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Massive spin-2 particle from a rank-2 tensor

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Here we obtain all possible second-order theories for a rank-2 tensor which describe a massive spin-2 particle. We start with a general second-order Lagrangian with ten real parameters. The absence of lower-spin modes and the existence of two local field redefinitions leads us to only one free parameter. The solutions are split into three one-parameter classes according to the local symmetries of the massless limit. In the class which contains the usual massive Fierz-Pauli theory, the subset of spin-1 massless symmetries is maximal. In another class where the subset of spin-0 symmetries is maximal, the massless theory is invariant under Weyl transformations and the mass term does not need to fit into the form of the Fierz-Pauli mass term. In the remaining third class neither the spin-1 nor the spin-0 symmetry is maximal and we have a new family of spin-2 massive theories.

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I. INTRODUCTION

In general relativity the gravitational interaction is mediated by an apparently massless spin-2 particle. In order to understand whether the graviton is really massless one investigates the consequences of turning on a (tiny) mass term. The mass discontinuity [1,2] and the appearance of ghosts [3] are two known longstanding problems of the massive theory. More recently, motivated also by experimental large-scale gravitational results, there has been intense work on massive gravity where the mass discontinuity problem has been addressed via the ideas of Ref. [4]; see, for instance, Refs. [5–7]. For a review, see Refs. [8,9]. Besides ghosts and mass discontinuity, there is also an ongoing discussion [10–14] on causality in massive gravity theories.

We would like to stress that all the above works are based on the usual description of massive spin-2 particles suggested long ago by Fierz and Pauli (FP) [15] where the basic field is a symmetric rank-2 tensor. It is certainly welcome to search for alternative descriptions of massive spin-2 particles. Among them, probably the most natural—especially if we have in mind a frame-like ($e_{\mu}^{\ a}$) description of gravity—is to allow for an arbitrary rank-2 tensor with a nonvanishing antisymmetric part $e_{[\mu a]} \neq 0$. This is the route we follow here, which has been followed before in, e.g., Refs. [16–20].

The conclusion of those works is that the only possibility, in the massive case, is the well-known symmetric description of FP. Regarding the massless spin-2 case, although Ref. [19] concludes in favor of the linearized Einstein-Hilbert theory (massless FP) as the only possibility, in Ref. [21] another theory was found which is invariant under transverse linearized reparametrizations and Weyl transformations and correctly describes a massless spin-2 particle in terms of a symmetric tensor. Moreover, in Ref. [22] there was a further description of a massless spin-2 particle in terms of an arbitrary rank-2 tensor. Back to the massive case, there are two exceptions to the conclusion of Refs. [16–20] recently found in Refs. [23,24]. In both Refs. [23,24], $e_{[\mu\nu]}$ does not decouple from the symmetric part $e_{(\mu\nu)}$ in any local way at the action level. In particular, in Ref. [24] the mass term $-m^2(e_{\mu\nu}e^{\nu\mu} + ce^2)$ does not need to fit in the usual FP form with $c = -1$. The real parameter c is arbitrary.

The above exceptions have prompted us to revisit Refs. [16–20] in order to achieve a complete classification of massive spin-2 particles in terms of an arbitrary rank-2 tensor. In Sec. II we start with a general second-order Lagrangian density with ten free real parameters. By requiring the existence of only one massive spin-2 pole in the propagator and using a trivial field redefinition [Eq. (34)] we get rid of eight parameters; see Eqs. (22), (24), (25), (28), (30), (31), and (42). There is another trivial field redefinition [Eq. (33)] which allows us to fix another undetermined parameter in Sec. III, where we also analyze the local symmetries of the corresponding massless theories. Based on those symmetries we end up with three classes of models, one of which is new. Its equations of motion are presented in Sec. IV where we also discuss the mass discontinuity problem. In Sec. V we draw our conclusions.

II. GENERAL SETUP

We start with a second-order Lagrangian density in $D = 4$ with ten real free parameters,[1]

[1]Throughout this work we use $\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+)$ and the abbreviations LEH (linearized Einstein-Hilbert) and GR (general relativity) among others defined in the text.

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$$\mathcal{L}_G[e_{\mu\nu}] = a_1(\partial^\alpha e_{\alpha\beta})^2 + a_2(\partial^\alpha e_{\alpha\nu})(\partial_\beta e^{\nu\beta}) + a_3(\partial^\nu e_{\mu\nu})^2 + b_1 e\Box e + b_2\partial^\mu e\partial^\alpha e_{\alpha\mu} + \frac{p_1}{2}e_{\mu\nu}\Box e^{\mu\nu} + \frac{p_2}{2}e_{\mu\nu}\Box e^{\nu\mu} + ce^2 + d_1 e_{\mu\nu}e^{\mu\nu} + d_2 e_{\mu\nu}e^{\nu\mu}. \quad (1)$$

It is convenient to rewrite $\mathcal{L}_G$ in terms of antisymmetric ($B_{\mu\nu}$) and symmetric ($h_{\mu\nu}$) tensors. Using $e_{\mu\nu} = B_{\mu\nu} + h_{\mu\nu}$ we obtain

$$\mathcal{L}_G[B_{\mu\nu}, h_{\mu\nu}] = S(\partial^\alpha h_{\alpha\beta})^2 + b_1 h\Box h + b_2\partial^\mu h\partial^\alpha h_{\alpha\mu} + ch^2 + p_+ h_{\mu\nu}\Box h^{\mu\nu} + d_+ h^2_{\mu\nu} + 2(a_1 - a_3)\partial^\mu B_{\mu\nu}\partial_\alpha h^{\alpha\nu} + (a_1 + a_3 - a_2)\partial^\mu B_{\mu\nu}\partial_\alpha B^{\alpha\nu} + p_- B_{\mu\nu}\Box B^{\mu\nu} + d_- B_{\mu\nu}B^{\mu\nu}, \quad (2)$$

where

$$S = a_1 + a_2 + a_3, \quad (3)$$

$$d_\pm = d_1 \pm d_2, \quad (4)$$

$$p_\pm = (p_1 \pm p_2)/2. \quad (5)$$

Our aim is to single out the regions in the ten-dimensional parameter space of Eq. (1) which correspond to only one massive spin-2 particle without tachyons and ghosts. We assume that all parameters in the second-derivative terms are dimensionless and in the massless limit all nonderivative terms vanish, i.e.,

$$\lim_{m\to 0}(c, d_1, d_2) = (0, 0, 0). \quad (6)$$

As far as we know there are three examples of massive spin-2 models in the literature [15,23,24] in agreement with our hypothesis. For all three models we have

$$(a_2, a_3) = (1/2, 1/4), \qquad p_1 = p_2 = 1/2, \qquad (d_1, d_2) = (0, -m^2/2). \quad (7)$$

The first model is the well-known massive FP theory [15] which can be entirely described by means of a symmetric tensor. Its massless limit is the linearized Einstein-Hilbert theory. It is invariant under linearized reparametrizations $\delta e_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$. The massive FP theory corresponds to

$$(a_1, b_1, b_2, c) = (1/4, -1/2, -1, m^2/2), \qquad S = 1. \quad (8)$$

Since $B_{\mu\nu}$ only appears now in the last term of Eq. (2), the parameter d_- is a free parameter in the FP model.

In the second model, defined in Ref. [23], there is a nontrivial coupling between the symmetric ($h_{\mu\nu}$) and antisymmetric ($B_{\mu\nu}$) parts of the rank-2 tensor. The massless limit of Ref. [23] is invariant under $\delta e_{\mu\nu} = \partial_\mu\xi_\nu$. For the massive model [23] we have

$$(a_1, b_1, b_2, c) = (-1/4, 0, 0, m^2/2), \qquad S = 1/2. \quad (9)$$

Regarding the third model defined in Ref. [24], the massless limit—previously studied in Ref. [22]—is invariant under $\delta e_{\mu\nu} = \partial_\mu\xi_\nu + \eta_{\mu\nu}\Lambda$. While the mass term of the models in Refs. [15,23] must be of the Fierz-Pauli type ($e_{\mu\nu}e^{\nu\mu} - e^2$), in the massive model [24] one can add a term proportional to the square of the trace of the rank-2 tensor ($e_{\mu\nu}e^{\nu\mu}/2 + fe^2$), where f is an arbitrary real constant. This is a consequence of the linearized Weyl symmetry of the massless theory. This symmetry can be extended to the whole massive theory if we choose $f = -1/4$ which allows us to use a traceless rank-2 tensor. The massive model of Ref. [24] is described in general by

$$(a_1, b_1, b_2) = (-1/12, -1/6, -1/3), \qquad S = 2/3, \quad (10)$$

while the parameter c remains arbitrary for reasons already mentioned.

The Lagrangian $\mathcal{L}_G$ can be written as $\mathcal{L}_G = e_{\mu\nu}G^{\mu\nu\alpha\beta}e_{\alpha\beta}$ where, suppressing the indices, we have the differential operator

$$G = (d_+ + p_+\Box)P^{(2)}_{TT} + (d_- + p_-\Box)P^{(0)}_{AA} + \sum_{s=0,1}\sum_{I,J} A^{(s)}_{IJ}P^{(s)}_{IJ}, \quad (11)$$

where the spin-s operators $P^{(s)}_{IJ}$, given in Appendix A, satisfy the algebra

$$P^{(s)}_{IJ}P^{(r)}_{KL} = \delta^{rs}\delta_{JK}P^{(s)}_{IL}. \quad (12)$$

The operators $P^{(1)}_{IJ}$ with $I, J = S, A$ form a subalgebra of Eq. (12) as well as the operators $P^{(0)}_{IJ}$ with $I, J = T, W$. The 2×2 matrices $A^{(s)}_{IJ}$ are given by

$$A^{(1)}_{AA} = d_+ + (p_+ - S/2)\Box, \quad (13)$$

$$A^{(1)}_{SS} = d_- + (2p_- + a_2 - a_1 - a_3)\frac{\Box}{2}, \quad (14)$$

$$A^{(1)}_{AS} = A^{(1)}_{SA} = (a_1 - a_3)\Box/2, \quad (15)$$

$$A^{(0)}_{TT} = d_+ + 3c + (p_+ + 3b_1)\Box, \quad (16)$$

$$A^{(0)}_{WW} = d_+ + c + (p_+ + b_1 - S - b_2)\Box, \quad (17)$$

$$A^{(0)}_{TW} = A^{(0)}_{WT} = \sqrt{3}[c + (b_1 - b_2/2)\Box]. \quad (18)$$

A key role will be played by the propagator which is proportional to the operator $G^{-1}_{\mu\nu\alpha\beta}$ which is given (again suppressing the indices) by

$$G^{-1} = \frac{P_{TT}^{(2)}}{d_+ + p_+\Box} + \frac{P_{AA}^{(0)}}{d_- + p_-\Box} + \sum_{s=0,1}\sum_{I,J}(A^{-1})^{(s)}_{IJ}P^{(s)}_{IJ}, \tag{19}$$

where the inverse matrix $(A^{-1})^{(s)}_{IJ}$ is given explicitly by

$$(A^{-1})^{(s)}_{11} = \frac{A^{(s)}_{22}}{K^{(s)}}, \qquad (A^{-1})^{(s)}_{22} = \frac{A^{(s)}_{11}}{K^{(s)}}, \qquad (A^{-1})^{(s)}_{12} = (A^{-1})^{(s)}_{21} = -\frac{A^{(s)}_{12}}{K^{(s)}}, \tag{20}$$

with the determinants

$$K^{(s)} = A^{(s)}_{11}A^{(s)}_{22} - [A^{(s)}_{12}]^2, \qquad s = 0, 1. \tag{21}$$

In G^{-1} we have four sources of poles, namely, the operators in the denominators below $P_{TT}^{(2)}$ and $P_{AA}^{(0)}$ and the determinants $K^{(s)}$ for $s = 0, 1$. We must determine the parameters of our model such that we only have one massive physical pole coming from the denominator below $P_{TT}^{(2)}$. The no-pole condition in the denominator of $P_{AA}^{(0)}$ requires

$$p_1 - p_2 = 0. \tag{22}$$

In the operator below $P_{TT}^{(2)}$ we have a massive particle with $m^2 = -d_+/p_+$. From the two-point amplitude saturated with external sources $T_{\mu\nu}$ we can calculate the residue at $k^2 \to -m^2$, whose imaginary part is given in momentum space by

$$\begin{aligned} I_{-m^2} &= \Im \lim_{k^2\to -m^2}(k^2+m^2)\left(-\frac{i}{2}\right)T^*_{\mu\nu}(k) \\ &\quad \times [G^{-1}(k,-k)]^{\mu\nu\alpha\beta}T_{\alpha\beta}(k) \\ &= \frac{1}{p_+}T^*_{\mu\nu}(k)[P_{TT}^{(2)}]^{\mu\nu\alpha\beta}T_{\alpha\beta}(k) \\ &\equiv \frac{1}{p_+}T^*P_{TT}^{(2)}T. \end{aligned} \tag{23}$$

Since (see, for instance, Ref. [24]) $T^*P_{TT}^{(2)}T > 0$ at $k^2 = -m^2$, a physical particle ($I_{-m^2} > 0$) requires $p_+ > 0$. After a dilatation $e_{\mu\nu} \to \Lambda e_{\mu\nu}$ we can set without loss of generality

$$p_+ = p_1 = p_2 = 1/2, \tag{24}$$

and the mass is fixed by d_+,

$$d_+ = -m^2/2. \tag{25}$$

Let us now examine the consequences of the no-pole condition on the determinants $K^{(s)}$. From Eqs. (21) and (13)–(18) we can write down

$$K^{(s)} = C_2^{(s)}\Box^2 + C_1^{(s)}\Box + C_0. \tag{26}$$

The existence of G^{-1} and the absence of poles in $K^{(s)}$ require

$$C_2^{(1)} = (a_1 - 1/4)(a_3 - 1/4) - (a_2 - 1/2)^2 = 0, \tag{27}$$

$$C_1^{(1)} = d_-(1-S)/2 + m^2(a_2 - a_1 - a_3)/4 = 0, \tag{28}$$

$$C_0^{(1)} = -m^2 d_-/2 \neq 0, \tag{29}$$

$$C_2^{(0)} = 3(b_1 + 1/6)(2/3 - S) - 3(b_2 + 1/3)^2/4 = 0, \tag{30}$$

$$C_1^{(0)} = m^2(S - 1 + b_2 - 4b_1)/2 + 3c(2/3 - S) = 0, \tag{31}$$

$$C_0^{(0)} = m^2(m^2/8 - c) \neq 0. \tag{32}$$

Henceforth we assume $d_- \neq 0$ and $c \neq m^2/8$. The seven conditions (22), (24), (25), (27), (28), (30), and (31) still leave three out of ten coefficients in Eq. (1) arbitrary. Part of this redundancy is due to the following two families of local field redefinitions:

$$e_{\mu\nu} \to e_{\mu\nu} + \frac{a}{2}\eta_{\mu\nu}e, \qquad a \neq -1/2, \tag{33}$$

$$e_{\mu\nu} \to Ae_{\mu\nu} + (1-A)e_{\nu\mu}, \qquad A \neq 1/2. \tag{34}$$

The restrictions $a \neq -1/2$ and $A \neq 1/2$ are needed for the existence of the inverse transformations. The transformations (34) are dilations in the antisymmetric part of the tensor, i.e., $(h_{\mu\nu}, B_{\mu\nu}) \to (h_{\mu\nu}, (2A-1)B_{\mu\nu})$.

From Eqs. (33) and (34) we can fix two more coefficients such that all solutions to the no-pole conditions (27), (28), (30), and (31) become one-parameter families. More specifically, the transformations (33) and (34) lead to

$$b_2 \to b_2 + a(S + 2b_2), \tag{35}$$

$$c \to c + 4a(a+1)(c - m^2/8), \tag{36}$$

$$b_1 \to b_1 + a[(a+1)(4b_1 + 1/2) - a/4S - (a + 1/2)b_2], \tag{37}$$

$$a_2 \to a_2 + 2A(1-A)(a_1 + a_3 - a_2), \tag{38}$$

$$a_1 \to A^2(a_1 + a_3 - a_2) + A(a_2 - 2a_3) + a_3, \tag{39}$$

$$a_3 \to A^2(a_1 + a_3 - a_2) + A(a_2 - 2a_1) + a_1, \tag{40}$$

$$d_- \to d_-(1-2A)^2; \qquad p_- \to p_-(1-2A)^2, \tag{41}$$

while d_+, p_+ and the constraints (27), (28), (30), and (31) are invariant. The sum $S = a_1 + a_2 + a_3$ is also invariant under Eqs. (35)–(41), it will play an important role in the classification of the solutions to Eqs. (27), (28), (30), and (31).

In the Appendix we prove that due to Eq. (34) we can choose without loss of generality the following solution to Eq. (27):

$$a_2 = 1/2, \qquad a_3 = 1/4. \tag{42}$$

The solution (42) is assumed henceforth unless otherwise stated. Due to the special role [see Eq. (35)] of the fixed point $S = -2b_2$ we postpone fixing b_2 by using Eq. (33).

In summary, any one-parameter family of solutions to Eqs. (28), (30), and (31) with Eqs. (22), (24), (25), (29), (32), and (42) describes one massive physical particle of spin-2. In the next section we split those solutions into three classes.

III. CLASSIFYING THE MASSIVE MODELS VIA THE MASSLESS LIMIT

As we will see in this section, the massless limit, defined in Eq. (6), of the massive spin-2 models which satisfy the constraints (27) and (30) necessarily have local symmetries which can help us classify the solutions to the constraints (28), (30), and (31).

First, it is clear from Eq. (11) and the algebra (12) that there can only be spin-1 and spin-0 local symmetries of Eq. (1). Moreover, since each of the sets of operators $\{P_{AA}^{(0)}\}$,$\{P_{IJ}^{(1)}\}$ and $\{P_{KL}^{(0)}\}$, with $(K, L) \neq (A, A)$, form subalgebras of the algebra (12), we can write down the most general symmetry transformation of $e_{\mu\nu}$ as follows:

$$\delta e_{\mu\nu} = \delta_{AA}^{(0)} e_{\mu\nu} + \delta^{(0)} e_{\mu\nu} + \delta^{(1)} e_{\mu\nu}, \tag{43}$$

where

$$\delta_{AA}^{(0)} e_{\mu\nu} = E[P_{AA}^{(0)}]_{\mu\nu\alpha\beta}\Lambda^{\alpha\beta}, \tag{44}$$

$$\delta^{(0)} e_{\mu\nu} = [AP_{TT}^{(0)} + BP_{WW}^{(0)} + C\sqrt{3}(P_{TW}^{(0)} + P_{WT}^{(0)})]_{\mu\nu\alpha\beta}\Lambda^{\alpha\beta}, \tag{45}$$

$$\delta^{(1)} e_{\mu\nu} = [\tilde{A}P_{SS}^{(1)} + \tilde{B}P_{AA}^{(1)} + \tilde{C}(P_{AS}^{(1)} + P_{SA}^{(1)})]_{\mu\nu\alpha\beta}\Lambda^{\alpha\beta}. \tag{46}$$

The parameter $\Lambda^{\alpha\beta}(x)$ is an arbitrary rank-2 tensor while $A, B, C, \tilde{A}, \tilde{B}, \tilde{C}, E$ are arbitrary real constants. Due to the fact that $P_{IJ}^{(s)}\Lambda$ is independent of $P_{KL}^{(r)}\Lambda$ unless $(I, J, s) = (K, L, r)$ we have from $\delta S_G = 2\int d^4x e_{\alpha\beta} G^{\alpha\beta\mu\nu}\delta e_{\mu\nu} = 0$ [with (a_2, a_3) being arbitrary] the following set of equations:

$$(b_1 + 1/6)A + (b_1 - b_2/2)C = 0, \tag{47}$$

$$(b_1 - b_2/2)A + (b_1 - b_2 - S + 1/2)C = 0, \tag{48}$$

$$(b_1 - b_2 - S + 1/2)B + 3(b_1 - b_2/2)C = 0, \tag{49}$$

$$(b_1 - b_2/2)B + 3(b_1 + 1/6)C = 0, \tag{50}$$

$$(1 - S)\tilde{A} + (a_1 - a_3)\tilde{C} = 0, \tag{51}$$

$$(a_1 - a_3)\tilde{A} + (a_2 - a_1 - a_3)\tilde{C} = 0, \tag{52}$$

$$(a_2 - a_1 - a_3)\tilde{B} + (a_1 - a_3)\tilde{C} = 0, \tag{53}$$

$$(a_1 - a_3)\tilde{B} + (1 - S)\tilde{C} = 0, \tag{54}$$

$$d_- E = 0. \tag{55}$$

For future purposes we note that if we use Eqs. (42) and (51)–(54) become equivalent to

$$(S - 1)(\tilde{A} - \tilde{C}) = 0, \qquad (S - 1)(\tilde{B} - \tilde{C}) = 0. \tag{56}$$

In order for Eqs. (47)–(50) to have a nontrivial solution for the pairs (A, C) and (B, C) a determinant must be zero. It turns out that such a determinant is exactly the constraint (30). Likewise, a nontrivial solution for Eqs. (51)–(54) is warranted by the constraint (27). Therefore, the absence of spin-0 and spin-1 poles in the propagator requires the existence of both spin-0 and spin-1 local symmetries in the corresponding massless theory. This is of course analogous to the massive spin-1 Maxwell-Proca theory. The Maxwell term, invariant under the usual spin-0 U(1) gauge symmetry, is singled out as the unique massless term by requiring the nonpropagation of the scalar mode $\partial^\mu A_\mu$.

Regarding Eq. (55), since in the massless limit $d_- \to 0$, the constant E is left arbitrary. Consequently, the derivative terms of Eq. (1) are invariant under the following transverse antisymmetric shifts (after redefining $\Lambda_{\mu\nu} \to \Box\Lambda_{\mu\nu}/E$):

$$\delta_{AA}^{(0)} e_{\alpha\beta} = [P_{AA}^{(0)}]_{\alpha\beta}{}^{\mu\nu}\Box\Lambda_{\mu\nu} = \partial^\mu\Omega_{[\mu\alpha\beta]} \equiv \lambda_{\alpha\beta}^T, \tag{57}$$

where $\partial^\alpha\lambda_{\alpha\beta}^T = 0$ and

$$\Omega_{[\mu\alpha\beta]} = \partial_\alpha\Lambda_{[\mu\beta]} - \partial_\beta\Lambda_{[\mu\alpha]} - \partial_\mu\Lambda_{[\alpha\beta]}. \tag{58}$$

The symmetry (57) is a consequence of the fact that the derivative terms in Eq. (2) only depend upon $B_{\mu\nu}$ through the derivatives $\partial^\mu B_{\mu\nu}$; see Eqs. (2) and (22).

It is clear from Eqs. (28), (30), (31), and (56) that $S = 1$ and $S = 2/3$ play a special role in the parameter space of the general model (1). From now on we split our analysis into three classes, namely, i) $S \neq 1; 2/3$, ii) $S = 1$, and iii) $S = 2/3$.

A. $S \neq 1; 2/3$

In the massive case, if $S \neq 1$ we have from Eq. (56) $\tilde{A} = \tilde{B} = \tilde{C}$. Back in Eq. (46) the spin-1 symmetry, after redefining $\Lambda \to \Box\Lambda/\tilde{A}$, becomes a transverse linearized reparametrization,[2]

$$\delta^{(1)} e_{\mu\nu} = [P_{SS}^{(1)} + P_{AA}^{(1)} + P_{AS}^{(1)} + P_{SA}^{(1)}]_{\mu\nu}{}^{\alpha\beta}\Box\Lambda_{\alpha\beta} = \partial_\nu C_\mu^T, \tag{59}$$

where $C_\mu^T = \partial^\beta(\Lambda_{\mu\beta} - \Lambda_{\beta\mu})$ satisfies $\partial^\mu C_\mu^T = 0$.

If $S \neq 2/3$ we can obtain $b_1 = b_1(b_2, S)$ from Eq. (30) and plug it back into Eqs. (47)–(50) in order to produce relationships between the constants A, B, C such that we can, after some rearrangements, write down the spin-0 symmetry,

$$\delta^{(0)} e_{\mu\nu} = (x\theta_{\mu\nu} - 3y\omega_{\mu\nu})(x\theta^{\alpha\beta} - 3y\omega^{\alpha\beta})\frac{\Lambda_{\alpha\beta}}{4(2/3 - S)} = (2S + b_2 - 1)\Box\Phi\eta_{\mu\nu} - 2(S + 2b_2)\partial_\mu\partial_\nu\Phi, \tag{60}$$

where the spin-0 and spin-1 projection operators $\omega_{\mu\nu}$ and $\theta_{\mu\nu}$ respectively are defined in Eq. (A1) and $x = 2S + b_2 - 1$, $y = b_2 + 1/2$, and $\Phi = \Box(x\theta^{\alpha\beta} - 3y\omega^{\alpha\beta})\Lambda_{\alpha\beta}/[4(2/3 - S)]$.

Therefore, if $S \neq 1$ and $S \neq 2/3$, the local symmetries of the massless theory become

$$\delta e_{\mu\nu} = \partial_\nu C_\mu^T + (2S + b_2 - 1)\Box\Phi\eta_{\mu\nu} - 2(S + 2b_2)\partial_\mu\partial_\nu\Phi + \lambda_{\mu\nu}^T. \tag{61}$$

The transformation (61) suggests that we split the analysis further into two subcases: $S = -2b_2$ and $S \neq -2b_2$. If we plug $S = -2b_2$ back into the massive constraints (30) and (31), recalling that $b_2 \neq -1/3$ due to $S \neq 2/3$, we must have $c = m^2/8$, which violates our hypothesis (32) and invalidates our particle content analysis in the massive case. Indeed, if $S = -2b_2$, with $b_2 \neq -1/2; -1/3$, it can be shown that the massive theory also contains a scalar particle in the spectrum besides the massive spin-2 mode. This is out of the scope of this work and $S = -2b_2$ will no longer be considered here except in the cases where $S = 1 (b_2 = -1/2)$ and $S = 2/3 (b_2 = -1/3)$, which are considered in the next subsections.

If $S \neq -2b_2$ we see from Eq. (35) that we can redefine b_2 as we wish. In particular, we can fix $b_2 = 1 - 2S$, which by the way holds for all models in the literature; see Eqs. (8)–(10). The symmetries (61) of the massless theory become linearized reparametrizations plus transverse antisymmetric shifts,

$$\delta e_{\mu\nu} = \partial_\nu \xi_\mu + \lambda_{\mu\nu}^T, \quad \text{if } b_2 = 1 - 2S. \tag{62}$$

In order to figure out the particle content of the massless theory we introduce an auxiliary vector field C_μ and rewrite Eq. (2) in the massless limit (6) as

$$\mathcal{L}_{m\to 0}(S) = (\partial^\mu h_{\mu\nu})^2 + (1 - 2S)\partial^\alpha h_{\alpha\mu}\partial^\mu h + \left(\frac{1}{2} - S\right)h\Box h + \frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} + (1 - S)[C^\mu C_\mu + 2C^\mu(\partial^\alpha B_{\alpha\mu} + \partial^\alpha h_{\alpha\mu})]. \tag{63}$$

If we perform the Gaussian integral over C_μ we recover our original model (2) in the massless limit. If, however, we first integrate over $B_{\mu\nu}$ in the path integral we have a functional constraint assuring that $C_\mu = \partial_\mu \phi$ for some scalar field ϕ. Plugging this back into Eq. (63) and changing variables $\phi = \varphi - h$ followed by $h_{\mu\nu} \to \tilde{h}_{\mu\nu} + (1 - S)\varphi\eta_{\mu\nu}$, we have the decoupled scalar-tensor theory,

$$\mathcal{L}_{m\to 0}(S) = \mathcal{L}_{\text{LEH}}[\tilde{h}_{\mu\nu}] - 3(S - 1)(S - 2/3)\partial^\mu\varphi\partial_\mu\varphi. \tag{64}$$

Here $\mathcal{L}_{\text{LEH}}$ is the usual linearized Einstein-Hilbert theory (or massless FP theory). Therefore, we have a physical massless spin-2 particle plus a physical massless scalar particle as far as $S < 2/3$ or $S > 1$. Otherwise, the scalar particle becomes a ghost or disappears at $S = 1$ or $S = 2/3$. In particular, the massive spin-2 model ($S = 1/2$ and $b_2 = 0$) of Ref. [23] has a healthy scalar-tensor massless limit. Regarding the symmetry (62), since the equations of motion of Eq. (63) imply $C_\mu = -\partial^\alpha e_{\alpha\mu}$, we deduce from Eq. (62) and $C_\mu = \partial_\mu\phi$ that $\delta\phi = -\partial\cdot\xi$. Consequently, $\delta\varphi = \delta(h + \Phi) = 0$. Actually, this was the guideline for the definition of φ in the first place. Moreover, from $h_{\mu\nu} = \tilde{h}_{\mu\nu} + (1 - S)\varphi\eta_{\mu\nu}$ we have $\delta\tilde{h}_{\mu\nu} = \delta h_{\mu\nu} = \frac{1}{2}(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu)$, which confirms the symmetry of Eq. (64) under Eq. (62). In Sec. IV we return to the $S \neq 1; 2/3$ family with nonzero mass.

B. $S = 1$

If $S = 1$, or equivalently $a_1 = 1/4$, we see from Eq. (28) that d_- is a free parameter and from Eqs. (30) and (31) we have, respectively,

$$b_1 = -(3b_2^2 + 2b_2 + 1)/4, \tag{65}$$

$$c = m^2[1 + 3(2b_2 + 1)^2]/8. \tag{66}$$

Given our constraint $c \neq m^2/8$ [see Eq. (29)], we see that $b_2 = -1/2$ plays a special role. This is also expected from the fact that $S = -2b_2$ at this point.

We see from Eq. (56) a peculiar feature of the $S = 1$ case, namely, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ are arbitrary which implies a maximal spin-1 symmetry in the massless limit, i.e., we have an independent symmetry generated by each of the operators $P_{AA}^{(1)}$, $P_{SS}^{(1)}$, and $P_{AS}^{(1)} + P_{SA}^{(1)}$. We also have the spin-0 symmetries (57) and (60) with $S = 1$. In particular, from the symmetry generated by $P_{AA}^{(1)}$ and $P_{AA}^{(0)}$ and the antisymmetric closure relation (A11), it is clear that we have symmetry

[2]The role of this symmetry for describing massless spin-2 particles has been discussed in Ref. [25] see also Ref. [26].

under arbitrary shifts in the antisymmetric sector ($\delta e_{\mu\nu} = \Lambda_{[\mu\nu]}$) which allows us to get rid of $e_{[\mu\nu]}$ in the $S = 1$ case. Moreover, since the transformations generated by $P^{(1)}_{AS}$ lie in the antisymmetric sector, which can be gauged away, we just need to worry about $P^{(1)}_{SS}$ and $P^{(1)}_{SA}$, which give rise to

$$\delta^{(1)}_{SS} e_{\mu\nu} + \delta^{(1)}_{SA} e_{\mu\nu} = \partial_\mu C^T_\nu + \partial_\nu C^T_\mu, \tag{67}$$

where the transverse vector is now given by

$$C^T_\mu = \Box\partial^\alpha(\Lambda_{\alpha\mu} + \Lambda_{\mu\alpha}) - 2\partial_\mu(\partial^\alpha\partial^\beta\Lambda_{\alpha\beta}) + \partial^\alpha(\Lambda_{\alpha\mu} - \Lambda_{\mu\alpha}). \tag{68}$$

Altogether, from Eqs. (60) and (67) we have the whole set of local symmetries of the massless $S = 1$ case given by

$$\delta e_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + (1 + b_2)\eta_{\mu\nu}\Box\Phi + \Lambda_{[\mu\nu]}, \tag{69}$$

where

$$\xi_\mu = C^T_\mu - (1 + 2b_2)\partial_\mu\Phi. \tag{70}$$

If $b_2 = -1/2$ ($S = -2b_2$) we can redefine $\Phi \to 2\Phi/\Box$ and write down the symmetry of the massless theory,

$$\delta e_{\mu\nu} = \partial_\mu C^T_\nu + \partial_\nu C^T_\mu + \eta_{\mu\nu}\Phi + \Lambda_{[\mu\nu]} \quad \text{if } b_2 = -1/2. \tag{71}$$

The above case is known as WTDIFF theory (see Ref. [21]), due to the Weyl symmetry and transverse (linearized) diffeomorphisms. It is the only possible description of a massless spin-2 particle in terms of one symmetric rank-2 tensor which differs from the usual massless Fierz-Pauli theory (linearized Einstein-Hilbert). It admits a nonlinear extension known as unimodular gravity [21]. If we add a mass term, the theory becomes unstable [21].

If $b_2 \neq -1/2$ ($S \neq -2b_2$) we can bring $b_2 \to -1$ ($b_2 = 1 - 2S$) and end up with the usual massless Fierz-Pauli theory describing an $m = 0$ spin-2 particle which admits the usual massive extension with mass term $m^2(e_{\mu\nu}e^{\nu\mu} - e^2)$. The symmetries of the massless theory (69) become the usual linearized diffeomorphisms plus arbitrary antisymmetric shifts $\delta e_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu + \Lambda_{[\mu\nu]}$.

C. $S = 2/3$

In this case we have $a_1 = -1/12$ and from Eqs. (30) and (31) we have $b_2 = -1/3$ and $b_1 = -1/6$ while c is a free parameter. The peculiar feature of this case is the maximal set of spin-0 symmetries in the massless theory with arbitrary A, B, C; see Eqs. (47)–(50). Since $d_- \to 0$ in the massless limit, each of the spin-0 operators $P^{(0)}_{AA}$, $P^{(0)}_{WW}$, $P^{(0)}_{SS}$, $P^{(0)}_{SW} + P^{(0)}_{WS}$ generates a symmetry. Altogether it can be shown that those spin-0 symmetries can be written as

$$\delta e_{\mu\nu} = \partial_\nu\xi_\mu + \eta_{\mu\nu}\phi + \Lambda^T_{\mu\nu}\phi. \tag{72}$$

The massless $S = 2/3$ theory was first analyzed in Ref. [22] and later in Ref. [24]. It describes one massless spin-2 particle in terms of a nonsymmetric tensor. The massive case was studied in Ref. [24] and describes one massive spin-2 particle. We can extend the Weyl symmetry to the massive case for the choice $c = m^2/8$. For more details see Refs. [22,24].

IV. A NEW FAMILY OF MASSIVE SPIN-2 MODELS

In the last section we have seen that all one-parameter models describing one massive spin-2 particle out of a rank-2 tensor can be classified in three classes. Given that two of them ($S = 1$ and $S = 2/3$) have already appeared in the literature, we now focus on the new family of models defined in terms of the free parameter a_1 which we call[3] $\mathcal{L}(a_1)$. The new family corresponds to the coefficients

$$d_1 = 0; \quad d_2 = -m^2/2, \quad a_2 = 1/2, \quad a_3 = 1/4, \quad b_1 = -(a_1 + 1/4), \quad b_2 = -2(a_1 + 1/4), \quad c = m^2/2. \tag{73}$$

Explicitly, from Eq. (1) we have

$$\mathcal{L}(a_1) = a_1(\partial^\alpha e_{\alpha\beta})^2 + \frac{1}{2}(\partial^\alpha e_{\alpha\nu})(\partial_\beta e^{\nu\beta}) + \frac{1}{4}(\partial^\nu e_{\mu\nu})^2 + \left(a_1 + \frac{1}{4}\right)\partial^\mu e(\partial_\mu e - 2\partial^\alpha e_{\alpha\mu}) + \frac{1}{4}e_{\mu\nu}\Box(e^{\mu\nu} + e^{\nu\mu}) - \frac{m^2}{2}(e_{\mu\nu}e^{\nu\mu} - e^2). \tag{74}$$

The equations of motion of Eq. (74) become

$$\Box e^{(\mu\nu)} - (2a_1 + 1/2)[\eta^{\mu\nu}(\Box e - \partial^\alpha\partial^\beta e_{\alpha\beta}) - \partial^\mu\partial^\nu e] = \partial^{(\mu}\partial^\alpha e^{\nu)}{}_\alpha + 2a_1\partial^\mu\partial^\alpha e_\alpha{}^\nu + \frac{1}{2}\partial^\nu\partial^\alpha e_\alpha{}^\mu - m^2(\eta^{\mu\nu}e - e^{\nu\mu}). \tag{75}$$

Applying ∂_ν to Eq. (75), we derive the constraint

$$\partial_\nu e^{\nu\mu} = \partial^\mu e. \tag{76}$$

By plugging back Eq. (76) into Eq. (75) the antisymmetric part of the resulting equation leads to another constraint,

$$e_{[\mu\nu]} = 0. \tag{77}$$

From the trace of Eq. (75), and using Eqs. (76) and (77), we derive

$$e = 0. \tag{78}$$

[3] Alternatively we could use the sum $S = a_1 + 3/4$ as a free parameter.

Therefore, back in Eq. (76) we have the transversality relation

$$\partial_\mu e^{\mu\nu} = 0 = \partial_\nu e^{\mu\nu}. \quad (79)$$

Finally, Eq. (75) becomes the Klein-Gordon equation,

$$(\Box - m^2)e^{(\mu\nu)} = 0. \quad (80)$$

In conclusion we have a massive spin-2 particle with the correct counting of degrees of freedom for arbitrary values of a_1. If $a_1 = 1/4$ ($S = 1$) we recover the massive FP theory, while $a_1 = -1/12$ ($S = 2/3$) and $a_1 = -1/4$ ($S = 1/2$) lead to the other two models from the literature which describe massive spin-2 particles via a rank-2 tensor (from Refs. [23,24], respectively). Thus, the one-parameter family $\mathcal{L}(a_1)$ intersects the other two classes $S = 1$ ($b_2 \neq -1/2$) and $S = 2/3$ at the specific points where the corresponding free parameters of those classes become, respectively, $d_- = m^2/2$ and $c = m^2/2$. So $\mathcal{L}(a_1)$ contains all known models for a massive spin-2 particle.

We finish this section by commenting on the van Dam-Veltman-Zakharov mass discontinuity for the $\mathcal{L}(a_1)$ family. Since in the massive case the only singular term of the propagator is of the same form as the massive FP theory (a_1 independent). Disregarding terms which are not important for the light beams' deviation by the sun we have for the massless limit of the massive propagator

$$\begin{aligned}(G^{-1}_{a_1})^{\text{sing}}_{\mu\nu\alpha\beta} &= \lim_{m\to 0}\frac{2}{\Box - m^2}(P^{(2)}_{SS})_{\mu\nu\alpha\beta} \\ &= \frac{2}{\Box}\left(\frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}}{2} - \frac{\eta_{\mu\nu}\eta_{\alpha\beta}}{3}\right) \\ &\quad + \cdots, \end{aligned} \quad (81)$$

where the dots (here and in the next formula) stand for unimportant terms for the light beams' deviation. The discrepancy of the deviation angle will be the same as for the massive FP theory $\theta(a_1) = \theta(FP) = (3/4)\theta_{\text{GR}}$.

On the other hand, for the massless model $\mathcal{L}_{m\to 0}(a_1)$ the relevant piece of the propagator is given by

$$\begin{aligned}(G_{a_1})^{\text{sing}}_{\mu\nu\alpha\beta} &= \left[\frac{2}{\Box}P^{(2)}_{SS} - \frac{4P^{(0)}_{SS}}{\Box(1+12a_1)} + \cdots\right]_{\mu\nu\alpha\beta} \\ &= \frac{2}{\Box}\left[\frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}}{2}\right. \\ &\quad \left. - \frac{(1+4a_1)}{(1+12a_1)}\eta_{\mu\nu}\eta_{\alpha\beta}\right] + \cdots. \end{aligned} \quad (82)$$

Therefore, we have the deviation angle

$$\theta_{m=0}(a_1) = \left(\frac{16a_1}{1+12a_1}\right)\theta_{\text{GR}}. \quad (83)$$

Thus, the massless theory only reproduces the GR result in the trivial case $a_1 = 1/4$ where $\mathcal{L}(a_1) = \mathcal{L}_{\text{LEH}}$. However, since $\theta_{m=0}(a_1)$ continuously approaches θ_{GR} from above as $a_1 \to (1/4)^+$ we may have a_1 close enough to $1/4$ such that the difference $\theta_{m=0}(a_1) - \theta_{\text{GR}}$ is below the experimental error bar. So we can not discard the massless scalar-tensor theory $\mathcal{L}_{m=0}(a_1)$ based on the light beams' deviation by the sun. Recall that $S = a_1 + 3/4$, and consequently the ghost-free bounds $S \geq 1$ or $S \leq 2/3$ contain the region where a_1 is slightly above $1/4$.

V. CONCLUSION

We have started with a second-order theory for a general rank-2 tensor with ten free parameters. Requiring that we only have one massive spin-2 particle in the spectrum we are left with only one free parameter up to the local field redefinitions (33) and (34). We have proved that in the massless limit we always have one spin-0 plus one spin-1 local symmetry. The use of spin projection and transition operators and their algebra (see appendix A) has helped us in splitting the one-parameter family of models into three classes: $S = 1$, $S = 2/3$, and $S \neq 1; 2/3$, where $S = a_1 + a_2 + a_3$ is invariant under the field redefinitions (33) and (34). In the massless limit, if $S = 1$ the local spin-1 symmetry is maximal while $S = 2/3$ corresponds to a maximal spin-0 symmetry.

In the class $S = 1$ the coefficient d_- in Eq. (1) is the free parameter. We can get rid of the antisymmetric part of the tensor via a local gauge symmetry and work with a purely symmetric tensor. If $S = 1$ and $b_2 \neq -1/2$ the one-parameter family of models is equivalent—after a field redefinition—to the well-known massive Fierz-Pauli theory, and thus describes a massive spin-2 particle. The special case $S = 1$ and $b_2 = -1/2$ is unstable [21]. Its massless limit is invariant under linearized Weyl and linearized transverse reparametrizations. It is the WTDIFF model of Ref. [21]. It describes a massless spin-2 particle and it is the linearized version of unimodular gravity. We remark that here we never have the so-called TDIFF theories which are invariant only under transverse linearized reparametrizations. This follows from our primary assumption that our massive theory only describes a physical massive spin-2 particle while in TDIFF theories there is always a scalar particle [21].

In the class $S = 2/3$ the coefficient c in Eq. (1) becomes a free parameter. The derivative terms are invariant under a linearized Weyl transformation such that the trace $e = \eta^{\mu\nu}e_{\mu\nu}$ only appears nondynamically in the mass term. Although the trace is nonzero off shell, as long as we remain at the quadratic level (free theory) it decouples and plays no role. So effectively we have a traceless (though nonsymmetric) description of a massive spin-2 particle [24]. In the massless limit we have one massless spin-2 particle (see Ref. [22] and also Ref. [24]).

In the third class of models, $S \neq 1; 2/3$, the parameter S itself becomes the free parameter. There is no further restriction on S in the massive theory. It does represent a new one-parameter family of models describing a massive spin-2 particle. At the point $S = 1/2$ we recover the model of Ref. [23]. The massive theory has the same mass discontinuity problem as the massive Fierz-Pauli theory and predicts the same incorrect deviation angle of the light beams by the sun which is independent of S. In the massless limit we have a scalar-tensor theory whose unitarity requires $S \leq 2/3$ or $S \geq 1$. For the scalar-tensor theory the deviation angle can be made consistent with experimental data if S is chosen slightly above one. General relativity is recovered at $S = 1$.

The one-parameter families $S = 2/3$ and $S \neq 1; 2/3$ are ghost- and tachyon-free although we have a coupling between the antisymmetric and symmetric parts of the rank-2 tensor. This is contrary to the claim of Ref. [19] that this kind of coupling will necessarily lead to ghosts. In the specific example of antisymmetric/symmetric coupling chosen in Ref. [19] there is in fact a ghost but it is not the general situation as shown here and in the earlier examples of Ref. [22] (massless case) and Refs. [23,24] (massive cases).

Finally, an arbitrary tensor $e_{\mu\nu}$ can be decomposed into a traceless symmetric field ($h^T_{\mu\nu}$), a pure trace piece ($h\eta_{\mu\nu}$) and an antisymmetric tensor ($B_{\mu\nu}$). Since a massive spin-2 particle requires on shell that $h = 0 = B_{\mu\nu}$, the fields h and $B_{\mu\nu}$ are auxiliary. It is well known (see, for instance, Ref. [17]) that we cannot set $h = 0 = B_{\mu\nu}$ off shell. So the next minimal possibility is to set only $B_{\mu\nu} = 0$ off shell, which is indeed possible and corresponds to the usual massive Fierz-Pauli theory or linearized Einstein-Hilbert theory. In our notation it corresponds to the $S = 1$ case with $b_2 \neq -1/2$. It was shown in Ref. [24] that the next simplest case with $h = 0$ and $B_{\mu\nu} \neq 0$, both off shell, is also possible; this is the $S = 2/3$ case. Here we have shown that we are allowed to keep both auxiliary fields h and $B_{\mu\nu}$ nonvanishing off shell ($S \neq 1$; $2/3$ case) without any inconsistency as far as we deal with the free theory. Since auxiliary fields may become dynamical when we turn on interactions and lead to troubles like incorrect counting of degrees of freedom (loss of constraints), ghosts, and acausality (see, e.g., Refs. [3,27] and more recently Refs. [10–13]), it is crucial to investigate the addition of interactions to the massive spin-2 models with $S \neq 1$. This is now in progress.

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APPENDIX A

After defining the spin-0 and spin-1 projection operators as, respectively,

$$\omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\Box}, \qquad \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}, \tag{A1}$$

one can define the projection and transition operators (see, e.g., Ref. [19]). First we present the symmetric operators

$$(P^{(2)}_{TT})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda{}_\alpha \theta^\mu{}_\beta + \theta^\mu{}_\alpha \theta^\lambda{}_\beta) - \frac{\theta^{\lambda\mu}\theta_{\alpha\beta}}{3}, \tag{A2}$$

$$(P^{(1)}_{SS})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda{}_\alpha \omega^\mu{}_\beta + \theta^\mu{}_\alpha \omega^\lambda{}_\beta + \theta^\lambda{}_\beta \omega^\mu{}_\alpha + \theta^\mu{}_\beta \omega^\lambda{}_\alpha), \tag{A3}$$

$$(P^{(0)}_{TT})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{3}\theta^{\lambda\mu}\theta_{\alpha\beta}, \qquad (P^{(0)}_{WW})^{\lambda\mu}{}_{\alpha\beta} = \omega^{\lambda\mu}\omega_{\alpha\beta}, \tag{A4}$$

$$(P^{(0)}_{TW})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{\sqrt{3}}\theta^{\lambda\mu}\omega_{\alpha\beta}, \qquad (P^{(0)}_{WT})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{\sqrt{3}}\omega^{\lambda\mu}\theta_{\alpha\beta}. \tag{A5}$$

They satisfy the symmetric closure relation

$$[P^{(2)}_{TT} + P^{(1)}_{SS} + P^{(0)}_{TT} + P^{(0)}_{WW}]_{\mu\nu\alpha\beta} = \frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}}{2}. \tag{A6}$$

The remaining antisymmetric and mixed symmetric-antisymmetric operators are given by

$$(P^{(1)}_{AA})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda{}_\alpha \omega^\mu{}_\beta - \theta^\mu{}_\alpha \omega^\lambda{}_\beta - \theta^\lambda{}_\beta \omega^\mu{}_\alpha + \theta^\mu{}_\beta \omega^\lambda{}_\alpha), \tag{A7}$$

$$(P^{(1)}_{SA})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda{}_\alpha \omega^\mu{}_\beta + \theta^\mu{}_\alpha \omega^\lambda{}_\beta - \theta^\lambda{}_\beta \omega^\mu{}_\alpha - \theta^\mu{}_\beta \omega^\lambda{}_\alpha), \tag{A8}$$

$$(P^{(1)}_{AS})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda{}_\alpha \omega^\mu{}_\beta - \theta^\mu{}_\alpha \omega^\lambda{}_\beta + \theta^\lambda{}_\beta \omega^\mu{}_\alpha - \theta^\mu{}_\beta \omega^\lambda{}_\alpha), \tag{A9}$$

$$(P^{(0)}_{AA})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda{}_\alpha \theta^\mu{}_\beta - \theta^\mu{}_\alpha \theta^\lambda{}_\beta). \tag{A10}$$

They satisfy the antisymmetric closure relation

$$[P^{(1)}_{AA} + P^{(0)}_{AA}]_{\mu\nu\alpha\beta} = \frac{\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\beta}\eta_{\nu\alpha}}{2}. \tag{A11}$$

Adding up Eqs. (A6) and (A11), we have

$$[P^{(2)}_{TT} + P^{(1)}_{SS} + P^{(0)}_{TT} + P^{(0)}_{WW} + P^{(1)}_{AA} + P^{(0)}_{AA}]_{\mu\nu\alpha\beta} = \eta_{\mu\alpha}\eta_{\nu\beta}. \tag{A12}$$

APPENDIX B

Here we prove, with the help of the field redefinition (34), that we can always choose $(a_2, a_3) = (1/2, 1/4)$ as a solution of Eq. (27) without loss of generality.

It is clear from Eqs. (38)–(40) that the combination $a_1 + a_3 - a_2$ plays a special role. It is convenient to rewrite Eq. (27) as

$$(S - 1)(a_1 + a_3 - a_2) = (a_1 - a_3)^2. \tag{B1}$$

If $a_1 + a_3 - a_2 = 0$ we must have $a_1 = a_3$. Back in Eq. (28) we have—since Eq. (29) demands $d_- \neq 0$—that $S = a_1 + a_2 + a_3 = 1$. Those equations fix $a_2 = 1/2$ and $a_1 = a_3 = 1/4$. Regarding the hypothesis $d_- \neq 0$ we note that, since $a_1 + a_3 - a_2 = 0 = a_1 - a_3$, the antisymmetric tensor $B_{\mu\nu}$ only appears in the Lagrangian (2) through the trivial term $d_- B_{\mu\nu} B^{\mu\nu}$. Therefore there will be no change in the physical content of the theory if we assume $d_- \neq 0$. We conclude that if $a_1 + a_3 - a_2 = 0$ we automatically have $(a_2, a_3) = (1/2, 1/4)$.

On the other hand, if $a_1 + a_3 - a_2 \neq 0$ we can always move a_2 from any given value to $a_2 = 1/2$ via Eq. (38) by choosing $A = (2a_1 - a_2)/[2(a_1 + a_3 - a_2)]$ and using Eq. (27). Back in Eq. (40) we have [again using Eq. (27)] the new value $a_3 = (4a_1 a_3 - a_2^2)/[4(a_1 + a_3 - a_2)] = 1/4$. This completes the proof of Eq. (42).

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