3}\int_0^\infty dp\,\frac{\vec{p}^2}{1 + \vec{p}^2/M^2}$$
$$\times \int_{-\infty}^\infty d\omega\left(\frac{1}{\omega^2 + \vec{p}^2} - \frac{1}{\omega^2 + \vec{p}^2 + A_\pm^2}\right)$$
$$A_\pm^2 = \frac{m_1^2 + m_2^2 + 2\mu^2}{2} \pm \frac{\sqrt{(m_1^2 - m_2^2)^2 + 4\mu^2(m_1 + m_2)^2}}{2}.$$

(17)

After the integration over the frequency ω and the momentum $\vec{p}$ in the integrals $J(A_\pm^2)$, we obtain for $M \gg m_1$, m_2, μ

$$I_1 \simeq \frac{1}{16\pi^2} \frac{1}{A_+^2 - A_-^2} \left[A_-^2 \ln\left(\frac{A_-^2}{M^2}\right) - A_+^2 \ln\left(\frac{A_+^2}{M^2}\right) \right],$$

$$I_2 \simeq \frac{1}{16\pi^2} \frac{1}{A_+^2 - A_-^2} \ln\left(\frac{A_-^2}{A_+^2}\right). \tag{18}$$

The four equations (15) do not have obvious solutions, since they must be satisfied by only three unknowns m_1, m_2, μ. In what follows, we study different solutions. The ones allowing for the generation of flavor oscillations must have $\mu \neq 0$.

C. Constraints

From the first two equations (15), one obtains for $e_1^2 e_2^2 \neq \epsilon^4$:

$$I_1 = \frac{1}{4 + \zeta} \frac{e_2^2 m_1^2 - e_1^2 m_2^2}{(e_1^2 e_2^2 - \epsilon^4)(m_1^2 - m_2^2)},$$

$$(\mu^2 - m_1 m_2) I_2 = \frac{1}{4 + \zeta} \frac{m_1 m_2 (e_1^2 - e_2^2) + \epsilon^2 (m_2^2 - m_1^2)}{(e_1^2 e_2^2 - \epsilon^4)(m_1^2 - m_2^2)}, \tag{19}$$

and the third and forth equations lead to the following constraints respectively

$$\mu(m_1 + m_2)(e_2 m_1 + e_1 m_2)(e_1 - e_2) = 0,$$

$$\epsilon(e_2 m_1 + e_1 m_2) = 0. \tag{20}$$

We are therefore left with different possibilities, that we study in the next section. Note that, although the denominators in Eq. (19) vanish when $m_1^2 = m_2^2$, we will see in the next section that no singularity arises, since the numerator then also vanishes, because $e_1 = e_2$.

III. SOLUTIONS OF THE GAP EQUATIONS-DYNAMICAL FERMION MASSES AND MIXING

We now detail the different solutions to the gap equations (15). The trivial solution corresponds to the situation where no dynamical mass is generated, $m_1 = m_2 = \mu = 0$, which is of no interest to us here. In what follows we focus on situations, in which fermion masses are generated with the constraints (20) satisfied.

A. The case $m_1 = m_2 = 0$ and $\mu \neq 0$

In this case, the eigenmasses are

$$\lambda_\pm = \pm \mu, \tag{21}$$

and the mass eigenstates are

$$\psi_\pm = \frac{1}{\sqrt{2}} (\psi_2 \pm \psi_1), \tag{22}$$

such that the mixing angle (10) is $\theta = -\pi/4$, in our conventions. This case does not include a mass hierarchy, hence there are no oscillations (11) among the fermion flavors either, since the energy eigenvalues $E = \sqrt{p^2 + \mu^2}$ are the same.

Among the four gap equations (15) in this case, only the third is not trivial, and leads to

$$\frac{1}{4 + \zeta} = (e_1 e_2 - \epsilon^2)(I_1 - \mu^2 I_2). \tag{23}$$

Since $A_\pm^2 = \mu^2$, the expressions (18) lead to

$$I_1 \simeq \frac{-1}{16\pi^2} \left(1 + \ln\left(\frac{\mu^2}{M^2}\right) \right), \tag{24}$$

$$I_2 \simeq \frac{-1}{16\pi^2} \frac{1}{\mu^2}, \tag{25}$$

and we obtain

$$\ln\left(\frac{\mu^2}{M^2}\right) = \frac{-16\pi^2}{(4 + \zeta)(e_1 e_2 - \epsilon^2)}. \tag{26}$$

We note that this expression has a meaning only if $e_1 e_2 > \epsilon^2$, otherwise $\mu^2 > M^2$. Assuming this constraint on the couplings, we finally obtain

$$\mu \simeq M \exp\left(\frac{-8\pi^2}{(4 + \zeta)(e_1 e_2 - \epsilon^2)}\right). \tag{27}$$

B. The case $e_2 m_1 + e_1 m_2 = 0$ and $m_1^2 \neq m_2^2$

In this situation, the first equation (19) leads to $I_1 = 0$. The expression (18) for I_1 leads then to

$$A_+^2 = A_-^2 = \exp(-1) M^2, \tag{28}$$

which is not physical, because the dynamical masses are then necessarily of the order M. This possibility is therefore disregarded, since we will eventually take the limit $M \to \infty$.

C. The case $m_1 = -m_2 \neq 0$

In order to have $m_1 = -m_2 \equiv m$, it can be seen from Eqs. (15) that necessarily $e_1 = e_2$, such that both constraints (20) are satisfied. Also, Eqs. (15) are equivalent to

$$\frac{1}{4 + \zeta} = (e^2 - \epsilon^2)[I_1 - (\mu^2 + m^2)I_2], \tag{29}$$

and $A_\pm^2 = m^2 + \mu^2$, such that we find

$$m^2 + \mu^2 = M^2 \exp\left(\frac{-16\pi^2}{(4 + \zeta)(e^2 - \epsilon^2)}\right), \tag{30}$$

which has a meaning only if $e^2 > \epsilon^2$. This condition allows one to take the limit $\epsilon \to 0$ without affecting the mass eigenvalues or mixing angles (see below). This is important, because, as already mentioned, a nonzero

flavor-mixing coupling ϵ might lead to dynamical generation of vector boson masses [19,20], thereby spoiling their nature as regulator fields.

We stress here that we cannot determine m and μ independently, and the eigenmasses are

$$\lambda_{\pm} = \pm\sqrt{m^2 + \mu^2}. \tag{31}$$

The mass eigenstates are

$$\psi_{\pm} = \frac{1}{N_{\pm}}\left(\psi_1 + \frac{\mu}{m \pm \sqrt{m^2 + \mu^2}}\psi_2\right), \tag{32}$$

where

$$N_{\pm}^2 = \frac{2m^2 + 2\mu^2 \pm 2m\sqrt{m^2 + \mu^2}}{2m^2 + \mu^2 \pm 2m\sqrt{m^2 + \mu^2}}, \tag{33}$$

and the mixing angle θ (10) is given by

$$\tan\theta = \frac{-\mu}{m + \sqrt{m^2 + \mu^2}}. \tag{34}$$

In order to fix the mixing angle, one would need an additional ingredient, since the present model does not fix μ, but only $m^2 + \mu^2$.

Again, there is no mass hierarchy due to (31) in this case, the energy eigenvalues are the same, and so no oscillations (11) among fermion flavors.

D. The case $m_1 = m_2 \neq 0$: Dynamical flavor oscillations

We find here from Eqs. (15) that necessarily $e_1 = e_2$, $\epsilon = 0$ and $\mu^2 = m^2$. We have then

$$\mu^2 = m_1 m_2 = m^2 \quad \text{and} \quad I_1 = \frac{1}{(4 + \zeta)e^2}, \tag{35}$$

where $e = e_1 = e_2$. We have $A_-^2 = 0$ and $A_+^2 = 4m^2$, such that the expressions (18) and (35) for I_1 lead to

$$-\frac{1}{16\pi^2}\ln\left(\frac{4m^2}{M^2}\right) = \frac{1}{(4 + \zeta)e^2}, \tag{36}$$

and the common dynamical mass is finally

$$m = \frac{M}{2}\exp\left(-\frac{8\pi^2}{(4 + \zeta)e^2}\right) \tag{37}$$

which, as expected, is not perturbative in e. In this situation, the mass matrix has identical elements, and has the eigenvalues

$$\lambda_+ = 2m = M\exp\left(-\frac{8\pi^2}{(4 + \zeta)e^2}\right), \qquad \lambda_- = 0, \tag{38}$$

and the corresponding mass eigenstates are also given by Eq. (22). The mixing angle (10) is $\theta = \mp\pi/4$, depending on the sign of $\mu = \pm m$, respectively.

In this case, one of the fermions is massless, and the other massive, with mass $2m$. There is a mass hierarchy

and thus *oscillations* (11) among the fermion flavors in this case. We note that because of the constraints (20), this is the only case in the model (3) where nontrivial oscillations among fermion flavors take place. As we have seen above, in this case necessarily the flavor-mixing gauge couplings $\epsilon \to 0$, so one does not have to worry about dynamical generation of gauge boson masses, and thus the latter play a consistent role as regulator fields.

E. The case $\epsilon = 0$, $\mu = 0$

This is a straightforward generalization of the original model of [4], which involved one fermion, to the two fermion-flavor case with no mixing at all. This case can be divided into two situations: (i) $m_1 \neq 0$ and $m_2 \neq 0$ and (ii) $m_1 = 0$ or $m_2 = 0$. As we shall discuss in Sec. IV, these may be relevant for Majorana neutrinos in the standard model or extensions thereof, involving right-handed neutrinos, respectively.

1. (i) $m_1 \neq 0$ and $m_2 \neq 0$

In this first situation, we obtain from (9) that the two eigenvalues of the mass matrix are

$$m_i = M\exp\left(\frac{-8\pi^2}{(4 + \zeta)e_i^2}\right), \qquad i = 1, 2, \tag{39}$$

so the dynamically generated mass matrix is diagonal with masses m_i among the two flavors. Hence there is no mixing (10) or oscillations (11) between the flavors ψ_i, $i = 1, 2$, in this case.

2. (ii) $m_1 = 0$ or $m_2 = 0$

In this case, we observe from the system of equations (15) that there is also a consistent solution, with either $m_1 = 0$ with $m_2 \neq 0$ or $m_2 = 0$ and $m_1 \neq 0$. The two cases are symmetric. For reasons that will become clear from our discussion on neutrinos in Sec. IV, we may concentrate for brevity in the former case, i.e. $m_1 = 0$. In that case, the solution of Eqs. (15) yields

$$I_1 = \frac{1}{(4 + \zeta)e_2^2}, \tag{40}$$

while from (18) and the definitions (17) we obtain that in this case $A_- = 0$, while $A_+^2 = m_2^2 \neq 0$, and thus from (40) we have

$$|A_+| = |m_2| \simeq M\exp\left(-\frac{8\pi^2}{(4 + \zeta)e_2^2}\right). \tag{41}$$

Note that, although I_2 diverges logarithmically as $\mu \to 0$, it enters the gap equations (15) only in the combination $(\mu^2 - m_1 m_2)I_2 = \mu^2 I_2$, which vanishes in this limit.

The mass eigenvalues are in this case, $\lambda_- = 0$ and $\lambda_+ = m_2 \neq 0$ given by (41). The mixing angle θ is though vanishing and thus there are no oscillations between the states.

F. Lorentz symmetric limit

In order to recover Lorentz invariance, we finally take the simultaneous limits

$$M \to \infty \quad \text{and} \quad e_1, e_2, \epsilon \to 0, \tag{42}$$

in such a way that the dynamical masses are *finite*, and we denote the corresponding "renormalized" mass matrix by $\mathbf{M}_R$. This procedure is independent of the gauge parameter ζ, and the resulting fermion mass is set to any desired value. In this limit, the gauge field decouples from fermions, and the only finite effect from Lorentz violation in the original model is the presence of finite dynamical masses for fermions.

We now check this statement by demonstrating that the fermion dispersion relations are relativistic in the limit (42). We focus here for concreteness on the solution described in subsection III D, with $\mu = +m$, but clearly the same conclusion holds for all the other solutions given in Sec. III. Because one of the eigenmasses vanishes, which leads to one-loop infrared (IR) divergence, we consider $m_1 = m_2 = m$ and $m - \mu = m\delta$ with $\delta \ll 1$. As will be seen, after the limit (42) is taken, the fermion self-energy won't depend on δ, such that the limit $\delta \to 0$ will not introduce any IR divergence. We calculate in Appendix B the one-loop fermion self energy, where we use the Feynman gauge since the limit (42) is gauge independent. To lowest order in momentum, we find then

$$\Sigma = \begin{pmatrix} Z^0_{\text{diag}} & Z^0_{\text{off}} \\ Z^0_{\text{off}} & Z^0_{\text{diag}} \end{pmatrix} \omega \gamma^0 - \begin{pmatrix} Z^1_{\text{diag}} & Z^1_{\text{off}} \\ Z^1_{\text{off}} & Z^1_{\text{diag}} \end{pmatrix} \vec{p} \cdot \vec{\gamma} - \mathbf{M},$$

(43)

where $(\omega, \vec{p})$ is the external 4-momentum and

$$Z^0_{\text{diag}} = \frac{e^2}{8\pi^2}\left(\frac{1}{4} - \frac{1}{2}\ln 2 + \frac{1}{2}\ln \delta + \ln\left(\frac{m}{M}\right)\right),$$

$$Z^1_{\text{diag}} = \frac{e^2}{8\pi^2}\left(-\frac{1}{12} - \frac{1}{2}\ln 2 + \frac{1}{2}\ln \delta + \ln\left(\frac{m}{M}\right)\right), \tag{44}$$

$$Z^0_{\text{off}} = Z^1_{\text{off}} = \frac{e^2}{16\pi^2}(\ln 2 - \ln \delta).$$

As expected, because of Lorentz-symmetry violation, $Z^0_{\text{diag}} \neq Z^1_{\text{diag}}$, but since

$$e^2 \ln\left(\frac{m}{M}\right) = -2\pi^2, \tag{45}$$

the limit (42) leads to

$$\Sigma \to -\frac{1}{4}(\omega \gamma^0 - \vec{p} \cdot \vec{\gamma})\mathbf{1} - \mathbf{M}_R. \tag{46}$$

Therefore the dispersion relations are relativistic, since time and space derivatives are dressed with the same corrections in the limit (42). These corrections can be absorbed in a fermion field redefinition, so that we are left with two free relativistic fermion flavors oscillating.

G. Energetics arguments

Among the different possibilities to generate masses dynamically, one can question the preference for the system to have nonvanishing masses, rather than no dynamical mass generated. We give here an energetics argument supporting the choice of nonvanishing dynamical masses [20]. This argument is based on the Feynman-Hellmann theorem [22], which states that, if there is a ground state $|\Psi_\lambda\rangle$ of a system with Hamiltonian that depends on a parameter λ, then for the energy E of this ground state we have

$$\frac{\partial E}{\partial \lambda} = \langle \Psi_\lambda | \frac{\partial \hat{H}}{\partial \lambda} | \Psi_\lambda \rangle, \tag{47}$$

where $\hat{H}$ is the Hamiltonian operator of the system. In our situation, the parameter λ can be chosen to be $\lambda = M^{-2}$, such that

$$\frac{\partial E}{\partial \lambda} = +\frac{1}{4} {}_M\langle 0| \int d^4x^E (F_{\mu\nu}\Delta F^{\mu\nu})_E |0\rangle_M, \qquad \lambda = M^{-2},$$

(48)

where the index E denotes Euclidean formalism, as a result of the fact that the Hamiltonian of the system is identified with minus the effective Euclidean action. One should expect that the Lorentz-violating nature of the vacuum $|0\rangle_M$ implies in general the nonvanishing of the right-hand side, implying a dependence of the vacuum energy on the dynamically generated mass. Using the cyclic Bianchi identity for the gauge bosons field strengths,

$$\partial_{[\mu} F_{\nu\rho]} = 0, \tag{49}$$

with the symbol $[\ldots]$ denoting symmetrization of the appropriate indices, we obtain

$$\frac{\partial E}{\partial \lambda} = -\frac{1}{4} {}_M\langle 0| \int d^4x^E (F_{\mu\nu}\partial_i[\partial^\mu F^{\nu i} + \partial^\nu F^{i\mu}])_E |0\rangle_M.$$

(50)

Integrating by part and assuming that the fields decay away at space-time infinity, one may write Eq. (48) in the form:

$$\frac{\partial E}{\partial \lambda} = +\frac{1}{2} {}_M\langle 0| \int d^4x^E (\partial^\mu F_{\mu\nu}\partial_i F^{\nu i})_E |0\rangle_M. \tag{51}$$

We write then the equations of motion for the vector fields, from the Lagrangian (3) where we neglect the operator Δ/M^2, and we obtain:

$$\frac{\partial E}{\partial \lambda} = +\frac{1}{2} {}_M\langle 0| \int d^4x^E ((J^0)^2 + \vec{J} \cdot \vec{J} - J_k \partial_0 F^{k0})_E |0\rangle_M,$$

(52)

where the current is $J^\mu = \bar{\Psi}\gamma^\mu \tau \Psi$ and the Euclidean formalism is used in (52). In the framework of the LIV model studied here, one might face a situation where nontrivial condensates of the covariant square of the stationary four-current J^μ are observed in the (rotationally invariant)

vacuum. For such stationary currents, where $\partial_0 F^{k0} = 0$, we have then from Eq. (52):

$$\frac{\partial E}{\partial \lambda} = \frac{1}{2} {}_M\langle 0| \int d^4 x^E (J^\mu J_\mu)_E |0\rangle_M \geq 0. \qquad (53)$$

This implies that the vacuum energy E in this case is a monotonically decreasing function of M^2, so that the energy goes to its minimum in the Lorentz symmetric limit $M \to \infty$ we are interested in.

The above argument in favor of the stability of the Lorentz invariant limit (2) can be turned into an argument in favor also of the dynamical fermion-mass generation as follows: in a finite $M < \infty$ situation, the gauge coupling e is considered as an independent quantity from M, and thus, in view of (1), the LIV mass scale is proportional to the fermion mass $m > 0$ (absolute value if $m < 0$). In this sense, one obtains from (53),

$$\frac{\partial E}{\partial m} = \frac{\partial M}{\partial m} \frac{\partial \lambda}{\partial M} \frac{\partial E}{\partial \lambda}$$
$$= -\frac{1}{mM^2} {}_M\langle 0| \int d^4 x^E (J^\mu J_\mu)_E |0\rangle_M \leq 0. \qquad (54)$$

Thus, the energy of the vacuum for any finite value of M is also a monotonically decreasing function of the fermion mass. In the Lorentz-symmetric limit (2), the energy exhibits a plateaux, as far as its dependence on the finite $m > 0$ is concerned, i.e. $\partial E / \partial m = 0$, but its value is lower than the case where $m = 0$.

We stress, however, that the above arguments rely on the formation of condensates for the covariant square of the current. The latter property is at present a conjecture, and its proof goes far beyond our considerations in this article.

IV. EXTENSION TO CHIRAL MAJORANA NEUTRINOS

Above we considered Dirac nonchiral fermions. However, if we wish to present the above-described dynamical mass generation scenario as a viable alternatives to standard seesaw mechanisms for neutrinos, and explain the neutrino oscillations as a dynamical phenomenon, then we should extend the above considerations to the case where the fermions are chiral and Majorana (as most likely is the case realized in nature).

Below we shall consider two separate cases. The first is the one in which the fermions correspond to Majorana mass eigenstates obtained from the left-handed flavor neutrino physical fields of the standard model, while the second case involves sterile right-handed neutrinos as in seesaw extensions of the standard model.

We shall discuss a connection of our previous findings on dynamical mass generation to both types of neutrino masses. In particular, we shall first review the underlying formalism, which is necessary for a better and more complete understanding of the details of such connections. More specifically, as explained below, it is because

Majorana fermions are *mass* eigenstates, involving *both* chiralities, that our results can be relevant to neutrino oscillations. In what follows, we shall first link our dynamical mass generation scenario described in Sec. III E 1 to the standard model left-handed neutrinos, and then we shall connect the dynamical mass generation scenario in Sec. III E 2 to a dynamical seesaw model, involving right-handed Majorana neutrinos that exist in extensions of the standard model. Since in our scenarios the values of the mass can be fixed phenomenologically, we can assume that any other mass contributions to neutrinos (e.g. due to a Higgs mechanism in conventional seesaw models) are subdominant. The advantage of our dynamical mass generation approach lies specifically with the possibility of being applied directly to left-handed standard model neutrinos, without the need of introducing right-handed ones (although there may be other reasons to introduce the latter, and this is why in this section we describe both cases).

A. Left-handed neutrino Majorana mass generation

For instructive purposes it is useful first to review some basic formalism. According to the standard theory [23] a [Majorana (M)] mass term for neutrinos, which involves only left-handed fields, reads

$$\mathcal{L}^M = -\frac{1}{2} \overline{\nu_L} M^M (\nu_L)^c + \text{H.c.,} \qquad (55)$$

where the normalization of $1/2$ will be understood in what follows. In the one generation case we focus upon here M^M is a c-number (In case of many generations, $\nu_\ell, \ell = e, \mu, \tau$ then M^M is a symmetric 3×3 matrix, as can be seen easily). The (physical) Majorana field ν^M, involving *both* chiralities, is defined as

$$\nu^M = \nu_L + (\nu_L)^c \qquad (56)$$

and is always an *eigenstate of the mass*, that is when expressed in terms of it the mass matrix is diagonal

$$\mathcal{L}^M = -\frac{1}{2} \overline{\nu^M} M^M \nu^M = -\frac{1}{2} \sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i, \qquad (57)$$

with m_i the mass eigenvalues. It satisfies the *Majorana condition*

$$(\nu^M)^c = \nu^M, \qquad (58)$$

which implies that a Majorana field is its own antiparticle.

The kinetic (Dirac) term of the Lagrangian with respect the left-handed ν_L fields, when expressed in terms of the Majorana mass eigenstate fields ν_i reads (up to an irrelevant total derivative):

$$\mathcal{L}_{\text{kin}} = \bar{\nu}_L i \not{\partial} \nu_L = \sum_j \frac{1}{2} \bar{\nu}_j i \not{\partial} \nu_j \qquad (59)$$

with $\quad \nu_j = (\nu_j)^c = \nu_{jL} + (\nu_{jL})^c,$

with the suffix j denoting mass eigenstate fields. In view of the extra $\frac{1}{2}$ normalization of the kinetic terms of the Majorana fields then, it is customary to define the corresponding mass terms with the same normalization, as we have done above, compared to other Dirac fields one encounters in the standard model.

In our approach we consider the coupling of a doublet of (mass eigenstate) Majorana fields to the regulator U(1) gauge field A_μ in the case discussed in subsection III E 1. The fact that a Majorana field contains both chiralities allows for a straightforward extension of the Dirac case discussed in previous sections to the current situation. In this way, we are able to generate dynamically different mass eigenvalues for the two species, *without mixing*, as implied by the corresponding solutions

$$m_i = M \exp\left(\frac{-8\pi^2}{(4+\zeta)e_i^2}\right), \qquad i = 1, 2. \quad (60)$$

This is a consistent way of discussing the dynamical appearance of a Majorana mass for left-handed neutrinos of the standard model. Nontrivial mixing of flavor neutrinos, coupled to the physical $SU(2)_L$ gauge fields of the standard model, can then be obtained in the case where the mass eigenvalues are different. In order to recover the Lorentz symmetric limit in this case, we need to take simultaneously $e_1, e_2 \to 0$ in such a way that their ratio is fixed to the phenomenologically desired value. It is important that in this approach we started from Majorana mass eigenstates coupled to the regulator gauge fields, with no mixing. The latter is obtained when one expresses the Majorana mass eigenstates in terms of the *flavor* neutrino eigenstates, which appear in nature.

B. Extensions of the standard model with right-handed (sterile) neutrinos

When there are right-handed (sterile) neutrino components present, ν_R, one can define two kinds of mass terms, Majorana (55) (M) and Dirac (D). The most general mass term, then, reads [23]:

$$\mathcal{L}^{M+D} = -\frac{1}{2}\bar{\nu}_L M_L^M (\nu_L)^c - \bar{\nu}_L M^D \nu_R$$
$$- \frac{1}{2}\bar{\nu}_R M_R^M (\nu_R)^c + \text{H.c.}, \quad (61)$$

where the mass matrices $M_{L,R}^M$ and M^D are in general different.

We consider below the mixed mass terms (61) in the one generation case, of relevance to our models discussed in this work. In this case we may assemble the left-handed neutrino fields and the conjugate of the right-handed one into a left-handed doublet field

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} \quad (62)$$

in which case the mass term (61) can be written in terms of a 2×2 mass matrix (6×6 in the case of three generations):

$$\mathcal{L}^{MD} = -\frac{1}{2}\bar{n}_L M^{M+D}(n_L)^c,$$
$$M^{M+D} = \begin{pmatrix} M_L^M & M_D \\ (M_D)^T & M_R^M \end{pmatrix} \quad (63)$$

where for the sake of generality we expressed here the mass matrix as a matrix with flavor components as well. For a single generation of neutrinos, we consider below, the elements of the above (2×2 in this case) matrix are c-numbers. For our toy purposes here we assume no *CP* violation in the lepton sector [23].

The matrix M^{M+D} can be diagonalized by a Hermitian matrix U:

$$M^{M+D} = U\tilde{\mathfrak{m}}U^T = O\tilde{\mathfrak{m}}\eta O^T$$

with [23]:

$$U = O\eta^{1/2}, \qquad O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad (64)$$

where the matrix η has eigenvalues $\eta_i = \pm 1$, which are related to the so-called *CP* parity of the Majorana neutrinos [23], and stem from the fact that the mass eigenvalues can be positive or negative

$$\mathfrak{m}'_{1,2} = \frac{1}{2}(M_R + M_L) \mp \frac{1}{2}\sqrt{(M_R - M_L)^2 + 4M_D^2} \quad (65)$$

so one can rewrite them as $\mathfrak{m}_i = |\mathfrak{m}_i|\eta_i \equiv \tilde{\mathfrak{m}}_i\eta_i$, $\eta_i = \pm 1$, $i = 1, 2$. The mixing angle θ being such that

$$\cos 2\theta = \frac{M_R - M_L}{\sqrt{(M_R - M_L)^2 + 4M_D^2}},$$
$$\tan 2\theta = \frac{2M_D}{M_R - M_L}. \quad (66)$$

The Majorana fields, involving *both* chiralities, are then defined in terms of U as

$$\nu^M = U^\dagger n_L + (U^\dagger n_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \quad \nu_i^c = \nu_i, \quad i = 1, 2. \quad (67)$$

These are the mass eigenstate fields with masses $\tilde{\mathfrak{m}}_{1,2}$ (65).

The original chiral (left-handed) neutrinos, appearing in the Lagrangian (55) are related therefore to these mass eigenstates as follows:

$$\nu_L = \cos\theta\sqrt{\eta_1}\nu_{1L} + \sin\theta\sqrt{\eta_2}\nu_{2L}$$
$$(\nu_R)^c = -\sin\theta\sqrt{\eta_1}\nu_{1L} + \cos\theta\sqrt{\eta_2}\nu_{2L} \quad (68)$$

In the standard seesaw scenarios [2], there are no masses for the left-handed fields, $M_L = 0$, and the right-handed neutrino (sterile) Majorana masses are assumed to be much heavier than the Dirac masses, $M_R \gg M_D$, the latter being

given by means of a Higgs mechanism by, e.g., Yukawa coupling terms of the form

$$F\phi^C \bar{\nu}_L \nu_R + \text{H.c.}, \tag{69}$$

where F is the Yukawa coupling and $\phi^c = i\sigma_2 \phi^\star$ is the dual of the Higgs doublet. In this limit, from (65) and (66) the mass eigenstates generated are of the form $\mathfrak{m}_1 \simeq \frac{M_D^2}{M_R} \ll M_D$ and $\mathfrak{m}_2 \simeq M_R \gg M_D$, while the mixing angle $\theta \simeq \frac{M_D}{M_R} \ll 1$, and also $\eta_1 = -1$, $\eta_2 = 1$, hence from (68) we do obtain:

$$\nu_L \simeq i\nu_{1L} + \frac{M_D}{M_R}\nu_{2L,} \qquad (\nu_R)^c \simeq -i\frac{M_D}{M_R}\nu_{1L} + \nu_{2L}. \tag{70}$$

The purpose of the remainder of this section is to adopt the previous procedure and generate *dynamically* masses for the Majorana fields by coupling them to gauge fields. To this end we view one of the flavors as a right-handed sterile neutrino, $N_R = \frac{1}{2}(1 + \gamma_5)N$ and the other flavor $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$, as an active neutrino of the standard model. Here, N, ψ are nonchiral spinors, which in the case of neutrino may be taken to be Majorana. This would be a toy (with one active and one sterile neutrino) version of the minimal (nonsupersymmetric) extension of the standard model of Ref. [24], termed νMSM. In this case we do not avoid right-handed neutrinos but we use the dynamical mass generation mechanism presented here to give masses to them. In this case the Lagrangian (3) is replaced by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}\left(1 - \frac{\Delta}{M^2}\right)F^{\mu\nu} + \bar{N}(i\slashed{\partial} - e_1\slashed{A})\frac{1}{2}(1 + \gamma_5)N$$
$$+ \bar{\psi}(i\slashed{\partial} - e_2\slashed{A})\frac{1}{2}(1 - \gamma_5)\psi, \tag{71}$$

Notice that in this case, due to the opposite chiralities of the two spinor fields, the off diagonal flavor mixing gauge couplings ϵ are irrelevant because the corresponding terms vanish identically.

According to our general discussion on combined Dirac and Majorana masses above, we may express this Lagrangian in terms of Majorana fields and view the initially massless ψ and N as the Majorana field doublet ν^M (67), $\nu^M = \binom{\psi}{N}$, which then couples to the vector fields. Dynamically generated mixing of the two should involve a small mixing angle in phenomenologically realistic situations in view of the discussion above, cf. Eq. (70).

Unfortunately, in our single gauge field toy models considered here, the only solution from the cases discussed in Sec. III that can be carried over to the case of Majorana neutrinos is the one discussed in subsection III E 2. In this case, the masses m_1, m_2 can be identified with the dynamically generated mass eigenvalues

$$m_1 = \lambda_- = 0 \qquad m_2 = \lambda_+ = M\exp\left(\frac{-8\pi^2}{(4+\zeta)e_2^2}\right), \tag{72}$$

where m_1 can be identified with the left-handed Majorana mass $M_L = 0$, which in the usual seesaw models is assumed zero, and m_2 is then identified with the heavy right-handed Majorana mass, M_R. In this way, the dynamically generated masses (72) correspond to a seesaw type mass matrix (63) of the form (2×2 in our one-generation example considered explicitly here):

$$M^{M+D} = \begin{pmatrix} 0 & 0 \\ 0 & m_2 \end{pmatrix} \tag{73}$$

for the Majorana neutrinos. The reader should note that there is no nontrivial Dirac mass μ, since the latter vanishes in the dynamical solution, as explained in subsection III E 2.

Nevertheless, the latter can be generated through the usual Yukawa couplings (69) with the Higgs field, which upon acquiring a vacuum expectation value via the Higgs mechanism would generate a Dirac mass term, as we shall discuss below. In this scenario it is the (heavy) right-handed mass that can be generated dynamically, due to the coupling with the LIV gauge sector. Since, as we have already mentioned, the finite mass in the Lorentz invariant limit (7) is arbitrary, we can arrange so that the latter is much heavier than the Higgs-generated Dirac mass, which leads to naturally light active neutrinos in the standard model sector. Let us now proceed to discuss in some detail this latter scenario.

In this case we can consider the Schwinger-Dyson equations in the background of a Higgs field [2] acquiring a v.e.v. $\langle\phi\rangle = v$. This will yield a bare Dirac mass term of the form Fv, where F is the pertinent Yukawa coupling. This affects the form of the bare fermion propagator S by Dirac-mass terms proportional to the Higgs-induced $\mu_0 = Fv$, while the dressed fermion propagator G will have a form similar to that in (13), but with the replacement of μ by the sum $\mu + \mu_0$, with μ the dynamically generated Dirac mass term. It can be readily seen then that the pertinent Schwinger-Dyson equations read:

$$I_1 = \frac{1}{4+\zeta}\frac{e_2^2 m_1^2 - e_1^2 m_2^2}{(e_1^2 e_2^2 - \epsilon^4)(m_1^2 - m_2^2)},$$
$$((\mu+\mu_0)^2 - m_1 m_2)I_2$$
$$= \frac{1}{4+\zeta}\frac{m_1 m_2(e_1^2 - e_2^2) + \epsilon^2(m_2^2 - m_1^2)}{(e_1^2 e_2^2 - \epsilon^4)(m_1^2 - m_2^2)}, \tag{74}$$

supplemented by the following constraints, similar to those given by Eqs. (20):

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[2]Any contributions of the fluctuations of the Higgs to the Schwinger-Dyson equations will be suppressed by the Higgs mass and will be ignored to our leading approximation adopted here.

$$(m_1 + m_2)[\mu(e_2 m_1 + e_1 m_2)(e_1 - e_2)$$
$$- \mu_0(m_1(\epsilon^2 + e_2^2) - m_2(\epsilon^2 + e_1^2))] = 0,$$
$$\epsilon(e_2 m_1 + e_1 m_2) = 0, \tag{75}$$

where we stress once again that μ is the dynamically generated Dirac mass term, and $\mu_0 = Fv$ is the bare (Higgs-induced) one. The integrals I_i, $i = 1$, 2 are given by the same expressions as in (18) but with the replacement of μ by $\mu + \mu_0$.

For consistency with our considerations above, we seek solutions of (74) in which $m_2 \neq 0$ and $m_1 = \mu = \epsilon = 0$, which on account of the constraints (75) imply $e_1 = 0$. We also make the physically relevant assumption that the Dirac mass $\mu_0 \ll m_2$ (which is consistent with light active neutrino species). To leading order in $x \equiv \frac{\mu_0}{m_2} \ll 1$, we then obtain

$$I_1 \simeq \frac{-1}{16\pi^2} \ln\left(\frac{m_2^2}{M^2}\right) + \mathcal{O}(x^2), \qquad \mu_0^2 I_2 \simeq \mathcal{O}(x^2 \ln x).$$

The solution of Eqs. (74), then, for the dynamically generated mass matrix of the Majorana neutrinos is the same as in (72) but with the mass matrix having bare $\mu_0 = Fv$ Dirac terms,

$$M^{M+D} = \begin{pmatrix} 0 & Fv \\ Fv & m_2 \end{pmatrix}, \qquad Fv \ll m_2, \tag{76}$$

with m_2 given by (72). So our dynamical mass generation scenario provides a novel way for generating heavy right-handed neutrino masses when applied to extensions of the standard model containing such states, such as the model of Ref. [24].

V. CONCLUSIONS AND OUTLOOK

In this work we have considered the coupling of flavored fermion fields to LIV vector gauge bosons, with Lorentz invariance being violated in the gauge sector at a mass scale M and studied the limiting case where the gauge couplings go to zero, while the LIV mass scale $M \to \infty$ simultaneously, in such a way that the Schwinger-Dyson dynamically generated fermion masses remain finite. No vector boson mass is generated due to an appropriate arrangement of the couplings. In this way, the LIV vector bosons are viewed as regulator fields, with the only remnant of the LIV the dynamical fermion mass. Unfortunately, the dynamical equations are sufficiently restricted so as to allow only one case where oscillation among fermion flavors is allowed and in this case one of the fermion mass eigenstates is massless, while the other is massive. The mixing angle is necessarily maximal in this case $\theta = \pm\pi/4$. One may hope that extension to a third flavor may lead to more phenomenologically realistic situations with arbitrary mixing and mass generated for all flavors.

Another possibility toward this result might be the inclusion of more than one regulator vector fields, along the lines of [20], where, however, not only a mass hierarchy is generated between the fermions, with nontrivial masses, but also one of the gauge bosons acquires a mass. In our case of regulators, unfortunately, this last mass would also be kept finite, but probably this would not be a problem, since the massive vector field decouples from the Lagrangian of the fermions in the zero gauge coupling "relativistic limit" (7). We hope to come back to this case in a future work.

Another aspect of our work, which was also the original motivation, is the one in which this method applies to chiral neutrinos of the standard model, in an attempt to discuss neutrino mass generation independently of the seesaw mechanism. We have discussed two scenarios in this respect.

In the first, we avoided the inclusion of sterile neutrinos altogether. In this case the two flavors considered above have been viewed as corresponding to Majorana mass eigenstates of two left-handed neutrino flavors, interacting with a LIV regulator gauge field with vanishing couplings. It was demonstrated that different mass eigenstates could be obtained in the Lorentz symmetric limit, which then leads to standard oscillations among the physical neutrino flavors coupled to the $SU(2)_L$ gauge fields of the toy standard model involving only two flavors. Extension to the physical case of three generations, including CP violation in the lepton sector, will constitute the subject of a forthcoming publication.

The second scenario, involved an extension of the model (3) to a toy version of the νMSM model of [24], in which one of fermion flavors of (3), say ψ_1, represented a right-handed neutrino field, and the other flavor ψ_2 a left-handed active neutrino of the standard model. In such a case our aim was to generate dynamically a mass hierarchy between active and right-handed (possibly sterile) neutrinos of the type needed in phenomenological approaches to dark matter, where a keV sterile neutrino may play the role of a dark matter field, consistently with current astrophysical and cosmological data [24]. In the context of our framework, we can only generate dynamically the right-handed neutrino mass, but not a Dirac mass term. It is interesting that the absence of a left-handed Majorana mass (standard assumption in seesaw models) appears naturally in our models. A Dirac mass then, coupling left(active) and right-handed(sterile) components can be generated by the standard Higgs mechanism of the standard model.

APPENDIX A: GAP EQUATIONS

The aim of this appendix is to present the main steps to obtain (15) from the Schwinger-Dyson equation (14) which is rewritten below:

$$G^{-1} - S^{-1} = \int_p D_{\mu\nu} \tau\gamma^\mu G\tau\gamma^\nu. \tag{A1}$$

The first step that we take is to commute the first τ and γ^μ in (A1), so that in the middle of the integrand we have a matrix product given by

$$\begin{aligned}
\tau G\tau &= X\begin{pmatrix} e_1 & -i\epsilon \\ i\epsilon & e_2 \end{pmatrix}\begin{pmatrix} \not{p} - m_2 & \mu \\ \mu & \not{p} - m_1 \end{pmatrix}\begin{pmatrix} e_1 & -i\epsilon \\ i\epsilon & e_2 \end{pmatrix} \\
&= X\begin{pmatrix} e_1^2(\not{p} - m_2) + \epsilon^2(\not{p} - m_1) & -Y + \mu(e_1 e_2 - \epsilon^2) \\ Y + \mu(e_1 e_2 - \epsilon^2) & \epsilon^2(\not{p} - m_2) + e_2^2(\not{p} - m_1) \end{pmatrix},
\end{aligned} \tag{A2}$$

where

$$\begin{aligned}
X &= i\frac{p^2 + \not{p}(m_1 + m_2) + m_1 m_2 - \mu^2}{(p^2 - m_1^2)(p^2 - m_2^2) - 2\mu^2(p^2 + m_1 m_2) + \mu^4} = i\frac{p^2 + \not{p}(m_1 + m_2) + m_1 m_2 - \mu^2}{(p^2 - A_-^2)(p^2 - A_+^2)}; \\
Y &= i\epsilon[e_1(\not{p} - m_2) + e_2(\not{p} - m_1)],
\end{aligned} \tag{A3}$$

with $A_\pm^2$ defined as in (17). If we identify individually each matrix element in the Schwinger-Dyson equation (A1), we obtain for the M_{11} element

$$\begin{aligned}
im_1 &= \int_p D_{\mu\nu}\gamma^\mu X[e_1^2(\not{p} - m_2) + \epsilon^2(\not{p} - m_1)]\gamma^\nu \\
&= \int_p \frac{(4 + \zeta)}{(1 + \vec{p}^2/M^2)} \frac{p^2(e_1^2 m_1 + \epsilon^2 m_2) + (\mu^2 - m_1 m_2)(e_1^2 m_2 + \epsilon^2 m_1)}{p^2(p^2 - A_-^2)(p^2 - A_+^2)},
\end{aligned} \tag{A4}$$

The last equation can be written as

$$\frac{m_1}{4 + \zeta} = (e_1^2 m_1 + \epsilon^2 m_2)I_1 + (\mu^2 - m_1 m_2)(e_1^2 m_2 + \epsilon^2 m_1)I_2, \tag{A5}$$

where

$$I_1 = -i\int_p \frac{1}{1 + \vec{p}^2/M^2} \frac{1}{(p^2 - A_-^2)(p^2 - A_+^2)}, \qquad I_2 = -i\int_p \frac{1}{(1 + \vec{p}^2/M^2)} \frac{1}{p^2(p^2 - A_-^2)(p^2 - A_+^2)}. \tag{A6}$$

The Wick rotation $p_0 \to i\omega$ leads to

$$\begin{aligned}
I_1 &= \frac{1}{4\pi^3}\int_0^\infty \frac{\vec{p}^2 d\vec{p}}{1 + \vec{p}^2/M^2}\int_{-\infty}^\infty \frac{d\omega}{(\omega^2 + \vec{p}^2 + A_+^2)(\omega^2 + \vec{p}^2 + A_-^2)}, \\
I_2 &= \frac{-1}{4\pi^3}\int_0^\infty \frac{\vec{p}^2 d\vec{p}}{1 + \vec{p}^2/M^2}\int_{-\infty}^\infty \frac{d\omega}{(\omega^2 + \vec{p}^2)(\omega^2 + \vec{p}^2 + A_+^2)(\omega^2 + \vec{p}^2 + A_-^2)}.
\end{aligned} \tag{A7}$$

The integrand of I_1 which depends on ω only can be written

$$\frac{1}{A_+^2 - A_-^2}\left[\left(\frac{1}{\omega^2 + \vec{p}^2} - \frac{1}{(\omega^2 + \vec{p}^2 + A_+^2)}\right) - \left(\frac{1}{\omega^2 + \vec{p}^2} - \frac{1}{(\omega^2 + \vec{p}^2 + A_-^2)}\right)\right], \tag{A8}$$

and similarly, the integrand of I_2 which depends on ω only can be expressed as

$$\frac{1}{A_+^2 - A_-^2}\left[\frac{1}{A_-^2}\left(\frac{1}{\omega^2 + \vec{p}^2} - \frac{1}{(\omega^2 + \vec{p}^2 + A_-^2)}\right) - \frac{1}{A_+^2}\left(\frac{1}{\omega^2 + \vec{p}^2} - \frac{1}{(\omega^2 + \vec{p}^2 + A_+^2)}\right)\right] \tag{A9}$$

Finally, substituting (A8) and (A9) into (A7) leads to the first equation of (15). Furthermore, due to the symmetry of our model, the second equation of (15) is obtained from the first one by exchanging m_1 and m_2. Finally, the left-hand side of (A1) is symmetric with nondiagonal elements given by $i\mu$, therefore, the nondiagonal elements of the right-hand side must also be equal. However, looking at (A2), we realize that it is only possible if the terms related with Y vanish. So, the nondiagonal elements give us the following equations

$$i\mu = \int_p \frac{(4+\zeta)}{(1+\vec{p}^2/M^2)} \mu(e_1 e_2 - \epsilon^2) \frac{p^2 + m_1 m_2 - \mu^2}{p^2(p^2 - A_-^2)(p^2 - A_+^2)},$$

$$0 = \int_p D_{\mu\nu} \gamma^\mu X Y \gamma^\nu = \epsilon \int_p \frac{(4+\zeta)}{(1+\vec{p}^2/M^2)} \frac{p^2(e_1 m_1 + e_2 m_2) + (\mu^2 - m_1 m_2)(e_1 m_2 + e_2 m_1)}{p^2(p^2 - A_-^2)(p^2 - A_+^2)}, \tag{A10}$$

where using Eq. (A6), leads to the last two equations (15).

APPENDIX B: ONE-LOOP FERMION SELF-ENERGY

We calculate here, in the Feynman gauge, the fermion wave function renormalization for the case $\{e_1 = e_2$ and $\epsilon = 0\}$. In order to avoid IR divergences obtained in the one-loop calculation for $m_1 = m_2 = \mu$, because one of the eigenmasses vanishes, we consider here the situation $m_1 = m_2 = m \neq \mu$. The fermion propagator is then given by

$$G(p) = i \frac{p^2 + 2m\slashed{p} + m^2 - \mu^2}{[p^2 - (m+\mu)^2][p^2 - (m-\mu)^2]} \begin{pmatrix} \slashed{p} - m & \mu \\ \mu & \slashed{p} - m \end{pmatrix}. \tag{B1}$$

We obtain the fermion wave function renormalization by differentiating the fermion self-energy with respect to the external momentum and then, set it to zero. Since the fermion propagator (B1) has two independent flavor components, we consider the one-loop diagonal self-energy $\Sigma_{\mathrm{diag}}^{(1)}$ and the one-loop off-diagonal part $\Sigma_{\mathrm{off}}^{(1)}$, where

$$\Sigma_{\mathrm{diag}}^{(1)}(\omega, \vec{p}) = \frac{-ie^2}{(2\pi)^4} \int \frac{d^4k}{1+\vec{k}^2/M^2} \left\{ \frac{\gamma^\mu \gamma_\mu [(p-k)^2 - (m^2 - \mu^2)]m}{k^2[(p-k)^2 - (m+\mu)^2][(p-k)^2 - (m-\mu)^2]} \right.$$

$$\left. + \frac{\gamma^\mu(\slashed{p} - \slashed{k})\gamma_\mu[(p-k)^2 - (m^2 + \mu^2)]}{k^2[(p-k)^2 - (m+\mu)^2][(p-k)^2 - (m-\mu)^2]} \right\}$$

$$\Sigma_{\mathrm{off}}^{(1)}(\omega, \vec{p}) = \frac{-ie^2}{(2\pi)^4} \int \frac{d^4k}{1+\vec{k}^2/M^2} \frac{\gamma^\mu \gamma_\mu [(p-k)^2 + m^2 - \mu^2]\mu + 2m\mu\gamma^\mu(\slashed{p} - \slashed{k})\gamma_\mu}{k^2[(p-k)^2 - (m+\mu)^2][(p-k)^2 - (m-\mu)^2]}. \tag{B2}$$

Differentiating these terms with respect to p_ρ and then setting $\omega = 0$ and $\vec{p} = 0$, we find

$$\frac{\partial \Sigma_{\mathrm{diag}}^{(1)}}{\partial p_\rho}\bigg|_{p=0} = \frac{ie^2}{8\pi^4} \int \frac{d^4k}{1+\vec{k}^2/M^2} \left\{ \frac{k^2\gamma^\rho - (m^2 + \mu^2)\gamma^\rho + 2k^\rho \slashed{k}}{k^2[k^2 - (m+\mu)^2][k^2 - (m-\mu)^2]} \right.$$

$$\left. - \frac{4k^\rho \slashed{k}k^4 - 8k^\rho \slashed{k}k^2(m^2 + \mu^2) + 4k^\rho \slashed{k}(m^2 + \mu^2)^2}{k^2[k^2 - (m+\mu)^2]^2[k^2 - (m-\mu)^2]^2} \right\}, \tag{B3}$$

$$\frac{\partial \Sigma_{\mathrm{off}}^{(1)}}{\partial p_\rho}\bigg|_{p=0} = -\frac{i\mu m e^2}{4\pi^4} \int \frac{d^4k}{1+\vec{k}^2/M^2} \left\{ \frac{-\gamma^\rho}{k^2[k^2 - (m+\mu)^2][k^2 - (m-\mu)^2]} + \frac{4k^\rho \slashed{k}k^2 - 4k^\rho \slashed{k}(m^2 + \mu^2)}{k^2[k^2 - (m+\mu)^2]^2[k^2 - (m-\mu)^2]^2} \right\}. \tag{B4}$$

We write then

$$\Sigma_{\mathrm{diag}}^{(1)} = -m + Z_{\mathrm{diag}}^0 \omega \gamma^0 - Z_{\mathrm{diag}}^1 \vec{p} \cdot \vec{\gamma}, \qquad \Sigma_{\mathrm{off}}^{(1)} = -\mu + Z_{\mathrm{off}}^0 \omega \gamma^0 - Z_{\mathrm{off}}^1 \vec{p} \cdot \vec{\gamma}, \tag{B5}$$

and since we are interested in the limit $\mu \to m$, we write $m - \mu = m\delta$, with $\delta \ll 1$ and approximate $m + \mu \approx 2m$. In terms of new variables $x = \sqrt{\vec{k}^2}/m$, $y = k_0/m$, $\lambda = m/M \ll 1$ and after a Wick rotation, we obtain

$$Z_{\text{diag}}^0 = \frac{e^2}{2\pi^3} \int_0^\infty \frac{x^2 dx}{1 + \lambda^2 x^2} \int_{-\infty}^\infty dy \left[\frac{-(x^2 + y^2) - 2y^2 - 2}{(x^2 + y^2)(x^2 + y^2 + 4)(x^2 + y^2 + \delta^2)} + \frac{4y^2(x^2 + y^2)^2 + 16y^2(x^2 + y^2) + 16y^2}{(x^2 + y^2)(x^2 + y^2 + 4)^2(x^2 + y^2 + \delta^2)^2} \right],$$

$$Z_{\text{off}}^0 = \frac{e^2}{\pi^3} \int_0^\infty \frac{x^2 dx}{1 + \lambda^2 x^2} \int_{-\infty}^\infty dy \left[\frac{1}{(x^2 + y^2)(x^2 + y^2 + 4)(x^2 + y^2 + \delta^2)} - \frac{4y^2(x^2 + y^2) + 8y^2}{(x^2 + y^2)(x^2 + y^2 + 4)^2(x^2 + y^2 + \delta^2)^2} \right],$$

(B6)

and

$$Z_{\text{diag}}^1 = \frac{e^2}{2\pi^3} \int_0^\infty \frac{x^2 dx}{1 + \lambda^2 x^2} \int_{-\infty}^\infty dy \left[\frac{-(x^2 + y^2) - 2x^2/3 - 2}{(x^2 + y^2)(x^2 + y^2 + 4)(x^2 + y^2 + \delta^2)} + \frac{4}{3} \frac{x^2(x^2 + y^2)^2 + 4x^2(x^2 + y^2) + 4x^2}{(x^2 + y^2)(x^2 + y^2 + 4)^2(x^2 + y^2 + \delta^2)^2} \right],$$

$$Z_{\text{off}}^1 = \frac{e^2}{\pi^3} \int_0^\infty \frac{x^2 dx}{1 + \lambda^2 x^2} \int_{-\infty}^\infty dy \left[\frac{1}{(x^2 + y^2)(x^2 + y^2 + 4)(x^2 + y^2 + \delta^2)} - \frac{4}{3} \frac{x^2(x^2 + y^2) + 2x^2}{(x^2 + y^2)(x^2 + y^2 + 4)^2(x^2 + y^2 + \delta^2)^2} \right].$$

(B7)

Finally, after solving these integrals, we find

$$Z_{\text{diag}}^0 = \frac{e^2}{8\pi^2} \left(\frac{1}{4} - \frac{1}{2} \ln 2 + \frac{1}{2} \ln \delta + \ln \lambda \right), \qquad Z_{\text{off}}^0 = \frac{e^2}{16\pi^2} (\ln 2 - \ln \delta),$$

(B8)

and

$$Z_{\text{diag}}^1 = \frac{e^2}{8\pi^2} \left(-\frac{1}{12} - \frac{1}{2} \ln 2 + \frac{1}{2} \ln \delta + \ln \lambda \right), \qquad Z_{\text{off}}^1 = \frac{e^2}{16\pi^2} (\ln 2 - \ln \delta).$$

(B9)

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