

Constraints on primordial black holes as dark matter candidates from capture by neutron starsFabio Capela,^{1,*} Maxim Pshirkov,^{2,3,4,†} and Peter Tinyakov^{1,‡}¹*Service de Physique Théorique, Université Libre de Bruxelles (ULB), CP 225 Boulevard du Triomphe, B-1050 Bruxelles, Belgium*²*Sternberg Astronomical Institute, Lomonosov Moscow State University, Universitetsky prospekt 13, 119992 Moscow, Russia*³*Pushchino Radio Astronomy Observatory, Astro Space Center, Lebedev Physical Institute Russian Academy of Sciences, 142290 Pushchino, Russia*⁴*Institute for Nuclear Research of the Russian Academy of Sciences, 117312 Moscow, Russia*

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We investigate constraints on primordial black holes (PBHs) as dark matter candidates that arise from their capture by neutron stars (NSs). If a PBH is captured by a NS, the star is accreted onto the PBH and gets destroyed in a very short time. Thus, mere observations of NSs put limits on the abundance of PBHs. High DM densities and low velocities are required to constrain the fraction of PBHs in DM. Such conditions may be realized in the cores of globular clusters if the latter are of a primordial origin. Assuming that cores of globular clusters possess the DM densities exceeding several hundred GeV/cm³ would imply that PBHs are excluded as comprising all of the dark matter in the mass range $3 \times 10^{18} \leq m_{\text{BH}} \leq 10^{24}$ g. At the DM density of 2×10^3 GeV/cm³ that has been found in simulations in the corresponding models, less than 5% of the DM may consist of PBH for these PBH masses.

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I. INTRODUCTION

The existence of the dark matter (DM) has been established so far only through its gravitational interaction. Consequently, little is known about the DM nature apart from the fact that it is nonbaryonic, nonrelativistic, weakly interacting and constitutes about 26.8% of the total energy budget of the Universe (for a recent review see, e.g., [1–3]).

Various candidates for the DM have been considered in the literature. In the context of particle physics they are associated with new stable particles beyond the Standard Model, a popular example being the so-called weakly interacting massive particles. However, candidates that do not require new stable particles also exist and are still viable. An attractive candidate of this type is primordial black holes (PBHs) [4,5]. This is the possibility we consider in this paper.

In the early Universe, some primordial density fluctuations could have collapsed producing a certain amount of black holes. These PBHs possess properties that make them viable DM candidates: they are nonrelativistic and have a microscopic size of the order $r \sim 10^{-8}$ cm ($m_{\text{BH}}/10^{20}$ g), which makes them effectively collisionless. The initial mass function of PBHs depends on their production mechanism in the early Universe and is, essentially, arbitrary.

There exist a number of observational constraints on the fraction of PBHs in the total amount of DM. First, PBHs with masses $m_{\text{BH}} \leq 5 \times 10^{14}$ g evaporate due to Hawking radiation [6] in a time shorter than the age of the Universe and cannot survive until today. At slightly larger masses,

even though the PBH lifetime is long enough, the Hawking evaporation still poses a problem: the PBHs emit γ rays with energies around 100 MeV [7] in the amount that contradicts the data on the extragalactic gamma-ray background. For instance, the Energetic Gamma Ray Experiment Telescope [8] has put an upper limit on the cosmological density $\Omega_{\text{PBH}} \leq 10^{-9}$ for $m_{\text{BH}} = 10^{15}$ g [9]. From such observations, one can infer that PBHs with masses $m_{\text{BH}} \leq 10^{16}$ g cannot constitute more than 1% of the DM. In the mass range between $\sim 10^{18}$ and $\sim 10^{20}$ g the PBH fraction is constrained to less than 10% by the femto-lensing of the gamma-ray bursts [10]. More massive PBHs were constrained by EROS microlensing survey and the MACHO Collaboration, which set an upper limit of 3% on the fraction of PBHs in the Galactic halo in the mass range $10^{26} < m_{\text{BH}} < 10^{30}$ g [11,12]. These constraints may be improved in the future [13,14]. At even larger masses $10^{33} < m_{\text{BH}} < 10^{40}$ g, the three-year Wilkinson Microwave Anisotropy Probe (WMAP3) data and the COBE Far Infrared Absolute Spectrophotometer (FIRAS) data have been used to put limits on the abundance of PBHs [15]. These constraints are summarized in Fig. 1. They leave open the windows of masses (a few) $\times 10^{16} < m_{\text{BH}} < 10^{18}$ g and $10^{20} < m_{\text{BH}} < 10^{26}$ g.

In order to put constraints on PBHs in the remaining allowed mass range, in Ref. [16] we have considered the capture of PBHs by a star during the star formation process and their further inheritance by the star's compact remnant, the neutron star (NS) or the white dwarf (WD). The presence of even a single PBH of a corresponding mass inside the remnant (NS or WD) leads to a rapid destruction of the latter by the accretion of the star matter onto the PBH [17–21]. Thus, mere observations of NSs and WDs in a DM-rich environment, such as could be present in the

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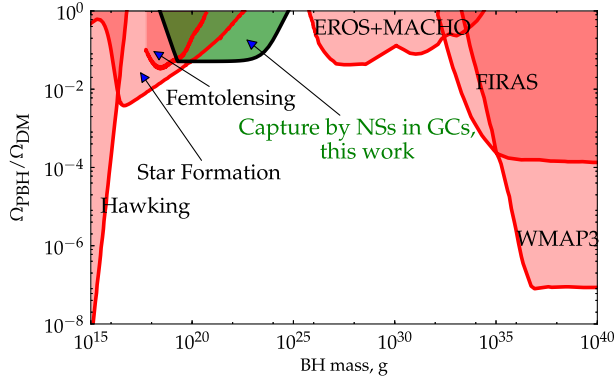


FIG. 1 (color online). Constraints on the fraction of PBHs in the total amount of DM from various observations as explained in Sec. I (red shaded regions). The green shaded region shows constraints derived in this paper, which would follow from observations of NSs in the cores of globular clusters if one assumes the DM density of $2 \times 10^3 \text{ GeV/cm}^3$ as obtained in numerical simulations.

centers of globular clusters, impose constraints on the fraction of PBHs in the DM and exclude PBHs as the only DM candidate in the range of masses $10^{16} < m_{\text{BH}} < 3 \times 10^{22} \text{ g}$. Still, a range of PBHs masses from 3×10^{22} to 10^{26} g remains unconstrained.

In this paper we derive constraints that arise from the direct capture of PBHs by NSs. The origin of the constraints is the same as in Ref. [16]: even a single PBH captured by a compact star rapidly destroys the latter, so the existing observations of the NSs and WDs require that the probability of capture is much less than 1. This implies constraints on the PBH abundance at the location of the compact star and may be translated into constraints on the fraction of PBHs in the total amount of DM.

Similarly to the constraints derived from the PBH capture during star formation in Ref. [16], the constraints that follow from the direct capture require a high DM density and low velocity dispersion, as may be present in the cores of metal-poor globular clusters if the latter are of a primordial origin. Within the same assumptions, the main one being that the cores of the globular clusters contain the DM density exceeding several hundred GeV/cm^3 as is expected from numerical simulations (see Sec. III for a detailed discussion), we find that the arguments based on the capture of PBHs by the NSs allow one to extend the constraints of Ref. [16] to higher PBH masses and exclude PBHs as comprising 100% of the DM up to $m_{\text{BH}} \lesssim (\text{a few}) \times 10^{24} \text{ g}$, leaving open only a small window of less than 2 orders of magnitude. Also, the constraints on the fraction $\Omega_{\text{PBH}}/\Omega_{\text{DM}}$ of PBHs in the total amount of DM at large PBH masses become tighter as compared to Ref. [16]. The final situation is summarized in Fig. 1.

The rest of this paper is organized as follows. In Sec. II we discuss the capture of PBHs by compact stars. In Sec. III we derive the constraints on the fraction of PBHs

in the DM from the capture in NSs. In Sec. IV we summarize the results and present our conclusions. The Appendix contains the calculation of the energy loss by a BH passing through a neutron star. Throughout the paper, we use the units $\hbar = c = 1$.

II. CAPTURE OF BLACK HOLES BY COMPACT STARS

A. Energy loss

A PBH is captured if, during its passage through a star, it loses its initial energy and becomes gravitationally bound. From this moment every subsequent PBH orbit will again pass through the star, so that finally the PBH will lose enough energy and will remain inside the star all the time. Therefore, the criterion of capture of a PBH is $E_{\text{loss}} > m_{\text{BH}} v_0^2/2$ with E_{loss} being the energy loss during the collision and v_0 the PBH asymptotic velocity. Two mechanisms of energy loss are operating during the collision: deceleration of the PBH due to the accretion of the star's material and the so-called dynamical friction [22,23]. In the relevant range of PBH masses the accretion is less efficient compared to the dynamical friction in the case of WDs, while the two mechanisms are competitive in the case of NSs.

As a PBH passes through the star, it transfers momentum and energy to the surrounding matter. The result, called the dynamical friction, is a net force that is opposite to the direction of motion of the PBH. As long as the PBH velocity v during the collision is larger than the velocity of the particles constituting the compact object (which is a good approximation in the case of compact stars), one may take the dynamical friction force to be

$$\mathbf{f}_{\text{dyn}} = -4\pi G^2 m_{\text{BH}}^2 \rho \ln \Lambda \frac{\mathbf{v}}{v^3}, \quad (1)$$

where ρ is the density of the star matter and the factor $\ln(\Lambda)$ is the so-called Coulomb logarithm [22,23] whose value is ~ 30 in the case of ordinary stars. Assuming a uniform flux of incoming PBHs across the star, the average energy loss can be written as follows:

$$E_{\text{loss}} = \frac{4G^2 m_{\text{BH}}^2 M}{R^2} \left\langle \frac{\ln \Lambda}{v^2} \right\rangle, \quad (2)$$

where M and R are the mass and the radius of the star, respectively, and $\langle \dots \rangle$ denotes the density-weighted average over the star volume:

$$\langle f(r) \rangle \equiv \frac{1}{M} \int_0^R 4\pi r^2 dr \rho(r) f(r). \quad (3)$$

When deriving Eq. (3) we have transformed the integral along the PBH trajectory inside the star and the integral over the orthogonal plane which comes from the averaging into a single integral over the star volume. We also accounted for the dependence of the velocity v on the distance r from the star center, and allowed for an analogous

dependence of the Coulomb logarithm $\ln \Lambda$, as will be important in what follows.

Taking into account that the PBHs velocity during the collision is of order $v = v_{\text{esc}} = \sqrt{2GM/R} \gg v_0$, and assuming that $\ln \Lambda$ is r -independent, the energy loss is parametrically given by $E_{\text{loss}} \propto Gm_{\text{BH}}^2/R$. Since E_{loss} is inversely proportional to the radius of the star, NSs induce a much larger energy loss during one collision compared to WDs. Thus, we will only consider the case of NSs from now on.

Several complications arise in the calculation of E_{loss} in the case of NS. First, the accretion of the nuclear matter onto the PBH contributes significantly into slowing it down. As far as the capture criterion is concerned, the effect of the accretion can be incorporated into Eq. (2) by adding an extra contribution to the Coulomb logarithm $\ln \Lambda \rightarrow \ln \Lambda(r) = \ln \Lambda + c(r)v^4$, where $c(r)$ is an r -dependent coefficient whose precise value is given in the Appendix.

Second, the core of a neutron star is comprised of the degenerate neutron gas, so the question arises to which extent Eq. (1) is still applicable. Here we note that by the time the falling PBH reaches the core of NS it picks a relativistic velocity $v \sim 0.6c$. This velocity is by a factor of a few larger than the velocity of sound, so the nucleons can be considered as free particles and the arguments leading to Eq. (1) apply. With this velocity, the PBH can transfer to neutrons the momentum of up to ~ 1.8 GeV, which is by a factor of a few larger than the Fermi momentum of neutrons in the center of the star, and much larger than the Fermi momentum away from the center. However, only neutrons with sufficiently small impact parameters—such that the momentum transfer is larger than their Fermi momentum—contribute to slowing the BH down. Thus, the Coulomb logarithm gets cut at a much smaller distance which, moreover, depends on the local density of neutrons through their Fermi momentum.

Both effects can be incorporated into Eq. (2) through the r -dependence of $\ln \Lambda$ and, finally, expressed in terms of the average value of $\langle \ln \Lambda/v^2 \rangle$. We have calculated this quantity numerically making use of a concrete NS density profile from Ref. [24] (see Appendix for details). We found

$$\left\langle \frac{\ln \Lambda}{v^2} \right\rangle = 14.7. \quad (4)$$

As we argue in the Appendix, this value depends weakly on the NS mass and radius. Making use of Eq. (4) one obtains

$$E_{\text{loss}}/m_{\text{BH}} = 6.3 \times 10^{-12} \left(\frac{m_{\text{BH}}}{10^{22} \text{ g}} \right), \quad (5)$$

where we have substituted $R = 12$ km and $M = 1.4M_{\odot}$ as typical NS parameters. These values for the radius and the mass of the NS are assumed throughout the rest of the paper except where the opposite is stated explicitly.

It remains to be checked that, once the PBH becomes gravitationally bound, multiple collisions bring the PBH inside the NS sufficiently fast. Assuming a radial orbit and denoting the apastron r_{max} , the half-period is

$$\Delta T = \frac{\pi r_{\text{max}}^{3/2}}{\sqrt{GM}}.$$

The energy loss in half a period (that is, during a single collision with NS) as a function of r_{max} is given by Eq. (2). Dividing the energy loss by the time and expressing the energy in terms of r_{max} one obtains the differential equation for the evolution of r_{max} as a function of time,

$$\dot{\xi} = -\frac{1}{\tau} \sqrt{\xi}, \quad (6)$$

where $\xi = r_{\text{max}}/R$ and

$$\tau = \frac{\pi R^{5/2}}{4Gm_{\text{BH}}\sqrt{GM}} \left\langle \frac{\ln \Lambda}{v^2} \right\rangle^{-1} \simeq 8 \times 10^6 \text{ s} \left(\frac{m_{\text{BH}}}{10^{22} \text{ g}} \right)^{-1}.$$

The corresponding energy loss time is

$$t_{\text{loss}} \simeq 2\tau\sqrt{\xi_0},$$

where the initial value ξ_0 can be estimated by requiring that the initial PBH energy is of the order of E_{loss} . Assembling all the factors one has

$$t_{\text{loss}} \simeq 4.1 \times 10^4 \text{ yr} \left(\frac{m_{\text{BH}}}{10^{22} \text{ g}} \right)^{-3/2}. \quad (7)$$

Thus, PBHs heavier than $m_{\text{PBH}} \gtrsim 2.5 \times 10^{18}$ g end up inside the NS in a time shorter than 10^{10} yr.

B. Capture rate

In order to calculate the capture rate, we assume that the PBHs follow a Maxwellian distribution in velocities with the dispersion \bar{v} ,

$$dn = n_{\text{BH}} \left(\frac{3}{2\pi\bar{v}^2} \right)^{3/2} \exp \left\{ \frac{-3v^2}{2\bar{v}^2} \right\} d^3v, \quad (8)$$

where $n_{\text{BH}} = \rho_{\text{BH}}/m_{\text{BH}}$, ρ_{BH} being the density of PBHs at the star location. It can be expressed in terms of the local DM density ρ_{DM} as follows:

$$\rho_{\text{BH}} = \frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}} \rho_{\text{DM}}. \quad (9)$$

Following [25], the capture rate takes the form

$$F = \frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}} F_0, \quad (10)$$

where

$$F_0 = \sqrt{6\pi} \frac{\rho_{\text{DM}}}{m_{\text{BH}}} \frac{R_g R}{\bar{v}(1 - R_g/R)} \left(1 - \exp \left(-\frac{3E_{\text{loss}}}{m_{\text{BH}}\bar{v}^2} \right) \right) \quad (11)$$

is the capture rate assuming PBHs comprise all of the DM, $R_g = 2GM$ is the Schwarzschild radius of the NS and E_{loss} is given by Eq. (5).

Two different regimes are possible depending on the PBH mass. In the case when the energy loss is small, $E_{\text{loss}} \ll m_{\text{BH}}\bar{v}^2/3$, the exponential can be expanded and one gets at the leading order

$$F_0 = 3\sqrt{6\pi} \frac{\rho_{\text{DM}}}{m_{\text{BH}}} \frac{R_g R}{\bar{v}^3(1 - R_g/R)} \frac{E_{\text{loss}}}{m_{\text{BH}}}. \quad (12)$$

In view of Eq. (5) the capture rate is independent of m_{BH} in this regime. In the opposite case $E_{\text{loss}} \gg m_{\text{BH}}\bar{v}^2/3$ the exponential in Eq. (11) can be neglected and

$$F_0 = \sqrt{6\pi} \frac{\rho_{\text{DM}}}{m_{\text{BH}}} \frac{R_g R}{\bar{v}(1 - R_g/R)}, \quad (13)$$

so that the capture rate decreases with increasing m_{BH} . In both cases the capture rate is inversely proportional to some power of velocity and is thus maximum for sites with high dark matter density ρ_{DM} and small velocity dispersion \bar{v} .

III. CONSTRAINTS

As previously mentioned, if a NS captures a PBH, the accretion of the NS material onto the PBH rapidly destroys the star. Therefore, observations of NSs imply constraints on the capture rate of PBHs which has to be such that the probability of the PBH capture is much less than 1. In view of Eq. (10) these constraints translate into constraints on the fraction of PBHs in the dark matter, $\Omega_{\text{PBH}}/\Omega_{\text{DM}}$.

Given a NS of age t_{NS} , the probability of its survival is $\exp(-t_{\text{NS}}F)$ with F given by Eqs. (10) and (11). Requiring that the survival probability is not small leads to the constraint

$$\frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}} \leq \frac{1}{t_{\text{NS}}F_0}. \quad (14)$$

Depending on the environment where the NS is located, F_0 may vary by many orders of magnitude. The most stringent constraints come from sites where F_0 is high. Among such sites, globular clusters (GCs) are the best candidates.

GCs are compact, nearly spherical collections of stars scattered over the Galactic halo. They have ages between 8 to 13.5 Gyr, and as such are the oldest substructures of our Galaxy. GCs are made of population II stars, WDs, NSs and black holes. A typical GC has an average radius of 30 pc, a core radius of 1 pc and a baryonic mass of (a few) $\times 10^5 M_\odot$ [26].

The DM content of GCs is a matter of an ongoing debate. The distribution of metallicity in GCs is bimodal, indicating two subpopulations formed by different mechanisms [27]. The metal-rich GCs are considered to be formed during gas-rich mergers in proto-galaxies [28–31].

These GCs contain very little DM, if any. Instead, as cosmological simulations show, metal-poor GCs could have been formed in low-mass dark matter halos at very high-redshift $z \sim 10\text{--}15$ [32–37]. Observations of GCs show no evidence of DM halos [38]. This is expected as the halos should have been tidally stripped due to interactions with the Galaxy [39]. The DM content would, however, be preserved in the cores of such GCs. In support of this picture, it has been found in Refs. [34,39], using high-resolution N-body simulations, that many properties of simulated GCs with DM halos are similar to those of observed GCs. In what follows we will focus on metal-poor GCs and assume that they have been formed in DM halos and thus possess DM-rich cores.

In Ref. [40] the DM density close to the core of such GCs has been estimated to be of the order $\rho_{\text{DM}} \sim 2 \times 10^3 \text{ GeV cm}^{-3}$. This result was concluded to be rather independent of the original halo mass and is in agreement with N-body simulations [34,39]. Therefore, we adopt this value in our estimates.

The velocity dispersion is another important parameter. Since stars are collisionless and therefore behave similarly to DM particles, this parameter can be extracted from observations. We adopt the value of $\bar{v} = 7 \text{ km s}^{-1}$. The velocity dispersion varies noticeably from cluster to cluster. The list of measured velocities of known GCs can be found in Ref. [41]; the adopted value is a median of this distribution. Finally, we adopt the NS radius $R_{\text{NS}} = 12 \text{ km}$ and mass $M_{\text{NS}} = 1.4M_\odot$ as stated above, and the lifetime $t_{\text{NS}} = 10^{10} \text{ yr}$ [42].

The constraints arising from observations of NSs in the core of a GC under these assumptions, as well as previously existing constraints, are summarized in Fig. 1. As one can see, the new constraints exclude the PBHs as the unique DM component for masses lower than $m_{\text{BH}} \sim (\text{a few}) \times 10^{24} \text{ g}$, thus extending by about 2 orders of magnitude the constraints derived in Ref. [16] to higher PBH masses.

In qualitative terms, the shape of the exclusion region in Fig. 1 is easy to understand from Eqs. (12) and (13). The horizontal part of the curves is due to Eq. (12) where the dependence on the PBH mass cancels out [cf. Eq. (5)]. The inclined part on the right results from Eq. (13). The transition between the two regimes is at the PBH mass such that $E_{\text{loss}} \sim m_{\text{BH}}\bar{v}^2/3$. The sharp cut at small masses occurs when the time needed for multiple collisions to bring the PBH inside the NS exceeds the NS lifetime.

Given the uncertain DM content of the GCs, in Fig. 2 we show the dependence of the constraints on the assumed DM density in the GC core. Apart from the cutoff at small masses, the constraints scale trivially with the DM density. The dependence on the velocity dispersion is similar, but not identical (not shown in Fig. 2): the horizontal part of the constraints scales like $1/\bar{v}^3$, while the inclined part at large masses scales like $1/\bar{v}$.

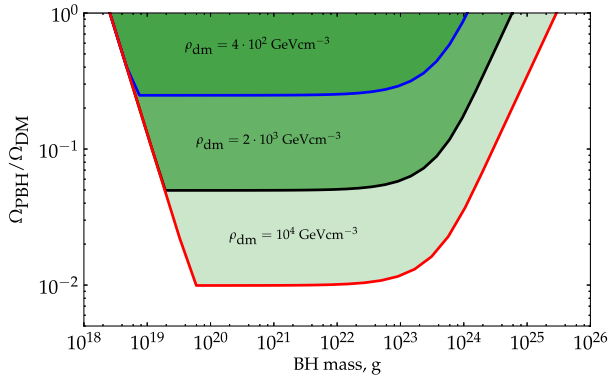


FIG. 2 (color online). The dependence of the constraints on the fraction $\Omega_{\text{PBH}}/\Omega_{\text{DM}}$ of PBHs in the total amount of DM on the assumed DM density in the core of a GC. Three cases are shown: $\rho_{\text{DM}} = 4 \times 10^2 \text{ GeV cm}^{-3}$, $\rho_{\text{DM}} = 2 \times 10^3 \text{ GeV cm}^{-3}$ (the same as in Fig. 1), and $\rho_{\text{DM}} = 10^4 \text{ GeV cm}^{-3}$.

IV. CONCLUSIONS

We have studied the constraints on the fraction of PBHs in the total amount of DM that arises from the requirement that PBHs be captured by NSs with probability much less than 1, since capture of even a single PBH leads to a rapid accretion of the star matter onto the PBH and eventual star destruction. High DM density in excess of several hundred GeV/cm^3 and low velocity dispersion are required to obtain meaningful constraints. Such conditions may be realized in the cores of metal-poor globular clusters if they are formed in low-mass DM halos at very high-redshift $z \sim 10\text{--}15$.

If the metal-poor globular clusters are indeed of a primordial origin, simulations predict that their cores have DM densities as high as $2 \times 10^3 \text{ GeV}/\text{cm}^3$ [40]. At this value, our constraints would exclude PBH as the only DM candidate in the mass range $3 \times 10^{18} \leq m_{\text{BH}} \leq 5 \times 10^{24} \text{ g}$. Together with the previously existing constraints, this would leave open only a small window of masses around 10^{25} g where PBHs can still constitute all of the DM. Note, however, that a viable PBH model would have to explain a very narrow PBH mass distribution of the width of less than 2 orders of magnitude.

As one can see in Fig. 1, the constraints derived here are complementary to those of Ref. [16]. The constrained region has been extended up to masses $\sim 5 \times 10^{24} \text{ g}$. While in Ref. [16] better constraints were achieved for masses $10^{16} \leq m_{\text{BH}} \leq 10^{20} \text{ g}$, here we obtain more competitive constraints for masses $m_{\text{BH}} \geq 10^{20} \text{ g}$. It is also important to note that different assumptions are required in the two cases: while the constraints of Ref. [16] are sensitive to the DM distribution at the epoch of the GC formation, for the constraints derived in this paper the present-epoch DM distribution in GCs is relevant.

We did not present the constraints that come from observations of the Galactic center, which is another relatively close region of high DM density. If the DM density in the Galactic center is comparable to that assumed above for the cores of the GCs, no new constraints arise from that region [43]. The reason is that the capture rate depends strongly on the PBH velocity dispersion, cf. Eq. (12), which is by more than an order of magnitude larger in the Galactic center than in the cores of GCs. It has been suggested, however, that the DM density in the Galactic center may be as high as $\rho_{\text{DM}} = 10^6 \text{ GeV cm}^{-3}$ [44]. If this were confirmed, the constraints from the Galactic center would become competitive to the ones presented here.

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APPENDIX: CALCULATION OF THE FRICTION FORCE

When the BH moves through a neutron star, it experiences a friction force that is the result of scattering and accretion of nucleons. In Sec. II we have written this force in the form (2) analogous to the dynamical friction [22] with all the effects combined in the single factor $\langle \ln \Lambda / v^2 \rangle$. Here we calculate this factor.

To make the calculations manageable, we make a number of simplifying assumptions: (i) We treat the motion of the BH through the NS in the Newtonian approximation (that is, we neglect the general relativity effects), but do not assume the BH to be nonrelativistic. In fact, the BH in the center of the star may attain velocities of up to about $0.6c$. (ii) Since the BH velocity exceeds the sound speed, we treat the nucleons as free particles and account only for their individual interactions with the BH. (iii) To determine which neutrons of the degenerate matter of the NS are excited and absorb momentum we use a simple criterion: we require that the momentum transferred to the neutron in the gravitational collision with the BH exceeds its Fermi momentum k_F .

In the BH reference frame, the scattering of a nucleon off the BH is described by the following expression [45] for the scattering angle $\phi(b)$ as a function of the impact parameter b :

$$\phi(b) = -\pi + 2\tilde{b} \int_0^{x_{\max}} \frac{dx}{\sqrt{\gamma^2 - (1 + \tilde{b}^2 x^2)(1 - x)}}, \quad (\text{A1})$$

where γ is the gamma factor of the nucleon; $\tilde{b} = bv\gamma/R_g$ is the rescaled impact parameter, R_g being the gravitational radius of the BH; and x_{\max} is the smallest zero of the denominator in Eq. (A1). The variable x is the inverse distance between the nucleon and the BH in units of R_g , so that in terms of the distance the integration range in Eq. (A1) is from infinity to the point of the closest approach. Equation (A1) includes all the GR effects.

The scattering is impossible below some critical value of the impact parameter b_{crit} which is determined by the set of equations

$$\gamma^2 = U(x), \quad \frac{\partial U}{\partial x} = 0, \quad (\text{A2})$$

where $U(x) = (1 + \tilde{b}^2 x^2)(1 - x)$. For smaller values $b < b_{\text{crit}}$ the nucleons get accreted onto the BH. The value of b_{crit} depends only on the relative asymptotic velocity of BH and nucleons v ; at $v = 0.6$ one has $b_{\text{crit}} = 3.79R_g$.

Consider the case of scattering, $b > b_{\text{crit}}$. In the reference frame of the NS the nucleons are initially at rest. After the collision they acquire the momentum

$$\Delta p = (mv\gamma^2(-1 + \cos \phi), mv\gamma \sin \phi, 0), \quad (\text{A3})$$

m being the neutron mass and we have assumed that the BH velocity is along the x -direction. The nucleons contribute to the friction force only up to some impact parameter b_{\max} which is determined by the equation

$$\begin{aligned} k_F^2 &\equiv \left(3\pi^2 \frac{\rho}{m_n}\right)^{2/3} \\ &= m^2 v^2 \gamma^2 \{(1 - \cos \phi(b))^2 \gamma^2 + \sin^2 \phi(b)\}, \end{aligned} \quad (\text{A4})$$

where ρ is the neutron density. Note that the resulting value of b_{\max} depends on the nucleon density through the first equality of Eq. (A4).

After the collisions with many nucleons the y -component of the transferred momentum averages away, while the x -component adds up and results in the friction force acting on the BH. Including the effect of the accreted nucleons, one can write this force as follows:

$$\frac{dE}{dr} = 4\pi\rho \frac{G^2 m_{\text{BH}}^2}{v^2} \ln \Lambda(r), \quad (\text{A5})$$

where

$$\ln \Lambda(r) = v^4 \gamma^2 \frac{b_{\text{crit}}^2}{R_g^2} + v^4 \gamma^2 \frac{2}{R_g^2} \int_{b_{\text{crit}}}^{b_{\max}} b db (1 - \cos \phi(b)). \quad (\text{A6})$$

The first term in this expression is due to the accretion, while the second to the scattering of nucleons. It is easy to check that in the nonrelativistic limit and assuming nondegenerate matter (that is, extending the integral to the size of the star), the second term dominates and reduces to the standard expression for the Coulomb logarithm. Making use of Eq. (A6) the density-weighted average in Eq. (2) reads

$$\begin{aligned} \left\langle \frac{\ln \Lambda}{v^2} \right\rangle &= \frac{4\pi}{MR_g^2} \int_0^{R_{\text{NS}}} r^2 dr \rho(r) v^2 \gamma^2 \\ &\times \left\{ b_{\text{crit}}^2 + 2 \int_{b_{\text{crit}}}^{b_{\max}} b db (1 - \cos \phi(b)) \right\}. \end{aligned} \quad (\text{A7})$$

Here v , γ , b_{crit} and b_{\max} all depend on r . Note that in view of Eqs. (A1), (A2), and (A4) this equation is independent of the BH mass m_{BH} .

We have calculated this expression numerically. As an input we used the tabulated NS density profile given in Ref. [24] which corresponds to the NS of mass $1.8M_\odot$ and radius 13.5 km. For a given value of r we have calculated v and γ in the Newtonian approximation, determined the critical impact parameter b_{crit} from Eqs. (A2) (the latter can be solved analytically), calculated the function $\phi(b)$ from Eq. (A1) and the maximum impact parameter b_{\max} . We considered the NS matter to be degenerate down to densities $\rho = 10^{14}$ g/cm³ which we took as the boundary of the NS crust [24]. Finally, we have calculated the integral in Eq. (A7) and found that it equals 14.7, which gives Eq. (4). The contributions of the accretion and dynamical friction [the first and the second terms in Eq. (A7)] are roughly equal.

In conclusion, an important remark is in order. Although we have performed the calculation for a concrete NS mass, the result depends very weakly on the latter. We have checked this by rescaling the density profile of Ref. [24] in such a way that the new NS mass and radius are $1.4M_\odot$ and 12 km, respectively. Repeating the above calculations, we have found that the average in Eq. (A7) changes by less than 4%. We neglect this difference and use the value given in Eq. (4) in our estimates.

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