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# Improved QCD sum rule study of $Z_c(3900)$ as a $\bar{D}D^*$ molecular state

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In the framework of QCD sum rules, we present an improved study of our previous work [Phys. Rev. D **80**, 056004 (2009)], particularly on the $\bar{D}D^*$ molecular state, to investigate the possibility that the newly observed $Z_c(3900)$ is a $S$-wave $\bar{D}D^*$ molecular state. To ensure the quality of QCD sum rule analysis, contributions of up to dimension nine are calculated to test the convergence of operator product expansion (OPE). We find that the two-quark condensate $\langle\bar{q}q\rangle$ is very large and makes the standard OPE convergence (i.e. the perturbative at least larger than each condensate contribution) happen at very large values of Borel parameters. By releasing the rigid OPE convergence criterion, one could find that the OPE convergence is still under control. We arrive at the numerical result $3.86 \pm 0.27$ GeV for $\bar{D}D^*$, which agrees with the mass of $Z_c(3900)$ and could support the explanation of $Z_c(3900)$ in terms of a $S$-wave $\bar{D}D^*$ molecular state.

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## I. INTRODUCTION

Very recently, the BESIII Collaboration studied the process $e^+e^- \to \pi^+\pi^- J/\psi$ at a center-of-mass energy of 4.26 GeV and reported the observation of a new charged charmonium-like structure $Z_c(3900)$ in the $\pi^\pm J/\psi$ invariant spectrum with a mass of $3899.0 \pm 3.6 \pm 4.9$ MeV and a width of $46 \pm 10 \pm 20$ MeV [1]. Before the BESIII's observation, Chen *et al.* predicted that a charged charmonium-like structure is observable in the $Y(4260) \to J/\psi\pi^+\pi^-$ process [2]. In the study of $Y(4260) \to \pi^+\pi^- J/\psi$ decays, the Belle Collaboration also observed a $Z(3895)^\pm$ state with a mass of $3894.5 \pm 6.6 \pm 4.5$ MeV and a width of $63 \pm 24 \pm 26$ MeV in the $\pi^\pm J/\psi$ mass spectrum [3]. Xiao *et al.* confirmed the charged state $Z_c(3900)$ in the analysis of data taken with the CLEO-c detector at $\psi(4160)$, and measured its mass and width to be $3885 \pm 5 \pm 1$ MeV and $34 \pm 12 \pm 4$ MeV, respectively [4].

The new experimental results have aroused theorists' great interest in comprehending the internal structures of $Z_c(3900)$. Soon after the observation of $Z_c(3900)$, it was proposed that these states are $S$-wave $\bar{D}D^*$ molecules [5,6]. Subsequently, there also appeared many other works to explain these exotic states [7–13]. Undoubtedly, it is interesting and significative to investigate whether $Z_c(3900)$ could be a $S$-wave $\bar{D}D^*$ state. To understand the inner structure of $Z_c(3900)$, it is very helpful and quite needed to determine their properties, like masses, quantitatively. Nowadays, QCD is widely believed to be the true theory of describing strong interactions. However, it is quite difficult to acquire the hadron spectrum from QCD first principles. The main reason is that low-energy QCD involves a regime where it is futile to attempt perturbative calculations and the strong interaction dynamics of hadronic systems is governed by nonperturbative QCD effects completely. Meanwhile, one has limited knowledge on nonperturbative QCD aspects, for there are still many questions that remain unanswered or realized only at a qualitative level.

The method of QCD sum rules [14] is a nonperturbative formulation firmly based on the basic theory of QCD, which has been successfully applied to conventional hadrons (for reviews see Refs. [15–18] and references therein) and multiquark states (e.g. see Ref. [19]). In particular, for the $S$-wave $\bar{D}D^*$ molecular state, we have definitely predicted its mass to be $3.88 \pm 0.10$ GeV with QCD sum rules several years ago in Ref. [20], in which mass spectra of molecular states with various $\{Q\bar{q}\}\{\bar{Q}^{(\prime)}q\}$ configurations have been systematically studied. Numerically, one could see that our prediction for the mass of the $\bar{D}D^*$ state agrees well with the experimental data of the newly observed $Z_c(3900)$. That result could support the explanation of $Z_c(3900)$ as a $S$-wave $\bar{D}D^*$ molecular state. At present, we would put forward an improved study of our previous work on the $\bar{D}D^*$ state in view of the following reasons. First, it is known that one can analyze the OPE convergence and the pole contribution dominance to determine the conventional Borel window in the standard QCD sum rule approach to ensure the validity of QCD sum rule analysis. However, we find that it may be difficult to find a conventional work window rigidly satisfying both of the two rules in some recent works [21], which actually has also been discussed in some other works (e.g., Refs. [22–24]). The main reason is that some high-dimension condensates are very large and play an important role on the OPE side, which makes the standard OPE convergence happen only at very large values of Borel parameters. By contrast, in the previous Ref. [20], we merely considered contributions of the operators up to dimension six in OPE, and the Borel windows are roughly taken to hold the same values for the similar class of states for simplicity and convenience. Thus, it may be more reliable to test the OPE convergence by including higher dimension condensate contributions than six and considering the work windows minutely, and then one could more safely extract the hadronic information from QCD sum rules. Second, even if higher condensate contributions may not radically influence the character of OPE convergence in some cases, one still could attempt to improve the theoretical result, because some higher condensates are helpful to stabilize the Borel curves. Particularly for the newly observed $Z_c(3900)$ states, they cannot be simple $c\bar{c}$ conventional mesons since they are electric charged. It may be a new hint for the existence of exotic hadrons, and $Z_c(3900)$ are some ideal candidates for them. Once exotic states can be confirmed by experiment, QCD will be further tested, and then one will understand QCD low-energy behaviors more deeply. Therefore, it is of importance and worth to make meticulous theoretical efforts to reveal the underlying structures of $Z_c(3900)$. All in all, we would like to improve our previous work to investigate that whether $Z_c(3900)$ could serve as a $\bar{D}D^*$ molecular state.

The rest of the paper is organized as follows: In Sec. II, QCD sum rules for the molecular states are introduced, and both the phenomenological representation and the QCD side are derived, followed by the numerical analysis and some discussions in Sec. III. The last section is a brief summary.

## II. MOLECULAR STATE QCD SUM RULES

The starting point of the QCD sum rule method is to construct a proper interpolating current to represent the studied state. One knows that the method of QCD sum rules has been widely applied to multiquark systems since the experimental observations of many new hadrons in recent years. At present, currents of molecular states and tetraquark states could be differentiated by their different ways of construction. Concretely, molecular currents are built up with the color-singlet currents of their composed hadrons to form hadron-hadron configurations of fields, which are different from currents of tetraquark states constructed by diquark-antidiquark configurations of fields. What is necessary to note is that these two types of currents can be related to each other by Fiertz rearrangement. However, the transformation relations are suppressed by corresponding color and Dirac factors [19], and one could obtain a reliable sum rule while choosing the appropriate current to represent the physical state. This means that if the physical state is a molecular state, it would be best to choose a meson-meson type of current so that it has a large overlap with the physical state. Similarly, for a tetraquark state, it would be best to choose a diquark-antidiquark type of current. When the sum rule reproduces a mass consistent with the physical value, one can infer that the physical state has a structure well represented by the chosen current. In this way, one can indirectly and commonly discriminate between the molecular and the tetraquark structures of observed states. One can expect that these judgements could be very effective for some ideal cases, e.g. the results obtained from different types of currents are very different, so that one could easily discriminate them. Note that in some exceptional cases, the final results from molecular currents and tetraquark currents may not be very different. For example, Narison *et al.* investigated both molecular and tetraquark currents associated with $X(3872)$, and they finally gained the same mass predictions within the accuracy of the QCD sum rule method in Ref. [25]. For the present work, in order to study the possibility of $Z_c(3900)$ as a $S$-wave $\bar{D}D^*$ molecular state, we thus construct the molecular current from corresponding currents of $\bar{D}$ and $D^*$ mesons to form meson-meson configurations of fields. In the full theory, the interpolating currents for heavy $D$ mesons can be found in Ref. [26]. Therefore, one can build the following form of current:

$$j^{\mu}_{\bar{D}D^*} = (\bar{Q}_a i\gamma_5 q_a)(\bar{q}_b\gamma^{\mu}Q_b)$$

for $\bar{D}D^*$ with $J^P = 1^+$, where $q$ indicates the light $u$ or $d$ quark, $Q$ denotes the heavy $c$ quark, and the subscripts $a$ and $b$ are color indices. Note that the quantum numbers of $Z_c(3900)$ have not been given experimentally for the moment, and $1^+$ is just one possible choice of their spin parities.

One can then write down the two-point correlator

$$\Pi^{\mu\nu}(q^2) = i\int d^4x e^{iq.x}\langle 0|T[j^{\mu}_{\bar{D}D^*}(x)j^{\nu+}_{\bar{D}D^*}(0)]|0\rangle. \quad (1)$$

Lorentz covariance implies that the correlator can be generally parameterized as

$$\Pi^{\mu\nu}(q^2) = \left(\frac{q^{\mu}q^{\nu}}{q^2} - g^{\mu\nu}\right)\Pi^{(1)}(q^2) + \frac{q^{\mu}q^{\nu}}{q^2}\Pi^{(0)}(q^2). \quad (2)$$

The term proportional to $g_{\mu\nu}$ will be chosen to extract the mass sum rule. Phenomenologically, $\Pi^{(1)}(q^2)$ can be expressed as

$$\Pi^{(1)}(q^2) = \frac{[\lambda^{(1)}]^2}{M^2_{\bar{D}D^*} - q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\Pi^{(1)\mathrm{phen}}(s)}{s - q^2} + \text{subtractions}, \quad (3)$$

where $M_{\bar{D}D^*}$ denotes the mass of the $\bar{D}D^*$ state, $s_0$ is the continuum threshold parameter, and $\lambda^{(1)}$ gives the coupling of the current to the hadron $\langle 0|j^{\mu}_{\bar{D}D^*}|\bar{D}D^*\rangle = \lambda^{(1)}\epsilon^{\mu}$. On the OPE side, $\Pi^{(1)}(q^2)$ can be written as

$$\Pi^{(1)}(q^2) = \int_{4m_Q^2}^{\infty} ds \frac{\rho^{\mathrm{OPE}}(s)}{s - q^2} + \Pi_1^{\mathrm{cond}}(q^2), \quad (4)$$

where the spectral density is given by $\rho^{\mathrm{OPE}}(s) = \frac{1}{\pi}\mathrm{Im}\Pi^{(1)}(s)$. Technically, we work at leading order in $\alpha_s$ and consider condensates up to dimension nine, employing similar techniques to Refs. [27,28]. To keep the heavy quark mass finite, one can use the momentum-space expression for the heavy quark propagator [26]:

$$\begin{aligned}
S_Q(p) = {}& \frac{i}{\not{p} - m_Q} - \frac{i}{4} g t^A G^A_{\kappa\lambda}(0) \frac{1}{(p^2 - m_Q^2)^2}[\sigma_{\kappa\lambda}(\not{p} + m_Q) + (\not{p} + m_Q)\sigma_{\kappa\lambda}] \\
& - \frac{i}{4} g^2 t^A t^B G^A_{\alpha\beta}(0) G^B_{\mu\nu}(0) \frac{\not{p} + m_Q}{(p^2 - m_Q^2)^5}[\gamma^\alpha(\not{p} + m_Q)\gamma^\beta(\not{p} + m_Q)\gamma^\mu(\not{p} + m_Q)\gamma^\nu \\
& + \gamma^\alpha(\not{p} + m_Q)\gamma^\mu(\not{p} + m_Q)\gamma^\beta(\not{p} + m_Q)\gamma^\nu + \gamma^\alpha(\not{p} + m_Q)\gamma^\mu(\not{p} + m_Q)\gamma^\nu(\not{p} + m_Q)\gamma^\beta](\not{p} + m_Q) \\
& + \frac{i}{48} g^3 f^{ABC} G^A_{\gamma\delta} G^B_{\delta\varepsilon} G^C_{\varepsilon\gamma} \frac{1}{(p^2 - m_Q^2)^6}(\not{p} + m_Q)[\not{p}(p^2 - 3m_Q^2) + 2m_Q(2p^2 - m_Q^2)](\not{p} + m_Q). \qquad (5)
\end{aligned}$$

The light quark part of the correlator can be calculated in the coordinate space, with the light quark propagator

$$\begin{aligned}
S_{ab}(x) = {}& \frac{i\delta_{ab}}{2\pi^2 x^4}\not{x} - \frac{m_q \delta_{ab}}{4\pi^2 x^2} - \frac{i}{32\pi^2 x^2} t^A_{ab} g G^A_{\mu\nu}(\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\not{x}) - \frac{\delta_{ab}}{12}\langle\bar{q}q\rangle + \frac{i\delta_{ab}}{48} m_q \langle\bar{q}q\rangle\not{x} - \frac{x^2\delta_{ab}}{3\cdot 2^6}\langle g\bar{q}\sigma\cdot Gq\rangle \\
& + \frac{ix^2\delta_{ab}}{2^7\cdot 3^2} m_q \langle g\bar{q}\sigma\cdot Gq\rangle\not{x} - \frac{x^4\delta_{ab}}{2^{10}\cdot 3^3}\langle\bar{q}q\rangle\langle g^2G^2\rangle, \qquad (6)
\end{aligned}$$

which is then Fourier transformed to the momentum space in $D$ dimensions. Since the masses of light $u$ and $d$ quarks are 3 orders of magnitude less than that of the heavy $c$ quark, they are neglected here following the usual treatment. The resulting light quark part is combined with the heavy quark part before it is dimensionally regularized at $D = 4$. After equating Eqs. (3) and (4), assuming quark-hadron duality, and making a Borel transform, the sum rule can be written as

$$[\lambda^{(1)}]^2 e^{-M^2_{\bar{D}D^*}/M^2} = \int_{4m_Q^2}^{s_0} ds \rho^{\mathrm{OPE}} e^{-s/M^2} + \hat{B}\Pi_1^{\mathrm{cond}}, \qquad (7)$$

with $M^2$ the Borel parameter. Making the derivative in terms of $M^2$ to the sum rule and then dividing by itself, we have the mass of the $\bar{D}D^*$ state:

$$M^2_{\bar{D}D^*} = \left\{\int_{4m_Q^2}^{s_0} ds \rho^{\mathrm{OPE}} s e^{-s/M^2} + \frac{d\hat{B}\Pi_1^{\mathrm{cond}}}{d(-\frac{1}{M^2})}\right\} \Big/ \left\{\int_{4m_Q^2}^{s_0} ds \rho^{\mathrm{OPE}} e^{-s/M^2} + \hat{B}\Pi_1^{\mathrm{cond}}\right\}, \qquad (8)$$

where

$$\begin{aligned}
\rho^{\mathrm{OPE}}(s) = {}& \rho^{\mathrm{pert}}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle g^2G^2\rangle}(s) + \rho^{\langle g\bar{q}\sigma\cdot Gq\rangle}(s) \\
& + \rho^{\langle\bar{q}q\rangle^2}(s) + \rho^{\langle g^3G^3\rangle}(s) + \rho^{\langle\bar{q}q\rangle\langle g^2G^2\rangle}(s),
\end{aligned}$$

where $\rho^{\mathrm{pert}}$, $\rho^{\langle\bar{q}q\rangle}$, $\rho^{\langle g^2G^2\rangle}$, $\rho^{\langle g\bar{q}\sigma\cdot Gq\rangle}$, $\rho^{\langle\bar{q}q\rangle^2}$, $\rho^{\langle g^3G^3\rangle}$, and $\rho^{\langle\bar{q}q\rangle\langle g^2G^2\rangle}$ are the perturbative, two-quark condensate, two-gluon condensate, mixed condensate, four-quark condensate, three-gluon condensate, and two-quark multiply two-gluon condensate spectral densities, respectively. In fact, the spectral densities up to dimension six have been given in our previous work [20] and are also enclosed here for the paper's completeness. Concretely, the spectral densities are

$$\begin{aligned}
\rho^{\mathrm{pert}}(s) &= \frac{3}{2^{12}\pi^6}\int_{\alpha_{min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3}\int_{\beta_{\min}}^{1-\alpha}\frac{d\beta}{\beta^3}(1-\alpha-\beta)(1+\alpha+\beta)r(m_Q,s)^4, \\
\rho^{\langle\bar{q}q\rangle}(s) &= -\frac{3\langle\bar{q}q\rangle}{2^7\pi^4} m_Q \int_{\alpha_{\min}}^{\alpha_{\max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{\min}}^{1-\alpha}\frac{d\beta}{\beta}(1+\alpha+\beta)r(m_Q,s)^2, \\
\rho^{\langle g^2G^2\rangle}(s) &= \frac{\langle g^2G^2\rangle}{2^{11}\pi^6} m_Q^2 \int_{\alpha_{\min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha} d\beta(1-\alpha-\beta)(1+\alpha+\beta)r(m_Q,s), \\
\rho^{\langle g\bar{q}\sigma\cdot Gq\rangle}(s) &= \frac{3\langle g\bar{q}\sigma\cdot Gq\rangle}{2^8\pi^4} m_Q \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\left\{\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta} r(m_Q,s) - \frac{2}{1-\alpha}[m_Q^2 - \alpha(1-\alpha)s]\right\}, \\
\rho^{\langle\bar{q}q\rangle^2}(s) &= \frac{\langle\bar{q}q\rangle^2}{2^4\pi^2} m_Q^2\sqrt{1-\frac{4m_Q^2}{s}},
\end{aligned}$$

$$\rho^{\langle g^3G^3\rangle}(s) = \frac{\langle g^3G^3\rangle}{2^{13}\pi^6}\int_{\alpha_{\min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha} d\beta(1-\alpha-\beta)(1+\alpha+\beta)[r(m_Q,s)+2m_Q^2\beta],$$

$$\rho^{\langle\bar{q}q\rangle\langle g^2G^2\rangle}(s) = -\frac{\langle\bar{q}q\rangle\langle g^2G^2\rangle}{2^{11}\pi^4}m_Q\left[\sqrt{1-\frac{4m_Q^2}{s}}+4\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{\min}}^{1-\alpha}d\beta\beta(1+\alpha+\beta)\right],$$

with $r(m_Q, s)$ defined as $(\alpha+\beta)m_Q^2 - \alpha\beta s$. The integration limits are given by $\alpha_{\min} = (1-\sqrt{1-4m_Q^2/s})/2$, $\alpha_{\max} = (1+\sqrt{1-4m_Q^2/s})/2$, and $\beta_{\min} = \alpha m_Q^2/(s\alpha - m_Q^2)$. The term $\hat{B}\Pi_1^{\mathrm{cond}}$ reads

$$\begin{aligned}\hat{B}\Pi_1^{\mathrm{cond}} =& \frac{\langle\bar{q}q\rangle\langle g^2G^2\rangle}{3\cdot 2^9\pi^4}m_Q^3\int_0^1 d\alpha\left[\frac{1}{\alpha^3}\int_0^{1-\alpha}d\beta(\alpha+\beta)(1+\alpha+\beta)e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}} - \frac{1}{1-\alpha}e^{-\frac{m_Q^2}{\alpha(1-\alpha)M^2}}\right]\\
&-\frac{\langle\bar{q}q\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}{2^5\pi^2}m_Q^2\int_0^1 d\alpha\int_0^{1-\alpha}d\beta\left[1+\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}\right]e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}}\\
&+\frac{\langle g^2G^2\rangle^2}{3^2\cdot 2^{15}\pi^6}m_Q^4\int_0^1\frac{d\alpha}{\alpha^2}\int_0^{1-\alpha}\frac{d\beta}{\beta^2}(1-\alpha-\beta)(1+\alpha+\beta)\frac{1}{M^2}e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}}\\
&+\frac{\langle\bar{q}q\rangle\langle g^3G^3\rangle}{3\cdot 2^{11}\pi^4}m_Q\int_0^1\frac{d\alpha}{\alpha^4}\int_0^{1-\alpha}d\beta(1+\alpha+\beta)\left[\alpha(\alpha+6\beta)-\frac{2(\alpha+\beta)m_Q^2}{M^2}\right]e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}}\\
&+\frac{\langle g^2G^2\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}{3\cdot 2^{11}\pi^4}m_Q\int_0^1\frac{d\alpha}{\alpha^3}\left\{2\left[3\alpha(1-\alpha)-\frac{m_Q^2}{M^2}\right]e^{-\frac{m_Q^2}{\alpha(1-\alpha)M^2}}+\int_0^{1-\alpha}d\beta\left[-3\alpha\beta+(\alpha+\beta)\frac{m_Q^2}{M^2}\right]e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}}\right\},\end{aligned}$$

with $\langle\bar{q}q\rangle\langle g\bar{q}\sigma\cdot Gq\rangle$, $\langle g^2G^2\rangle^2$, $\langle\bar{q}q\rangle\langle g^3G^3\rangle$, and $\langle g^2G^2\rangle\times\langle g\bar{q}\sigma\cdot Gq\rangle$ denoting the two-quark multiply mixed condensate, four-gluon condensate, two-quark multiply three-gluon condensate, and two-gluon multiply mixed condensate, respectively.

## III. NUMERICAL ANALYSIS AND DISCUSSIONS

In this section, the sum rule [Eq. (8)] will be numerically analyzed. The input values are taken as $m_c = 1.23 \pm 0.05$ GeV, $\langle\bar{q}q\rangle = -(0.23\pm 0.03)^3$ GeV$^3$, $\langle g\bar{q}\sigma\cdot Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $m_0^2 = 0.8\pm 0.1$ GeV$^2$, $\langle g^2G^2\rangle = 0.88$ GeV$^4$, and $\langle g^3G^3\rangle = 0.045$ GeV$^6$ [16]. In the standard procedure of sum rule analysis, one should analyze the OPE convergence and the pole contribution dominance to determine the conventional Borel window for $M^2$: on the one side, the lower constraint for $M^2$ is obtained by considering that the perturbative contribution should be larger than each condensate contribution to have a good convergence on the OPE side; on the other side, the upper bound for $M^2$ is obtained by the consideration that the pole contribution should be larger than the continuum state contributions. At the same time, the threshold $\sqrt{s_0}$ is not arbitrary but characterizes the beginning of continuum states. Hence, the most expected case is that one could naturally find the conventional Borel windows for studied states to make QCD sum rules work well.

In order to test the convergence of OPE, its various contributions, i.e. the perturbative, two-quark, two-gluon, mixed, four-quark, three-gluon, two-quark multiply two-gluon, two-quark multiply mixed, four-gluon, two-quark multiply three-gluon, and two-gluon multiply mixed condensate contributions, are compared as a function of $M^2$ and shown in Fig. 1. Graphically, one could see that on the OPE side there exists a problem similar to one which has been discussed in some of our recent works [21] and others, e.g. Refs. [22–24]. Concretely, here some

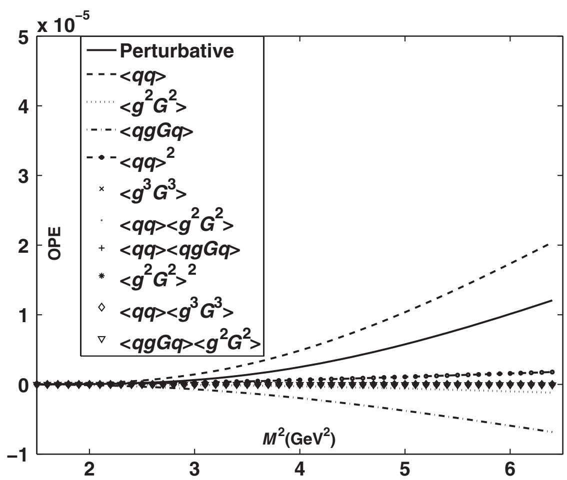


FIG. 1. The OPE contribution in the sum rule of Eq. (7) for $\sqrt{s_0} = 4.4$ GeV. The OPE convergence is shown by comparing the perturbative, two-quark condensate, two-gluon condensate, mixed condensate, four-quark condensate, three-gluon condensate, two-quark multiply two-gluon condensate, two-quark multiply mixed condensate, four-gluon condensate, two-quark multiply three-gluon condensate, and mixed multiply two-gluon condensate contributions.

condensates, especially two-quark condensate $\langle\bar{q}q\rangle$, are very large and play an important role on the OPE side, which makes the standard OPE convergence (i.e. the perturbative at least larger than each condensate contribution) happen only at very large values of $M^2$. The consequence is that it is difficult to find a conventional Borel window where both the pole dominates over the continuum and the OPE converges well. Following the similar treatment in Ref. [21], we could try releasing the rigid convergence criterion of the perturbative contribution larger than each condensate contribution here. It is not too bad for the present case—there are two main condensates, i.e. $\langle\bar{q}q\rangle$ and $\langle g\bar{q}\sigma\cdot Gq\rangle$, and they could cancel out each other to some extent since they have different signs. What is also very important is that most of other condensates calculated are very small, which means that they could not radically influence the character of OPE convergence. Therefore, one could find that the OPE convergence is still under control here. On the phenomenological side, the comparison between pole and continuum contributions of the sum rule in Eq. (7) as a function of the Borel parameter $M^2$ for the threshold value $\sqrt{s_0} = 4.4$ GeV is shown in Fig. 2, which shows that the relative pole contribution is approximate to 50% at $M^2 = 2.7\ \text{GeV}^2$ and decreases with $M^2$. Similarly, the upper bound values of Borel parameters are $M^2 = 2.6\ \text{GeV}^2$ for $\sqrt{s_0} = 4.3$ GeV and $M^2 = 2.9\ \text{GeV}^2$ for $\sqrt{s_0} = 4.5$ GeV. Thus, the Borel window for $\bar{D}D^*$ is taken as $M^2 = 2.1 \sim 2.7\ \text{GeV}^2$ for $\sqrt{s_0} = 4.4$ GeV. Similarly, the proper ranges of $M^2$ are $2.1 \sim 2.6\ \text{GeV}^2$ for $\sqrt{s_0} = 4.3$ GeV and $2.1 \sim 2.9\ \text{GeV}^2$ for $\sqrt{s_0} = 4.5$ GeV. The mass of the $\bar{D}D^*$ molecular state as a function of $M^2$ from the sum rule in Eq. (8) is shown in Fig. 3, and it is numerically calculated to be $3.86 \pm 0.13$ GeV in the above chosen work windows. Considering the uncertainty rooting in the variation of quark masses and condensates, we gain $3.86 \pm 0.13 \pm 0.14$ GeV (the first error reflects the uncertainty due to variation of $\sqrt{s_0}$ and $M^2$, and the second error results from the variation of QCD parameters), or concisely $3.86 \pm 0.27$ GeV for the $S$-wave $\bar{D}D^*$.

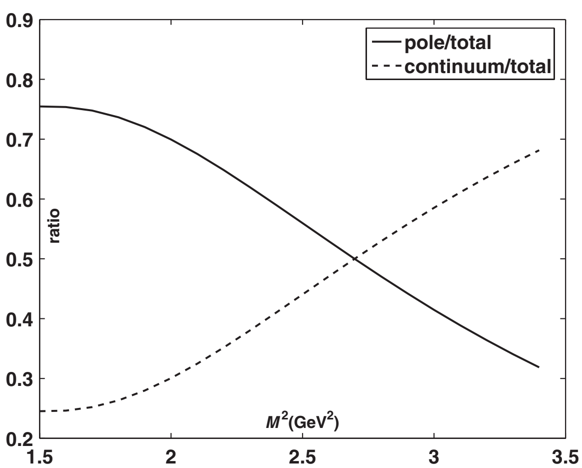


FIG. 2. The phenomenological contribution in the sum rule of Eq. (7) for $\sqrt{s_0} = 4.4$ GeV. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of $M^2$, and the dashed line is the relative continuum contribution.

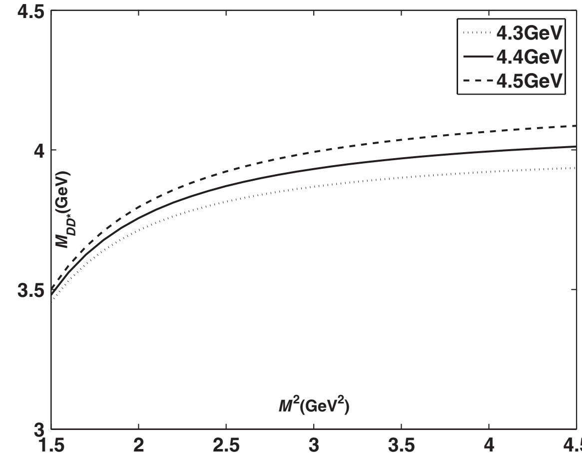


FIG. 3. The mass of the $\bar{D}D^*$ molecular state as a function of $M^2$ from the sum rule in Eq. (8). The continuum thresholds are taken as $\sqrt{s_0} = 4.3 \sim 4.5$ GeV. The ranges of $M^2$ are $2.1 \sim 2.6\ \text{GeV}^2$ for $\sqrt{s_0} = 4.3$ GeV, $2.1 \sim 2.7\ \text{GeV}^2$ for $\sqrt{s_0} = 4.4$ GeV, and $2.1 \sim 2.9\ \text{GeV}^2$ for $\sqrt{s_0} = 4.5$ GeV.

## IV. SUMMARY

Stimulated by the newly observed charged charmonium-like structure $Z_c(3900)$, which cannot be simple $c\bar{c}$ conventional mesons and presents some ideal candidates for exotic hadrons, we present an improved QCD sum rule study of our previous work on the $\bar{D}D^*$ molecular state to investigate whether it could be a $S$-wave $\bar{D}D^*$ molecular state. In order to ensure the quality of QCD sum rule analysis, contributions of up to dimension nine are calculated to test the convergence of OPE. We find that some condensates in particular $\langle\bar{q}q\rangle$ play an important role and make the standard OPE convergence (i.e. the perturbative at least larger than each condensate contribution) happen at very large values of Borel parameters $M^2$. By releasing the rigid OPE convergence criterion, one could find that the OPE convergence is still under control, and the final result $3.86 \pm 0.27$ GeV is obtained for the $S$-wave $\bar{D}D^*$ molecular state, which coincides with the experimental data of $Z_c(3900)$. From the final result, one could assuredly state that it could provide some support to the $\bar{D}D^*$ molecular explanation of $Z_c(3900)$. At the same time, one should note that the $\bar{D}D^*$ molecular state is just one possible theoretical interpretation of $Z_c(3900)$, and it does not mean that one could arbitrarily exclude some other possible explanations (e.g. tetraquark states) at the present time just from the result here. In fact, more minute information on the nature structures of $Z_c(3900)$ could be revealed by the future contributions of both experimental observations and theoretical studies.

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*Note added.*—A recent paper from our colleague also analyzes $Z_c(3900)$ as a $\bar{D}D^*$ molecular state with QCD sum rules [29], but then they consider contributions up to the same dimension (six) as our previous work [20].

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