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Cosmological parameter estimation with free-form primordial power spectrum

Dhiraj Kumar Hazra*

Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 790-784, Korea

Arman Shafieloo†

Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 790-784, Korea
and Department of Physics, POSTECH, Pohang, Gyeongbuk 790-784, Korea

Tarun Souradeep‡

Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411 007, India

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Constraints on the main cosmological parameters using cosmic microwave background (CMB) or large scale structure data are usually based on the power-law assumption of the primordial power spectrum (PPS). However, in the absence of a preferred model for the early Universe, this raises a concern that current cosmological parameter estimates are strongly prejudiced by the assumed power-law form of PPS. In this paper, for the first time, we perform cosmological parameter estimation allowing the free form of the primordial spectrum. This is in fact the most general approach to estimate cosmological parameters without assuming any particular form for the primordial spectrum. We use a direct reconstruction of the PPS for any point in the cosmological parameter space using the recently modified Richardson-Lucy algorithm; however, other alternative reconstruction methods could be used for this purpose as well. We use WMAP 9 year data in our analysis considering the CMB lensing effect, and we report, for the first time, that the flat spatial universe with no cosmological constant is ruled out by more than a 4σ confidence limit without assuming any particular form of the primordial spectrum. This would be probably the most robust indication for dark energy using CMB data alone. Our results on the estimated cosmological parameters show that higher values of the baryonic and matter density and a lower value of the Hubble parameter (in comparison to the estimated values by assuming power-law PPS) is preferred by the data. However, the estimated cosmological parameters by assuming a free form of the PPS have an overlap at 1σ confidence level with the estimated values assuming the power-law form of PPS.

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I. INTRODUCTION

The observables of the perturbed Universe, such as CMB anisotropy, galaxy surveys, and weak lensing, all depend on a set of cosmological parameters assuming a background model describing the current Universe as well as the parameters characterizing the presumed nature of the initial perturbations. While certain characteristics of the initial perturbations, such as the adiabatic nature and tensor contribution, can and are being tested independently, the shape of the primordial power spectrum (PPS) remains, at best, a well-motivated assumption. It is important to distinguish between the cosmological parameters within a model that describes the present Universe from that characterizing the initial conditions, specifically the PPS, $P(k)$. The standard model of cosmology, which is the most popular and widely used cosmological model, is the spatially flat ΛCDM model, which incorporates a power-law form of the primordial power spectrum. The model is described by six parameters. Four of them describe the background ΛCDM, represented by Ω_b (baryon density), $\Omega_{\rm CDM}$ (cold dark matter density), H_0 (present rate of expansion of the Universe),[1] and the reionization optical depth τ. We should mention that dark energy density Ω_Λ is directly obtained as a remainder of baryon and cold dark matter density from the total density as we have assumed a spatially flat model of the Universe. The other two parameters in the model describe the form of the primordial power spectrum, which is assumed to be the power law defined by $P(k) = A_s[\frac{k}{k_*}]^{ns-1}$, where A_s is the amplitude[2] and the tilt is given by the spectral index $n_{\rm S}$. The imposed form of the primordial spectrum allows us to provide tight constraints on the four background parameters; however, these tight constraints are basically the result of the rigidness of the model and a certain assumption of the primordial spectrum. In other words, choosing different assumptions for the form of the primordial spectrum results in different constraints on the background cosmological parameters [1].

*dhiraj@apctp.org
†arman@apctp.org
‡tarun@iucaa.ernet.in

[1]Sometimes the ratio of the sound horizon to the angular diameter distance at decoupling, θ, is considered to be a parameter instead of H_0

[2]Note that this amplitude is defined at some pivot scale k_*.

In this paper, for the first time we study the complete Markov Chain Monte Carlo parameter estimation assuming a free form of the primordial spectrum through a direct reconstruction of the PPS for each point in the background cosmological parameter space using WMAP 9 year data [2,3]. There have been different interesting attempts to directly reconstruct the form of the primordial spectrum [1,4–25], and in our analysis we use the recently modified Richardson-Lucy algorithm [25]. We show that it is indeed possible to do the cosmological parameter estimation allowing the free form of the primordial spectrum, and, for the first time, we report that a spatially flat universe without a cosmological constant is ruled out by more than a 4σ confidence limit using cosmic microwave background (CMB) data alone. This is without putting any prior constraints on the Hubble parameter or using any other cosmological observation. We show that, assuming a free form primordial spectrum, the confidence limits of the background cosmological parameters are larger, as expected, than those we get by assuming the power-law form of the PPS, and we present that the data prefers larger values of baryonic and matter densities for the free form of the primordial spectrum in comparison with the power-law assumption. In the next section we discuss the methodology of reconstruction and the parameter estimation followed by our demonstration of our results. We close with a brief discussion at the end.

It should be noted that the aim of this paper is not to relax a parameter of an underlying cosmological model and investigate the effects on other cosmological parameters as usually done to study the cosmographical degeneracies. In this paper we perform a cosmological parameter estimation analysis with the free form primordial power spectrum, which directly indicates that we do not consider any assumptions on underlying inflationary models (or any theoretical model of the early Universe). We have been able to compare the free form spectra because our method [25] is able to identify PPS functions with a very large improvement to the WMAP likelihood at any point in cosmological parameter space. This allows the estimation of cosmological parameters optimized over the PPS functional degree of freedom.

II. FORMALISM

In this work, we have used the recently modified version of Richardson-Lucy algorithm [25] (we call it hereafter MRL) to reconstruct the optimal form of the primordial spectrum for each point in the background cosmological parameter space. The Richardson-Lucy algorithm [26–29] has been used previously in this context to reconstruct the primordial spectrum [6,13,14], and in the recently modified version, one can use the combination of unbinned and binned data in the analysis [25]. The modified algorithm can be formulated as

$$P_k^{(i+1)} - P_k^{(i)} = P_k^{(i)} \times \Bigg[\sum_{\ell=2}^{\ell=900} \tilde{G}_{\ell k}^{\text{un-binned}} \Bigg\{ \Bigg(\frac{C_\ell^{\mathrm{D}} - C_\ell^{\mathrm{T}(i)}}{C_\ell^{\mathrm{T}(i)}} \Bigg) \tanh^2[Q_\ell (C_\ell^{\mathrm{D}} - C_\ell^{\mathrm{T}(i)})] \Bigg\}_{\text{un-binned}} + \sum_{\ell_{\text{binned}}>900} \tilde{G}_{\ell k}^{\text{binned}} \Bigg\{ \Bigg(\frac{C_\ell^{\mathrm{D}} - C_\ell^{\mathrm{T}(i)}}{C_\ell^{\mathrm{T}(i)}} \Bigg) \tanh^2 \Bigg[\frac{C_\ell^{\mathrm{D}} - C_\ell^{\mathrm{T}(i)}}{\sigma_\ell^{\mathrm{D}}} \Bigg]^2 \Bigg\}_{\text{binned}} \Bigg], \tag{1}$$

where $P_k^{(i+1)}$ and $P_k^{(i)}$ are the power spectrum evaluated in iterations $i+1$ and i, respectively, and the quantity $\tilde{G}_{\ell k}$ is the normalized radiative transport kernel $G_{\ell k}$. C_ℓ^{D} and σ_ℓ^{D} are the observed C_ℓ^{TT} data points and the corresponding diagonal terms of the inverse covariance matrix. $C_\ell^{\mathrm{T}(i)}$ is a theoretically calculated angular power spectrum in the ith iteration. Q_ℓ is given by the following expression:

$$Q_\ell = \sum_{\ell'} (C_{\ell'}^{\mathrm{D}} - C_{\ell'}^{\mathrm{T}(i)}) \mathrm{COV}^{-1}(\ell, \ell'), \tag{2}$$

where $\mathrm{COV}^{-1}(\ell, \ell')$ is the inverse of the error covariance matrix.

The radiative transport kernel, $G_{\ell k}$, which depends on the background cosmological parameters, satisfies the following equation:

$$C_\ell = \sum_i G_{\ell k_i} P_{k_i}. \tag{3}$$

We should mention that, as has been indicated in Eq. (1), we have used the unbinned data until $\ell = 900$ and used the binned data thereafter because of the increasing noise in higher multipoles. For more discussion on this procedure see, Ref. [25].

We use the publicly available CAMB [30,31] to calculate the kernel and the C_ℓ's and CosmoMC [32,33] to perform the Markov Chain Monte Carlo (MCMC) analysis on the cosmological parameters. The most recent WMAP 9 year observational data [3] has been used in this analysis. We have taken into account the effect of gravitational lensing through approximating the lensing effect on different background models. Basically, we assume that for each point in the background cosmological parameter space, the lensing effect on the observed angular power spectrum would be the same if we assume the power-law PPS or we use the reconstructed PPS. To do so, for each point in the background parameter space, one can identify the contribution of gravitational lensing to C_ℓ^{TT}'s for the best-fit power-law primordial spectrum from the computed lensing potential power spectrum and using the curved-sky correlation function method. We subtract the contribution from the WMAP-unbinned temperature anisotropy data and follow the reconstruction from the modified data. Finally, from the reconstructed primordial spectrum, we calculate the CMB temperature and polarization C_ℓ's, and we lens all the C_ℓ's again using the curved-sky correlation through CAMB [30,31] and compare them with WMAP data. We should mention that for the multipole range covered by WMAP, the effect of lensing on the temperature and polarization spectrum is not substantial, and we find that without lensing, we also recover our conclusions of this paper. In order to have a complete analysis we have included the effect of lensing.

We have performed the MRL up to 40 iterations. However, we have checked the consistency of our results allowing the MRL to work up to different iteration numbers.

III. RESULTS

Following the methodology described in the previous section, here we shall discuss the result of our MCMC analysis with the 9 year data from WMAP. We find the best-fit model provides a χ^2 $(-2\ln\mathcal{L})$ of 7441.4, which is about 115 better than what we get from the best-fit power-law spectrum. We compare the bounds on the background cosmological parameters with the power-law results in Fig. 1.

As we are allowing a free form primordial power spectrum, hence having larger effective degrees of freedom, it is expected to have bigger confidence contours compared to the confidence limits derived by assuming a power-law

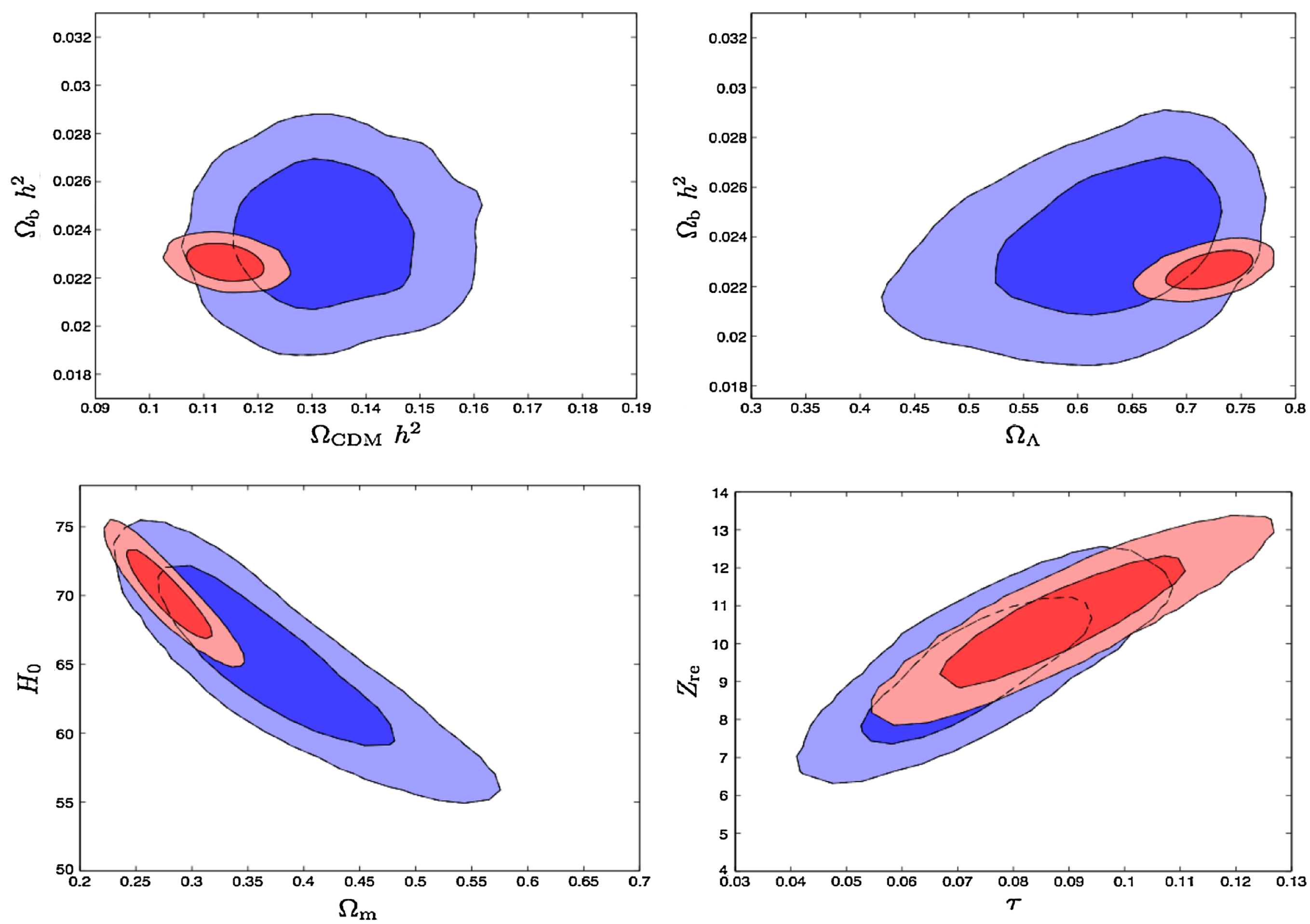

FIG. 1 (color online). 2-dimensional confidence contours of the background cosmological parameters. The blue contours (the contours lying behind) represent the results of the analysis allowing the free form of the primordial spectrum while the red contours (the contours in the front) are derived by assuming the power-law form of the primordial spectrum. As expected the free form of the primordial spectrum relaxes the bounds on the parameters. Throughout the analysis we have assumed a spatially flat universe, and one can see that a universe with zero density for the cosmological constant is ruled out with high confidence even with no assumption for the primordial spectrum. In comparison with the results from power-law assumption of the primordial spectrum, the data prefer higher values of baryonic and matter densities when we allow the free form of the primordial spectrum.

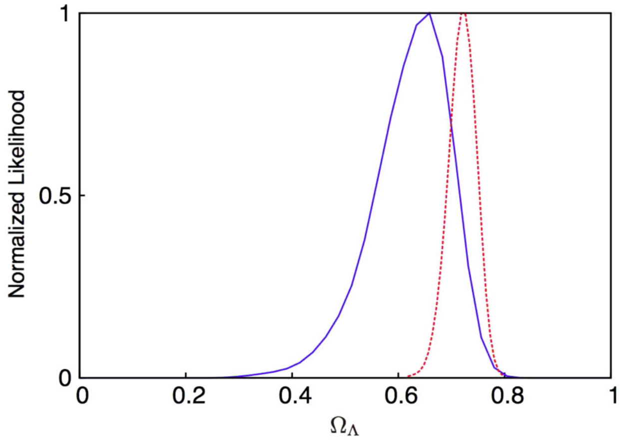

FIG. 2 (color online). The 1-dimensional marginalized likelihood of the dark energy density Ω_Λ obtained using the free form of the primordial spectrum (in the solid blue line) and using the power law (in the dashed red line). $\Omega_\Lambda = 0$ is clearly not favored by the data even if we allow a power spectra free of forms. Quantitatively, in 4σ the data rule out $\Omega_\Lambda < 0.25$. This is probably the first indication toward the presence of dark energy with a very high confidence using CMB data alone.

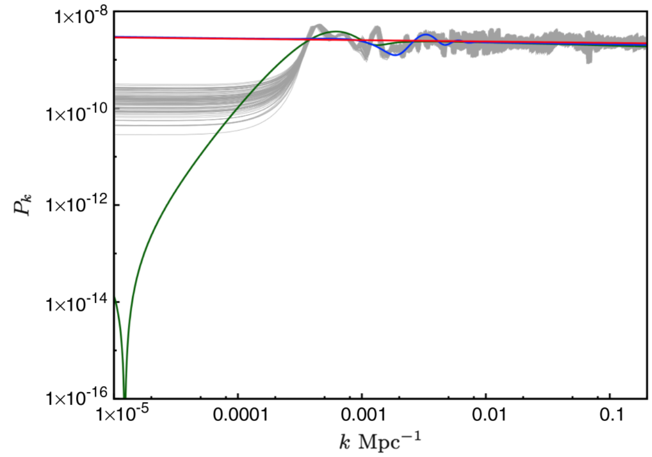

FIG. 3 (color online). Reconstructed power spectrum (in grey) obtained from parameters lying within the 2σ range of the best-fit likelihood. Over the sample of the reconstructed spectra, we have plotted the best-fit spectra from the step model [36] (in blue) and the punctuated model [35] (in green) of inflation. The best-fit power-law power spectrum (in red) is plotted as well for comparison. Note that, barring the low-k region (in which data has low sensitivity), the sample of reconstructed spectra incorporates nearly all the models within its 2σ variation.

PPS, and Fig. 1 clearly illustrates this fact. The results from Fig. 1 suggest that allowing the free form PPS, data prefer higher baryon and dark matter densities compared to the canonical results. Best-fit values of $\Omega_b h^2$, $\Omega_{CDM} h^2$, H_0, and τ are 0.0232, 0.132, 64, and 0.077, respectively. It should be pointed out that our analysis allows a lower value of optical depth (τ), which, in turn, allows the low redshift of reionization ($Z_{re} \sim 7$) compared to the power-law results (see the last plot of Fig. 1). Our results also indicate that, independent of the form of the PPS, a flat model of the universe with no cosmological constant is ruled out with a very high confidence. To our knowledge this is the first direct indication toward dark energy with high certainty from CMB temperature and polarization data analysis alone assuming spatial flatness. In Fig. 2 we plot the 1-dimensional marginalized likelihoods of the parameter Ω_Λ obtained using the power law (in the dashed red line) and allowing the free form of the primordial spectrum. The plot clearly demonstrates that a low value of dark energy density is ruled out. The obtained result suggests that values of $\Omega_\Lambda < 0.25$ are ruled out at 4σ, which implies a strong exclusion of $\Omega_\Lambda = 0$ with a very high confidence. In this context it should be noted that the no-dark-energy model including curvature was previously ruled out by 3.2σ using the Atacama Cosmology Telescope lensing measurements [34] but within the assumption of a power-law form of the PPS.

In Fig. 3 we plot a few (nearly 100) power spectra (in grey) reconstructed from the WMAP 9 year data with the kernels corresponding to the cosmological parameters within 95% (2σ) limits of the best fit. We also show the best-fit power spectrum from the punctuated inflation model [35] in green and the step models of inflation in blue [36] for comparison. The best-fit power-law power spectrum is also plotted (in red). We should mention that, while in the reconstruction process we have used only temperature data, in the likelihood analysis, the polarization data is included. As discussed in Ref. [25], considering the WMAP polarization data does not significantly improve the reconstruction procedure due to the low quality of the polarization data.

We should mention that to check the validity of our approximation regarding the lensing contribution, we have repeated our analysis without subtracting the power-law lensing effects from the data, and we find that the latter comparison provides a χ^2 worse by 6 than the actual analysis, which in turn indicates that our approximation on the lensing contribution works well.

To check the robustness of our analysis and the validity of the obtained results, we performed some tests with simulated data. We have synthesized a number of C_ℓ^{TT} data from the angular power spectrum obtained using the power-law and ΛCDM model with some fixed parameters. With the reconstructed free form spectrum, we perform a MCMC on the synthesized data sets and calculated the likelihood assuming χ^2 distribution.[3] We find that in most of the cases (more than 90%), the obtained confidence contours of the cosmological parameter contain the fiducial parameter values within the 2σ region. This indicates, with the reconstruction, we get our fiducial

[3]We have shown recently [25] that this likelihood estimator is robust and can be used as an approximation to the complete WMAP likelihood.

model back in almost all the cases, which in turn proves the robustness of our analysis.

IV. DISCUSSION

In this paper we have estimated the cosmological parameters assuming the free form of the primordial spectrum. The primordial spectrum for each point in the background cosmological parameter space is obtained using the MRL reconstruction procedure using the WMAP 9 year combined data of the unbinned and binned angular power spectrum. We should mention that for this analysis and instead of MRL, one can use trivially other alternative methods of nonparametric reconstruction of the primordial spectrum. In fact the MRL method serves just as a possible method of reconstruction to get a PPS that improves the fit of a cosmological model at different points in the cosmological parameter space to the CMB data. One is free to choose any other method to do this task. The background model is assumed to be a spatially flat ΛCDM model. Performing the MCMC analysis using CosmoMC, we obtained the bounds on the background parameters, and we find out that the data prefer higher baryonic and matter densities (hence lower Ω_Λ) and a lower Hubble parameter when we assume the free form of the PPS in comparison with the case of the power-law assumption. We should mention here that earlier efforts [24,37,38] have indicated that allowing deviation from simple power-law PPS prefers a lower Hubble constant, and our result, too, agrees with that. However, with the ever-increasing quality of CMB data from WMAP, we find our result does not agree with Ref. [37] anymore, where it has been shown that allowing deviations from the power law PPS, zero dark energy model fits the data as well as the ΛCDM model. Our results indicates that, independent of the form of the primordial spectrum and without any prior on the value of the Hubble parameter, the spatially flat universe with no cosmological constant is ruled out with a very high confidence using the WMAP 9 year data alone. This is the first direct evidence of dark energy with a very high certainty from the CMB data alone and with no prior on the Hubble parameter or assuming the form of the primordial spectrum. We expect to get tighter constraints on the background parameters assuming the free form of the primordial spectrum using upcoming Planck data [39].

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