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Systematic approach to $\Delta L = 1$ processes in thermal leptogenesis

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In this work we study the contribution to leptogenesis from $\Delta L = 1$ decay and scattering processes mediated by the Higgs with quarks in the initial and final states using the formalism of nonequilibrium quantum field theory. Starting from fundamental equations for correlators of the quantum fields we derive quantum-corrected Boltzmann and rate equations for the total lepton asymmetry improved in that they include quantum-statistical effects and medium corrections to the quasiparticle properties. To compute the collision term we take into account one- and two-loop contributions to the lepton self-energy and use the extended quasiparticle approximation for the Higgs two-point function. The resulting *CP*-violating and washout reaction densities are numerically compared to the conventional ones.

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I. INTRODUCTION

The standard model (SM) of particle physics [1–3] has successfully passed the numerous experimental tests performed so far. The recent observation of the Higgs particle [4] at the LHC [5,6] also seems to confirm the mechanism of spontaneous symmetry breaking, which is responsible for masses of the known gauge bosons and fermions. On the other hand, we know that the SM is not complete. First, it does not provide a viable dark matter candidate. Second, it predicts that the active neutrinos are strictly massless, which contradicts the results of neutrino oscillation experiments. A simple yet elegant way to generate small but nonzero neutrino masses is to add three right-handed Majorana neutrinos to the model:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{2}\bar{N}_i(i\not{\partial} - M_i)N_i - h_{\alpha i}\bar{\ell}_\alpha\tilde{\phi}P_R N_i - h^\dagger_{i\alpha}\bar{N}_i\tilde{\phi}^\dagger P_L \ell_\alpha, \tag{1}$$

where $N_i = N_i^c$ are the heavy Majorana fields, ℓ_α are the lepton doublets, and $\tilde{\phi} \equiv i\sigma_2\phi^*$ is the conjugate of the Higgs doublet. After the electroweak symmetry breaking the active neutrinos receive naturally small masses through the type-I seesaw mechanism. This scenario has even more far-reaching consequences as it can explain another beyond-the-SM observation, the baryon asymmetry of the Universe. The Majorana mass term in (1) violates lepton number. In the early Universe a decay of the Majorana neutrino into a lepton-Higgs pair increases the total lepton number of the Universe by one unit, and a decay into the corresponding antiparticles decreases the total lepton number by one unit. If there is *CP* violation then, on average, the number of leptons produced in those decays is not equal to the number of antileptons and a net lepton asymmetry is produced. It is also known that whereas the difference of the lepton and baryon numbers is conserved in the SM, any other linear combination is not [7]. This implies that the lepton asymmetry produced by the Majorana neutrinos is partially converted to the baryon asymmetry [8]. This mechanism, which is referred to as baryogenesis via leptogenesis, naturally explains the observed baryon asymmetry of the Universe. For a more detailed review of leptogenesis see e.g., [9–11].

The state-of-the-art analysis of the asymmetry generation uses Boltzmann equations with the decay and scattering amplitudes calculated in vacuum. Their applicability in the hot and expanding early universe is questionable and can be cross-checked using a first-principle approach based on the use of nonequilibrium quantum field theory (QFT). One of the most important processes for the generation of the asymmetry is the decay of the Majorana neutrino. Thermal effects enhancing *CP* violation in the decay have been studied in [12–16]. The role of the flavor effects has been addressed in [17]. A first-principle analysis of the asymmetry generation in the very interesting regime of resonant leptogenesis has been presented in [18,19]. The effect of next-to-leading order corrections from the gauge interactions of lepton and Higgs doublets on the production and decay rate of right-handed neutrinos at finite temperature has been recently studied in [20,21].

The asymmetry generated in the Majorana decay is partially washed out by the inverse decay and scattering processes. The latter can be classified into two categories. The first category includes $\Delta L = 2$ scattering processes mediated by the Majorana neutrinos. A first-principle analysis of such processes free of the notorious double-counting problem has been presented in [22]. The second category includes $\Delta L = 1$ decay and scattering processes

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mediated by the Higgs. The latter processes are also known to play an important role in the asymmetry generation and are addressed in the present paper.

The outline of the paper is as follows. In Sec. II we briefly review the canonical approach to the analysis of the $\Delta L = 1$ processes and derive the corresponding amplitudes and reduced cross sections. In Sec. III we derive quantum-generalized Boltzmann equations for the lepton asymmetry, calculate the effective amplitudes of the Higgs-mediated scattering processes, and compare them with the canonical ones. The obtained Boltzmann equations are used in Sec. V to derive a simple system of rate equations for the total lepton asymmetry. In Sec. VI we present a numerical comparison of the corresponding reaction densities with the ones obtained using the canonical approach. A summary of the results is presented in Sec. VII.

II. CONVENTIONAL APPROACH

In the scenario of thermal leptogenesis lepton asymmetry is generated in the lepton number and *CP*-violating decay of the heavy Majorana neutrinos. The corresponding *CP*-violating parameters receive contributions from the interference of the tree-level amplitude with the vertex [8,12] and self-energy [13,23–26] amplitudes, see Fig. 1. The contribution of the loop diagrams can be accounted for by effective Yukawa couplings [26]. If thermal masses of the SM particles are negligible, they are given by

$$h_{+,\alpha i} \equiv h_{\alpha i} - i h_{\alpha j}(h^\dagger h)^*_{ji} g_{ij}, \tag{2a}$$

$$h_{-,\alpha i} \equiv h^*_{\alpha i} - i h^*_{\alpha j}(h^\dagger h)_{ji} g_{ij}, \tag{2b}$$

where the loop-function g_{ij} is defined as

$$g_{ij} \equiv \frac{1}{16\pi}\frac{M_i M_j}{M_i^2 - M_j^2} + \frac{1}{16\pi}\frac{M_j}{M_i}\left[1 - \left(1 + \frac{M_j^2}{M_i^2}\right)\ln\left(1 + \frac{M_i^2}{M_j^2}\right)\right]. \tag{3}$$

Note that this expression is valid only for on-shell final states. The first term in (3) is related to the self-energy and the second term to the vertex contribution. This expression is applicable for a mildly or strongly hierarchical

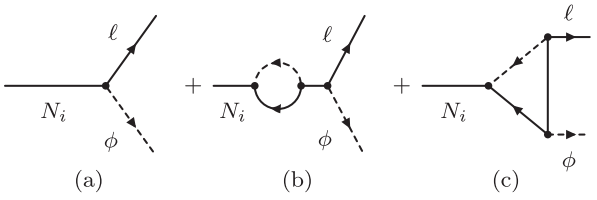

FIG. 1. Tree-level (a), one-loop self-energy (b), and one-loop vertex (c) contributions to the decay of the heavy Majorana neutrino.

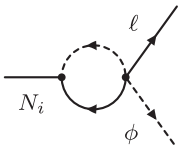

FIG. 2. Effective one-loop diagram for the self-energy and vertex contributions to the decay of the lightest Majorana neutrino for a strongly hierarchical mass spectrum.

mass spectrum of the Majorana neutrinos. In both cases most of the asymmetry is typically generated by the lightest Majorana neutrino, whereas the asymmetry generated by the heavier ones is almost completely washed out. For a strongly hierarchical mass spectrum, $M_i \ll M_j$, the intermediate Majorana line in Figs. 1(b) and 1(c) contracts to a point, see Fig. 2, and the structure of the self-energy and vertex contributions is the same. In this limit,

$$g_{ij} \approx -\frac{3}{32\pi}\frac{M_i}{M_j}. \tag{4}$$

Note that in this approximation the loop integral leading to (4) depends only on the momentum of the initial state and is independent of the momenta of the final states. This implies in particular that this expression can also be used for off-shell final states.

Using the effective couplings (2) we find for the decay amplitudes (squared) [22,26]:

$$\Xi_{N_i\to\ell\phi} = g_N g_w (h_+^\dagger h_+)_{ii}(qp), \tag{5a}$$

$$\Xi_{N_i\to\bar\ell\bar\phi} = g_N g_w (h_-^\dagger h_-)_{ii}(qp), \tag{5b}$$

where we have summed over flavors of the leptons in the final state as well as over the Majorana spin ($g_N = 2$) and the $SU(2)_L$ group ($g_w = 2$) degrees of freedom. Here q and p are momenta of the heavy neutrino and lepton, respectively. The decay amplitudes (5) can be traded for the total decay amplitude and *CP*-violating parameter:

$$\Xi_{N_i} \equiv \Xi_{N_i\to\ell\phi} + \Xi_{N_i\to\bar\ell\bar\phi}, \tag{6a}$$

$$\epsilon_i \equiv \frac{\Xi_{N_i\to\ell\phi} - \Xi_{N_i\to\bar\ell\bar\phi}}{\Xi_{N_i\to\ell\phi} + \Xi_{N_i\to\bar\ell\bar\phi}}. \tag{6b}$$

Combining (5) and (6) we then find for the (unflavored) *CP*-violating parameter:

$$\epsilon_i^{\rm vac} \approx \frac{{\rm Im}(h^\dagger h)^2_{ij}}{(h^\dagger h)_{ii}} \times 2 g_{ij}, \quad j \neq i. \tag{7}$$

The asymmetry generated by the Majorana decay is partially washed out by the inverse decay and scattering processes violating lepton number. An important role is played by the $\Delta L = 2$ scattering processes mediated by

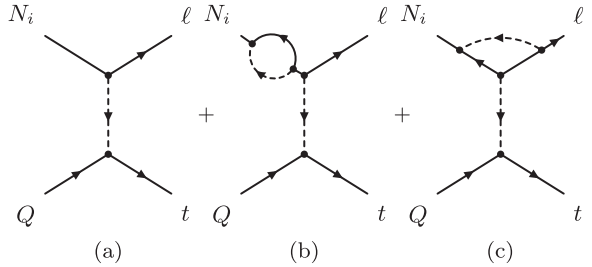

FIG. 3. Tree-level (a), self-energy (b), and vertex (c) contributions to the scattering processes $N_i Q \to \ell t$. Similar diagrams for the scattering process $N_i \bar{t} \to \ell \bar{Q}$ are obtained by replacing Q with $\bar{t}$ and t with $\bar{Q}$ as well as inverting the direction of the arrows.

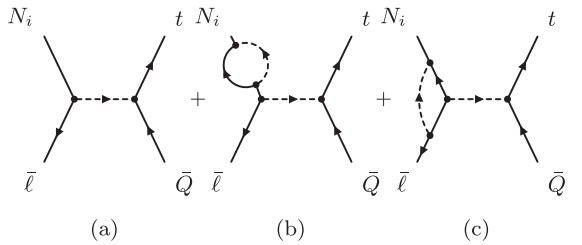

FIG. 4. Tree-level (a), self-energy (b), and vertex (c) contributions to the scattering processes $N_i \bar{\ell} \to \bar{Q} t$.

the heavy Majorana neutrinos [22,26,27]. In addition, there are $\Delta L = 1$ scattering process mediated by the Higgs doublet with quarks (or the gauge bosons) in the initial and final states [26,27], see Figs. 3 and 4. The Higgs coupling to the top is considerably larger than to the other quarks of the three generations. For this reason we do not consider the latter here. The corresponding Lagrangian reads

$$\mathcal{L}_{\rm SM} \supset -\lambda \bar{Q}\, \tilde{\phi}\, P_R t - \lambda^* \bar{t} P_L \tilde{\phi}^\dagger Q, \tag{8}$$

where Q and t are the $SU(2)_L$ doublet and singlet of the third quark generation. The $\Delta L = 1$ processes are also CP violating. The CP violation is generated by the same self-energy and vertex diagrams. Strictly speaking, since the Higgs is no longer on shell the effective couplings (2) are not applicable in this case. On the other hand, for a strongly hierarchical mass spectrum the intermediate Majorana lines in Figs. 3 and 4 again contract to a point and the momenta of the Higgs and lepton play no role. In other words, for a strongly hierarchical mass spectrum we still can use the effective couplings (2) supplemented with (4) to calculate the CP-violating scattering amplitudes.

Summing over flavors and colors of the quarks and leptons in the initial and final states as well as over the corresponding $SU(2)_L$ and spin degrees of freedom we find for the amplitude of $N_i Q \to \ell t$ scattering:

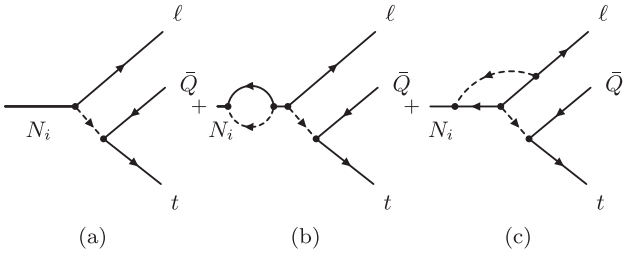

FIG. 5. Tree-level (a), self-energy (b), and vertex (c) contributions to the amplitude of the three-body decay processes $N_i \to \ell \bar{Q} t$.

$$\Xi_{N_i Q \to \ell t} = \Xi_{N_i \to \ell \phi} \times \Delta_T^2(p - q) \times \Xi_{\phi Q \to t}, \tag{9}$$

where $\Delta_T(k) \approx 1/(k^2 - m_\phi^2)$ is the Feynman (or time-ordered) propagator[1] of the intermediate Higgs and we have defined

$$\Xi_{\phi Q \to t} = 2 g_s |\lambda|^2 (p_Q p_t). \tag{10}$$

Here $g_s = 3$ is the $SU(3)_C$ factor, and p_t and p_Q are the momenta of the singlet and the doublet respectively. For the charge-conjugate process we find an expression similar to (9). As can be inferred from (10) in this work we neglect CP-violation in the quark sector, which is known to be small. Defining CP-violating parameter in scattering as

$$\epsilon_{X \to Y} \equiv \frac{\Xi_{X \to Y} - \Xi_{\bar{X} \to \bar{Y}}}{\Xi_{X \to Y} + \Xi_{\bar{X} \to \bar{Y}}}, \tag{11}$$

we then obtain for $\epsilon_{N_i Q \to \ell t}$ the same expression as for the Majorana decay, see (7). In the same approximation amplitude and CP-violating parameter for $N_i \bar{t} \to \ell \bar{Q}$ scattering coincide with those for the $N_i Q \to \ell t$ process. Proceeding in a similar way we find for the scattering amplitude of the $N_i \bar{\ell} \to \bar{Q} t$ process:

$$\Xi_{N_i \bar{\ell} \to \bar{Q} t} = \Xi_{N_i \bar{\ell} \to \phi} \times \Delta_T^2(p + q) \times \Xi_{\phi \to \bar{Q} t}, \tag{12}$$

where $\Xi_{\phi \to \bar{Q} t} = \Xi_{\phi Q \to t}$ because we neglect CP violation in the quark sector. Furthermore, for a strongly hierarchical mass spectrum $\Xi_{N_i \bar{\ell} \to \phi} = \Xi_{N_i \to \ell \phi}$. The resulting expression for the CP-violating parameter then coincides with (7).

If the lepton and both quarks are in the final state then instead of a scattering process we deal with a three-body Majorana decay, see Fig. 5. In complete analogy with the scattering processes we can write its amplitude in the form

[1]In the kinematic region of interest the decay width term in the Feynman propagator of the Higgs plays no role and can be neglected.

$$\Xi_{N_i\to\ell\bar{Q}t} = \Xi_{N_i\to\ell\phi}\times\Delta_T^2(p_Q+p_t)\times\Xi_{\phi\to\bar{Q}t}. \quad (13)$$

Evidently, the CP-violating parameter for this process coincides with that for the two-body Majorana decay.

To compute the generated lepton asymmetry, the conventional approach uses the generalized Boltzmann equation for the total lepton abundance, $Y_L \equiv n_L/s$, with s being the comoving entropy density [28]. In the Friedmann-Robertson-Walker universe the contribution of the Higgs-mediated processes to the right-hand side of the Boltzmann equation simplifies to

$$\begin{aligned}\frac{s\mathcal{H}}{z}\frac{dY_L}{dz} = \cdots &- \sum_i\int d\Pi^{qppQp_t}_{N_i\ell Qt}(2\pi)^4\delta(q+p-p_Q-p_t)[\Xi_{N_i\ell\to Q\bar{\imath}}f_{N_i}f_\ell(1-f_Q)(1-f_{\bar{\imath}})-\text{inverse}]\\ &+\sum_i\int d\Pi^{qpQpp_t}_{N_iQ\ell t}(2\pi)^4\delta(q+p_Q-p-p_t)[\Xi_{N_iQ\to\ell t}f_{N_i}f_Q(1-f_\ell)(1-f_t)-\text{inverse}]\\ &+\sum_i\int d\Pi^{qp_tppQ}_{N_i\bar{\imath}\ell\bar{Q}}(2\pi)^4\delta(q+p_t-p-p_Q)[\Xi_{N_i\bar{\imath}\to\ell\bar{Q}}f_{N_i}f_{\bar{\imath}}(1-f_\ell)(1-f_{\bar{Q}})-\text{inverse}]\\ &+\sum_i\int d\Pi^{qppQp_t}_{N_i\ell\bar{Q}t}(2\pi)^4\delta(q-p-p_Q-p_t)[\Xi_{N_i\to\ell\bar{Q}t}f_{N_i}(1-f_\ell)(1-f_{\bar{Q}})(1-f_t)-\text{inverse}]\\ &-CP\text{ conjugate processes},\end{aligned} \quad (14)$$

where we have introduced the dimensionless inverse temperature $z = M_1/T$, the Hubble rate $\mathcal{H} = H|_{T=M_1}$, and $d\Pi^{p_a p_b\cdots p_i p_j\cdots}_{ab\ldots ij\ldots}$ stands for the product of the invariant phase space elements, $d\Pi^p_a \equiv d^3p/[(2\pi)^3 2E_p]$. Note that to ensure vanishing of the asymmetry in thermal equilibrium one should also include CP-violating $2\leftrightarrow 3$ processes [10]. Since there is no need for that in the nonequilibrium quantum field theory approach we will not consider these processes here.

III. NONEQUILIBRIUM QFT APPROACH

The formalism of nonequilibrium quantum field theory provides a powerful tool for the description of out-of-equilibrium quantum fields and is therefore well suited for the analysis of leptogenesis. In this section we briefly review results obtained in [22] and introduce notation that will be used in the rest of the paper. As has been argued in [22], the equation of motion for the lepton asymmetry can be derived by considering the divergence of the lepton current. In the Friedmann-Robertson-Walker universe $j^\mu_L = (n_L, \mathbf{0})$ and therefore it is related to the total lepton abundance by

$$\mathcal{D}_\mu j^\mu_L = \frac{s\mathcal{H}}{z}\frac{dY_L}{dz}. \quad (15)$$

Using the formalism of nonequilibrium quantum field theory, one can express it through propagators and self-energies of leptons. After a transformation to the Wigner space we obtain [22]

$$\begin{aligned}\mathcal{D}_\mu j^\mu_L(t,p) = g_w\int d\Pi^4_p\,\mathrm{tr}[&\hat{\Sigma}_<(t,p)\hat{S}_>(t,p)\\ &-\hat{\Sigma}_>(t,p)\hat{S}_<(t,p)],\end{aligned} \quad (16)$$

where $d\Pi^4_p \equiv d^4p/(2\pi)^4$ and the hats denote matrices in flavor space. In (16) we have taken into account that the $SU(2)_L$ symmetry is unbroken at the epoch of leptogenesis. As a consequence, the $SU(2)_L$ structure of the propagator is trivial, $S^{\alpha\beta}_{ab} = \delta_{ab}S^{\alpha\beta}$, and summation over the $SU(2)_L$ components simply results in the overall factor $g_w = 2$. Furthermore, in this work we restrict ourselves to the analysis of unflavored leptogenesis. Therefore, the lepton propagator can be approximated by $S^{\alpha\beta} = \delta^{\alpha\beta}S$. A similar relation also holds for the lepton self-energy. Then the equation for the divergence of the lepton current takes the form

$$\begin{aligned}\mathcal{D}_\mu j^\mu_L = g_w\int_0^\infty\frac{dp^0}{2\pi}\int\frac{d^3p}{(2\pi)^3}\,\mathrm{tr}[&(\Sigma_<S_> - \Sigma_>S_<)\\ &-(\bar{\Sigma}_<\bar{S}_> - \bar{\Sigma}_>\bar{S}_<)],\end{aligned} \quad (17)$$

where $\Sigma \equiv \Sigma^{\alpha\alpha}$ and we have suppressed the argument (t,p) of the two-point functions. Note that the trace in (17) acts now in spinor space only. To convert the integration over positive and negative frequencies into the integration over positive frequencies only, we have introduced in (17) CP-conjugate two-point functions and self-energies which are denoted by the bar. According to the extended quasiparticle approximation (eQP) [29–31] the Wigthmann propagators can be split into on- and off-shell parts:

$$S_{\gtrless} = \tilde{S}_{\gtrless} - \frac{1}{2}(S_R\Sigma_{\gtrless}S_R + S_A\Sigma_{\gtrless}S_A). \quad (18)$$

The off-shell parts of the lepton propagators exactly cancel out in the lepton current as they are lepton number conserving. On the other hand, as we will see later, the off-shell part of the Higgs two-point functions is crucial for a correct description of the scattering processes. The on-shell part of the Wightman propagators is related to the eQP spectral function and one-particle distribution function f_ℓ by the Kadanoff-Baym (KB) ansatz:

$$\tilde{S}_> = (1 - f_\ell)\tilde{S}_\rho, \qquad \tilde{S}_< = -f_\ell \tilde{S}_\rho, \tag{19}$$

where

$$\tilde{S}_\rho = -\frac{1}{2} S_R \Sigma_\rho S_R \Sigma_\rho S_A \Sigma_\rho S_A. \tag{20}$$

In the limit of vanishing width the eQP spectral function $\tilde{S}_\rho$ approaches the Dirac delta function [22],

$$\begin{aligned}\tilde{S}_\rho &\approx (2\pi)\mathrm{sign}(p^0)\delta(p^2 - m_\ell^2) P_L \not{p} P_R \\ &\equiv \mathbf{S}_\rho(p) P_L \not{p} P_R,\end{aligned} \tag{21}$$

where we have extracted the "scalar" part $\mathbf{S}_\rho$ for notational convenience. In (21) we have approximately taken the gauge interactions into account in the form of effective masses of the leptons. Note that we will not attempt a fully consistent inclusion of the gauge interactions here. In the used approximation the spectral function is CP symmetric. This implies that the spectral properties, in particular the masses, of the particles and antiparticles are the same.

To evaluate the right-hand side of (17) we need to specify the form of the lepton self-energy. It can be obtained by functional differentiation of the 2PI effective action with respect to the lepton propagator. Loosely speaking, this means that the self-energies are obtained by cutting one line of the 2PI contributions to the effective action. The two- and three-loop contributions are presented in Figs. 6(a) and 6(c). The one-loop contribution to the lepton self-energy, see Fig. 6(b), is given by [22]

$$\begin{aligned}\Sigma^{(1)}_{\gtrless}(t, p) = -\int d\Pi^4_{qk} (2\pi)^4 \delta(p + k - q) \\ \times (h^\dagger h)_{ji} P_R \mathbb{S}^{ij}_{\gtrless}(t, q) P_L \Delta_{\lessgtr}(t, k),\end{aligned} \tag{22}$$

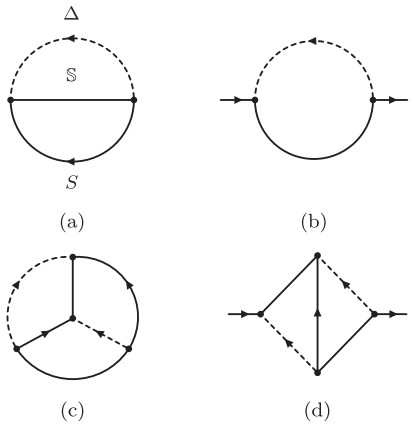

FIG. 6. Two- and three-loop contributions to the 2PI effective action [(a) and (c)] and the corresponding contributions to the lepton self-energy [(b) and (d)].

where $\mathbb{S}$ and Δ denote the Majorana and Higgs propagators respectively, and $d\Pi^4_{qk} \equiv d\Pi^4_q d\Pi^4_k$. The expression for the two-loop contribution, see Fig. 6(d), is rather lengthy. Here we will only need a part of it:

$$\begin{aligned}\Sigma^{(2)}_{\gtrless}(t, p) = \int d\Pi^4_{qk} (2\pi)^4 \delta(p + k - q)[(h^\dagger h)_{in} \\ \times (h^\dagger h)_{jm} \Lambda_{mn}(t, q, k) P_L C \mathbb{S}^{ij}_{\gtrless}(t, q) P_L \Delta_{\gtrless}(t, k) \\ + (h^\dagger h)_{ni} (h^\dagger h)_{mj} P_R \mathbb{S}^{ji}_{\gtrless}(t, q) C P_R V_{nm} \\ \times (t, q, k) \Delta_{\gtrless}(t, k)],\end{aligned} \tag{23}$$

where we have introduced two functions containing loop corrections:

$$\begin{aligned}\Lambda_{mn}(t, q, k) \equiv \int d\Pi^4_{k_1 k_2 k_3} (2\pi)^4 \delta(q + k_1 + k_2)(2\pi)^4 \delta(k + k_2 - k_3)[P_R \mathbb{S}^{mn}_R(t, -k_3) \\ \times C P_R S^T_F(t, k_2) \Delta_A(t, k_1) + P_R \mathbb{S}^{mn}_F(t, -k_3) C P_R S^T_R(t, k_2) \\ \times \Delta_A(t, k_1) + P_R \mathbb{S}^{mn}_R(t, -k_3) C P_R S^T_A(t, k_2) \Delta_F(t, k_1)],\end{aligned} \tag{24}$$

and $V_{nm}(t, q, k) \equiv P \Lambda^\dagger_{nm}(t, q, k) P$ to shorten the notation. Here $P = \gamma^0$ is the parity conjugation operator. The remaining terms of the two-loop self-energy can be found in [22]. As has been demonstrated in the same reference, CP conjugates of the above self-energies can be obtained by replacing the Yukawa couplings by the complex conjugated ones and the propagators by the CP-conjugated ones.

Comparing (22) and (23) we see that the two self-energies have a very similar structure. First, the integration is over momenta of the Higgs and Majorana neutrino and the delta function contains the same combination of the momenta. Second, they both include one Wightman propagator of the Higgs field and one Wightman propagator of the Majorana field. These can be interpreted as cut propagators which describe on-shell particles created from or absorbed by the plasma [32]. The retarded and advanced propagators can be associated with the off-shell intermediate states. We therefore conclude that the two self-energies describe CP-violating decay of the heavy neutrino into a lepton-Higgs pair. Note that this interpretation only holds for the "particle" part of the eQP ansatz. The inclusion of the off-shell part of the Higgs Wightman propagator gives rise to the Higgs mediated scattering processes and three-body decay, see Sec. IV.

To evaluate (22) and (23) we need to know the form of the Higgs and Majorana propagators. For the Higgs field we will adopt in this section a leading-order approximation:

$$\Delta_> = (1 + f_\phi)\Delta_\rho, \qquad \Delta_< = f_\phi \Delta_\rho, \tag{25}$$

and a simple quasiparticle approximation for the spectral function,

$$\Delta_\rho(t,k) = (2\pi)\mathrm{sign}(k^0)\delta(k^2 - m_\phi^2), \tag{26}$$

where m_ϕ is the effective thermal Higgs mass. Close to thermal equilibrium the full resummed Majorana propagator is given by [22]

$$\hat{\mathbb{S}}_{\gtrless} = \hat{\Theta}_R[\hat{\tilde{\mathcal{S}}}_{\gtrless} - \hat{\mathcal{S}}_R\hat{\Pi}'_{\gtrless}\hat{\mathcal{S}}_A - \frac{1}{2}(\hat{\mathcal{S}}_R\hat{\Pi}^d_{\gtrless}\hat{\mathcal{S}}_R + \hat{\mathcal{S}}_A\hat{\Pi}^d_{\gtrless}\hat{\mathcal{S}}_A)]\hat{\Theta}_A, \tag{27}$$

where $\hat{\Pi}^d$ and $\hat{\Pi}'$ denote the diagonal and off-diagonal components of the Majorana self-energy respectively, $\hat{\mathcal{S}}_R$ and $\hat{\mathcal{S}}_A$ are given by

$$\hat{\mathcal{S}}_{R(A)} = -(\not{q} - \hat{M} - \hat{\Pi}^d_{R(A)})^{-1}, \tag{28}$$

and we have introduced

$$\hat{\Theta}_R \equiv (\mathbb{1} + \hat{\mathcal{S}}_R\hat{\Pi}'_R)^{-1}, \qquad \hat{\Theta}_A \equiv (\mathbb{1} + \hat{\Pi}'_A\hat{\mathcal{S}}_A)^{-1}, \tag{29}$$

to shorten the notation. The first term in the square brackets of (27) describes (inverse) decay of the Majorana neutrino, whereas the remaining three terms describe two-body scattering processes mediated by the Majorana neutrino. For the "particle" part of the eQP diagonal Wightman propagators of the Majorana neutrino, one can use the KB approximation:

$$\tilde{\mathcal{S}}^{nn}_{>} = (1 - f_{N_n})\tilde{\mathcal{S}}^{nn}_\rho, \qquad \tilde{\mathcal{S}}^{nn}_{<} = -f_{N_n}\tilde{\mathcal{S}}^{nn}_\rho, \tag{30}$$

with the spectral function given by an expression identical to (20). Substituting (28) we find in the limit of small decay width:

$$\tilde{\mathcal{S}}^{nn}_\rho = (2\pi)\mathrm{sign}(q^0)\delta(q^2 - M_n^2)(\not{q} + M_n) \equiv \tilde{\boldsymbol{\mathcal{S}}}^{nn}_\rho(\not{q} + M_n). \tag{31}$$

Inserting (22) and (23) into the divergence of the lepton current (17) and integrating over the frequencies, we then obtain an expression that strongly resembles the Boltzmann equation:

$$\frac{sH}{z}\frac{dY_L}{dz} = \sum_i \int d\Pi^{qpk}_{N_i\ell\phi}[\Xi_{N_i\leftrightarrow\ell\phi}\mathcal{F}^{q;pk}_{N_i\leftrightarrow\ell\phi} - \Xi_{N_i\leftrightarrow\bar{\ell}\bar{\phi}}\mathcal{F}^{q;pk}_{N_i\leftrightarrow\bar{\ell}\bar{\phi}}], \tag{32}$$

where we have introduced

$$\mathcal{F}^{p_a p_b\cdots;p_i p_j\cdots}_{ab\ldots\leftrightarrow ij\ldots} \equiv (2\pi)^4\delta(p_a + p_b + \cdots - p_i - p_j - \cdots) \times [f_a^{p_a}f_b^{p_b}\ldots(1 \pm f_i^{p_i})(1 \pm f_j^{p_j})\ldots - f_i^{p_i}f_j^{p_j}\ldots(1 \pm f_a^{p_a})(1 \pm f_b^{p_b})\ldots], \tag{33}$$

with the plus (minus) sign corresponding to bosons (fermions). Note that $\mathcal{F}^{p_a p_b\cdots;p_i p_j\cdots}_{ab\ldots\leftrightarrow ij\ldots}$ vanishes in equilibrium due to detailed balance. This implies that in accordance with the third Sakharov condition [33] no asymmetry is generated in equilibrium. In the Kadanoff-Baym formalism this result is obtained automatically and no need for the real intermediate state subtraction arises.

The effective decay amplitudes Ξ are given by a sum of the tree-level, one-loop self-energy, and one-loop vertex contributions. The first two,

$$\Xi^T_{N_i\leftrightarrow\ell\phi} + \Xi^S_{N_i\leftrightarrow\ell\phi} \equiv g_w\sum_{mn}(h^\dagger h)_{mn}\mathrm{tr}[\Theta^{ni}_R(q)(\not{q} + M_i)\Theta^{im}_A(q)P_L\not{p}P_R], \tag{34a}$$

$$\Xi^T_{N_i\leftrightarrow\bar{\ell}\bar{\phi}} + \Xi^S_{N_i\leftrightarrow\bar{\ell}\bar{\phi}} \equiv g_w\sum_{mn}(h^\dagger h)^*_{mn}\mathrm{tr}[\bar{\Theta}^{ni}_R(q)(\not{q} + M_i)\bar{\Theta}^{im}_A(q)P_L\not{p}P_R], \tag{34b}$$

emerge from the one-loop lepton self-energy (22). The third one,

$$\Xi^V_{N_i\leftrightarrow\ell\phi} \equiv -g_w(h^\dagger h)^2_{ij}M_i\,\mathrm{tr}[\Lambda_{jj}(q,k)CP_L\not{p}P_R] - g_w(h^\dagger h)^2_{ji}M_i\,\mathrm{tr}[CV_{jj}(q,k)P_L\not{p}P_R], \tag{35a}$$

$$\Xi^V_{N_i\leftrightarrow\bar{\ell}\bar{\phi}} \equiv -g_w(h^\dagger h)^2_{ij}M_i\,\mathrm{tr}[CV_{jj}(q,k)P_L\not{p}P_R] - g_w(h^\dagger h)^2_{ji}M_i\,\mathrm{tr}[\Lambda_{jj}(q,k)CP_L\not{p}P_R], \tag{35b}$$

is generated by the two-loop lepton self-energy (23). Substituting (34) and (35) into (6) we find to leading order in the couplings that the total decay amplitude summed over the Majorana spin degrees of freedom is given by $\Xi_{N_i} = 2g_N g_w(h^\dagger h)_{ii}(pq)$. The self-energy CP-violating parameter reads [22]

$$\epsilon^S_i \approx -\sum\frac{\mathrm{Im}(h^\dagger h)^2_{ij}}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}}\frac{M_i\Gamma_j}{M_j^2}\frac{pL_S}{qp}\cdot M_j^2\boldsymbol{\mathcal{S}}^{jj}_h(q), \tag{36}$$

where the "scalar" part of the diagonal Hermitian Majorana propagator is given by [22]

$$\boldsymbol{\mathcal{S}}^{jj}_h(q) \equiv \frac{1}{2}[\boldsymbol{\mathcal{S}}^{jj}_R(q) + \boldsymbol{\mathcal{S}}^{jj}_A(q)] \approx -\frac{q^2 - M_j^2}{(q^2 - M_j^2)^2 + (\Gamma_j/M_j\cdot qL_S)^2}. \tag{37}$$

It describes the intermediate Majorana neutrino line in Fig. 1(b). Note that (36) has been obtained assuming a hierarchical mass spectrum of the heavy neutrinos and is not applicable for a quasidegenerate spectrum. For positive q^0 and q^2 the self-energy loop function L_S is given by [22]

$$L_S^\mu = 16\pi \int d\Pi^{\phi\ell}_{k_1 p_1} (2\pi)^4 \delta(q - k_1 - p_1) p_1^\mu [1 + f_\phi^{k_1} - f_\ell^{p_1}]. \tag{38}$$

Simplifying (35) we find for the vertex CP-violating parameter [22]:

$$\epsilon_i^V = -\frac{1}{2} \sum \frac{\mathrm{Im}(h^\dagger h)^2_{ij}}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{M_i \Gamma_j}{M_j^2} \frac{pL_V}{qp}. \tag{39}$$

The vertex loop function is given by

$$\begin{aligned} L_V^\mu(q,p) = 16\pi M_j^2 \int d\Pi^4_{q_1 p_1 k_1} (2\pi)^4 \delta(q + k_1 + p_1)(2\pi)^4 \delta(q - p + p_1 - q_1) p_1^\mu [\Delta_\rho(k_1) \mathbf{S}_F(p_1) \boldsymbol{\mathcal{S}}_h^{jj}(q_1) \\ + \Delta_F(k_1) \mathbf{S}_\rho(p_1) \boldsymbol{\mathcal{S}}_h^{jj}(q_1) - \Delta_h(k_1) \mathbf{S}_\rho(p_1) \boldsymbol{\mathcal{S}}_h^{jj}(q_1) + \Delta_h(k_1) \mathbf{S}_F(p_1) \boldsymbol{\mathcal{S}}_\rho^{jj}(q_1) + \Delta_\rho(k_1) \mathbf{S}_h(p_1) \boldsymbol{\mathcal{S}}_F^{jj}(q_1) \\ + \Delta_F(k_1) \mathbf{S}_h(p_1) \boldsymbol{\mathcal{S}}_\rho^{jj}(q_1)], \end{aligned} \tag{40}$$

where $\boldsymbol{\mathcal{S}}_F = (\boldsymbol{\mathcal{S}}_> + \boldsymbol{\mathcal{S}}_<)/2$ is the "scalar" part of the corresponding statistical propagator of the heavy neutrino. For the lepton and Higgs fields the definitions are similar. The three lines in the square brackets in (39) correspond to different cuts through two of the three internal lines of the vertex loop. The first line corresponds to cutting the propagators of the Higgs and lepton and can be simplified to [16]

$$\begin{aligned} pL_V^{\ell\phi}(q,p) = 16\pi \int d\Pi^{\phi\ell}_{k_1 p_1} (2\pi)^4 \delta(q - p_1 - k_1)(p p_1) \\ \times [1 + f_\phi^{k_1} - f_\ell^{p_1}] \frac{M_j^2}{M_j^2 - (q - p_1 - p)^2}. \end{aligned} \tag{41}$$

The other two are cuts through the Majorana and lepton and the Majorana and Higgs lines, respectively [15]. For the second cut we obtain

$$\begin{aligned} pL_V^{N_j\ell}(q,p) = 16\pi \int d\Pi^{N_j\ell}_{q_1 p_1} (2\pi)^4 \delta(q - p + p_1 - q_1) \\ \times (p p_1)[f_{N_j}^{q_1} - f_\ell^{p_1}] \frac{M_j^2}{m_\phi^2 - (q + p_1)^2} \\ + 16\pi \int d\Pi^{N_j}_{q_1} d\Pi^{\ell}_{p_1} (2\pi)^4 \delta(q - p - p_1 + q_1) \\ \times (p p_1)[f_{N_j}^{q_1} - f_\ell^{p_1}] \frac{M_j^2}{m_\phi^2 - (q - p_1)^2}, \end{aligned} \tag{42}$$

whereas contribution of the third cut is given by

$$\begin{aligned} pL_V^{N_j\phi}(q,p) = 16\pi \int d\Pi^{N_j\phi}_{q_1 k_1} (2\pi)^4 \delta(q_1 - p - k_1) \\ \times (pq + pk_1)[f_\phi^{k_1} + f_{N_j}^{q_1}] \frac{M_j^2}{m_\ell^2 - (q + k_1)^2}, \end{aligned} \tag{43}$$

where we have assumed $M_i < M_j$ so that the (inverse) decay $N_i \leftrightarrow N_j \ell\ell$ is kinematically forbidden. In (42) the second term vanishes for the decay process $N_i \leftrightarrow \ell\phi$ but gives a nonzero contribution for the scattering processes, see Sec. IV. If the intermediate Majorana neutrino is much heavier than the decaying one, the last two cuts are strongly Boltzmann suppressed. Furthermore, comparing (38) and (41) we observe that in this case $pL_V \approx pL_S$. In the same approximation we can also neglect the "regulator" term in the denominator of (37). The two contributions to the CP-violating parameter then have the same structure and their sum can be written in the form

$$\epsilon_i = \epsilon_i^{\mathrm{vac}} \frac{pL_S}{qp}. \tag{44}$$

In the vacuum limit $L_S^\mu = q^\mu$ and we recover (7). At finite temperatures the CP-violating parameter is moderately enhanced by the medium effects [22].

IV. HIGGS MEDIATED SCATTERING

In the previous section we have approximated the full resummed Higgs propagator by leading-order expressions (25) and (26). In this section we will use a more accurate eQP approximation. As we will see, it allows one to describe Higgs-mediated $\Delta L = 1$ two-body scattering and three-body decay processes.

Similarly to (18), the extended quasiparticle approximation for the Higgs propagator reads

$$\Delta_{\gtrless} = \tilde{\Delta}_{\gtrless} - \frac{1}{2}(\Delta_R^2 + \Delta_A^2)\Omega_{\gtrless}. \tag{45}$$

Its graphic interpretation is presented in Fig. 7. For the first term on the right-hand side of (45) we can again use approximations (25) and (26). To analyze the second term we have to specify the Higgs self-energy. At the one-loop level it reads

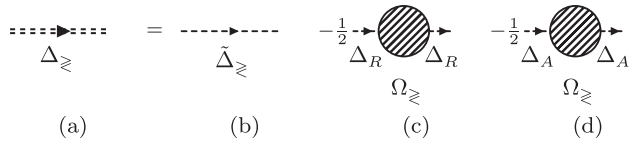

FIG. 7. Schematic representation of the eQP approximation for the Higgs field. (a) Corresponds to the full propagator, (b) to the free propagator, and (c),(d) to the one-loop corrections.

$$\Omega_{\gtrless}(t,k) = g_s|\lambda|^2 \int d\Pi^4_{p_Q p_t}(2\pi)^4\delta(k - p_t + p_Q) \times \mathrm{tr}[S_{Q\gtrless}(t,p_Q)P_R S_{t\gtrless}(t,p_t)P_L], \quad (46)$$

see Appendix A for more details. As is evident from (46), here we limit our analysis to contributions generated by the quarks of the third generations. Let us note that in the SM the gauge contribution to the Higgs self-energy is of the same order of magnitude and should not be dismissed in a fully consistent approximation. Using the KB ansatz for the eQP propagators of the quarks with effective thermal mass,

$$\tilde{S}_{t>} = (1 - f_t)\tilde{S}_{t\rho}, \qquad \tilde{S}_{t<} = -f_t\tilde{S}_{t\rho}, \quad (47a)$$

$$\tilde{S}_{Q>} = (1 - f_Q)\tilde{S}_{Q\rho}, \qquad \tilde{S}_{Q<} = -f_Q\tilde{S}_{Q\rho}, \quad (47b)$$

with

$$\tilde{S}_{Q\rho} = (2\pi)\mathrm{sign}({p_Q}^0)\delta({p_Q}^2 - m_Q^2)P_L\not{p}_Q P_R \equiv \mathbf{S}_{Q\rho}P_L\not{p}_Q P_R, \quad (48a)$$

$$\tilde{S}_{t\rho} = (2\pi)\mathrm{sign}({p_t}^0)\delta({p_t}^2 - m_t^2)P_R\not{p}_t P_L \equiv \mathbf{S}_{t\rho}P_R\not{p}_t P_L, \quad (48b)$$

and neglecting their off-shell parts, which are lepton number conserving, we can write the Higgs self-energy in the form

$$\Omega_>(t,k) = -2g_s|\lambda|^2 \int d\Pi^4_{p_Q p_t}(2\pi)^4\delta(k + p_Q - p_t)f_Q(1 - f_t)(p_Q p_t)\mathbf{S}_{Q\rho}(p_Q)\mathbf{S}_{t\rho}(p_t), \quad (49a)$$

$$\Omega_<(t,k) = -2g_s|\lambda|^2 \int d\Pi^4_{p_Q p_t}(2\pi)^4\delta(k + p_Q - p_t)(1 - f_Q)f_t(p_Q p_t)\mathbf{S}_{Q\rho}(p_Q)\mathbf{S}_{t\rho}(p_t). \quad (49b)$$

Substituting the one-loop lepton self-energy (22) with the Higgs propagator given by (45) into the divergence of the lepton current (17), we obtain

$$\frac{s\mathcal{H}}{z}\frac{dY_L}{dz} = \sum \int d\Pi^4_{q p_Q p p_t}(2\pi)^4\delta(q + p_Q - p - p_t)\tilde{\mathcal{S}}^{ii}_\rho(q)\mathbf{S}_\rho(p)\mathbf{S}_{Q\rho}(p_Q)\mathbf{S}_{t\rho}(p_t)\Xi_{N_i\to\ell\phi}(q,p)\Delta^2_{R+A}(p_t - p_Q) \times \Xi_{\phi Q\to t}(p_t,p_Q)[f^q_{N_i}f^{p_Q}_Q(1 - f^p_\ell)(1 - f^{p_t}_t) - f^p_\ell f^{p_t}_t(1 - f^q_{N_i})(1 - f^{p_Q}_Q)], \quad (50)$$

where we have introduced a combination of the retarded and advanced propagators,

$$\Delta^2_{R+A}(k) \equiv \frac{1}{2}[\Delta^2_R(k) + \Delta^2_A(k)], \quad (51)$$

which describes the intermediate Higgs line in Figs. 3 and 4. Note that in (50) the momenta are not restricted to the mass shell. In particular, the zeroth components of the momenta can have either sign. Because of the Dirac deltas in the spectral functions the frequency integration is trivial. Each of the spectral functions can be decomposed into a sum of two delta functions, one with positive and one with negative frequency, leading to 2^4 terms. These different terms correspond to $1 \leftrightarrow 3$ (inverse) decay, $2 \leftrightarrow 2$ scattering, and to the (unphysical) $0 \leftrightarrow 4$ process. An additional constraint comes from the delta function ensuring energy conservation. In the regime $M_i > m_\ell + m_Q + m_t$ only eight terms satisfy the energy conservation. Using the relation

$$1 \pm f_a(t,-p) = \mp f_{\bar{a}}(t,p), \quad (52)$$

where $f_{\bar{a}}$ denotes the distribution function of the antiparticles, we can then recast (50) in the form

$$\frac{s\mathcal{H}}{z}\frac{dY_L}{dz} = \sum_i \int d\Pi^{q p p_Q p_t}_{N_i\ell Q_t}([\mathcal{F}^{q p_Q; p p_t}_{N_i Q\leftrightarrow\ell t}\Xi_{N_i Q\leftrightarrow\ell t} - \mathcal{F}^{q p_Q; p p_t}_{N_i\bar{Q}\leftrightarrow\bar{\ell}\bar{t}}\Xi_{N_i\bar{Q}\leftrightarrow\bar{\ell}\bar{t}}] + [\mathcal{F}^{q p_t; p p_Q}_{N_i\bar{t}\leftrightarrow\ell\bar{Q}}\Xi_{N_i\bar{t}\leftrightarrow\ell\bar{Q}} - \mathcal{F}^{q p_t; p p_Q}_{N_i t\leftrightarrow\bar{\ell}Q}\Xi_{N_i t\leftrightarrow\bar{\ell}Q}] + [\mathcal{F}^{q p; p_Q p_t}_{N_i\bar{\ell}\leftrightarrow\bar{Q}t}\Xi_{N_i\bar{\ell}\leftrightarrow\bar{Q}t} - \mathcal{F}^{q p; p_Q p_t}_{N_i\ell\leftrightarrow Q\bar{t}}\Xi_{N_i\ell\leftrightarrow Q\bar{t}}] + [\mathcal{F}^{q; p p_Q p_t}_{N_i\leftrightarrow\ell\bar{Q}t}\Xi_{N_i\leftrightarrow\ell\bar{Q}t} - \mathcal{F}^{q; p p_Q p_t}_{N_i\leftrightarrow\bar{\ell}Q\bar{t}}\Xi_{N_i\leftrightarrow\bar{\ell}Q\bar{t}}]). \quad (53)$$

The effective scattering amplitudes in (53) correspond to different assignments for the sign of the four-momenta in (50), reflecting the usual crossing symmetry. For the tree-level and self-energy contributions to the effective scattering and decay amplitudes we obtain

$$\Xi^{T+S}_{N_i Q\leftrightarrow \ell t} = \Xi^{T+S}_{N_i \bar{t}\leftrightarrow \ell \bar{Q}} = \Xi^{T+S}_{N_i\leftrightarrow \ell\phi}\Delta^2_{R+A}(p_t - p_Q)\Xi_{\phi Q\leftrightarrow t}, \tag{54a}$$

$$\Xi^{T+S}_{N_i \bar{\ell}\leftrightarrow \bar{Q}t} = \Xi^{T+S}_{N_i \bar{\ell}\leftrightarrow \phi}\Delta^2_{R+A}(p_t + p_Q)\Xi_{\phi\leftrightarrow \bar{Q}t}, \tag{54b}$$

$$\Xi^{T+S}_{N_i\leftrightarrow \ell\bar{Q}t} = \Xi^{T+S}_{N_i\leftrightarrow \ell\phi}\Delta^2_{R+A}(p_t + p_Q)\Xi_{\phi\leftrightarrow \bar{Q}t}, \tag{54c}$$

and similar expressions for the CP-conjugate ones. Note that $\Xi^{T+S}_{N_i\leftrightarrow\ell\phi}$ and $\Xi^{T+S}_{N_i\bar{\ell}\leftrightarrow\phi}$ are given by the same expression since the CP-violating loop term in (34) depends only on the momentum of the Majorana neutrino. In vacuum these scattering amplitudes reduce to (9) and (12) respectively but with the Feynman propagator Δ^2_T replaced by Δ^2_{R+A}. In the latter the contribution of the real intermediate state is subtracted by construction [22]. However, in the regime $m_\phi < m_Q + m_t$ the intermediate Higgs cannot be on shell such that the vacuum and in-medium amplitudes become numerically equal. Since the amplitudes $\Xi_{\phi\leftrightarrow\bar{Q}t}$ and $\Xi_{\phi Q\leftrightarrow t}$ factorize in (54) and are CP conserving, the self-energy CP-violating parameter in these processes is the same as in the Majorana decay, see (36). However, since the decay and scattering processes have different kinematics the averaged decay and scattering CP-violating parameters are not identical.

Next we consider the two-loop lepton self-energy (23). Proceeding as above we find for the divergence of the lepton current an expression of the form (53) with the amplitudes given by

$$\Xi^{V}_{N_i Q\leftrightarrow \ell t} = \Xi^{V}_{N_i \bar{t}\leftrightarrow \ell \bar{Q}} = \Xi^{V}_{N_i\leftrightarrow \ell\phi}\Delta^2_{R+A}(p_t - p_Q)\Xi_{\phi Q\leftrightarrow t}, \tag{55a}$$

$$\Xi^{V}_{N_i \bar{\ell}\leftrightarrow \bar{Q}t} = \Xi^{V}_{N_i \bar{\ell}\leftrightarrow \phi}\Delta^2_{R+A}(p_t + p_Q)\Xi_{\phi\leftrightarrow \bar{Q}t}, \tag{55b}$$

$$\Xi^{V}_{N_i\leftrightarrow \ell\bar{Q}t} = \Xi^{V}_{N_i\leftrightarrow \ell\phi}\Delta^2_{R+A}(p_t + p_Q)\Xi_{\phi\leftrightarrow \bar{Q}t}. \tag{55c}$$

Since the vertex contribution to the Majorana decay amplitude depends on the momentum of the Higgs, the amplitude $\Xi^V_{N_i\leftrightarrow\ell\phi}$ does not coincide with $\Xi^V_{N_i\bar{\ell}\leftrightarrow\phi}$ and we can define two inequivalent vertex CP-violating parameters [34]. For the scattering processes $N_i Q \leftrightarrow \ell t$ and $N_i \bar{t} \leftrightarrow \ell \bar{Q}$ as well as for the three-body decay $N_i \leftrightarrow \ell \bar{Q} t$, the CP-violating parameter coincides with (39) with the contributions of the three possible cuts given by (41)–(43), respectively. For the $N_i\bar{\ell} \leftrightarrow \bar{Q}t$ process, the CP-violating parameter still has the form (39), but since the lepton is in the initial state the loop integral must be evaluated at $(q, -p)$ instead of (q, p). For the first cut we obtain

$$pL^{\ell\phi}_V(q,p) = 16\pi\int d\Pi^{\phi\ell}_{k_1 p_1}(2\pi)^4\delta(q - p_1 - k_1)(pp_1) \times [1 + f^{k_1}_\phi - f^{p_1}_\ell]\frac{M_j^2}{M_j^2 - (q - p_1 + p)^2}. \tag{56}$$

Contributions of the second and third cuts are given by

$$pL^{N_j\ell}_V(q,p) = 16\pi\int d\Pi^{N_j\ell}_{q_1 p_1}(2\pi)^4\delta(q + p - p_1 - q_1) \times (pp_1)[1 - f^{q_1}_{N_j} - f^{p_1}_\ell]\frac{M_j^2}{(q - p_1)^2 - m_\phi^2} - 16\pi\int d\Pi^{N_j\ell}_{q_1 p_1}(2\pi)^4\delta(q + p + p_1 - q_1) \times (pp_1)[f^{q_1}_{N_j} - f^{p_1}_\ell]\frac{M_j^2}{(q + p_1)^2 - m_\phi^2}, \tag{57}$$

and by

$$pL^{N_j\phi}_V(q,p) = 16\pi\int d\Pi^{N_j\phi}_{q_1 k_1}(2\pi)^4\delta(q_1 - p - k_1) \times (pq - pk_1)[f^{k_1}_\phi + f^{q_1}_{N_j}]\frac{M_j^2}{(q - k_1)^2 - m_\ell^2}, \tag{58}$$

respectively. As follows from (56) and (57), the CP-violating parameter in the $N_i\bar{\ell} \leftrightarrow \bar{Q}t$ scattering receives two vacuum contributions [34]. One is the usual cut through $\ell\phi$, and the second one is given by the first term in the cut through $N_j\ell$. The kinematics of the second contribution corresponds to $N_i\ell \leftrightarrow N_j\ell$ t-channel scattering and therefore requires the initial center-of-mass energy $s = q + p$ to be greater than the final masses $M_j + m_\ell$, meaning that contribution of this term to the reaction density is suppressed for a hierarchical mass spectrum.

V. RATE EQUATIONS

Solving a system of Boltzmann equations in general requires the use of numerical codes capable of treating large systems of stiff differential equations for the different momentum modes. This is a difficult task if one wants to study a wide range of model parameters. A commonly employed simplification is to approximate the Boltzmann equations by the corresponding system of "rate equations" for the abundances Y_a. In [35] it was shown that the two approaches, Boltzmann or the rate equations, give approximately equal results for the final asymmetry, up to $\sim$10% correction.

Starting from a quantum Boltzmann equation of the type (53) we derive here the rate equation for the lepton asymmetry which includes the (usually neglected) quantum statistical factors. In our analysis we are closely following [22]. The contribution of various processes to the generation of the lepton asymmetry can be represented in the form

$$\mathcal{D}_\mu j^\mu = \sum_{i,\{a\},\{j\}} \int d\Pi^{q p_a p_b \cdots ; p_j p_k \cdots}_{N_i a b \ldots j k} [\mathcal{F}^{q p_a p_b \cdots ; p_j p_k \cdots}_{N_i a b \ldots \leftrightarrow j k \ldots} \Xi_{N_i a b \ldots \leftrightarrow i j \ldots} - \mathcal{F}^{q p_a p_b \cdots ; p_j p_k \cdots}_{N_i \bar{a} \bar{b} \ldots \leftrightarrow \bar{j} \bar{k} \ldots} \Xi_{N_i \bar{a} \bar{b} \ldots \leftrightarrow \bar{i} \bar{j} \ldots}], \quad (59)$$

compare with (53), where the sum runs over each possible particle state with $\ell \in \{j\}$ or $\bar{\ell} \in \{a\}$. We assume that the SM particles are maintained in kinetic equilibrium by the fast gauge interactions. This means that their distribution function takes the form

$$f_a(t, E_a) = (e^{\frac{E_a - \mu_a}{T}} \mp 1)^{-1}, \quad (60)$$

with a time-(or temperature-)dependent chemical potential $\mu_a = \mu_a(t)$. Here the upper (lower) sign corresponds to bosons (fermions). It is also useful to define the equilibrium distribution function,

$$f_a^{\mathrm{eq}} = (e^{E_a/T} \mp 1)^{-1}. \quad (61)$$

The fast SM interactions relate chemical potentials of the leptons, quarks, and the Higgs, such that only one of them is independent. We can therefore express the chemical potential of the quarks as a function of the lepton chemical potential [36–38],

$$\mu_t = \frac{5}{21}\mu_\ell \equiv c_{t\ell}\mu_\ell, \qquad \mu_Q = -\frac{1}{3}\mu_\ell \equiv c_{Q\ell}\mu_\ell. \quad (62)$$

Chemical potentials of the antiparticles: $\mu_{\bar{a}} = -\mu_a$. The lepton chemical potential is related to the abundance by

$$\frac{\mu_\ell}{T} \approx c_\ell \frac{Y_L}{2Y_\ell^{\mathrm{eq}}}, \quad (63)$$

where c_ℓ depends on the thermal mass of the lepton. For $m_\ell/T \approx 0.2$ it can be very well approximated by the zero mass limit, $c_\ell \approx 9\xi(3)/\pi^2 \approx 1.1$.

Using the identity $1 \pm f_a = e^{(E_a - \mu_a)/T} f_a$ and energy conservation we can rewrite the combinations of distribution functions appearing in (59) as

$$\mathcal{F}^{q p_a p_b \cdots ; p_j p_k \cdots}_{N_i a b \ldots \leftrightarrow j k \ldots} = (2\pi)^4 \delta\Big(q + \sum_a p_a - \sum_j p_j\Big) \times \frac{\prod_a f_a \prod_j (1 \pm f_j)}{1 - f_{N_i}^{\mathrm{eq}}} \Big[f_{N_i} - f_{N_i}^{\mathrm{eq}} - f_{N_i}^{\mathrm{eq}}(1 - f_{N_i}) \times \Big\{e^{\sum_j \mu_j/T - \sum_a \mu_a/T} - 1\Big\}\Big], \quad (64)$$

where we have suppressed the momentum argument in the distribution functions for notational convenience. We can then expand (64) in the small chemical potential μ_a. Assuming the Majorana neutrino to be close to equilibrium, $f_{N_i} - f_{N_i}^{\mathrm{eq}} \sim \mathcal{O}(\mu_a)$, we see that the term in the square bracket in (64) is already of the first order in the chemical potential. We can therefore replace the distribution functions in the second line of (64) by the equilibrium ones,

$$\frac{\prod_a f_a^{\mathrm{eq}} \prod_j (1 \pm f_j^{\mathrm{eq}})}{1 - f_{N_i}^{\mathrm{eq}}} = \prod_a f_a^{\mathrm{eq}} \prod_j (1 \pm f_j^{\mathrm{eq}}) + \prod_j f_j^{\mathrm{eq}} \prod_a (1 \pm f_a^{\mathrm{eq}}), \quad (65)$$

and expand the exponential to first order in the chemical potential. Since we assume small departure from equilibrium the Majorana distribution function that multiplies the chemical potential should also be replaced by the equilibrium one. The corresponding equation for the antiparticles is obtained from the above equation by replacing $\mu_a \to -\mu_a$. The last step is to assume that the Majorana distribution function is proportional to its equilibrium value,

$$f_{N_i} \approx \frac{Y_{N_i}(t)}{Y_{N_i}^{\mathrm{eq}}(t)} f_{N_i}^{\mathrm{eq}}. \quad (66)$$

Putting everything together we get the conventional form of the rate equation,

$$\frac{s\mathcal{H}}{z}\frac{dY_L}{dz} = \sum_{i,\{a\},\{j\}} \Big[\langle \epsilon_{jk\ldots}^{N_i a b \ldots} \gamma_{jk\ldots}^{N_i a b \ldots} \rangle \Big(\frac{Y_{N_i}}{Y_{N_i}^{\mathrm{eq}}} - 1\Big) - \langle \gamma_{jk\ldots}^{N_i a b \ldots} \rangle c_\ell c_{a b \ldots \leftrightarrow j k \ldots} \frac{Y_L}{2Y_L^{\mathrm{eq}}}\Big], \quad (67)$$

where we have defined the production and washout reaction densities:

$$\Big\langle \epsilon_{jk\ldots}^{N_i a b \ldots} \gamma_{jk\ldots}^{N_i a b \ldots} \Big\rangle \equiv \int d\Pi^{q p_a p_b \cdots ; p_j p_k \cdots}_{N_i a b \ldots j k \ldots} (2\pi)^4 \delta\Big(q + \sum_a p_a - \sum_j p_j\Big) \epsilon_{N_i a b \ldots \to j k \ldots} (\Xi_{N_i a b \ldots \leftrightarrow j k \ldots} + \Xi_{N_i \bar{a} \bar{b} \ldots \leftrightarrow \bar{j} \bar{k} \ldots}) f_{N_i}^{\mathrm{eq}} \times \Big(\prod_a f_a^{\mathrm{eq}} \prod_j (1 \pm f_j^{\mathrm{eq}}) + \prod_j f_j^{\mathrm{eq}} \prod_a (1 \pm f_a^{\mathrm{eq}})\Big), \quad (68a)$$

$$\Big\langle \gamma_{jk\ldots}^{N_i a b \ldots} \Big\rangle \Big|_W \equiv \int d\Pi^{q p_a p_b \cdots ; p_j p_k \cdots}_{N_i a b \ldots j k \ldots} (2\pi)^4 \delta\Big(q + \sum_a p_a - \sum_j p_j\Big) (\Xi_{N_i a b \ldots \leftrightarrow j k \ldots} + \Xi_{N_i \bar{a} \bar{b} \ldots \leftrightarrow \bar{j} \bar{k} \ldots}) f_{N_i}^{\mathrm{eq}} (1 - f_{N_i}^{\mathrm{eq}}) \times \Big(\prod_a f_a^{\mathrm{eq}} \prod_j (1 \pm f_j^{\mathrm{eq}}) + \prod_j f_j^{\mathrm{eq}} \prod_a (1 \pm f_a^{\mathrm{eq}})\Big), \quad (68b)$$

and the numerical factor,

$$c_{ab\ldots\leftrightarrow jk\ldots} \equiv \frac{\sum_j \mu_j - \sum_a \mu_a}{\mu_\ell}. \tag{69}$$

Equation (67) must be supplemented by an equation for the heavy neutrino abundance,

$$\frac{s\mathcal{H}}{z}\frac{dY_{N_i}}{dz} = -\sum_{\{a\},\{j\}} \langle \gamma^{N_i ab\ldots}_{jk\ldots}\rangle|_P \left(\frac{Y_{N_i}}{Y^{\rm eq}_{N_i}} - 1\right), \tag{70}$$

with the reaction density given by an expression similar to (68b):

$$\langle \gamma^{N_i ab\ldots}_{jk\ldots}\rangle|_P \equiv \int d\Pi^{q p_a p_b \cdots p_j p_k \cdots}_{N_i ab\ldots jk\ldots} (2\pi)^4 \delta\left(q + \sum_a p_a - \sum_j p_j\right)(\Xi_{N_i ab\ldots\leftrightarrow jk\ldots} + \Xi_{N_i \bar{a}\, \bar{b}\ldots\leftrightarrow \bar{j}\,\bar{k}\ldots}) f^{\rm eq}_{N_i}$$
$$\times \left(\prod_a f^{\rm eq}_a \prod_j (1 \pm f^{\rm eq}_j) + \prod_j f^{\rm eq}_j \prod_a (1 \pm f^{\rm eq}_a)\right). \tag{71}$$

Note that these expressions are valid for two-body scattering processes with Majorana neutrino in the initial or final state as well as for Majorana (inverse) decay processes. The definitions (68) and (71) are somewhat different from the ones obtained in the framework of thermal field theory; see [39] for the definitions and explicit expressions for the averaged decay and scattering reaction densities.

If the quantum-statistical corrections are neglected, i.e., if the $1 \pm f$ terms are replaced by unity and the equilibrium fermionic and bosonic distributions are approximated by the Maxwell-Boltzmann one, then (68b) and (71) are equal. For the case of a $2 \leftrightarrow 2$ scattering process they read

$$\langle \gamma^{N_i a}_{jk}\rangle \equiv \int d\Pi^{q p_a p_j p_k}_{N_i a jk} (2\pi)^4 \delta(q + p_a - p_j - p_k)$$
$$\times (\Xi_{N_i a\leftrightarrow jk} + \Xi_{N_i \bar{a}\leftrightarrow \bar{j}\,\bar{k}}) f^{\rm eq}_{N_i} f^{\rm eq}_a. \tag{72}$$

Part of the integrations in (72) can be performed analytically and we obtain

$$\langle \gamma^{N_i a}_{jk}\rangle \approx \frac{T}{64\pi^4}\int_{s_{\rm min}}^{\infty} ds \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right)\hat{\sigma}^{N_i a}_{jk}(s). \tag{73}$$

Here $s_{\rm min} = (M_i + m_a)^2$ (assuming $M_i + m_a > m_j + m_k$) and $\hat{\sigma}(s)$ is the so-called reduced cross section:

$$\hat{\sigma}^{N_i a}_{jk} \equiv \frac{1}{8\pi}\int_0^{2\pi}\frac{d\varphi_{ai}}{2\pi}\int_{t^-}^{t^+}\frac{dt}{s}(\Xi_{N_i a\leftrightarrow jk} + \Xi_{N_i \bar{a}\leftrightarrow \bar{j}\,\bar{k}}), \tag{74}$$

where s and t are the usual Mandelstam variables. The integration limits are given by [22]

$$t^{\pm} = M_i^2 + m_j^2 - \frac{s}{2}[(1 + M_i^2/s - m_a^2/s)(1 + m_j^2/s - m_k^2/s)$$
$$\mp \lambda^{\frac{1}{2}}(1, M_i^2/s, m_a^2/s)\lambda^{\frac{1}{2}}(1, m_j^2/s, m_k^2/s)], \tag{75}$$

where $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the usual kinematical function. If effective thermal masses of the SM particles are neglected then the integration limits simplify to $t^+ = 0$ and $t^- = -(s - M_i^2)$. Integrating (9) and (12) over t and neglecting the effective masses of the initial and final lepton and quarks, we obtain the standard expressions (see, e.g., [40,41]) for the reduced cross sections of the Higgs-mediated scattering processes:

$$\hat{\sigma}^{N_i Q}_{\ell t} = \sigma^{N_i \bar{t}}_{\ell \bar{Q}}$$
$$= \frac{g_w g_s}{4\pi}(h^\dagger h)_{ii}|\lambda|^2 \frac{x - a_i}{x}\left[\frac{x - 2a_i + 2a_\phi}{x - a_i + a_\phi} + \frac{a_i - 2a_\phi}{x - a_i}\ln\left(\frac{x - a_i + a_\phi}{a_\phi}\right)\right], \tag{76a}$$

$$\hat{\sigma}^{N_i \bar{\ell}}_{\bar{Q} t} = \frac{g_w g_s}{4\pi}(h^\dagger h)_{ii}|\lambda|^2 \frac{(x - a_i)^2}{(x - a_\phi)^2}, \tag{76b}$$

where we have replaced s by $x \equiv s/M_1^2$ and introduced dimensionless quantities $a_i \equiv M_i^2/M_1^2$ and $a_\phi \equiv m_\phi^2/M_1^2$. Combined with (73), expressions (76) give the conventional reaction densities of the Higgs-mediated scattering processes.

Since in the conventional approach the CP-violating parameter is calculated in vacuum it is momentum independent and therefore can be taken out of the integral. The CP-violating reaction densities are thus proportional to the washout ones:

$$\langle \epsilon^{N_i a}_{jk}\gamma^{N_i a}_{jk}\rangle = \epsilon^{\rm vac}_i \langle \gamma^{N_i a}_{jk}\rangle, \tag{77}$$

where we have again assumed a strongly hierarchical mass spectrum of the heavy neutrinos. When the medium corrections are taken into account, the CP-violating parameter depends on the momenta of the initial and final states and this simple relation is violated.

VI. NUMERICAL RESULTS

To illustrate the effect of the quantum-statistical corrections and effective thermal masses, we present in this section ratios of the reaction densities to the conventional ones assuming a strongly hierarchical mass spectrum of the Majorana neutrinos.

Let us first consider the scattering processes. Ratios of the improved reaction densities to the conventional ones are presented in Fig. 8. Note that the Majorana (as well as

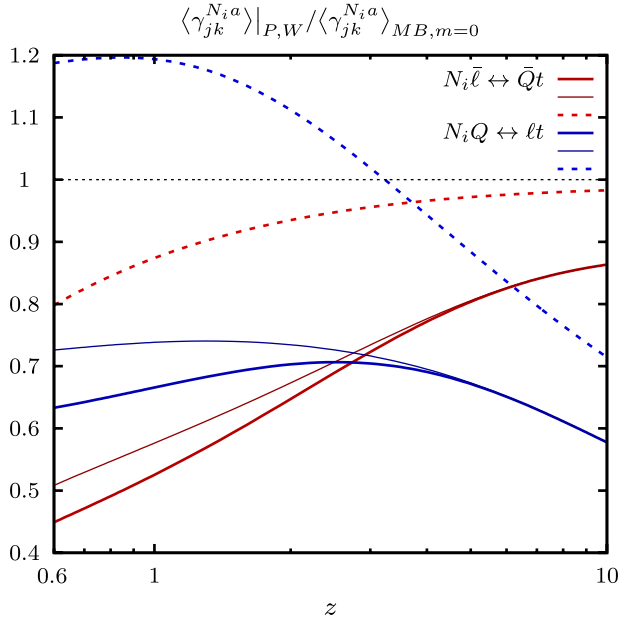

FIG. 8 (color online). Ratios of the scattering reaction densities obtained taking into account the thermal masses (dashed lines) and the thermal masses plus quantum-statistical effects (solid lines) to the conventional ones. The thick solid lines correspond to (68b) whereas the thin ones to (71).

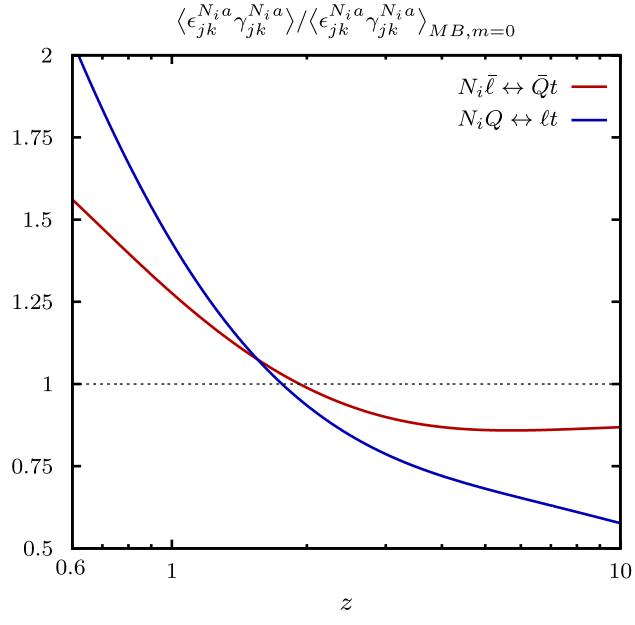

FIG. 9 (color online). Ratio of the CP-violating reaction densities to the ones computed using Boltzmann statistics and neglecting the thermal masses of the initial and final states.

the quark) Yukawa couplings cancel out in these ratios and for this reason we do not specify them here. The dashed lines show the ratio of the reaction density computed using (73)–(75), i.e., taking into account the effective thermal masses but neglecting the quantum-statistical corrections, to the conventional ones. For the $N_i\bar{\ell}\leftrightarrow\bar{Q}t$ process (dashed red line), the effective masses decrease the available phase space and lead to a suppression of the reaction density in the whole range of temperatures. Note that the ratio does not approach unity at low temperatures. Qualitatively this behavior can be understood from (72). Let us assume for a moment that the scattering amplitude is momentum independent. The reaction density at low temperatures can then be estimated by evaluating the distribution functions at the average momenta $\langle p_i\rangle$ and $\langle p_a\rangle\sim 3T$. In the ratio of the reaction densities the Majorana distribution function cancels out and

$$\frac{\langle\gamma_Y^X\rangle_{MB,m\neq 0}}{\langle\gamma_Y^X\rangle_{MB,m=0}}\approx\frac{\exp\left(-E_a/T\right)}{\exp\left(-\langle p_a\rangle/T\right)}\approx\exp\left(-m_a^2/2\langle p_a\rangle T\right).$$

A more accurate estimate for the ratio of $\langle\gamma_Y^X\rangle_{MB,m\neq 0}$ and $\langle\gamma_Y^X\rangle_{MB,m=0}$ is $\sim\exp\left(-m_a^2/T^2\right)$. Since to a good approximation $m_a\propto T$ we conclude that this ratio is a constant smaller than unity. In other words, despite the fact that at low temperatures the quark masses become small compared to the Majorana mass this ratio is *not* expected to approach unity as the temperature decreases. Note also that (in a very good agreement with the numerical cross-check) this ratio does not depend on the masses of the final states.

Of course, the momentum dependence of the scattering amplitude somewhat changes the low-temperature behavior of the reaction density. Interestingly enough, for the $N_iQ\leftrightarrow\ell t$ process the inclusion of the thermal masses actually enhances the reaction density at high temperatures (dashed blue line). This occurs because the induced increase of the amplitude turns out to be larger than the phase-space suppression. At low temperatures the effective masses become negligible in the scattering amplitude but still play an important role in the kinematics. As a result, the ratio becomes smaller than unity and continues to decrease as the temperature decreases. Let us note that for a (moderately) strong washout regime most of the asymmetry is typically produced at $z\lesssim 10$ and the low-temperature behavior of the reaction densities does not affect the generation of the asymmetry. Since all particles in the initial and final states are fermions, the quantum-statistical effects further suppress the reaction densities (solid blue and red lines) and render the ratio of the improved and conventional reaction densities smaller than unity for both $N_iQ\leftrightarrow\ell t$ and $N_i\bar{\ell}\leftrightarrow\bar{Q}t$ in the whole range of temperatures. Ratios of the improved CP-violating scattering reaction densities to the conventional ones are presented in Fig. 9. For both scattering processes the improved CP-violating reaction densities are enhanced at high temperatures. This is explained by the enhancement of the CP violation in the Majorana decay observed in [16,22]. At the intermediate temperatures the relative enhancement of the CP-violating parameters gets smaller and is overcompensated by the effective mass and Fermi-statistics induced suppression of the washout reaction densities that we have observed in

Fig. 8. The low-temperature behavior is somewhat different for the two scattering processes. For the $N_i\bar{\ell} \leftrightarrow \bar{Q}t$ process the effective mass and quantum-statistical effects get smaller in both the (unintegrated) CP-violating parameter and the washout reaction density, such that the ratio of the CP-violating reaction density to the conventional one slowly approaches a constant value. On the other hand, for the $N_iQ \leftrightarrow \ell t$ process the suppression of the washout reaction density induced by the effective masses of the initial and final states that we observed in Fig. 8 also leads to a suppression of the CP-violating reaction density that gets stronger at low temperatures.

Next we consider the three-body decay. Neglecting the quantum-statistical effects and using vacuum approximation for the $N_i \leftrightarrow \ell\bar{Q}t$ decay amplitude in (68) and (71), we recover the conventional expression for the decay reaction density:

$$\langle\gamma^{N_i}_{\ell\bar{Q}t}\rangle \approx \frac{g_N}{2\pi^2} T M_i^2 \Gamma_{N_i\to\ell\bar{Q}t} K_1(M_i/T). \tag{78}$$

Note that it is important to retain the effective thermal masses of the quarks in the calculation of $\Gamma_{N_i\to\ell\bar{Q}t}$. The four-momentum of the intermediate Higgs, see Fig. 5, varies in the range $(m_Q + m_t)^2 \le k^2 \le (M_i - m_\ell)^2$. The relation $m_\phi < m_Q + m_t$, which is fulfilled in the SM, ensures that the intermediate Higgs remains off shell and prevents a singularity in the Higgs propagator. The ratio of the reaction density computed taking into account the quantum-statistical effects and effective masses to the one computed taking into account only the effective masses is presented in Fig. 10. Note that since $m_Q \approx m_t \approx 0.4T$ and $m_\ell \approx 0.2T$ this three-body decay is kinematically allowed only at $T \lesssim M_i$. As one would expect, at high temperatures the fermionic nature of the initial and final states leads to a suppression as compared to the Boltzmann approximation. At low temperatures the quantum-statistical effects play no role and the ratio slowly approaches unity. The ratio of the CP-violating reaction density for the $N_i \leftrightarrow \ell\bar{Q}t$ process is presented in Fig. 11. At high and intermediate temperatures the medium enhancement of the (unintegrated) CP-violating parameter is overcompensated by the suppression of the washout decay reaction density that we have observed in Fig. 10. At low temperatures the effective mass and quantum-statistical effects get smaller in both the (unintegrated) CP-violating parameter and the washout reaction density, such that the CP-violating reaction density slowly approaches the conventional one.

To conclude this section we present the ratio of the three-body decay and $2 \leftrightarrow 2$ scattering processes to the reaction density of $N_i \leftrightarrow \ell\phi$ process, see Fig. 12. As can be inferred from this plot, the three-body decay is subdominant in the whole range of temperatures and can be safely neglected. The inclusion of the effective masses has a very similar effect on the two-body decay and scattering

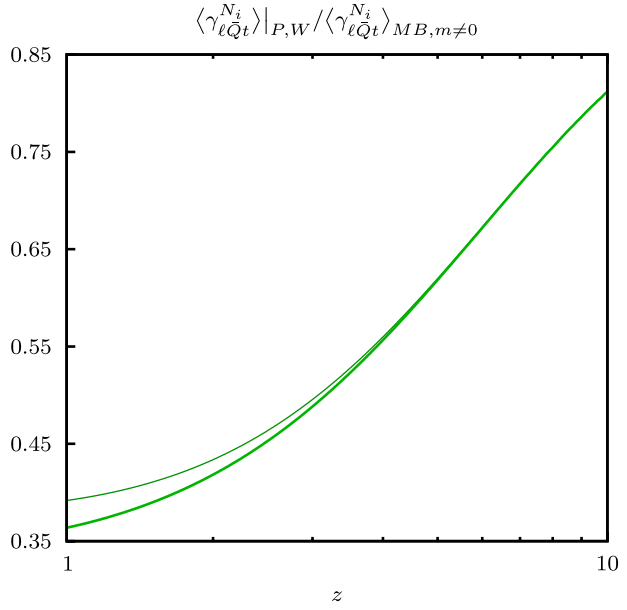

FIG. 10 (color online). Ratio of the $N_i \leftrightarrow \ell\bar{Q}t$ decay reaction density obtained taking into account effective thermal masses and quantum-statistical effects to the conventional one computed taking into account only the effective thermal masses of the final and intermediate states. The thick solid line corresponds to (68) whereas the thin one to (71).

reaction densities such that the ratios of the two almost do not change as compared to the one computed in the massless approximation. The inclusion of the quantum-statistical corrections has a stronger effect on the scattering

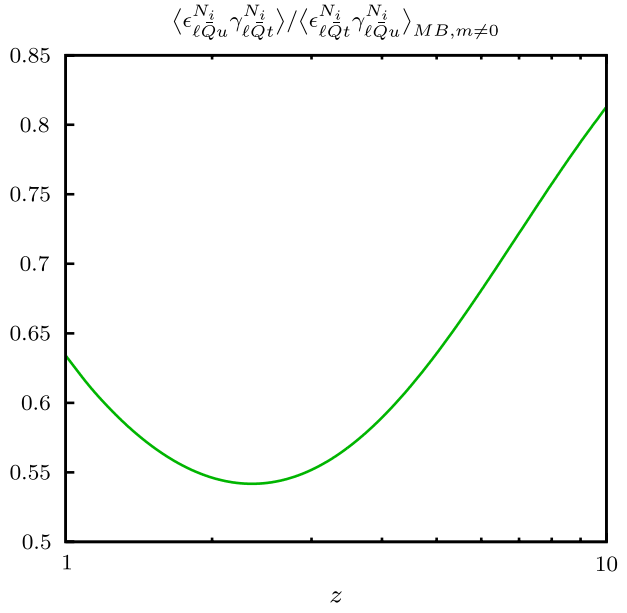

FIG. 11 (color online). Ratio of the CP-violating reaction density of the $N_i \leftrightarrow \ell\bar{Q}t$ process obtained taking into account effective thermal masses and quantum-statistical effects to the ones computed taking into account only the effective thermal masses.

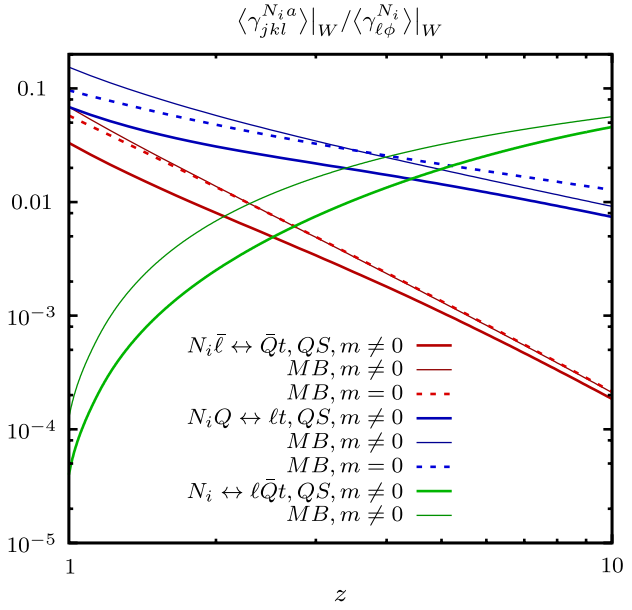

FIG. 12 (color online). Ratio of the washout scattering and three-body decay reaction densities to the reaction density of the $N_i \leftrightarrow \ell\phi$ process. The dashed lines denote the ratios of the conventional reaction densities, the thin solid lines the ratios computed taking into account only the effective masses in all the reaction densities, and finally the thick solid lines the ratios computed taking into account the effective masses and quantum-statistical corrections in all the reaction densities. The reaction density $\langle\gamma^{N_i}_{\ell\phi}\rangle_W$ is computed using (5) and the definition (68b), see also [22].

processes such that the ratio of the reaction densities is smaller than the ratio of the conventional ones. Let us also note that the scattering processes are very important at high temperatures but become subdominant at low temperatures.

VII. SUMMARY

In this work we have studied $\Delta L = 1$ decay and scattering processes mediated by the Higgs with quarks in the initial and final states using the formalism of nonequilibrium quantum field theory.

Starting from the Kadanoff-Baym equations for the lepton propagator we have derived the corresponding quantum-corrected Boltzmann and rate equations for the total lepton asymmetry. As compared to the canonical ones, the latter are free of the notorious double-counting problem and ensure that the asymmetry automatically vanishes in thermal equilibrium. To compute the collision term we have taken into account one- and two-loop contributions to the lepton self-energy and used the extended quasiparticle approximation for the Higgs propagator. The impact of the SM gauge interactions on the collision term has been approximately taken into account in the form of effective thermal masses of the Higgs, leptons and quarks.

We find that the inclusion of the effective masses and quantum-statistical terms suppresses the washout reaction densities of the decay and scattering processes with respect to the conventional ones, where these effects are neglected, in the whole relevant range of temperatures. For the $N_i\bar{\ell} \leftrightarrow \bar{Q}t$ process the ratio of the improved and conventional washout reaction densities slowly approaches a constant value close to unity at low temperatures. Interestingly enough, for the $N_iQ \leftrightarrow \ell t$ processes this ratio decreases even at low temperatures. Finally, for the $N_i \leftrightarrow \ell\bar{Q}t$ process the ratio slowly approaches unity at low temperatures. As far as the CP-violating reaction densities are concerned, we find that for the scattering processes the ratio of the improved and the conventional ones is greater than unity at high temperatures but is smaller than unity at intermediate and low temperatures because of the thermal masses and quantum-statistical effects. For the three-body decay this ratio is smaller than unity in the whole relevant range of temperatures.

We expect that the effects studied here can induce a $\mathcal{O}(10\%)$ correction to the total generated asymmetry. For a detailed phenomenological analysis it is necessary to include further phenomena such as flavor effects and process with gauge bosons in the initial and final states.

ACKNOWLEDGMENTS

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APPENDIX A: HIGGS SELF-ENERGY

The top quark contribution to the Higgs self-energy is derived from the 2PI effective action. At one-loop the contribution of the top quark is given by

$$i\Gamma_2 = g_s|\lambda|^2 \int_{\mathcal{C}} d^4u d^4v \,\mathrm{tr}[S_{Qba}(v,u)P_R S_t(u,v)] \times \epsilon^*_{bc}\Delta_{cd}(v,u)\epsilon^T_{da}, \tag{A1}$$

where the factor $g_s = 3$ comes from the summation over color indices and $\epsilon = i\sigma_2$. In a $SU(2)_L$ symmetric state, the Higgs and lepton propagators are proportional to the identity in the $SU(2)_L$ space, and so is the Higgs self-energy,

$$\Omega_{ab}(x,y) \equiv \Omega(x,y)\delta_{ab} = \frac{\delta i\Gamma_2}{\Delta_{ba}(y,x)} = g_s|\lambda|^2 \mathrm{tr}[S_Q(y,x)P_R S_t(x,y)P_L]\delta_{ab}. \tag{A2}$$

Its Wightman components are given by

$$\Omega_{\gtrless}(x,y) = g_s|\lambda|^2 \mathrm{tr}[S_{Q\gtrless}(y,x)P_R S_{t\gtrless}(x,y)P_L]. \tag{A3}$$

Finally, performing a Wigner transform of the above equation, we find

$$\Omega_{\gtrless}(t,k) = g_s|\lambda|^2 \int d\Pi^4_{p_Q p_t}(2\pi)^4\delta(k - p_t + p_Q) \times \mathrm{tr}[S_{Q\gtrless}(t,p_Q)P_R S_{t\gtrless}(t,p_t)P_L]. \quad \text{(A4)}$$

APPENDIX B: REACTION DENSITY OF $1 \to 3$ DECAY

For $N_i \to \ell\bar{Q}t$ decay the general expression (71) takes the form

$$\langle\gamma^{N_i}_{\ell\bar{Q}t}\rangle = \int d\Pi^{q p p_Q p_t}_{N_i \ell \bar{Q} t}(2\pi)^4\delta(q - p - p_Q - p_t) \times \Xi_{N_i\to\ell\phi} \times \Delta^2_{R+A}(p_Q + p_t)\Xi_{\phi\to\bar{Q}t} f^{\mathrm{eq}}_{N_i}[(1 - f^{\mathrm{eq}}_{\ell}) \times (1 - f^{\mathrm{eq}}_{\bar{Q}})(1 - f^{\mathrm{eq}}_{t}) + f^{\mathrm{eq}}_{\ell} f^{\mathrm{eq}}_{\bar{Q}} f^{\mathrm{eq}}_{t}], \quad \text{(B1)}$$

where we have used the explicit form of the decay amplitude (13). To reduce it to a form suitable for the numerical analysis we insert an identity:

$$1 = \int ds \int d^4k\,\delta(p_Q + p_t - k)\delta_+(k^2 - s), \quad \text{(B2)}$$

where k is the four-momentum of the intermediate Higgs. Approximating furthermore Δ^2_{R+A} by Δ^2_T we can rewrite the reaction density in the form

$$\langle\gamma^{N_i}_{\ell\bar{Q}t}\rangle = \int d\Pi^q_{N_i} f^{\mathrm{eq}}_{N_i} \int \frac{ds}{2\pi}\Delta^2_T(s) \int d\Pi^{pk}_{\ell\phi}(2\pi)^4 \times \delta(q - p - k)\Xi_{N_i\to\ell\phi} \int d\Pi^{p_Q p_t}_{\bar{Q}t}(2\pi)^4 \times \delta(k - p_Q - p_t)\Xi_{\phi\to\bar{Q}t}[(1 - f^{\mathrm{eq}}_{\ell})(1 - f^{\mathrm{eq}}_{\bar{Q}}) \times (1 - f^{\mathrm{eq}}_{t}) + f^{\mathrm{eq}}_{\ell} f^{\mathrm{eq}}_{\bar{Q}} f^{\mathrm{eq}}_{t}]. \quad \text{(B3)}$$

Note that in the regime $m_\phi < m_Q + m_t$ realized in the considered case the Higgs is always off shell and its width can be neglected in Δ_T. For the second line in (B3) we can use [22]

$$\int d\Pi^{pk}_{\ell\phi}(2\pi)^4\delta(q - k - p) \to \frac{1}{8\pi|\mathbf{q}|}\int_{E_p^-}^{E_p^+} dE_p \int_0^{2\pi}\frac{d\varphi}{2\pi}. \quad \text{(B4)}$$

The integration limits are given by

$$E_p^{\pm} = \frac{1}{2}[E_q(1 + x_\ell - x_\phi) \pm |\mathbf{q}|\lambda^{\frac{1}{2}}(1, x_\ell, x_\phi)], \quad \text{(B5)}$$

where $x_\ell \equiv m_\ell^2/M_i^2$, $x_\phi \equiv s/M_i$ and $\lambda(x,y,z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the usual kinematical function. For the third line we can use a similar expression with $x_Q = m_Q^2/s$ and $x_t = m_t^2/s$.

Expressed in terms of the integration variables the amplitudes take the form

$$\Xi_{N_i\to\ell\phi} = g_w(h^\dagger h)_{ii}(M_i^2 + m_\ell^2 - s), \quad \text{(B6a)}$$

$$\Xi_{\phi\to\bar{Q}t} = g_s|\lambda|^2(s - m_Q^2 - m_t^2). \quad \text{(B6b)}$$

Since they do not depend on the angles between the quarks and leptons, the integration over φ is trivial and the reaction density takes the form

$$\langle\gamma^{N_i}_{\ell\bar{Q}t}\rangle = \int d\Pi^q_{N_i} f^{\mathrm{eq}}_{N_i} \int \frac{ds}{2\pi}\Delta^2_T(s) \int_{E_p^-}^{E_p^+}\frac{dE_p}{8\pi|\mathbf{q}|} \times \Xi_{N_i\to\ell\phi}\int_{E_{pQ}^-}^{E_{pQ}^+}\frac{dE_{p_Q}}{8\pi|\mathbf{k}|}\Xi_{\phi\to\bar{Q}t} \times [(1 - f^{\mathrm{eq}}_{\ell})(1 - f^{\mathrm{eq}}_{\bar{Q}})(1 - f^{\mathrm{eq}}_{t}) + f^{\mathrm{eq}}_{\ell} f^{\mathrm{eq}}_{\bar{Q}} f^{\mathrm{eq}}_{t}]. \quad \text{(B7)}$$

The three-momentum of the intermediate Higgs is given by $|\mathbf{k}| = (E_k^2 - s)^{\frac{1}{2}}$ and $E_k = E_q - E_p$. Note that if we neglect the quantum-statistical factors in (B7) the reaction density takes the standard form.

APPENDIX C: MAJORANA SPECTRAL SELF-ENERGY

We compute here the Majorana spectral self-energy. In a CP-symmetric medium it reads [22]

$$\Pi^{ij}_\rho = -\frac{g_w}{16\pi}[(h^\dagger h)_{ij}P_L + (h^\dagger h)^*_{ij}P_R]L_S, \quad \text{(C1)}$$

where we have defined the loop function $L_S(q)$,

$$L_S(q) = 16\pi\int d\Pi^4_{pk}(2\pi)^4\delta(q - p - k)\not{p} \times [\Delta_F(k)\mathbf{S}_{\rho(h)}(p) + \Delta_{\rho(h)}(k)\mathbf{S}_F(p)]. \quad \text{(C2)}$$

Using the eQP for the Higgs, see (45), one can split the function L_S into a decay part, identical to the one computed in [22],

$$L_S^d(q) = 16\pi\int d\Pi^{\ell\phi}_{pk}\tilde{\mathcal{F}}^{q;pk}_{(N_i)\leftrightarrow\ell\phi}\not{p}, \quad \text{(C3)}$$

where we have assumed $q^0 > 0$, and defined

$$\tilde{\mathcal{F}}^{p_a p_b\cdots;p_i p_j\cdots}_{(a)b\ldots\leftrightarrow ij\ldots} \equiv (2\pi)^4\delta(p_a + p_b + \cdots - p_i - p_j - \cdots) \times [f_b^{p_b}\ldots(1 \pm f_i^{p_i})(1 \pm f_j^{p_j})\ldots + f_i^{p_i} f_j^{p_j}\ldots(1 \pm f_b^{p_b})\ldots], \quad \text{(C4)}$$

and a scattering part,

$$L_S^s(q) = 16\pi\int d\Pi^4_{p p_Q p_t}(2\pi)^4\delta(q + p_Q - p - p_t) \times \mathbf{S}_\rho(p)\mathbf{S}_{Q\rho}(p_Q)S_{t\rho}(p_t)\Delta^2_{R+A}(p_t - p_Q) \times \Xi_{\phi\bar{\imath}\to\bar{Q}}\not{p}[f_Q^{p_Q}(1 - f_\ell^p)(1 - f_t^{p_t}) + f_\ell^p f_t^{p_t}(1 - f_Q^{p_Q})]. \quad \text{(C5)}$$

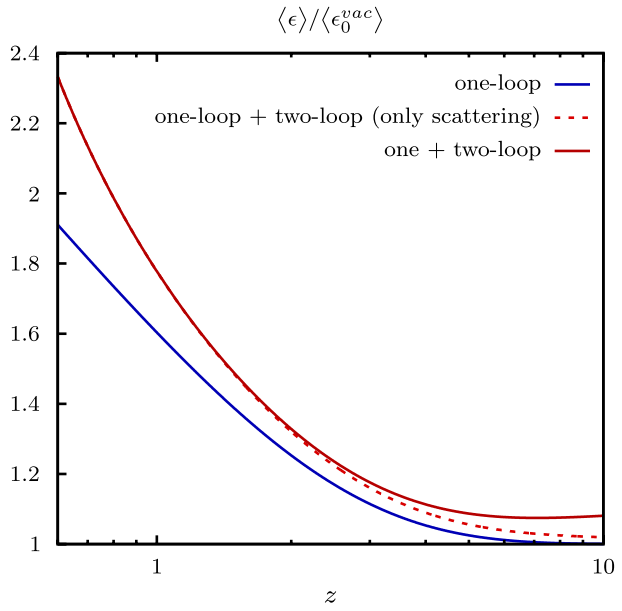

FIG. 13 (color online). Ratio of the thermally averaged CP-violating parameter to the one calculated in vacuum neglecting the contribution of the Higgs-mediated processes. The blue line corresponds to (C3), whereas the red lines to the sum of (C3) and (C6). The dashed red line is obtained by omitting the contribution of the three-body decay in (C6).

Performing the frequency integration as explained above, see (53), we can rewrite (C5) as a sum of four terms, corresponding to the three scattering and one three-body decay process as well as their CP conjugates. Assuming $q^0 > 0$ we obtain

$$\begin{aligned} L_S^s(q) = 16\pi \int d\Pi_{\ell Q t}^{p p_Q p_t} [& \tilde{\mathcal{F}}_{(N_i) Q \leftrightarrow \ell t}^{q p_Q; p p_t} \Delta_{R+A}^2(p_t - p_Q) \\ & \times \Xi_{\phi Q \to t} + \tilde{\mathcal{F}}_{(N_i) \bar{t} \leftrightarrow \ell \bar{Q}}^{q p_t; p p_Q} \Delta_{R+A}^2(p_t - p_Q) \\ & \times \Xi_{\phi \bar{t} \to \bar{Q}} + \tilde{\mathcal{F}}_{(N_i) \bar{\ell} \leftrightarrow \bar{Q} t}^{q p; p_Q p_t} \Delta_{R+A}^2(p_t + p_Q) \\ & \times \Xi_{\phi \to \bar{Q} t} + \tilde{\mathcal{F}}_{(N_i) \leftrightarrow \ell \bar{Q} t}^{q; p p_Q p_t} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi Q \to t}] \not{p}. \end{aligned} \tag{C6}$$

In the regime $m_\phi < m_Q + m_t$ the intermediate Higgs cannot be on shell. Therefore one can neglect the Higgs width in Δ_{R+A}^2 and approximate it by Δ_T^2.

As can be inferred from the definition (C4) for the scattering terms $\tilde{\mathcal{F}}$ vanishes in vacuum, whereas for the decay term it does not. Because of Lorentz covariance in vacuum both L_S^d and L_S^s must be proportional to the four-vector q. Using (B4) and (B5) we find that the coefficient of proportionality is equal to unity for the decay contribution, i.e., $L_S^d = q$, if thermal masses of the Higgs and leptons are neglected. Using (B7) we find that for the scattering contribution the coefficient of proportionality reads

$$\begin{aligned} & \frac{g_s |\lambda|^2}{16\pi^2} \int_{(m_Q + m_t)^2}^{(M_i - m_\ell)^2} \frac{ds}{M_i^2} \lambda^{\frac{1}{2}}(1, x_Q, x_t) \lambda^{\frac{1}{2}}(1, x_\ell, x_\phi) \\ & \quad \times \frac{(s - m_t^2 - m_Q^2)(M_i^2 + m_\ell^2 - s)}{(s - m_\phi^2)^2}. \end{aligned} \tag{C7}$$

Note that since we have omitted the Higgs decay width, this expression is convergent only if $m_\phi < m_Q + m_t$. The vacuum result (C7) provides also a very good approximation for nonzero temperatures provided that $M/T \gg 1$. The thermal masses of the quarks then ensure that the Higgs remains off shell and therefore that (C7) is finite. It is important to note that due to the temperature dependence of the effective masses the coefficient (C7) is temperature dependent as well. A numerical analysis shows that it grows as the temperature decreases.

Using L_S we can calculate the in-medium CP-violating parameter in $N_i \leftrightarrow \ell \phi$ process. For a hierarchical mass spectrum [22],

$$\epsilon = \epsilon_0^{\mathrm{vac}} \frac{p L_S}{q p}, \tag{C8}$$

where $\epsilon_0^{\mathrm{vac}}$ denotes the vacuum CP-violating parameter calculated neglecting contributions of the Higgs-mediated processes, i.e., using only L_S^d. As has been mentioned above, if thermal masses of the Higgs and leptons are neglected then $L_S^d = q$ in vacuum and we recover $\epsilon = \epsilon_0^{\mathrm{vac}}$. Once the contribution of the Higgs-mediated processes is taken into account $\epsilon^{\mathrm{vac}} \neq \epsilon_0^{\mathrm{vac}}$. To estimate the size of the corrections induced by (C6) we plot the ratio of thermally averaged CP-violating parameter, $\langle \epsilon \rangle \equiv \langle \epsilon \gamma_N^D \rangle / \langle \gamma_N^D \rangle$, to $\epsilon_0^{\mathrm{vac}}$, see Fig. 13. Note that we have neglected thermal masses of the final-state Higgs and lepton in the numerics. The blue line corresponds to the CP-violating parameter computed using (C3). In agreement with the above discussion the ratio reaches unity at low temperatures. The dashed red line corresponds to the CP-violating parameter computed using the sum of (C3) and the *scattering* (lines two to four) contributions to (C6). As expected, at high temperatures we observe an enhancement of the ratio, whereas at low temperatures it reaches unity. The solid red line is obtained by considering the sum of (C3) and all of the terms in (C6). Since the three-body process is kinematically suppressed at high temperatures, the dashed and solid lines overlap for $z \lesssim 1$. At lower temperatures the quantum-statistical effects are small. However, in agreement with the discussion below (C7), the effective thermal masses of the Higgs and quarks lead to a slow rise of the ratio at low temperatures.

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