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CPT violation and triple-product correlations in *B* decays

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The T-odd triple-product (TP) asymmetries in B decays to a pair of vector mesons are treated as a good probe of CP violation because of the CPT symmetry. If CPT is no longer a good symmetry, such correlations between T-odd and CP-odd observables do not exist, and one might get unexpected nonzero TP asymmetries as a signal for CPT violation. We give a general formalism of TP asymmetries in the presence of CPT violation, either in decay or in neutral meson mixing. We also discuss how the observables depending on the transversity amplitudes are modified, and compare our expressions with the LHCb results, showing that the study of TP asymmetries might turn out to be one of the best probes for CPT violation.

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I. INTRODUCTION

Triple-product (TP) correlations are known to be a good probe of CP violation in B decays [1–5]. Consider a B meson decaying into two vector mesons V_1 and V_2:

$$B(p) \to V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2), \tag{1}$$

where k and ϵ are respectively the four-momentum and polarization of the vector mesons. Suppose one constructs an observable $\alpha \equiv \vec{k}_1.(\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$, where we have taken out the spatial components of the respective four-vectors. The asymmetry

$$\frac{\Gamma(\alpha > 0) - \Gamma(\alpha < 0)}{\Gamma(\alpha > 0) + \Gamma(\alpha < 0)} \tag{2}$$

is odd under the time-reversal operator T as α itself is T-odd. As CPT is supposed to be a good symmetry of the Hamiltonian, the asymmetry is CP-odd too, and can be taken as a probe and measure of CP violation.

TP asymmetries are also an excellent probe of new physics (NP) beyond the Standard Model (SM). There are many TP asymmetries which are either zero or tiny in the SM but can go up to observable range under some new physics (NP) dynamics. Also, true TP asymmetries, unlike direct CP asymmetries, are nonzero even if the strong phase difference between two competing amplitudes is small or even zero. Of course, TP asymmetries can be faked by a sizable strong phase difference. The authors of Ref. [4] have discussed in detail the conditions for observation of TP asymmetries, and also the feasibility of measuring such asymmetries for different decay channels. The analysis has been extended by the authors of Ref. [5] for 4-body final states.

A crucial ingredient of extracting CP-violating signals from TP asymmetries is the CPT theorem: the combined discrete symmetry CPT, taken in any order, is an exact symmetry of any local axiomatic quantum field theory [6]. Experiments have put stringent limits on CPT violation (CPTV), as all tests performed so far to probe CPTV [7] yielded null results [8]. Still, one should try to measure CPTV in B systems in as many ways as possible, irrespective of the theoretical dogma, as CPTV can be a flavor-dependent phenomenon, and the constraints obtained from the K system [9] may not be applicable to the B systems. One might also want to know whether any tension between data and the SM expectation is due to CPT conserving canonical NP, or just due to CPTV.

The issue of CPTV has started to receive significant attention due to the growing phenomenological importance of CPTV scenarios in neutrino physics and cosmology [10]. A comprehensive study of CPTV in the neutral K meson system, with a formulation that is closely analogous to that in the B system, may be found in Ref. [11]. CPTV in the B systems and its possible signatures, including differentiation from CPT conserving NP models, have been already investigated by several authors [12–15]. It was shown that the lifetime difference of the two mass eigenstates, or the direct CP asymmetries and semileptonic observables, may be affected by such new physics. The experimental limits are set by both *BABAR*, who looked for diurnal variations of CP-violating observables [16], and Belle, who looked for lifetime differences of B_d mass eigenstates [17]. This makes it worthwhile to look for possible CPTV effects in the B_s system (by B_s we generically mean both B_s^0 and $\overline{B_s^0}$ mesons).

In this paper, we would like to develop the formalism of TP asymmetries with possible CPTV terms in the Lagrangian. Thus, T violation and CP violation are no longer correlated. We will show, in detail, how and where deviations occur from the standard CPT conserving cases. In particular, it will be shown that some decay channels where TP asymmetries are not expected might show some new surprises. We will also relate the TP-violating observables with the transversity amplitudes [4], and discuss the implications of the LHCb results [18] on $B_s \to \phi\phi$.

At this point, we note that violations of different conservation rules lead to different signals. For example, violation of $\Delta B = \Delta Q$ keeping CPT invariant would lead to some interesting time-integrated dilepton asymmetries [19]. While a systematic study of the inverse problem (i.e. going from the signal to the underlying model) in the B sector is worthwhile, it is outside the ambit of this paper. We would like to refer the reader to [15] for ways to differentiate between CPT-conserving and CPT-violating NP under certain conditions; such a differentiation is not always possible.

The paper is arranged as follows. In Sec. II, we discuss the essential formalism of TP asymmetries when CPTV terms are present in the decay amplitudes. In Sec. III, we show how the transversity amplitudes are modified by the CPTV terms. Section IV is devoted to the case where CPTV terms are present in the neutral B meson mixing Hamiltonian but not in the subsequent decay processes. In Sec. V, we correlate the expressions with the data from LHCb. In Sec. VI, we summarize and conclude. Some calculational details and a compendium of relevant expressions, not strictly necessary to catch the main flow of the paper, have been relegated to the two appendixes.

II. FORMALISM

Following Ref. [4], we can write the decay amplitude for $B(p) \to V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2)$ as

$$\begin{aligned} M &= \mathbf{a}\mathcal{S} + \mathbf{b}\mathcal{D} + i\mathbf{c}\mathcal{P} \\ &= \mathbf{a}\varepsilon_1^* \cdot \varepsilon_2^* + \frac{\mathbf{b}}{m_B^2}(p\cdot\varepsilon_1^*)(p\cdot\varepsilon_2^*) \\ &\quad + i\frac{\mathbf{c}}{m_B^2}\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}, \end{aligned} \quad (3)$$

where $q \equiv k_1 - k_2$. Terms are normalized with a factor m_B^2, so that each of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ is expected to be of the same order of magnitude. The $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ terms correspond to combinations of s-, d- and p-wave amplitudes for the final state, denoted by $\mathcal{S}$, $\mathcal{D}$, and $\mathcal{P}$ respectively. The quantities $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are complex and will in general contain both CP-conserving strong phases and CP-violating weak phases.

Similarly, the amplitude for the CP-conjugate process $\bar{B}(p) \to \bar{V}_1(k_1, \varepsilon_1) + \bar{V}_2(k_2, \varepsilon_2)$ can be expressed as

$$\begin{aligned} \bar{M} &= \bar{\mathbf{a}}\varepsilon_1^* \cdot \varepsilon_2^* + \frac{\bar{\mathbf{b}}}{m_B^2}(p\cdot\varepsilon_1^*)(p\cdot\varepsilon_2^*) \\ &\quad - i\frac{\bar{\mathbf{c}}}{m_B^2}\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}, \end{aligned} \quad (4)$$

where, considering CPT conservation, $\bar{\mathbf{a}}$, $\bar{\mathbf{b}}$ and $\bar{\mathbf{c}}$ can be obtained from $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ by changing the sign of the weak phases.

In that case, one can write

$$\begin{aligned} \mathbf{a} &= \sum_i a_i e^{i\phi_i^a} e^{i\zeta_i^a}, & \bar{\mathbf{a}} &= \sum_i a_i e^{-i\phi_i^a} e^{i\zeta_i^a}, \\ \mathbf{b} &= \sum_i b_i e^{i\phi_i^b} e^{i\zeta_i^b}, & \bar{\mathbf{b}} &= \sum_i b_i e^{-i\phi_i^b} e^{i\zeta_i^b}, \\ \mathbf{c} &= \sum_i c_i e^{i\phi_i^c} e^{i\zeta_i^c}, & \bar{\mathbf{c}} &= \sum_i c_i e^{-i\phi_i^c} e^{i\zeta_i^c}, \end{aligned} \quad (5)$$

where $\phi_i^{a,b,c}$ ($\zeta_i^{a,b,c}$) are weak (strong) phases of the respective amplitudes. The relevant quantities for true T-violating TP asymmetries are $[\mathrm{Im}(\mathbf{a}\mathbf{c}^*) - \mathrm{Im}(\bar{\mathbf{a}}\bar{\mathbf{c}}^*)]$ and $[\mathrm{Im}(\mathbf{b}\mathbf{c}^*) - \mathrm{Im}(\bar{\mathbf{b}}\bar{\mathbf{c}}^*)]$, which we get by adding T-odd asymmetries in $|M|^2$ and $|\bar{M}|^2$. One can show [4] that TPs would be nonzero in $B \to V_1 V_2$ decays as long as $\mathrm{Im}(\mathbf{a}\mathbf{c}^*)$ or $\mathrm{Im}(\mathbf{b}\mathbf{c}^*)$ is nonzero. For that, both $B \to V_1$ and $B \to V_2$ channels must be present with different weak phases, following a naive factorization argument, detailed in Appendix A following Ref. [4].

There are two ways to introduce CPT violation in the formalism, namely,

(1) CPTV in the decay amplitude, and
(2) CPTV in the mixing amplitude.

We will discuss the former here and postpone the latter for Sec. IV. However, note that even if CPTV is present in the decay amplitudes, one can still have a mixing-induced CPT violation, characterized by time-dependent TP asymmetries, as discussed below.

A. CPTV in decay

Let us start with the first option, which can be subdivided into two categories.

1. Type I: CPTV present only in the p-wave amplitude

We introduce the CPTV parameter $f \equiv \mathrm{Re}(f) + i\mathrm{Im}(f)$ in the following way:

$$\mathbf{c} = \sum_i c_i e^{i\phi_i^c} e^{i\zeta_i^c}(1-f), \qquad \bar{\mathbf{c}} = \sum_i c_i e^{-i\phi_i^c} e^{i\zeta_i^c}(1+f^*), \quad (6)$$

and other amplitudes remain the same. This is the simplest way to introduce CPTV; a channel-dependent CPTV parameter f_i would only complicate the calculation without giving any extra insight.

The relevant quantity for TP is

$$\begin{aligned} &\frac{1}{2}[\mathrm{Im}(\mathbf{a}\mathbf{c}^*) - \mathrm{Im}(\bar{\mathbf{a}}\bar{\mathbf{c}}^*)] \\ &\quad = \sum_{i,j} a_i c_j [\sin(\phi_i^a - \phi_j^c)\cos(\zeta_i^a - \zeta_j^c) \\ &\qquad - \mathrm{Re}(f)\cos(\phi_i^a - \phi_j^c)\sin(\zeta_i^a - \zeta_j^c) \\ &\qquad - \mathrm{Im}(f)\sin(\phi_i^a - \phi_j^c)\sin(\zeta_i^a - \zeta_j^c)]. \end{aligned} \quad (7)$$

A similar expression is obtained for $\frac{1}{2}[\mathrm{Im}(\mathbf{b}\mathbf{c}^*) - \mathrm{Im}(\bar{\mathbf{b}}\bar{\mathbf{c}}^*)]$. Even if the weak phase difference vanishes, these are still nonzero because of the second term, so the TP asymmetry will essentially probe Re(f).

2. Type II: Universal CPTV present in all amplitudes

In this case, the coefficients from Eqs. (3) and (4) are modified as

$$(\mathbf{a},\mathbf{b},\mathbf{c}) \to (\mathbf{a},\mathbf{b},\mathbf{c})(1-f),$$
$$(\bar{\mathbf{a}},\bar{\mathbf{b}},\bar{\mathbf{c}}) \to (\bar{\mathbf{a}},\bar{\mathbf{b}},\bar{\mathbf{c}})(1+f^*). \tag{8}$$

Thus, the relevant expression for TP becomes

$$\frac{1}{2}[\mathrm{Im}(\mathbf{a}\mathbf{c}^*) - \mathrm{Im}(\bar{\mathbf{a}}\bar{\mathbf{c}}^*)] = \sum_{i,j} a_i c_j [\sin(\phi_i^a - \phi_j^c)\cos(\zeta_i^a - \zeta_j^c) - 2\,\mathrm{Re}(f)\cos(\phi_i^a - \phi_j^c)\sin(\zeta_i^a - \zeta_j^c)]. \tag{9}$$

Here too, only the second term remains in absence of weak phase.

Following Eq. (A5) taken from [4], one finds the cases where no TP asymmetry is expected in the SM. On the other hand, introduction of CPTV may induce nonzero TP asymmetries for some of the cases as follows:

(1) In order to have a TP correlation in a given decay, both of the amplitudes in Eq. (A2) must be present; otherwise either X or Y becomes zero. This remains true for CPTV of type II, but for type I, even in the absence of either X or Y, TPs can be generated.

(2) For the same reason as above, CPTV of type I can produce nonzero TPs even if V_1 and V_2 have identical flavor wave functions (same meson, or an excited state). Such nonzero TPs are not allowed in the SM as then **a**, **b**, and **c** are all proportional to the same factor and there is no relative phase.

(3) In the SM (or in any NP model with *CPT* conservation), two kinematical amplitudes must have different weak phases for a nonzero TP asymmetry. Thus, if the quark-level decay is dominated by a single decay amplitude, a nonzero TP can never be generated. This is again not necessarily true for CPTV of either type I or type II, as we have seen from Eqs. (7) and (9) that even in the absence of weak phase difference, one of the terms in the relevant expressions can have a nonzero value.

3. Effects of type I and type II CPTV in mixing

There could be another way to induce CPTV. Let us suppose CPTV to be present only for $B \to V_1$ and not for $B \to V_2$. As can be seen from Eq. (A3), this changes only the terms with the same phase in the expressions for **a**, **b**, and **c**. Thus, $|f|$ is absorbed in the form factors and arg (f) in the phase. Obviously, this scenario does not produce any TP even if CPTV is present.

Now let us consider the special case where V_1 can be accessed from B but not from $\bar{B}$, and vice versa. Let us also take, for simplicity, $B \to V_1$ and $\bar{B} \to V_2$ to be single-amplitude processes. For $B = B_{d,s}$, there will be a mixing-induced TP because the B meson can oscillate into $\bar{B}$ and hence decay to V_2, thus providing the second amplitude. The relevant T-violating terms, as shown in Ref. [4], are proportional to the **a**-**c** (and **b**-**c**) interference contributions, and are given by

$$\begin{aligned}|M|^2_{\mathbf{ac}} + |\bar{M}|^2_{\mathbf{ac}} &\sim \mathrm{Im}(\mathbf{a}\mathbf{c}^*) - \mathrm{Im}(\bar{\mathbf{a}}\bar{\mathbf{c}}^*)\\ &= \cos^2\left(\frac{\Delta M t}{2}\right)\mathrm{Im}(\mathbf{a}_1\mathbf{c}_1^* - \bar{\mathbf{a}}_1\bar{\mathbf{c}}_1^*)\\ &\quad + \sin^2\left(\frac{\Delta M t}{2}\right)\mathrm{Im}(\mathbf{a}_2\mathbf{c}_2^* - \bar{\mathbf{a}}_2\bar{\mathbf{c}}_2^*)\\ &\quad + \sin\left(\frac{\Delta M t}{2}\right)\cos\left(\frac{\Delta M t}{2}\right)\mathrm{Re}[e^{-2i\phi_M}\mathbf{a}_2\mathbf{c}_1^*\\ &\quad - e^{2i\phi_M}\bar{\mathbf{a}}_2\bar{\mathbf{c}}_1^* - e^{2i\phi_M}\mathbf{a}_1\mathbf{c}_2^* + e^{-2i\phi_M}\bar{\mathbf{a}}_1\bar{\mathbf{c}}_2^*],\end{aligned} \tag{10}$$

where ΔM is the mass difference of the two eigenstates, and following Eq. (3),

$$\begin{aligned}A(B \to V_1V_2) &= \mathbf{a}_1\mathcal{S} + \mathbf{b}_1\mathcal{D} + i\mathbf{c}_1\mathcal{P},\\ A(\bar{B} \to V_1V_2) &= \mathbf{a}_2\mathcal{S} + \mathbf{b}_2\mathcal{D} + i\mathbf{c}_2\mathcal{P},\\ A(B \to \bar{V}_1\bar{V}_2) &= \bar{\mathbf{a}}_2\mathcal{S} + \bar{\mathbf{b}}_2\mathcal{D} - i\bar{\mathbf{c}}_2\mathcal{P},\\ A(\bar{B} \to \bar{V}_1\bar{V}_2) &= \bar{\mathbf{a}}_1\mathcal{S} + \bar{\mathbf{b}}_1\mathcal{D} - i\bar{\mathbf{c}}_1\mathcal{P},\end{aligned} \tag{11}$$

so that

$$\begin{aligned}M &\equiv A(B(t) \to V_1V_2) = e^{-i(M-\frac{i}{2}\Gamma)t}[\mathbf{a}\mathcal{S} + \mathbf{b}\mathcal{D} + i\mathbf{c}\mathcal{P}],\\ \bar{M} &\equiv A(\bar{B}(t) \to \bar{V}_1\bar{V}_2) = e^{-i(M-\frac{i}{2}\Gamma)t}[\bar{\mathbf{a}}\mathcal{S} + \bar{\mathbf{b}}\mathcal{D} - i\bar{\mathbf{c}}\mathcal{P}],\end{aligned} \tag{12}$$

with

$$\begin{aligned}\mathbf{a} &= \mathbf{a}_1\cos\left(\frac{\Delta M t}{2}\right) - ie^{-2i\phi_M}\sin\left(\frac{\Delta M t}{2}\right)\mathbf{a}_2,\\ \bar{\mathbf{a}} &= \bar{\mathbf{a}}_1\cos\left(\frac{\Delta M t}{2}\right) - ie^{2i\phi_M}\sin\left(\frac{\Delta M t}{2}\right)\bar{\mathbf{a}}_2,\\ \mathbf{b} &= \mathbf{b}_1\cos\left(\frac{\Delta M t}{2}\right) - ie^{-2i\phi_M}\sin\left(\frac{\Delta M t}{2}\right)\mathbf{b}_2,\\ \bar{\mathbf{b}} &= \bar{\mathbf{b}}_1\cos\left(\frac{\Delta M t}{2}\right) - ie^{2i\phi_M}\sin\left(\frac{\Delta M t}{2}\right)\bar{\mathbf{b}}_2,\\ \mathbf{c} &= \mathbf{c}_1\cos\left(\frac{\Delta M t}{2}\right) - ie^{-2i\phi_M}\sin\left(\frac{\Delta M t}{2}\right)\mathbf{c}_2,\\ \bar{\mathbf{c}} &= \bar{\mathbf{c}}_1\cos\left(\frac{\Delta M t}{2}\right) - ie^{2i\phi_M}\sin\left(\frac{\Delta M t}{2}\right)\bar{\mathbf{c}}_2.\end{aligned} \tag{13}$$

Note that amplitudes like $\mathbf{a}_1$ are complex, with relevant weak and strong phases:

$$\mathbf{a}_1 = a_1 e^{i\phi_1^a} e^{i\zeta_1^a}. \tag{14}$$

The first term in Eq. (10) describes the time evolution of the TP in $B \to V_1 V_2$ and the second term, generated due to B-$\bar{B}$ mixing, describes the time evolution of the TP in $\bar{B} \to V_1 V_2$. The third term can potentially generate a TP due to B-$\bar{B}$ mixing even in the absence of TP in $B \to V_1 V_2$. This term can be rewritten after explicitly writing down $\mathbf{a}_1$, $\mathbf{a}_2$ etc. following Eq. (5):

$$-(\sin\Delta Mt)[a_2c_1\sin(\phi_2^a-\phi_1^c-2\phi_M)\sin(\zeta_2^a-\zeta_1^c) - a_1c_2\sin(\phi_1^a-\phi_2^c+2\phi_M)\sin(\zeta_1^a-\zeta_2^c)]. \tag{15}$$

This expression goes to zero in the absence of strong phase differences, which is intuitively obvious as strong phase differences are related in part to kinematics, and the TP vanishes if the kinematics of $\bar{B} \to V_2$ is identical to $B \to V_1$.

However, in the presence of CPTV of type I, the expression in (15) is modified to

$$\begin{aligned}
&-(\sin\Delta Mt)[a_2c_1\sin(\phi_2^a-\phi_1^c-2\phi_M)\sin(\zeta_2^a-\zeta_1^c) - a_1c_2\sin(\phi_1^a-\phi_2^c+2\phi_M)\sin(\zeta_1^a-\zeta_2^c)]\\
&\quad -2\,\mathrm{Re}(f)[a_2c_1\cos(\phi_2^a-\phi_1^c-2\phi_M)\cos(\zeta_2^a-\zeta_1^c) - a_1c_2\cos(\phi_1^a-\phi_2^c+2\phi_M)\cos(\zeta_1^a-\zeta_2^c)]\\
&\quad -\mathrm{Im}(f)[a_2c_1\sin(\phi_2^a-\phi_1^c-2\phi_M)\cos(\zeta_2^a-\zeta_1^c) - a_1c_2\sin(\phi_1^a-\phi_2^c+2\phi_M)\cos(\zeta_1^a-\zeta_2^c)],
\end{aligned} \tag{16}$$

while for CPTV of type II, the same expression takes the form

$$\begin{aligned}
&-(\sin\Delta Mt)[a_2c_1\sin(\phi_2^a-\phi_1^c-2\phi_M)\sin(\zeta_2^a-\zeta_1^c)\\
&\quad - a_1c_2\sin(\phi_1^a-\phi_2^c+2\phi_M)\sin(\zeta_1^a-\zeta_2^c)]\\
&\quad -2\,\mathrm{Re}(f)[a_2c_1\cos(\phi_2^a-\phi_1^c-2\phi_M)\cos(\zeta_2^a-\zeta_1^c)\\
&\quad - a_1c_2\cos(\phi_1^a-\phi_2^c+2\phi_M)\cos(\zeta_1^a-\zeta_2^c)].
\end{aligned} \tag{17}$$

The last two equations show that in the presence of CPTV, we can get a nonzero TP from mixing, even if the strong phase differences vanish. Only if the final state is self-conjugate, the third term in Eq. (10) is zero and the first two terms add up, so the TP in $B \to V_1 V_2$ is time-independent and this remains true even in the presence of CPTV.

III. RELATION TO TRANSVERSITY AMPLITUDES

The angular momentum amplitudes are related to the tranversity amplitudes by the following relations [4]:

$$A_\parallel = \sqrt{2}\mathbf{a}, \qquad A_0 = -\mathbf{a}x - \frac{m_1 m_2}{m_B^2}\mathbf{b}(x^2-1),$$
$$A_\perp = 2\sqrt{2}\frac{m_1 m_2}{m_B^2}\mathbf{c}\sqrt{x^2-1}. \tag{18}$$

Let us consider, following Ref. [5], the channels in which each of the two vector mesons in $B \to V_1 V_2$ further decays into two pseudoscalar mesons. The decay angular distribution in three dimensions is given in terms of the three transversity amplitudes. We take θ_1 (θ_2) to be the angle between the direction of motion of P_1 (P_2) in the V_1 (V_2) rest frame and that of V_1 (V_2) in the B rest frame. The angle between the planes defined by P_1P_1' and P_2P_2' in the B rest frame is denoted by φ. One obtains [5]

$$\begin{aligned}
\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\varphi} &= N\bigg[|A_0|^2\cos^2\theta_1\cos^2\theta_2 + \frac{|A_\parallel|^2}{2}\sin^2\theta_1\sin^2\theta_2\cos^2\varphi + \frac{|A_\perp|^2}{2}\sin^2\theta_1\sin^2\theta_2\sin^2\varphi\\
&\quad + \frac{\mathrm{Re}(A_0A_\parallel^*)}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\cos\varphi - \frac{\mathrm{Im}(A_\perp A_0^*)}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\sin\varphi - \frac{\mathrm{Im}(A_\perp A_\parallel^*)}{2}\sin^2\theta_1\sin^2\theta_2\sin 2\varphi\bigg],\\
\frac{d\bar\Gamma}{d\cos\bar\theta_1 d\cos\bar\theta_2 d\bar\varphi} &= N\bigg[|\bar A_0|^2\cos^2\bar\theta_1\cos^2\bar\theta_2 + \frac{|\bar A_\perp|^2}{2}\sin^2\bar\theta_1\sin^2\bar\theta_2\sin^2\bar\varphi + \frac{|\bar A_\parallel|^2}{2}\sin^2\bar\theta_1\sin^2\bar\theta_2\cos^2\bar\varphi\\
&\quad + \frac{\mathrm{Re}(\bar A_0\bar A_\parallel^*)}{2\sqrt{2}}\sin 2\bar\theta_1\sin 2\bar\theta_2\cos\bar\varphi + \frac{\mathrm{Im}(\bar A_\perp \bar A_0^*)}{2\sqrt{2}}\sin 2\bar\theta_1\sin 2\bar\theta_2\sin\bar\varphi + \frac{\mathrm{Im}(\bar A_\perp \bar A_\parallel^*)}{2}\sin^2\bar\theta_1\sin^2\bar\theta_2\sin 2\bar\varphi\bigg].
\end{aligned} \tag{19}$$

Integrating these over θ_1 and θ_2 gives a T-odd asymmetry involving $\sin 2\varphi$ [4]

$$A_T^{(2)} \equiv \frac{\Gamma(\sin 2\varphi > 0) - \Gamma(\sin 2\varphi < 0)}{\Gamma(\sin 2\varphi > 0) + \Gamma(\sin 2\varphi < 0)} = -\frac{4}{\pi}\frac{\mathrm{Im}(A_\perp A_\parallel^*)}{|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2}. \tag{20}$$

Similarly, we may define an asymmetry with respect to the values of $\sin\varphi$, assigning it the sign of $\cos\theta_1\cos\theta_2$ and integrating over all angles,

$$A_T^{(1)} \equiv \frac{\Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\varphi>0]-\Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\varphi<0]}{\Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\varphi>0]+\Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\varphi<0]} = -\frac{2\sqrt{2}}{\pi}\frac{\mathrm{Im}(A_\perp A_0^*)}{|A_0|^2+|A_\perp|^2+|A_\parallel|^2}. \tag{21}$$

One can define similar asymmetries $\bar{A}_T^{(1)}$ and $\bar{A}_T^{(2)}$ by integrating the second part of Eq. (19) and proceeding in a similar manner. As the p-wave amplitude in $\bar{M}$ changes sign relative to that of M [Eqs. (3) and (4)], the sign of the T-odd asymmetry in $|\bar{M}|^2$ is opposite that in $|M|^2$. The true T-violating asymmetry is therefore found by *adding* the T-odd asymmetries in $|M|^2$ and $|\bar{M}|^2$ [2]:

$$\mathcal{A}_T \equiv \frac{1}{2}(A_T + \bar{A}_T). \tag{22}$$

This essentially means that instead of $\mathrm{Im}(A_\perp A_i^*)$, we should look for expressions involving $\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*)$ in search of true TP-violating asymmetries. If we consider specifically the decay $B_s \to \phi\phi$, following Ref. [5], we notice that final states are flavorless and accessible to both B_s and $\bar{B}_s$. As a result of B_s-$\bar{B}_s$ oscillation, the angular decay distributions become time-dependent. Using standard notations for B_s-$\bar{B}_s$ mixing, and assuming no *CP* violation in mixing ($|q/p| = 1$) and decay ($|\bar{A}_k| = |A_k|$), one has [20]

$$\frac{q}{p}\frac{\bar{A}_k}{A_k} = \eta_k e^{-2i\phi_k}. \tag{23}$$

Here η_k is the *CP* parity for a state of transversity k ($\eta_0 = \eta_\parallel = -\eta_\perp = +1$), while ϕ_k is the weak phase involved in an interference between mixing and decay amplitudes. Denoting the *CP* conserving strong phase of A_k by ζ_k, one can write $A_k = |A_k| e^{i\zeta_k} e^{i\phi_k}$, so that $\bar{A}_k = (p/q)\eta_k e^{i\zeta_k} e^{-i\phi_k}$. One thus has for $i = 0, \parallel$:

$$\begin{aligned}\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*) &= |A_\perp||A_i|\mathrm{Im}[e^{i\zeta^-}(e^{i\phi^-} - e^{-i\phi^-})] \\ &= 2|A_\perp||A_i|\cos(\zeta^-)\sin(\phi^-),\end{aligned} \tag{24}$$

where we define the notations for our future references:

$$\zeta^- = \zeta_\perp - \zeta_i, \qquad \phi^- = \phi_\perp - \phi_i, \qquad \phi^+ = \phi_\perp + \phi_i. \tag{25}$$

One finds from Eq. (18) that expressions such as $\mathrm{Im}(A_\perp A_0^*)$ are proportional to linear combinations of terms like $\mathrm{Im}(\mathbf{a}^*\mathbf{c})$ and $\mathrm{Im}(\mathbf{b}^*\mathbf{c})$. Now, as per Eq. (A5), they are all zero for decays like $B_s \to \phi\phi$; thus, $A_T^{(1)}$, $A_T^{(2)}$, and consequently all of their combinations are zero. This can also be seen from Eq. (24) if the weak phases for all the tranversity amplitudes are the same. So, any nonzero values to any of these observables unambiguously point to new physics.

Let us assume the NP to be *CPT* violating in nature, and parametrize the amplitudes following Eqs. (5) and (18):

$$\begin{aligned}A_\perp &= \sum_l |A_\perp^l| e^{i\phi_\perp^l} e^{i\zeta_\perp^l}(1-f), \\ A_i &= \sum_m |A_i^m| e^{i\zeta_i^m} e^{i\phi_i^m}, \\ \bar{A}_\perp &= \eta_\perp \sum_l |A_\perp^l| e^{-i\phi_\perp^l} e^{i\zeta_\perp^l}(1+f^*), \\ \bar{A}_i &= \sum_m |A_i^m| e^{i\zeta_i^m} e^{-i\phi_i^m}, \qquad (i = 0, \parallel).\end{aligned} \tag{26}$$

Using the notation $\zeta_{l,m}^- = (\zeta_\perp^l - \zeta_i^m)$ and $\phi_{l,m}^- = (\phi_\perp^l - \phi_i^m)$, we obtain

$$\begin{aligned}\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*) = 2\sum_{l,m}|A_\perp^l||A_i^m|[&\sin(\phi_{l,m}^-)\cos(\zeta_{l,m}^-) \\ &- \mathrm{Re}(f)\sin(\zeta_{l,m}^-)\cos(\phi_{l,m}^-) \\ &+ \mathrm{Im}(f)\sin(\phi_{l,m}^-)\sin(\zeta_{l,m}^-)].\end{aligned} \tag{27}$$

For $f = 0$ this reduces to Eq. (24). On the other hand, even if $\phi_{l,m}^- = 0$, we still get a nonzero result:

$$\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*) = -2\sum_{l,m}|A_\perp^l||A_i^m|\mathrm{Re}(f)\sin(\zeta_{l,m}^-). \tag{28}$$

A. Time dependence of the transversity amplitudes

Next, let us consider the time dependence of transversity amplitudes; we will use a formalism closely following Ref. [5]. The states B and $\bar{B}$ evolve in time as

$$\begin{aligned}B(t) &= f_+(t)B + (q/p)f_-(t)\bar{B}, \\ \bar{B}(t) &= (p/q)f_-(t)B + f_+(t)\bar{B},\end{aligned} \tag{29}$$

where

$$\begin{aligned}f_+(t) &= \frac{1}{2}(e^{-i\lambda_1^{(q)}t} + e^{-i\lambda_2^{(q)}t}) \\ &= \frac{1}{2}(e^{-im_1t-(\Gamma_1t/2)} + e^{-im_2t-(\Gamma_2t/2)}) \\ f_-(t) &= \frac{1}{2}(e^{-i\lambda_1^{(q)}t} - e^{-i\lambda_2^{(q)}t}) \\ &= \frac{1}{2}(e^{-im_1t-(\Gamma_1t/2)} - e^{-im_2t-(\Gamma_2t/2)}), \\ |f_\pm(t)|^2 &= (e^{-\Gamma t}/2)[\cosh(\Delta\Gamma t/2) \pm \cos(\Delta M t)], \\ f_+^*(t)f_-(t) &= (e^{-\Gamma t}/2)[\sinh(\Delta\Gamma t/2) - i\sin(\Delta M t)],\end{aligned} \tag{30}$$

ΔM and $\Delta\Gamma$ being the mass and width differences of the stationary states respectively.

Time dependence of transversity amplitudes, $A_k \equiv \langle k|B\rangle$, $\bar{A}_k \equiv \langle k|\bar{B}\rangle$ ($k = 0, \parallel, \perp$), is given by

$$\begin{aligned} A_k(t) &\equiv \langle k|B(t)\rangle = f_+(t)A_k + (q/p)f_-(t)\bar{A}_k, \\ \bar{A}_k(t) &\equiv \langle k|\bar{B}(t)\rangle = (p/q)f_-(t)A_k + f_+(t)\bar{A}_k. \end{aligned} \tag{31}$$

Let us calculate the interference terms $A_i^*(t)A_k(t)$ and $\bar{A}_i^*(t)\bar{A}_k(t)$, where $i = 0, \parallel$, $k = \perp$. Inserting $A_i^*A_k = |A_i||A_k|(1-f)\exp[i(\zeta_k - \zeta_i)]\exp[i(\phi_k - \phi_i)]$, and $\bar{A}_i^*\bar{A}_k = \eta_i\eta_k|A_i||A_k|(1+f^*)\exp[i(\zeta_k-\zeta_i)]\exp[-i(\phi_k-\phi_i)]$, one gets, using Eq. (25),

$$\begin{aligned} \mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] = 2|A_\perp||A_i|e^{-\Gamma t}[&\{\cos(\zeta^-)\sin(\phi^-) - \sin(\zeta^-)(\mathrm{Re}(f)\cos(\phi^-) \\ &- \mathrm{Im}(f)\sin(\phi^-))\}\cosh(\Delta\Gamma t/2) + \{\cos(\zeta^-)\sin(\phi^+) + \sin(\zeta^-)(\mathrm{Re}(f)\cos(\phi^+) \\ &+ \mathrm{Im}(f)\sin(\phi^+))\}\sinh(\Delta\Gamma t/2)]. \end{aligned} \tag{32}$$

This, again, agrees with Eq. (24) at $t = 0$, $f = 0$. When *CPT* is conserved, it shows the variation of a genuine *CP*-violating quantity with time which requires no strong phase differences. The CPTV contribution is nonzero even if the weak phase difference vanishes but the strong phase difference ζ^- must be nonzero.

If there is more than one decay channel contributing to the transversity amplitudes, Eq. (32) can be generalized to

$$\begin{aligned} \mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] = \sum_{l,m} 2|A_\perp^l||A_i^m|e^{-\Gamma t}[&\{\cos(\zeta_{l,m}^-)\sin(\phi_{l,m}^-) - \sin(\zeta_{l,m}^-)[\mathrm{Re}(f)\cos(\phi_{l,m}^-) \\ &- \mathrm{Im}(f)\sin(\phi_{l,m}^-)]\}\cosh(\Delta\Gamma t/2) + \{\cos(\zeta_{l,m}^-)\sin(\phi_{l,m}^+) \\ &+ \sin(\zeta_{l,m}^-)[\mathrm{Re}(f)\cos(\phi_{l,m}^+) + \mathrm{Im}(f)\sin(\phi_{l,m}^+)]\}\sinh(\Delta\Gamma t/2)]. \end{aligned} \tag{33}$$

The two "true" *CP*-violating time-integrated triple product asymmetries ($i = 0, \parallel$) for untagged decays are proportional to

$$\begin{aligned} \Gamma\int_0^\infty \mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)]dt = \sum_{l,m} 2|A_\perp^l||A_i^m|[&\{\cos(\zeta_{l,m}^-)\sin(\phi_{l,m}^-) - \sin(\zeta_{l,m}^-)(\mathrm{Re}(f)\cos(\phi_{l,m}^-) - \mathrm{Im}(f)\sin(\phi_{l,m}^-))\} \\ &+ \{\cos(\zeta_{l,m}^-)\sin(\phi_{l,m}^+) + \sin(\zeta_{l,m}^-)(\mathrm{Re}(f)\cos(\phi_{l,m}^+) + \mathrm{Im}(f)\sin(\phi_{l,m}^+))\}(\Delta\Gamma/2\Gamma) \\ &+ \mathcal{O}[(\Delta\Gamma/2\Gamma)^2]]. \end{aligned} \tag{34}$$

In the limit $\Delta\Gamma \ll \Gamma$, one can neglect everything apart from the first term in Eq. (34) and find

$$\begin{aligned} \mathcal{A}_T^{(1)\mathrm{untagged}} &= -\frac{4\sqrt{2}}{\pi}\sum_{l,m}\frac{|A_\perp^l||A_0^m|[\cos(\zeta_{l,m}^{0-})\sin(\phi_{l,m}^{0-}) - \sin(\zeta_{l,m}^{0-})(\mathrm{Re}(f)\cos(\phi_{l,m}^{0-}) - \mathrm{Im}(f)\sin(\phi_{l,m}^{0-}))]}{(|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2) + (|\bar{A}_0|^2 + |\bar{A}_\perp|^2 + |\bar{A}_\parallel|^2)} + \mathcal{O}[(\Delta\Gamma/2\Gamma)] \\ \mathcal{A}_T^{(2)\mathrm{untagged}} &= -\frac{8}{\pi}\sum_{l,m}\frac{|A_\perp^l||A_\parallel^m|[\cos(\zeta_{l,m}^{\parallel-})\sin(\phi_{l,m}^{\parallel-}) - \sin(\zeta_{l,m}^{\parallel-})(\mathrm{Re}(f)\cos(\phi_{l,m}^{\parallel-}) - \mathrm{Im}(f)\sin(\phi_{l,m}^{\parallel-}))]}{(|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2) + (|\bar{A}_0|^2 + |\bar{A}_\perp|^2 + |\bar{A}_\parallel|^2)} + \mathcal{O}[(\Delta\Gamma/2\Gamma)], \end{aligned} \tag{35}$$

where $\zeta_{l,m}^{i-} = (\zeta_\perp^l - \zeta_i^m)$ and $\phi_{l,m}^{i-} = (\phi_\perp^l - \phi_i^m)$ for $i = 0, \parallel$, and the coefficients of the $\Delta\Gamma/2\Gamma$ terms can be easily found out from Eq. (34).

In the absence of weak phase difference, $\phi_\perp = \phi_0 = \phi_\parallel$, i.e. $\phi_{l,m}^{i-} = 0$, the asymmetries vanish in the leading order if *CPT* is conserved [5] but are nonzero if *CPT* is violated. Again, a nonzero strong phase difference $\zeta_{l,m}^{i-}$ is obligatory for this.

In the SM, all three transversity amplitudes have approximately equal and very small weak phases. Thus, one expects the asymmetries to be quite small. On the other hand, if CPTV is present, these asymmetries, measured in self-tagged decays to final *CP* eigenstates, need not be nonzero; thus, measurements of such asymmetries may either put stringent limits on the *CPT*-violating parameter f, or indicate physics beyond SM.

IV. *CPT* VIOLATION IN MIXING

One can also consider the case where CPTV is present not in decay but in B-$\bar{B}$ mixing, and parametrize the 2×2 Hamiltonian matrix with the introduction of an extra complex parameter δ which incorporates *CPT* violation [14]:

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}}, \quad (36)$$

so that

$$\mathcal{M} = \left[\begin{pmatrix} M_0 - \delta' & M_{12} \\ M_{12}^* & M_0 + \delta' \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_0 & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 \end{pmatrix}\right], \quad (37)$$

where δ' is defined by

$$\delta = \frac{2\delta'}{\sqrt{H_{12}H_{21}}}. \quad (38)$$

We work within the Wigner-Weisskopf approximation which is a reliable one after a time scale of $\sim 1/M_B$. Violation of this approximation, which has nevertheless been considered in the literature [21], would change all the subsequent expressions, and we refrain from considering such a possibility. This will give, akin to the Bell-Steinberger analysis [22], a way to measure the *CPT*-violating parameter δ in terms of the interference amplitudes which are supposed to be good probes of *CP* violation.

Eq. (31) can be written as

$$\begin{aligned} A_i(t) &\equiv \langle k|B(t)\rangle = f_+(t)A_i + \eta_1 f_-(t)\bar{A}_i, \\ \bar{A}_i(t) &\equiv \langle k|\bar{B}(t)\rangle = \frac{f_-(t)}{\eta_2}A_i + \bar{f}_+(t)\bar{A}_i, \end{aligned} \quad (39)$$

where $f_\pm(t)$, $\bar{f}_+(t)$ and $\eta_{(1,2)}$ are defined in Appendix B. Using Eq. (25), one gets

$$\begin{aligned} \mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] = 2e^{-\Gamma t}|A_i||A_\perp|\Big[&\cosh(\Delta\Gamma t/2)\Big\{\cos\zeta^- \sin\phi^- - \frac{1}{4}\mathrm{Im}\delta\cos\phi^+(1 + \sin\zeta^-)\Big\} \\ &+ \sinh(\Delta\Gamma t/2)\Big\{\cos\zeta^-\Big(\sin\phi^+ - \frac{1}{2}\mathrm{Re}\delta\sin\phi^-\Big) - \frac{1}{2}\mathrm{Re}\delta\sin\zeta^-\cos\phi^-\Big\} \\ &+ \frac{1}{2}\cos(\Delta M t)\mathrm{Im}\delta\cos\zeta^-\cos\phi^+ - \frac{1}{2}\sin(\Delta M t)\mathrm{Im}\delta\sin\zeta^-\cos\phi^-\Big]. \end{aligned} \quad (40)$$

If there are multiple decay channels, one can generalize the above expression, by replacing ζ^-, ϕ^-, ϕ^+ with $\zeta^-_{l,m}$ etc., $|A_i||A_\perp|$ with $|A_i^m||A_\perp^l|$ and then taking a summation over l and m.

Then the two "true" *CP*-violating time-integrated triple product asymmetries ($i = 0, \parallel$) for untagged decays are proportional to

$$\begin{aligned} \Gamma\int_0^\infty \mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] = \sum_{l,m} 2|A_i^m||A_\perp^l|\Big[&\Big\{\cos\zeta^-_{l,m}\sin\phi^-_{l,m} - \frac{1}{4}\mathrm{Im}\delta\cos\phi^+_{l,m}(1 + \sin\zeta^-_{l,m})\Big\} \\ &+ \Big(\frac{\Delta\Gamma}{2\Gamma}\Big)\Big\{\cos\zeta^-_{l,m}\Big(\sin\phi^+_{l,m} - \frac{1}{2}\mathrm{Re}\delta\sin\phi^-_{l,m}\Big) - \frac{1}{2}\mathrm{Re}\delta\sin\zeta^-_{l,m}\cos\phi^-_{l,m}\Big\} \\ &+ \frac{1}{2}\Big(\frac{1}{1 + (\frac{\Delta M}{\Gamma})^2}\Big)\mathrm{Im}\delta\cos\zeta^-_{l,m}\cos\phi^+_{l,m} - \frac{1}{2}\Big(\frac{\frac{\Delta M}{\Gamma}}{1 + (\frac{\Delta M}{\Gamma})^2}\Big)\mathrm{Im}\delta\sin\zeta^-_{l,m}\cos\phi^-_{l,m}\Big]. \end{aligned} \quad (41)$$

In the limit $\Delta M/\Gamma \ll 1$, one can neglect the last term and simplify the expression considerably.

We also note that even in the case $\zeta^-_{l,m} = \phi^-_{l,m} = 0$, i.e. when all strong and weak phase differences cancel out individually, there is a nonzero TP asymmetry that gives a clean measurement of $\mathrm{Im}\delta$:

$$\begin{aligned} \Gamma\int_0^\infty &\mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] \\ &\approx \sum_{l,m}\frac{1}{2}|A_i^m||A_\perp^l|\mathrm{Im}\delta\cos\phi^+_{l,m}, \end{aligned} \quad (42)$$

where we have used $\Delta M/\Gamma \approx 0$ and neglected the subleading $\Delta\Gamma/\Gamma$ terms.

V. $B_s \to \phi\phi$ AT LHCB

The LHCb collaboration has recently measured the transversity amplitudes for the decay $B_s \to \phi\phi$ [18], which is a pure penguin process and hence dominated by a single amplitude in the SM. Thus, for all l, m, $A_i^l = A_i^m$ (for $i = 0, \parallel, \perp$). The analysis also assumes that the weak phases of the three polarization amplitudes are all equal; thus, all $\phi^{i-}_{l,m}$ (for $i = 0, \parallel$) in our notation become zero. The correspondence between our notation and that of Ref. [18] is as follows:

$$\begin{aligned} &\mathcal{A}_T^{(2)\mathrm{untagged}} \to A_U, \qquad \mathcal{A}_T^{(1)\mathrm{untagged}} \to A_V \\ &(\zeta_\perp - \zeta_\parallel) \to \delta_1, \qquad (\zeta_\perp - \zeta_0) \to \delta_2, \\ &(\zeta_\parallel - \zeta_0) \to \delta_\parallel \equiv (\delta_2 - \delta_1). \end{aligned} \quad (43)$$

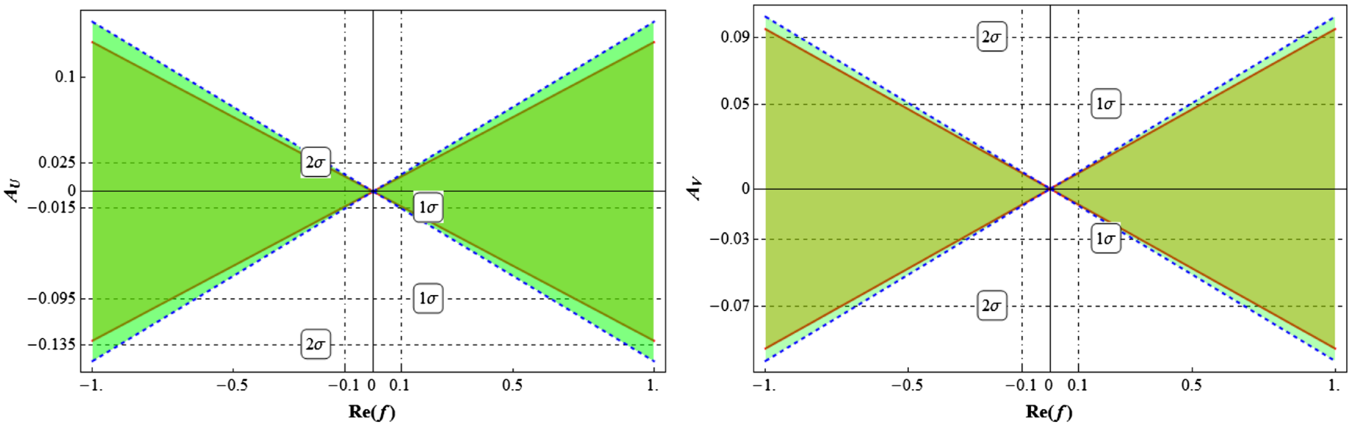

FIG. 1 (color online). Left panel: Allowed values of A_U for $-1 \le \mathrm{Re}(f) \le 1$. The inner wedge is for the input parameters varied in their 1σ ranges; the outer wedge is for 2σ variation. Also shown are the 1σ and 2σ experimental bands for A_U, and the allowed region for a smaller range of Re(f), namely, $|\mathrm{Re}(f)| \le 0.1$. Right panel: Same plot for A_V.

With the standard normalization of the transversity amplitudes, viz. $|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2 = |\bar{A}_0|^2 + |\bar{A}_\perp|^2 + |\bar{A}_\parallel|^2 = 1$, Eq. (35) becomes

$$\begin{aligned} A_V = &-\frac{2\sqrt{2}}{\pi}|A_\perp||A_0|[-\sin(\delta_2)(\mathrm{Re}(f)) \\ &+\{\cos(\delta_2)\sin(2\phi_s) + \sin(\delta_2)(\mathrm{Re}(f)\cos(2\phi_s) \\ &+\mathrm{Im}(f)\sin(2\phi_s))\}(\Delta\Gamma/2\Gamma)] + \mathcal{O}[(\Delta\Gamma/2\Gamma)^2] \\ A_U = &-\frac{4}{\pi}|A_\perp||A_\parallel|[-\sin(\delta_1)\mathrm{Re}(f) + \{\cos(\delta_1)\sin(2\phi_s) \\ &+\sin(\delta_1)(\mathrm{Re}(f)\cos(2\phi_s) \\ &+\mathrm{Im}(f)\sin(2\phi_s))\}(\Delta\Gamma/2\Gamma)] + \mathcal{O}[(\Delta\Gamma/2\Gamma)^2]. \end{aligned} \tag{44}$$

We will use the following numbers from Ref. [18]:

$$\begin{aligned} |A_0|^2 &= 0.365 \pm 0.022(\mathrm{stat}) \pm 0.012(\mathrm{syst}), \\ |A_\perp|^2 &= 0.291 \pm 0.024(\mathrm{stat}) \pm 0.010(\mathrm{syst}), \\ |A_\parallel|^2 &= 0.344 \pm 0.024(\mathrm{stat}) \pm 0.014(\mathrm{syst}), \\ \cos(\delta_\parallel) &= -0.844 \pm 0.068(\mathrm{stat}) \pm 0.029(\mathrm{syst}), \\ A_U &= -0.055 \pm 0.036(\mathrm{stat}) \pm 0.018(\mathrm{syst}) \\ A_V &= 0.010 \pm 0.036(\mathrm{stat}) \pm 0.018(\mathrm{syst}). \end{aligned} \tag{45}$$

For our analysis, we use Eqs. (43)–(45), and keep terms only up to the first order in $\Delta\Gamma/\Gamma$. Even for the B_s system, this is a good approximation. All $\phi^{i+}_{l,m}$s in Eq. (34) (for $i = 0, \parallel$) are now equal to $2\phi_s$, where ϕ_s is the weak CP-violating phase which is the same for the three polarization amplitudes, and very small in the SM ($\phi_s \sim 0.02$ [23,24] based on QCD factorization).[1] Even if there is some new physics making ϕ_s large, the effects will be suppressed by $\Delta\Gamma/\Gamma$, so we do not expect much sensitivity on the precise value of ϕ_s. One may note that this phase has recently been measured by the LHCb Collaboration [25] to be between −2.46 and −0.76 rad with 68% confidence level, which is not exactly in total conformity with the SM prediction.

As is evident from Eq. (44), if we neglect higher order terms in $\Delta\Gamma/\Gamma$, both A_U and A_V are zero in the SM; thus, any definite nonzero value for these observables would point to the presence of some NP. Considering CPT violation as the source of NP, one sees that there is a definite deviation from zero even at the zeroth order of $\Delta\Gamma/\Gamma$; unfortunately, the shift depends only on Re(f), as Im(f) comes as a coefficient of $\sin(2\phi_s)$ in the subleading order. Figure 1 shows the allowed ranges for A_U and A_V when the input parameters are varied over their experimental ranges. We have varied the three transversity amplitudes over their allowed ranges keeping the normalization to unity fixed, and also varied the strong phase differences δ_1 and δ_2 over the entire range of $[0:2\pi]$ keeping the constraint on $\cos(\delta_\parallel)$. This gives a bound on A_U and A_V, although this is quite weak at present (however, note that if we take the 1σ region on A_U seriously, small values of Re(f) are ruled out, as is the SM). The allowed region will shrink considerably with more data.

In Fig. 2 we show the allowed region in the A_U-A_V plane for large and small values of Re(f), varying all other input parameters as above. Again, with more data, the elliptic figures are bound to shrink, as well as the horizontal and vertical bands, constraining CPT violation. If finally the intersection of the bands settles outside the ellipses, that

[1]This should not be confused with the phase ϕ_s relevant for B_s-$\bar{B}_s$ mixing and defined as $\phi_s = \arg(-M_{12}/\Gamma_{12})$.

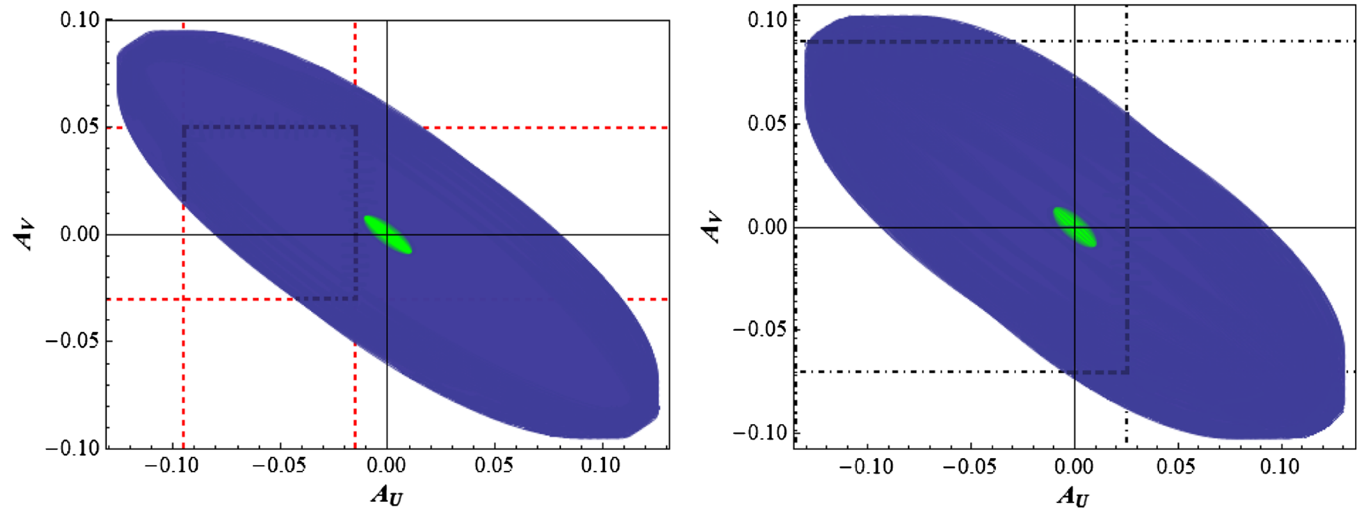

FIG. 2 (color online). Left panel: Allowed region in the A_U-A_V plane when all the input parameters are varied over their 1σ ranges. The outer ellipse is for $-1 \le \mathrm{Re}(f) \le 1$ and the inner green ellipse is for $-0.1 \le \mathrm{Re}(f) \le 0.1$. The 1σ bands for A_U and A_V are shown as dashed lines. Right panel: The same plot when the input parameters are varied over 2σ; also, the 2σ bands are shown. The left edge of the A_U band coincides with the left edge of the plot.

will rule out *CPT* violation in this channel at least, but that will also rule out the pure-SM explanation and call for some other NP.

VI. CONCLUSIONS

The role of TP asymmetries as a probe of *CP* violation crucially hinges on the *CPT* theorem which relates a possible T-violating observable to a *CP*-violating one. If *CPT* is not conserved, there is no such relationship, and observables that are not supposed to show any TP asymmetries in the SM might do so. For example, if *CPT* violation is present in one or more decay amplitudes, there will be a nonzero TP asymmetry even if the weak phases of all the amplitudes are equal. The same trend persists in the time dependence of the TP asymmetries.

One might trade the *s*-, *p*-, and *d*-wave amplitudes with the transversity amplitudes, which are directly accessible to the experiments. Some of the interference terms between these amplitudes are *CP* violating only if the corresponding weak phases are different; in the presence of *CPT* violation, we again observe that a nonzero signal can be observed even if all the weak phases are equal. The observables A_U and A_V, as measured by LHCb, are supposed to be zero in the SM for channels like $B_s \to \phi\phi$. We show how one gets nonzero and possibly large values for these observables with *CPT* violation; a more canonical NP that contributes only to the B-$\bar{B}$ mixing and hence modifies the weak *CP*-violating phase ϕ_s in the decay can hardly generate such large values as all ϕ_s-dependent terms are suppressed by $\Delta\Gamma/\Gamma$. The other side of the coin is that with more data, one can successfully constrain the parameter space for the *CPT*-violating parameters.

ACKNOWLEDGMENTS

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APPENDIX A: FACTORIZATION

Following Ref. [4], we briefly describe the main results of naive factorization. The prediction of naive factorization, that most TP asymmetries with ground state vector mesons are expected to be small in the SM, will necessarily hold in perturbative QCD (PQCD) or QCD factorization too.

The starting point for factorization is the SM effective Hamiltonian for B decays [26]:

$$H^q_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}\Big[V_{fb}V^*_{fq}(c_1 O^q_{1f} + c_2 O^q_{2f}) - \sum_{i=3}^{10}(V_{ub}V^*_{uq}c^u_i + V_{cb}V^*_{cq}c^c_i + V_{tb}V^*_{tq}c^t_i)O^q_i\Big] + \mathrm{H.c.}, \tag{A1}$$

where the superscripts u, c, t indicate the internal quark, f can be the u or c quark, and q can be either a d or s quark.

Within factorization, the amplitude for $B \to V_1 V_2$ can be written as

$$\mathcal{A}(B \to V_1 V_2) = \sum_{\mathcal{O},\mathcal{O}'}\{\langle V_1|\mathcal{O}|0\rangle\langle V_2|\mathcal{O}'|B\rangle + \langle V_2|\mathcal{O}|0\rangle\langle V_1|\mathcal{O}'|B\rangle\}, \tag{A2}$$

where $\mathcal{O}$ and $\mathcal{O}'$ are some relevant four-fermion operators. The first amplitude, $\langle V_1|\mathcal{O}|0\rangle$, is proportional to the polarization vector of V_1, namely, ε_1^*. The second amplitude, $\langle V_2|\mathcal{O}'|B\rangle$, can be written in terms of the usual vector and axial-vector form factors. Thus, the first term of Eq. (A2) is given by

$$\sum_{\mathcal{O},\mathcal{O}'}\langle V_1|\mathcal{O}|0\rangle\langle V_2|\mathcal{O}'|B\rangle = -(m_B+m_2)m_1 g_{V_1} X A_1^{(2)}(m_1^2)\varepsilon_1^*\cdot\varepsilon_2^* + 2\frac{m_1}{m_B+m_2}g_{V_1}XA_2^{(2)}(m_1^2)\varepsilon_2^*\cdot p\varepsilon_1^*\cdot p - i\frac{m_1}{(m_B+m_2)}g_{V_1}XV^{(2)}(m_1^2)\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu\varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}. \tag{A3}$$

All phase information is contained within the factor X, which is common to all three independent amplitudes. Thus, these quantities must have the same phase.

A similar treatment for the second term in Eq. (A2) gives

$$\sum_{\mathcal{O},\mathcal{O}'}\langle V_2|\mathcal{O}|0\rangle\langle V_1|\mathcal{O}'|B\rangle = -(m_B+m_1)m_2 g_{V_2} Y A_1^{(1)}(m_2^2)\varepsilon_1^*\cdot\varepsilon_2^* + 2\frac{m_2}{m_B+m_1}g_{V_2}YA_2^{(1)}(m_2^2)\varepsilon_2^*\cdot p\varepsilon_1^*\cdot p - i\frac{m_2}{(m_B+m_1)}g_{V_2}YV^{(1)}(m_2^2)\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu\varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}, \tag{A4}$$

where the phase information is contained in the common factor Y, which need not be the same as X.

We can now express the quantities a, b and c of Eq. (3) as follows:

$$\begin{aligned} a &= -m_1 g_{V_1}(m_B+m_2)A_1^{(2)}(m_1^2)X - m_2 g_{V_2}(m_B+m_1)A_1^{(1)}(m_2^2)Y \\ b &= 2m_1 g_{V_1}\frac{m_B}{(m_B+m_2)}m_B A_2^{(2)}(m_1^2)X + 2m_2 g_{V_2}\frac{m_B}{(m_B+m_1)}m_B A_2^{(1)}(m_2^2)Y \\ c &= -m_1 g_{V_1}\frac{m_B}{(m_B+m_2)}m_B V^{(2)}(m_1^2)X - m_2 g_{V_2}\frac{m_B}{(m_B+m_1)}m_B V^{(1)}(m_2^2)Y. \end{aligned} \tag{A5}$$

Thus, nonzero TP asymmetries are generated from $\mathrm{Im}(ac^*)$ or $\mathrm{Im}(bc^*)$ if and only if both X and Y are present with different phases. Thus, if $V_1 = V_2$, there cannot be any TP asymmetry in the SM.

APPENDIX B: *CPT* VIOLATION IN MIXING

This closely follows Ref. [14] with a couple of typographical errors corrected. Consider the 2×2 Hamiltonian matrix with an explicit *CPT*-violating term δ. Let us define

$$\eta_1 = \frac{q_1}{p_1} = \left(y+\frac{\delta}{2}\right)\alpha; \quad \eta_2 = \frac{q_2}{p_2} = \left(y-\frac{\delta}{2}\right)\alpha; \quad \omega = \frac{\eta_1}{\eta_2}, \tag{B1}$$

and

$$\begin{aligned} f_-(t) &= \frac{1}{(1+\omega)}(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}), \\ f_+(t) &= \frac{1}{(1+\omega)}(e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t}), \\ \bar{f}_+(t) &= \frac{1}{(1+\omega)}(\omega e^{-i\lambda_1 t} + e^{-i\lambda_2 t}). \end{aligned} \tag{B2}$$

Thus,

$$\begin{aligned} |f_-(t)|^2 &= \frac{2e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \cos(\Delta M t)\right] \approx \frac{e^{-\Gamma t}(1-\mathrm{Re}\delta)}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \cos(\Delta M t)\right], \\ |f_+(t)|^2 &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(1+|\omega|^2) + \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1-|\omega|^2) + 2\mathrm{Re}(\omega)\cos(\Delta M t) - 2\mathrm{Im}(\omega)\sin(\Delta M t)\right], \\ &\approx \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \sinh\left(\frac{\Delta\Gamma t}{2}\right)\mathrm{Re}\delta + \cos(\Delta M t) - \mathrm{Im}\delta\sin(\Delta M t)\right], \\ |\bar{f}_+(t)|^2 &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(1+|\omega|^2) - \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1-|\omega|^2) + 2\mathrm{Re}(\omega)\cos(\Delta M t) + 2\mathrm{Im}(\omega)\sin(\Delta M t)\right], \\ &\approx \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \sinh\left(\frac{\Delta\Gamma t}{2}\right)\mathrm{Re}\delta + \cos(\Delta M t) + \mathrm{Im}\delta\sin(\Delta M t)\right], \end{aligned}$$

$$
\begin{aligned}
f_+^*(t)f_-(t) &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\bigg[\cosh\Big(\frac{\Delta\Gamma t}{2}\Big)(1-\omega^*) + \sinh\Big(\frac{\Delta\Gamma t}{2}\Big)(1+\omega^*) + \cos(\Delta M t)(-1+\omega^*) - i\sin(\Delta M t)(1+\omega^*)\bigg],\\
&\approx \frac{e^{-\Gamma t}}{4}\bigg[\cosh\Big(\frac{\Delta\Gamma t}{2}\Big)(-\mathrm{Re}\delta + i\mathrm{Im}\delta) + \sinh\Big(\frac{\Delta\Gamma t}{2}\Big)(2-\mathrm{Re}\delta - i\mathrm{Im}\delta) + \cos(\Delta M t)(\mathrm{Re}\delta - i\mathrm{Im}\delta)\\
&\quad - \sin(\Delta M t)(\mathrm{Im}\delta + i(2-\mathrm{Re}\delta))\bigg],\\
\bar f_+(t)f_-^*(t) &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\bigg[\cosh\Big(\frac{\Delta\Gamma t}{2}\Big)(\omega-1) + \sinh\Big(\frac{\Delta\Gamma t}{2}\Big)(1+\omega) + \cos(\Delta M t)(1-\omega) + i\sin(\Delta M t)(1+\omega)\bigg]\\
&\approx \frac{e^{-\Gamma t}}{4}\bigg[\cosh\Big(\frac{\Delta\Gamma t}{2}\Big)(\mathrm{Re}\delta + i\mathrm{Im}\delta) + \sinh\Big(\frac{\Delta\Gamma t}{2}\Big)(2-\mathrm{Re}\delta + i\mathrm{Im}\delta) - \cos(\Delta M t)(\mathrm{Re}\delta + i\mathrm{Im}\delta)\\
&\quad + i\sin(\Delta M t)(-\mathrm{Im}\delta + i(2-\mathrm{Re}\delta))\bigg],
\end{aligned}
\tag{B3}
$$

where we take $y \approx 1$, $\eta_{1(2)} \approx (1 + (-)\frac{\delta}{2})$, $\omega \approx (1+\delta)$, $|\omega|^2 \approx (1 + 2\mathrm{Re}\delta)$, $|1+\omega|^{-2} \approx \frac{1}{4}(1-\mathrm{Re}\delta)$, $|\eta_{1(2)}|^2 \approx (1 + (-)\mathrm{Re}\delta)$.

This gives

$$
\begin{aligned}
A_i^*(t)A_k(t) &= [f_+^*A_i^* + \eta_1^* f_-^* \bar A_i^*][f_+A_k + \eta_1 f_- \bar A_k]\\
&= A_i^*A_k[|f_+|^2 + \eta_1(\bar A_k/A_k)f_+^*f_-] + \bar A_i^*\bar A_k[|\eta_1|^2|f_-|^2 + \eta_1^*(A_k/\bar A_k)f_+f_-^*]\\
&= \frac{e^{-\Gamma t}}{2}\bigg[A_i^*A_k\{\cosh(\Delta\Gamma t/2) + \cos(\Delta m t) - \mathrm{Re}\delta\sinh(\Delta\Gamma t/2) - \mathrm{Im}\delta\sin(\Delta m t)\}\\
&\quad + \frac{\eta_k e^{-2i\phi_k}}{2}A_i^*A_k\{2\sinh(\Delta\Gamma t/2) - 2i\sin(\Delta M t) + (-\mathrm{Re}\delta + i\mathrm{Im}\delta)\cosh(\Delta\Gamma t/2) + (\mathrm{Re}\delta - i\mathrm{Im}\delta)\cos(\Delta M t)\}\\
&\quad + \bar A_i^*\bar A_k\{\cosh(\Delta\Gamma t/2) - \cos(\Delta M t)\} + \frac{\eta_k e^{2i\phi_k}}{2}\bar A_i^*\bar A_k\{2\sinh(\Delta\Gamma t/2) + 2i\sin(\Delta M t)\\
&\quad + \cosh(\Delta\Gamma t/2)(-\mathrm{Re}\delta - i\mathrm{Im}\delta) + (\mathrm{Re}\delta + i\mathrm{Im}\delta)\cos(\Delta m t)\}\bigg],
\end{aligned}
$$

$$
\begin{aligned}
\bar A_i^*(t)\bar A_k(t) &= \bigg[\frac{f_-^*}{\eta_2^*}A_i^* + \bar f_+^*\bar A_i^*\bigg]\bigg[\bar f_+\bar A_k + \frac{f_-}{\eta_2}A_k\bigg]\\
&= A_i^*A_k\bigg[\frac{|f_-|^2}{|\eta_2|^2} + (\bar A_k/A_k)\frac{\bar f_+f_-^*}{\eta_2^*}\bigg] + \bar A_i^*\bar A_k\bigg[|\bar f_+|^2 + (A_k/\bar A_k)\frac{f_-\bar f_+^*}{\eta_2}\bigg]\\
&= \frac{e^{-\Gamma t}}{2}\bigg[A_i^*A_k\{\cosh(\Delta\Gamma t/2) - \cos(\Delta M t)\} + \frac{\eta_k e^{-2i\phi_k}}{2}A_i^*A_k\{2\sinh(\Delta\Gamma t/2) + 2i\sin(\Delta M t)\\
&\quad + (\mathrm{Re}\delta + i\mathrm{Im}\delta)\cosh(\Delta\Gamma t/2) - (\mathrm{Re}\delta + i\mathrm{Im}\delta)\cos(\Delta M t)\} + \bar A_i^*\bar A_k\{\cosh(\Delta\Gamma t/2) + \cos(\Delta M t)\\
&\quad + \mathrm{Re}\delta\sinh(\Delta\Gamma t/2) + \mathrm{Im}\delta\sin(\Delta M t)\} + \frac{\eta_k e^{2i\phi_k}}{2}\bar A_i^*\bar A_k\{2\sinh(\Delta\Gamma t/2) - 2i\sin(\Delta M t)\\
&\quad + (\mathrm{Re}\delta - i\mathrm{Im}\delta)\cosh(\Delta\Gamma t/2) - (\mathrm{Re}\delta - i\mathrm{Im}\delta)\cos(\Delta M t)\}\bigg].
\end{aligned}
\tag{B4}
$$

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