

Radiative natural supersymmetry: Reconciling electroweak fine-tuning and the Higgs boson massHoward Baer,^{1,*} Vernon Barger,^{2,†} Peisi Huang,^{2,‡} Dan Mickelson,^{1,§} Azar Mustafayev,^{3,||} and Xerxes Tata^{3,¶}¹*Department of Physics and Astronomy, University of Oklahoma, Norman, Oklahoma 73019, USA*²*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706, USA*³*Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822, USA*

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Models of natural supersymmetry seek to solve the little hierarchy problem by positing a spectrum of light Higgsinos $\lesssim 200\text{--}300$ GeV and light top squarks $\lesssim 600$ GeV along with very heavy squarks and TeV-scale gluinos. Such models have low electroweak fine-tuning and satisfy the LHC constraints. However, in the context of the minimal supersymmetric standard model, they predict too low a value of m_h , are frequently in conflict with the measured $b \rightarrow s\gamma$ branching fraction, and the relic density of thermally produced Higgsino-like weakly interacting massive particles (WIMPs) falls well below dark matter measurements. We propose a framework dubbed *radiative natural supersymmetry* (RNS), which can be realized within the minimal supersymmetric standard model (avoiding the addition of extra exotic matter) and which maintains features such as gauge coupling unification and radiative electroweak symmetry breaking. The RNS model can be generated from supersymmetry (SUSY) grand unified theory type models with nonuniversal Higgs masses. Allowing for high-scale soft SUSY breaking Higgs mass $m_{H_u} > m_0$ leads to *automatic cancellations* during renormalization group running and to *radiatively-induced low fine-tuning* at the electroweak scale. Coupled with large mixing in the top-squark sector, RNS allows for fine-tuning at the 3%–10% level with TeV-scale top squarks and a 125 GeV light Higgs scalar h . The model allows for at least a partial solution to the SUSY flavor, CP , and gravitino problems since first-/second-generation scalars (and the gravitino) may exist in the 10–30 TeV regime. We outline some possible signatures for RNS at the LHC, such as the appearance of low invariant mass opposite-sign isolated dileptons from gluino cascade decays. The smoking gun signature for RNS is the appearance of light Higgsinos at a linear e^+e^- collider. If the strong CP problem is solved by the Peccei-Quinn mechanism, then RNS naturally accommodates mixed axion-Higgsino cold dark matter, where the light Higgsino-like WIMPs—which in this case make up only a fraction of the measured relic abundance—should be detectable at upcoming WIMP detectors.

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I. INTRODUCTION

The recent discovery by Atlas and CMS of a Higgs-like resonance at the CERN LHC [1,2] adds credence to supersymmetric (SUSY) models of particle physics in that the mass value $m_h \approx 125$ GeV falls squarely within the narrow window predicted by the minimal supersymmetric standard model (MSSM): $m_h \sim 115\text{--}135$ GeV [3]. At the same time, the lack of a SUSY signal at LHC7 and LHC8 implies $m_{\tilde{g}} \gtrsim 1.4$ TeV (for $m_{\tilde{g}} \sim m_{\tilde{q}}$) and $m_{\tilde{g}} \gtrsim 0.9$ TeV (for $m_{\tilde{g}} \ll m_{\tilde{q}}$) [4,5]. While weak scale SUSY [6] provides a solution [7] to the gauge hierarchy problem via the cancellation of quadratic divergences, the apparently multi-TeV sparticle masses required by LHC searches seemingly exacerbate *the little hierarchy problem*: How do multi-TeV values of SUSY model parameters conspire to yield a Z boson (Higgs boson) mass of just 91.2 (125) GeV?

Models of *natural supersymmetry* [8] address the little hierarchy problem by positing a spectrum of light Higgsinos $\lesssim 200$ GeV and light top and bottom squarks with $m_{\tilde{t}_{1,2}, \tilde{b}_1} \lesssim 600$ GeV along with very heavy first-/second-generation squarks and TeV-scale gluinos [9–11]. Such a spectrum allows for low electroweak fine-tuning (EWFT) while at the same time keeping sparticles safely beyond LHC search limits. Because third-generation scalars are in the few hundred GeV range, the radiative corrections to m_h , which increase logarithmically with $m_{\tilde{t}_i}^2$, are never very large, and these models have great difficulty in accommodating a light SUSY Higgs scalar with mass $m_h \sim 125$ GeV [11,12]. Thus, we are faced with a new conundrum: How does one reconcile low EWFT with such a large value of m_h [13]? A second problem occurs in that the predicted branching fraction for $b \rightarrow s\gamma$ decay is frequently at odds with the measured value due to the very light third-generation squarks [11]. A third issue appears in that the light Higgsino-like weakly interacting massive particles (WIMPs) predicted by models of natural SUSY lead to a thermally generated relic density, which is typically a factor of 10–15 below [11,14] the WMAP measured value of $\Omega_{\text{CDM}} h^2 \approx 0.11$.

*baer@nhn.ou.edu

†barger@pheno.wisc.edu

‡phuang7@wisc.edu

§mickelso@nhn.ou.edu

||azar@phys.hawaii.edu

¶tata@phys.hawaii.edu

One solution to the fine-tuning/Higgs problem is to add extra fields to the theory, thus moving beyond the MSSM [13]. For example, adding an extra singlet as in the next-to-minimal supersymmetric standard model (NMSSM) permits a new quartic coupling in the Higgs potential, thus allowing for an increased value of m_h [15]. Alternatively, one may add extra vectorlike matter to increase m_h while maintaining light top squarks [16]. In the former case of the NMSSM, adding extra gauge singlets may lead to the reintroduction of destabilizing divergences into the theory [17]. In the latter case, one might wonder about the *ad hoc* introduction of extra weak scale matter multiplets and how they might have avoided detection. A third possibility, which is presented below, is to reexamine EWFT and to ascertain if there do indeed exist sparticle spectra *within the MSSM* that lead to $m_h \sim 125$ GeV while maintaining modest levels of electroweak fine-tuning.

A. Electroweak fine-tuning

One way to evaluate EWFT in SUSY models is to examine the minimization condition from the Higgs sector scalar potential, which determines the Z -boson mass. (Alternatively, one may examine the mass formula for m_h and arrive at similar conclusions.) Minimization of the one-loop effective potential $V_{\text{tree}} + \Delta V$ leads to

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2, \quad (1.1)$$

where Σ_u^u and Σ_d^d are radiative corrections that arise from the derivatives of ΔV evaluated at the minimum. Equation (1.1) reduces to the familiar tree-level expression [6] for M_Z^2 when radiative correction terms are ignored. As we will discuss in detail below, Σ_u^u and Σ_d^d include contributions, listed in the appendix, from various particles and sparticles with sizeable Yukawa and/or gauge couplings to the Higgs sector. To obtain a *natural* value of M_Z on the left-hand side, one would like each term C_i [with $i = H_d, H_u, \mu$ as well as $\Sigma_u^u(k), \Sigma_d^d(k)$, where k denotes the various contributions to the Σ s that we just mentioned] on the right-hand side to have an absolute value of order $M_Z^2/2$. Noting that all entries in Eq. (1.1) are defined at the weak scale, we are led to define the *electroweak fine-tuning parameter*¹ by [20]

$$\Delta_{\text{EW}} \equiv \max_i (C_i)/(M_Z^2/2), \quad (1.2)$$

where $C_{H_u} = |-m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)|$, $C_{H_d} = |m_{H_d}^2/(\tan^2\beta - 1)|$, and $C_\mu = |-\mu^2|$, along with analogous

¹Barbieri and Giudice [18] (and, even earlier, Ellis *et al.* [19]) define a fine-tuning measure $\Delta_{\text{BG}} = \max |(a_i/M_Z^2)\partial M_Z^2/\partial a_i|$ for input parameters a_i . Our definition coincides with theirs when M_Z^2 depends linearly on input parameters (such as $\mu^2, m_{H_u}^2$, or $m_{H_d}^2$ using electroweak scale parameters) but differs when the parameter dependence is nonlinear. For electroweak scale parameters, the nonlinear dependence only occurs in the radiative correction terms Σ_u^u and Σ_d^d and in $\tan\beta$.

definitions for $C_{\Sigma_u^u(k)}$ and $C_{\Sigma_d^d(k)}$. Low Δ_{EW} means less fine-tuning. Since C_{H_d} and $C_{\Sigma_d^d(k)}$ terms are suppressed by $\tan^2\beta - 1$, for even moderate $\tan\beta$ values, this expression reduces approximately to

$$\frac{M_Z^2}{2} \simeq -(m_{H_u}^2 + \Sigma_u^u) - \mu^2. \quad (1.3)$$

We see that to get low Δ_{EW} , we require $|-m_{H_u}^2| \sim M_Z^2/2$ and $\mu^2 \sim M_Z^2/2$. The question then arises: What is the model, and can we find a set of model parameters such that $\Delta_{\text{EW}} \sim 1-30$, corresponding to better than $\Delta_{\text{EW}}^{-1} = 3\%$ EWFT? Note that Δ_{EW} depends only on the weak scale parameters of the theory and hence is essentially fixed by the particle spectrum, independent of how superpartner masses arise.

To understand how the underlying framework for superpartner masses may be relevant, consider a model with input parameters defined at some high scale $\Lambda \gg M_{\text{SUSY}}$, where M_{SUSY} is the SUSY breaking scale ~ 1 TeV and Λ may be as high as M_{GUT} or even the reduced Planck mass M_P . Then

$$m_{H_u}^2(M_{\text{SUSY}}) = m_{H_u}^2(\Lambda) + \delta m_{H_u}^2, \quad (1.4)$$

where

$$\delta m_{H_u}^2 \simeq -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \log\left(\frac{\Lambda}{M_{\text{SUSY}}}\right). \quad (1.5)$$

Requiring $\delta m_{H_u}^2 \leq \Delta \times \frac{m_{H_u}^2}{2}$ then leads for $m_h = 125$ GeV to

$$\sqrt{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2} \lesssim 600 \text{ GeV} \frac{\sin\beta}{\sqrt{1+R_t^2}} \left(\frac{\log\frac{\Lambda}{\text{TeV}}}{3}\right)^{-1/2} \left(\frac{\Delta}{5}\right)^{1/2}, \quad (1.6)$$

where $R_t = A_t/\sqrt{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}$. Taking $\Delta = 10$ and Λ as low as 20 TeV corresponds to [8–10]:

- (i) $|\mu| \lesssim 200$ GeV,
- (ii) $m_{\tilde{t}_i}, m_{\tilde{b}_1} \lesssim 600$ GeV,
- (iii) $m_{\tilde{g}} \lesssim 1.5-2$ TeV.

The last of these conditions arises because the squark radiative corrections $\delta m_{\tilde{t}_i}^2 \sim (2g_s^2/3\pi^2)m_{\tilde{g}}^2 \times \log\Lambda$. Setting the log to unity and requiring $\delta m_{\tilde{t}_i}^2 < m_{\tilde{t}_i}^2$ then implies $m_{\tilde{g}} \lesssim 3m_{\tilde{t}_i}$, or $m_{\tilde{g}} \lesssim 1.5-2$ GeV for $\Delta \lesssim 10$. Taking Λ as high as M_{GUT} leads to even tighter constraints: $m_{\tilde{t}_{1,2}}, m_{\tilde{b}_1} \lesssim 200$ GeV and $m_{\tilde{g}} \lesssim 600$ GeV, almost certainly in violation of LHC sparticle search constraints. Since (degenerate) first-/second-generation squarks and sleptons enter into Eq. (1.1) only at the two-loop level, these can be much heavier, beyond the LHC reach and also possibly heavy enough to provide a (partial) decoupling solution to the SUSY flavor and CP problems. In gravity mediation where $m_{\tilde{q}} \sim m_{3/2}$, then one also solves the

cosmological gravitino problem [21,22], and in grand unified theories (GUTs) one also suppresses proton decay. Then we may also have

$$(i) \ m_{\tilde{q},\tilde{\ell}} \sim 10\text{--}50 \text{ TeV}.$$

The generic natural SUSY (NS) solution reconciles the lack of a SUSY signal at the LHC with allowing for electroweak naturalness. It also predicts that the $\tilde{t}_{1,2}$ and \tilde{b}_1 may soon be accessible to LHC searches. New limits from direct top- and bottom-squark pair production searches, interpreted within the context of simplified models, have begun to bite into the NS parameter space [23]. Of course, if $m_{\tilde{t}_{1,2}}, m_{\tilde{b}_1} \simeq m_{\tilde{z}_1}$, then the visible decay products from stop and sbottom production will be soft and difficult to see at the LHC.

A more worrisome problem comes from the newly discovered value of the Higgs mass $m_h \simeq 125 \text{ GeV}$. In the MSSM, one obtains [3] (assuming that the t squarks are not very split)

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3g^2}{8\pi^2} \frac{m_t^4}{m_W^2} \left[\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} \left(1 - \frac{X_t^2}{12m_{\tilde{t}}^2} \right) \right], \quad (1.7)$$

where $X_t = A_t - \mu \cot \beta$ and $m_{\tilde{t}}^2 \simeq m_{Q_3} m_{U_3}$. For a given $m_{\tilde{t}}^2$, this expression is maximal for large mixing in the top-squark sector with $X_t^{\max} = \sqrt{6} m_{\tilde{t}}$. With top-squark masses below about 500 GeV, the radiative corrections to m_h are not large enough to yield $m_h \simeq 125 \text{ GeV}$ even with maximal mixing [11,13]. This situation has been used to argue that additional multiplets beyond those of the MSSM must be present in order to raise up m_h while maintaining very light third-generation squarks [13]. Added to these are the two issues mentioned earlier: 1) the very light third-generation squarks [11] endemic to NS lead to a predicted branching fraction for $b \rightarrow s\gamma$ decay, which is frequently much lower than the measured value [11], and 2) the relic abundance of Higgsino-like WIMPs inherent in NS, calculated in the standard MSSM-only cosmology, is typically a factor of 10–15 below measured values [11]. These issues have led to increasing skepticism of weak scale SUSY as realized in the natural SUSY incarnation described above.

A possible resolution to the above issues associated with a NS spectrum is to simply invoke a SUSY particle spectrum at the weak scale (or some other nearby scale [24]), as in the phenomenological MSSM model [25] so that large logarithms associated with a high value of Λ are absent. In this case, $\Lambda \sim M_{\text{SUSY}}$ and $\delta m_{H_u}^2$ is not enhanced by large logarithms and we may select parameters $m_{H_u}^2 \sim \mu^2 \sim M_Z^2 \sim m_h^2$. Of course, heavy top squarks are needed to obtain the observed value of m_h . While a logical possibility, this solution loses several attractive features of models which are valid up to scales as high as $\Lambda \sim M_{\text{GUT}}$, such as gauge coupling unification and radiative electroweak symmetry breaking driven by a large top-quark mass.

Another alternative is to use Δ_{EW} defined above as a fine-tuning measure even for models defined at the high scale. This use of weak scale parameters to define the fine-tuning criterion is a *weaker condition* since it allows for possible cancellations in Eq. (1.4). Indeed this is precisely what happens in what is known as the hyperbolic branch or focus point region (HB/FP) of minimal supergravity model (mSUGRA) [26]: $m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 \sim m_{H_u}^2(M_{\text{SUSY}}) \sim \mu^2 \sim M_Z^2$. The HB/FP region of mSUGRA occurs, however, only for small values of A_0/m_0 [27] and yields $m_h < 120 \text{ GeV}$, well below the Atlas/CMS measured value of $m_h \simeq 125 \text{ GeV}$. Scans over parameter space show that the HB/FP region is nearly excluded if one requires both low $|\mu|$ and $m_h \sim 123\text{--}127 \text{ GeV}$ [27,28].

To obtain a viable high-scale model, we see that we clearly need to go beyond mSUGRA. The small value of $-m_{H_u}^2(M_{\text{SUSY}})$ that we require for low EWFT can be obtained in several ways. For instance, we could introduce nonuniversality of gaugino masses and adopt very high GUT scale values of the $SU(2)$ gaugino mass parameters [29] or a low value of the $SU(3)$ gaugino mass parameter [30]. Both choices would lead to a larger chargino to gluino mass ratio than in models with universal gaugino masses [31] and, since charginos couple directly to the Higgs sector, potentially significant contributions to the radiative corrections for gluinos that satisfy the LHC bound. The other way of obtaining small values of $-m_{H_u}^2(M_{\text{SUSY}})$ without undue cancellations in Eq. (1.1) is to introduce nonuniversality in the scalar Higgs sector. To facilitate our analysis, we use the two parameter nonuniversal Higgs mass (NUHM2) extension [32] of the mSUGRA model where $m_{H_u}^2(M_{\text{GUT}})$ and $m_{H_d}^2(M_{\text{GUT}})$, or, equivalently, the weak scale parameters μ and m_A , are chosen independently of matter scalar mass parameters, and the model is completely specified by the parameter set,

$$m_0, m_{1/2}, \mu, m_A, A_0, \tan \beta \quad (\text{NUHM2}). \quad (1.8)$$

Modest electroweak fine-tuning is then obtained due to large cancellations between $m_{H_u}^2(\Lambda = M_{\text{GUT}})$ and $\delta m_{H_u}^2$.

Along with $-m_{H_u}^2(M_{\text{SUSY}}) \sim M_Z^2/2$, low Δ_{EW} also requires $\mu^2 \sim M_Z^2/2$. In gravity-mediated SUSY breaking models—where the μ problem is solved by the Giudice-Masiero mechanism [33]—one expects that $\mu \sim \lambda m_{3/2}$, where λ is a hidden-visible sector coupling. For small values of λ , then we expect $|\mu| \ll m_{3/2}$, which is then significantly smaller than the typical soft-SUSY breaking (SSB) masses.

Later in our analysis, we will also allow for the possibility of split matter generations where the third-generation mass parameter $m_0(3)$ is independent from the corresponding parameter $m_0(1, 2)$ for the first two generations. We refer to this case as the NUHM3 model. The NUHM3 model allows for an improved decoupling solution to the SUSY flavor problem. Finally, we mention that a small magnitude of μ is also possible in the one-parameter

nonuniversal Higgs model (NUHM1) where we take the two GUT scale Higgs mass parameters to have a common value m_ϕ^2 which can be raised above m_0 until $m_{H_u}^2(M_{\text{SUSY}})$ becomes comparable to M_Z^2 [34]. In this case, m_A is of course determined. We consider this case briefly. In all cases, intrageneration splitting is avoided since, as noted in Ref. [27], it can lead to large fine-tuning if scalars are very heavy. We emphasize that Δ_{EW} is determined by physical sparticle masses and couplings so that *our results can be applied to any model that yields a similar spectrum*, irrespective of how sparticles acquire their SUSY breaking mass.

At this point the reader may legitimately wonder about the validity of using Δ_{EW} as a measure of fine-tuning in the NUHM2 model which is, by construction, assumed to be a description of physics up to energy scales as large as M_{GUT} . The introduction of the Δ_{HS} in Ref. [27] was precisely to include the impact of these large logarithms—which also appear in Eq. (1.6)—on the fine-tuning. The use of Δ_{EW} as a fine-tuning measure allows for the possibility of large cancellation between $m_{H_{u,d}}^2(\Lambda)$ and the term $\delta m_{H_{u,d}}^2$ (which may include large logarithms). For instance, special regions of parameter space of some models (e.g., focus-point SUSY, the mixed-modulus-anomaly-mediation model for special values of the ratio α [35], or particular regions of parameter space of nonuniversal Higgs mass models) enjoy nearly complete cancellations between the terms with large logarithms and $m_{H_{u,d}}^2(\Lambda)$. In a more encompassing framework that includes the origin of soft SUSY breaking parameters, such cancellations might not only be allowed but might be automatic [35,36]. We note, however, that Δ_{HS} , as we have defined it, does *not* take such a cancellation into account; under these circumstances Δ_{EW} is the appropriate fine-tuning measure to use.

The fine-tuning measure Δ_{EW} introduced in Refs. [20,27] has several attractive features that merit consideration:

- (i) *Model independent* (within the context of models which reduce to the MSSM at the weak scale): Δ_{EW} is essentially determined by the sparticle spectrum [27] and—unlike Δ_{HS} and other measures of fine-tuning—does not depend on the mechanism by which sparticles acquire masses. Since Δ_{EW} is determined only from weak scale Lagrangian parameters, the phenomenological consequences which may be derived by requiring low Δ_{EW} will apply not only for the NUHM2 model considered here but also for other possibly more complete (or less complete, such as phenomenological MSSM) models which lead to look-alike spectra at the weak scale.
- (ii) *Conservative*: Δ_{EW} captures the minimal fine-tuning that is necessary for any given sparticle spectrum and so leads to the *most conservative conclusions* regarding fine-tuning considerations.

- (iii) *Measureable*: Δ_{EW} is in principle measurable in that it can be evaluated if the underlying weak scale parameters can be extracted from data.
- (iv) *Unambiguous*: Fine-tuning measures which depend on high-scale parameter choices, such as the Barbieri-Guidice measure Δ_{BG} discussed previously, are highly sensitive to exactly which set of model input parameters one adopts; for example, it is well-known that significantly different values of Δ_{BG} result depending on whether the high-scale top-Yukawa coupling is or is not included as an input parameter [37]. There is no such ambiguity in the fine-tuning sensitivity as measured by both Δ_{EW} and Δ_{HS} .
- (v) *Predictive*: While Δ_{EW} is less restrictive than Δ_{HS} , it still remains highly restrictive. The requirement of low Δ_{EW} highly disfavors models such as mSUGRA/constrained MSSM (CMSSM) [27], while allowing for very distinct predictions from more general models such as NUHM2.
- (vi) *Falsifiable*: The most important prediction from requiring low Δ_{EW} is that $|\mu|$ cannot be too far removed from M_Z . This implies the existence of light Higgsinos $\sim 100\text{--}300$ GeV, which are hard to see at hadron colliders but which are easily detected at a linear e^+e^- collider with $\sqrt{s} \gtrsim 2|\mu|$. If no Higgsinos appear at ILC1000, then the idea of electroweak naturalness in SUSY models is dead.
- (vii) *Simple to calculate*: Δ_{EW} is extremely simple to encode in sparticle mass spectrum programs, even if one adopts models with very large numbers of input parameters.

To illustrate how a low value of $m_{H_u}^2(M_{\text{SUSY}})$ is obtained, in Fig. 1 we show the running of various SUSY parameters vs the renormalization scale Q for the radiative natural SUSY (RNS)2 benchmark point from Ref. [20]. The RNS2 point has parameters $m_0 = 7025$ GeV, $m_{1/2} = 568.3$ GeV, $A_0 = -11426.6$ GeV, $\tan \beta = 8.55$ with $\mu = 150$ GeV, and $m_A = 1000$ GeV. The gaugino and matter scalar mass parameters evolve from $m_{1/2}$ and m_0 to their weak scale values, resulting in a pattern of masses very similar to that in mSUGRA. The parameter μ hardly evolves, and for such a low value of $\tan \beta$, $m_{H_d}^2$ also suffers little evolution. Of most interest to us here is the renormalization group (RG) evolution of $m_{H_u}^2$. As is well-known, the SUSY breaking parameters $m_{Q_3}^2$, $m_{U_3}^2$, and $m_{H_u}^2$ of the scalar fields that couple via the large top-quark Yukawa coupling are driven down with reducing values of the scale Q . The reduction is the greatest for $m_{H_u}^2$, which, in fact, is driven negative, triggering the radiative breakdown of electroweak symmetry [38]. We see from the figure that the weak scale value of $-m_{H_u}^2$ has a magnitude $\sim M_Z^2$ and is much smaller than the weak scale value of other mass parameters. This is not an accident because *the NUHM2 model provides us the flexibility to adjust the GUT scale value of $m_{H_u}^2$ so that it barely runs to negative values at the*

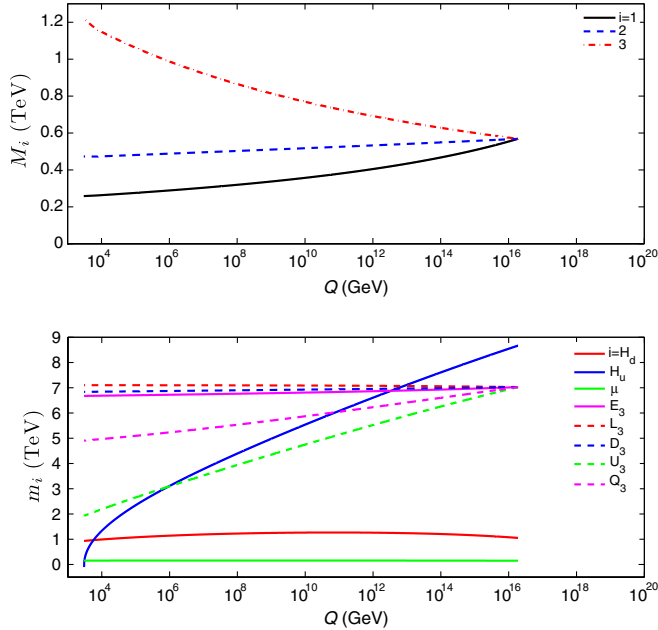


FIG. 1 (color online). Evolution of SSB parameters from M_{GUT} to M_{weak} for the RNS2 benchmark point taken from Ref. [20] whose parameters are given in the text. The graph extends to values below $Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$ where the Higgs mass parameters are extracted.

weak scale. Since $m_{H_u}^2$ is driven radiatively to $\sim -M_Z^2$ at the weak scale, this scenario has been dubbed *radiative natural SUSY*.

In Fig. 2 we scan over parameter space of the NUHM2 model—while enforcing $m_h = 125 \pm 2$ GeV and LHC sparticle mass limits—and plot the value of Δ_{HS} vs the high-scale matter scalar mass parameter m_0 . We see that the smallest value of Δ_{HS} is 10^3 for the lowest values of m_0

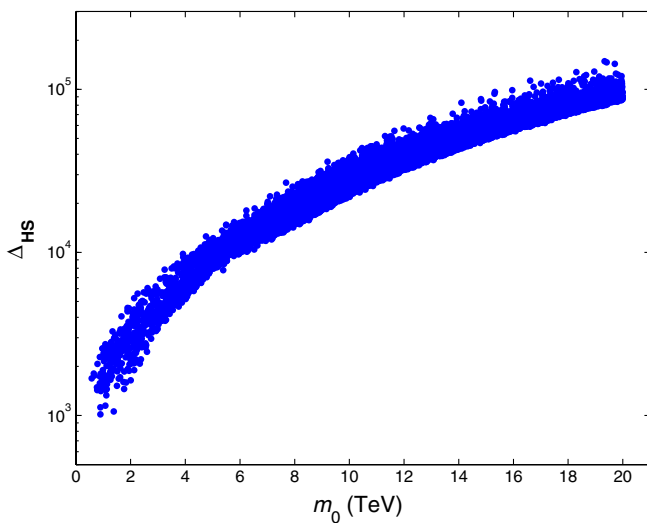


FIG. 2 (color online). Plot of Δ_{HS} vs m_0 from a scan over NUHM2 model parameters, accepting only points where $m_h = 125 \pm 2$ GeV and which obey LHC sparticle mass constraints.

allowed. This is because of the large logarithms that we discussed above. The NUHM2 would be fine-tuned to at least 0.1%, and usually even higher fine-tuning is necessary. We will see below that much smaller values of Δ_{EW} are possible in some parts of parameter space. An underlying high-scale theory that automatically leads to a NUHM2-like spectrum in this parameter region would then not be so fine-tuned. In contrast, there are no analogous regions of mSUGRA/CMSSM parameters for which we have shown that $\Delta_{\text{EW}} \geq 100$ [27].

In the remainder of this paper, we will explore what parameter choices lead to low values of Δ_{EW} . While Δ_{EW} seems bounded from below by about 100 in mSUGRA/CMSSM [27], we will find that Δ_{EW} as low as ~ 10 can be obtained in NUHM2. In addition, requiring low $\Delta_{\text{EW}} \lesssim 30$ places strong restrictions on the allowed sparticle mass spectra, leading to distinctive predictions for collider and dark matter searches.

B. Radiative natural supersymmetry

Motivated by the possibility of cancellations occurring in $m_{H_u}^2(M_{\text{SUSY}})$, we return to the electroweak symmetry breaking minimization condition (1.1), which was introduced earlier, and examine more carefully the radiative corrections embodied in Σ_u^u and Σ_d^d that we have not discussed up to now. These affect the minimization condition in an important way when $m_{H_u}^2(M_{\text{SUSY}})$ and μ^2 are much smaller than the scale of other weak scale SUSY breaking parameters. At the one-loop level, Σ_u^u contains the contributions [39,40] $\Sigma_u^u(\tilde{t}_{1,2})$, $\Sigma_u^u(\tilde{b}_{1,2})$, $\Sigma_u^u(\tilde{\tau}_{1,2})$, $\Sigma_u^u(\tilde{W}_{1,2})$, $\Sigma_u^u(\tilde{Z}_{1-4})$, $\Sigma_u^u(h, H)$, $\Sigma_u^u(H^\pm)$, $\Sigma_u^u(W^\pm)$, $\Sigma_u^u(Z)$, and $\Sigma_u^u(t)$. Σ_d^d contains similar terms along with $\Sigma_d^d(b)$ and $\Sigma_d^d(\tau)$ while $\Sigma_d^d(t) = 0$. The complete set of one-loop contributions to these is listed in the appendix. There are additional contributions from first-/second-generation sparticles from their D -term couplings to Higgs scalars. If these squarks, and separately sleptons, are degenerate, then these contributions cancel within each generation because the sum of weak isospins/hypercharges of squarks/sleptons totals zero [27]. In the parameter space region where RNS is realized, i.e., where $-m_{H_u}^2(M_{\text{SUSY}}) \sim \mu^2 \sim M_Z^2$, the radiative correction terms from Σ_u^u may give the largest contributions to Δ_{EW} .

The largest of the Σ_u^u terms almost always comes from top squarks, for which we find

$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \times \left[f_t^2 - g_Z^2 + \frac{f_t^2 A_t^2 - 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_W)\Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right], \quad (1.9)$$

where $\Delta_t = (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)/2 + M_Z^2 \cos 2\beta(\frac{1}{4} - \frac{2}{3}x_W)$, $g_Z^2 = (g^2 + g'^2)/8$, $x_W \equiv \sin^2 \theta_W$, and $F(m^2) = m^2(\log(m^2/Q^2) - 1)$, with $Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$. In Ref. [20] it is shown that

for the case of the \tilde{t}_1 contribution, as $|A_t|$ gets large there is a suppression of $\Sigma_u^u(\tilde{t}_1)$ due to a cancellation between terms in the square brackets of Eq. (1.9). The \tilde{t}_2 contribution is suppressed if there is a sizeable splitting between $m_{\tilde{t}_2}$ and $m_{\tilde{t}_1}$ due to a large cancellation within $F(m_{\tilde{t}_2}^2)$ because $\log(m_{\tilde{t}_2}^2/Q^2) = \log(m_{\tilde{t}_2}/m_{\tilde{t}_1}) \simeq 1$. The large $|A_t|$ values suppress both top-squark contributions to Σ_u^u and at the same time lift up the value of m_h , which is near maximal for large negative A_t . Combining all effects, one sees that the same mechanism responsible for boosting the value of m_h into accord with LHC measurements can also suppress the Σ_u^u contributions to EWFT, leading to a model with low EWFT.

To display the quality of EWFT explicitly, we show in Fig. 3(a) the various *signed* contributions to $M_Z^2/2$ that enter Eq. (1.1) for the RNS2 point from Fig. 1 and Ref. [20]. In this figure, we label these signed contributions by C_i where $i = H_u, H_d, \mu, \Sigma_u^u, \Sigma_d^d$. The largest contributions come from $C_{\Sigma_u^u} \sim 0.04 \text{ TeV}^2$ and $C_{H_u} \sim -0.03 \text{ TeV}^2$. In Fig. 3(b) we show these same quantities for the mSUGRA model (where μ and m_A are outputs instead of input parameters). Here, the maximal contributions $C_{H_u} \sim 15 \text{ TeV}^2$ and $C_\mu \sim -15 \text{ TeV}^2$. Figure 3(c) compares results from the two models using a common scale. Here, it is clearly seen that the mSUGRA model is enormously fine-tuned compared to the RNS2 benchmark point.

Our goal in this paper is to provide a rather complete characterization of radiative natural SUSY. This should provide a comprehensive picture as to where in model parameter space we can find 1) $m_h \sim 125 \text{ GeV}$ along with 2) low EWFT $\Delta_{\text{EW}} \lesssim 30$ while at the same time 3) respecting LHC constraints on sparticle masses. With this goal in

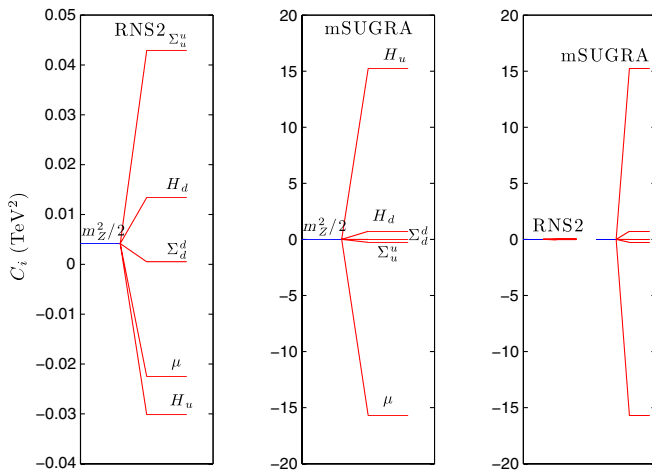


FIG. 3 (color online). Signed contributions to $M_Z^2/2$ from terms in the electroweak symmetry breaking minimization condition Eq. (1.1) from a) the RNS2 benchmark point defined in the text and b) the corresponding mSUGRA model as RNS2 with μ and m_A as outputs rather than inputs. In frame c), the results for both models are plotted on a common scale.

mind, in Sec. II we show parameter space regions leading to low Δ_{EW} from scans over the 2-parameter nonuniversal Higgs model NUHM2, which allow for radiative natural SUSY. In Sec. III we extend the results to include the split generation nonuniversal Higgs model NUHM3, wherein high-scale third-generation scalar masses $m_0(3)$ need not equal first-/second-generation scalar masses $m_0(1, 2)$. While the former implementation allows for fewer parameters, the additional freedom in the NUHM3 model allows for a more robust decoupling solution to the SUSY flavor and CP problems because heavier multi-TeV first-/second-generation sfermion masses are then possible. In Sec. IV we show that constraints from B physics—especially $\text{BF}(b \rightarrow s\gamma)\nu$ —are much more easily respected in RNS than in generic NS models. In Sec. V we discuss prospects for detecting RNS at the LHC. We also show the new RNS m_0 vs $m_{1/2}$ parameter plane, which offers a template for future searches for RNS at the LHC. Searches for RNS at the ILC are discussed in Sec. VI while direct and indirect detection of Higgsino-like WIMPs is discussed in Sec. VII. In an appendix, we present formulas needed for the implementation of our measure of electroweak fine-tuning Δ_{EW} .

II. RADIATIVE NATURAL SUSY FROM THE NUHM MODELS

The direct supersymmetrization of the standard model—augmented by weak scale soft supersymmetry breaking terms—leads to the MSSM. Since the mass scale of the MSSM is stable to radiative corrections even when the MSSM is embedded into a high-scale framework, it is tempting to speculate that the MSSM arises as the low energy limit of an underlying SUSY grand unified theory with a unification scale $M_{\text{GUT}} \simeq 2 \times 10^{16} \text{ GeV}$. Indeed, the MSSM [possibly with additional gauge singlets and/or additional complete $SU(5)$ multiplets] receives some indirect support from experiment in that 1) the measured weak scale gauge couplings nearly unify at M_{GUT} under MSSM RG evolution, 2) radiative corrections due to the large top-quark Yukawa coupling—consistent with $m_t \sim 173 \text{ GeV}$ —dynamically break electroweak symmetry, and 3) a light standard model (SM)-like Higgs boson has been discovered to be lying squarely within the narrow mass window predicted by the MSSM.

Motivated by these successes, the interesting question arises as to whether a natural SUSY sparticle mass spectrum, i.e., one with a modest value of Δ_{EW} , can be consistently generated from a model with parameters defined at the high-scale $Q = M_{\text{GUT}}$. Naturalness requires $|\mu| \sim M_Z \sqrt{\Delta_{\text{EW}}/2}$, while the recently measured [41] value of the branching fraction $\text{BF}(B_s \rightarrow \mu^+ \mu^-)$ qualitatively agrees with the predicted SM value, which in turn requires the CP odd boson A to be relatively heavy. We are thus led to adopt the NUHM2 [32], wherein weak scale values of μ and m_A may be used as inputs in lieu of GUT scale values of $m_{H_u}^2$

and $m_{H_d}^2$.² In Sec. II A for simplicity we take a common GUT scale mass parameter m_0 for *all* the matter scalars. Motivated by grand unification, we also assume a common GUT scale gaugino mass parameter. Later in Sec. III we also explore the possibility of split first-/second- vs third-generation matter scalars where we allow the third-generation GUT scale mass parameter $m_0(3)$ to differ from $m_0(1, 2)$ for the first-/second-generation scalars. Universality within each generation is well-motivated by $SO(10)$ GUT symmetry, since all matter multiplets of a single generation belong to a 16-dimensional spinor representation of $SO(10)$. We can also envisage some degree of nonuniversality between $m_0(1)$ and $m_0(2)$ as long as both lie in the tens of TeV regime: such a scenario invokes a partial-decoupling-partial-degeneracy solution to the SUSY flavor and CP problems (for constraints from flavor changing neutral current processes [42], see, e.g., Ref. [43]). For convenience, we will take $m_0(1) = m_0(2)$.

A. RNS from the NUHM2 model

The NUHM2 model is defined by the NUHM2 parameter set (1.8) introduced earlier. We take $m_t = 173.2$ GeV throughout this paper. For our calculations, we use the Isajet 7.83 [44] SUSY spectrum generator Isasugra [45]. Isasugra begins the calculation of the sparticle mass spectrum with input \overline{DR} gauge couplings and f_b, f_τ Yukawa couplings at the scale $Q = M_Z$ (f_i running begins at $Q = m_t$) and evolves the 6 couplings up in energy to scale $Q = M_{\text{GUT}}$ (defined as the value Q where $g_1 = g_2$) using two-loop RG equations (RGEs). We do not enforce the exact unification condition $g_3 = g_1 = g_2$ at M_{GUT} , since a few percent deviation from unification can be attributed to unknown GUT-scale threshold corrections [46]. Next, we use the SSB boundary conditions at $Q = M_{\text{GUT}}$ and evolve the set of 26 coupled two-loop MSSM RGEs [47,48] back down in scale to $Q = M_Z$. Full two-loop MSSM RGEs are used for soft term evolution, and the gauge and Yukawa coupling evolution includes threshold effects in the one-loop beta functions, so the gauge and Yukawa couplings transition smoothly from the MSSM to SM effective theories as different mass thresholds are passed. In Isasugra, the values of SSB terms which mix are frozen out at the scale $Q = M_{\text{SUSY}} = \sqrt{\overline{m_{\tilde{L}} \overline{m_{\tilde{R}}}}$, while nonmixing SSB terms are frozen out at their own mass scale [45]. The scalar potential is minimized using the RG-improved one-loop MSSM effective potential evaluated at an optimized scale $Q = M_{\text{SUSY}}$ to account for leading two-loop effects [49]. Once the tree-level sparticle mass spectrum is obtained, one-loop radiative corrections are calculated for all sparticle and Higgs boson masses, including complete one-loop weak scale threshold corrections for the top, bottom, and tau

masses at scale $Q = M_{\text{SUSY}}$ [50]. Since Yukawa couplings are modified by the threshold corrections, the solution must be obtained iteratively, with successive up-down running until a convergence at the required level is found. Since Isasugra uses a ‘‘tower of effective theories’’ approach to RG evolution, we expect a more accurate evaluation of the sparticle mass spectrum for models with split spectra than with programs such as SuSpect, SoftSUSY, or Spheno, which make an all-at-once transition from the MSSM to SM effective theories.

Our goal in this section is to find parameter ranges of the NUHM2 model which satisfy LHC sparticle and Higgs boson mass constraints while maintaining a low level of EWFT. We will also calculate the allowed mass range for various sparticles in low fine-tuned/phenomenologically viable parameter space. Toward this end, we search for regions of the NUHM2 parameter space with $\Delta_{\text{EW}} \lesssim 30$, where fine-tuning is better than about 3%. We will also require that our calculated light Higgs scalar mass lies within the range $m_h = 125 \pm 2$ GeV to allow for an estimated uncertainty in our calculation of m_h . We will also require that the parameters m_0 and $m_{1/2}$ respect the recent LHC limits on squark and gluino masses obtained within the mSUGRA model [4,5].

We search for radiative natural SUSY solutions by first performing a random scan over the following NUHM2 parameter ranges:

$$\begin{aligned}
 m_0 &: 0\text{--}20 \text{ TeV}, \\
 m_{1/2} &: 0.3\text{--}2 \text{ TeV}, \\
 -3 &< A_0/m_0 < 3, \\
 \mu &: 0.1\text{--}1.5 \text{ TeV}, \\
 m_A &: 0.15\text{--}1.5 \text{ TeV}, \\
 \tan \beta &: 3\text{--}60.
 \end{aligned} \tag{2.1}$$

We require of our solutions that:

- (i) electroweak symmetry be radiatively broken;
- (ii) the neutralino \tilde{Z}_1 is the lightest MSSM particle;
- (iii) the light chargino mass obeys the model independent LEP2 limit, $m_{\tilde{W}_1} > 103.5$ GeV [51];
- (iv) the LHC search bounds on $m_{\tilde{g}}$ and $m_{\tilde{q}}$ are respected;
- (v) $m_h = 125 \pm 2$ GeV.

To begin our investigation of NUHM2 model parameters leading to low Δ_{EW} , in Fig. 4 we plot each scan point as a red ‘‘+’’ in frames of Δ_{EW} vs a) m_0 , b) $m_{1/2}$, c) A_0/m_0 , d) $\tan \beta$, e) μ , and f) m_A . Since low Δ_{EW} solutions are only possible for low values of μ , we have performed a separate narrow scan, but this time with μ restricted between 100 and 300 GeV. The results of this second scan is shown by the blue crosses in the figure.

We see from the plots that Δ_{EW} varies from as low as ~ 10 ($\Delta_{\text{EW}}^{-1} = 10\%$ EWFT) to over 1000. While the bulk of points shown is fine-tuned with large $\Delta_{\text{EW}} \gtrsim 100$, there do

²Since the Higgs fields belong to different multiplets from matter fields, it is easy to envisage models with independent SUSY breaking mass parameters for Higgs and matter scalars.

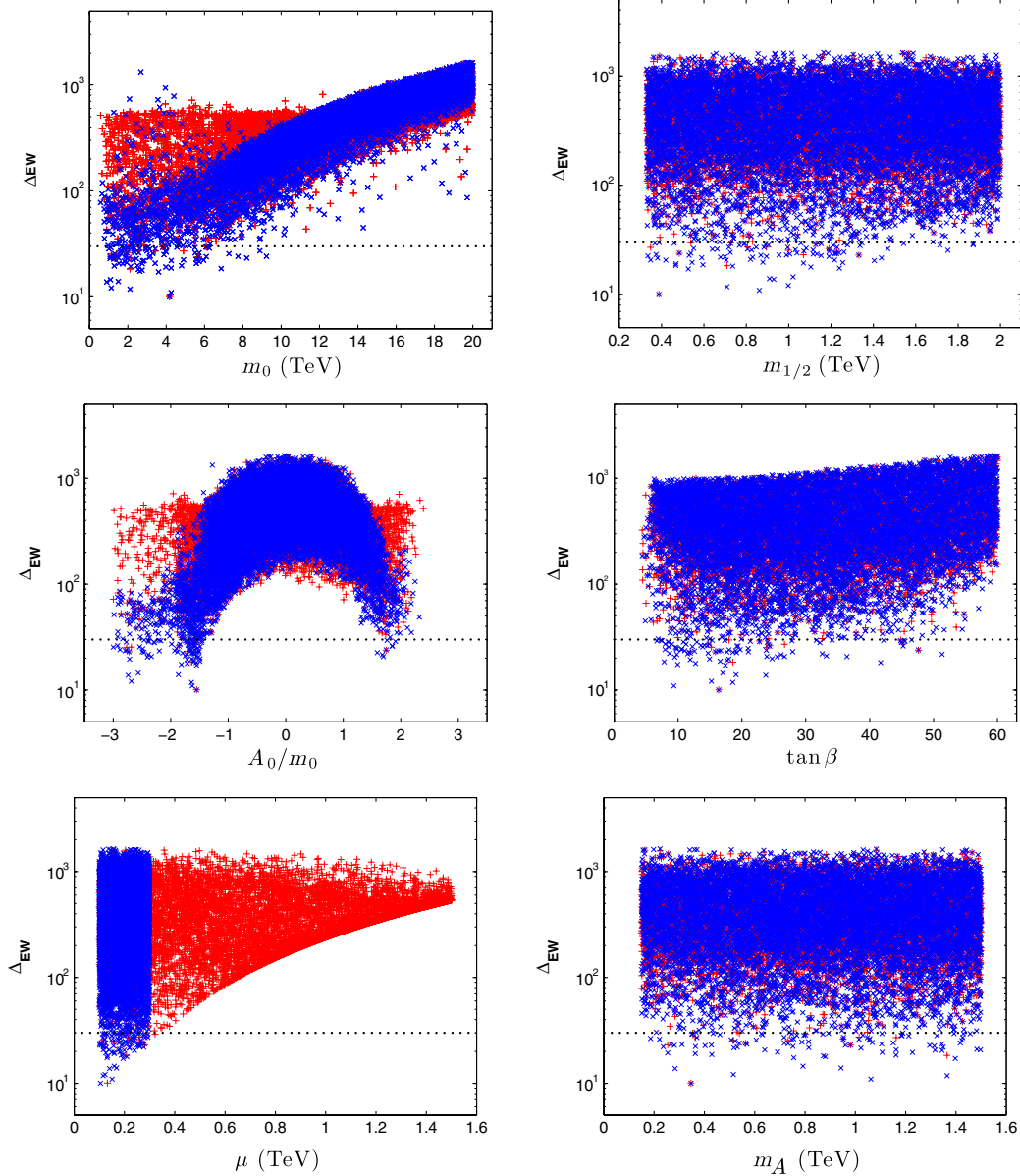


FIG. 4 (color online). The dependence of Δ_{EW} on various NUHM2 parameters from a scan (2.1) over parameter space (red pluses) and for the dedicated scan with $100 \text{ GeV} < \mu < 300 \text{ GeV}$ (blue crosses). The line at $\Delta_{EW} = 30$ is to guide the eye.

exist many solutions with $\Delta_{EW} \lesssim 30$, corresponding to better than 3% EWFT. The RNS solutions with $\Delta_{EW} \lesssim 30$ are obtained for values of $m_0 \sim 1\text{--}8 \text{ TeV}$. In the cases where m_0 is as high as 5–10 TeV, the top-squark masses are driven to much lower values via 1) the large top-quark Yukawa coupling f_t which suppresses top-squark soft masses during RG evolution, 2) large mixing effects which can suppress $m_{\tilde{t}_1}$ and yield a large $m_{\tilde{t}_1} - m_{\tilde{t}_2}$ splitting, and 3) two-loop RGE suppression of diagonal top-squark mass terms arising from large first-/second-generation sfermion masses [52–54]. If m_0 is too large—in this case above $\sim 10 \text{ TeV}$ —then these suppression mechanisms are insufficient to drive $m_{\tilde{t}_{1,2}}$ to low enough values to allow for low EWFT. Thus, the span of points shown in Fig. 4(a) trends

upward in Δ_{EW} as m_0 increases past about 8 TeV. We also see that for the red pluses in frame (a) Δ_{EW} has an upper bound close to about 500 if $m_0 \lesssim 10 \text{ TeV}$. For still larger values of m_0 , then Δ_{EW} increases with m_0 . This is because while μ^2 (or equivalently $-m_{H_u}^2$) is the largest of the quantities in Eq. (1.1) for the lower range of m_0 , for very large values of m_0 , then Σ_u^u begins to dominate. The blue crosses from the narrow scan with small μ have a different shape from the red broad scan since the upper edge is mostly determined by Σ_u^u , and so increases with m_0 .

In Fig. 4(b), we show Δ_{EW} vs $m_{1/2}$. Here, the low values of Δ_{EW} span a wide range of $m_{1/2}$ values from 0.3 to 1.5 TeV. Since $m_{\tilde{g}} \sim (2.5\text{--}3)m_{1/2}$, we expect $\Delta_{EW} \lesssim 30$ for $m_{\tilde{g}}$ values up to about 4 TeV. For the entire parameter

space (red pluses) Δ_{EW} is roughly evenly distributed with respect to the gaugino mass parameter. In Fig. 4(c), we show Δ_{EW} vs A_0/m_0 . We see a clear trend for low values of EWFT when $|A_0/m_0| \sim 1.5$ –2. The reason is that the hole at low magnitudes of A_0/m_0 and small values of Δ_{EW} occurs because of the Higgs mass constraint. Large magnitudes of GUT scale A_0 lead to a correspondingly large weak scale A_t parameters, which, in turn, provide large mixing in the top-squark sector. This leads to low EWFT and also heightened values of $m_h \sim 125$ GeV. Figure 4(d) shows Δ_{EW} vs $\tan\beta$. We see a slight preference for low $\tan\beta \sim 10$ –20 but otherwise no structure to speak of. Figure 4(e) shows Δ_{EW} vs the weak scale value of μ . The parabolic lower edge of the span of points reflects the upper bound on μ necessary for low EWFT. From the plot, bounds on μ can be conveniently read off; for instance, requiring $\Delta_{EW} \lesssim 30$ then requires $\mu \lesssim 350$ GeV. Of course, models with low $\mu \sim 100$ GeV but multi-TeV top squarks can still be very fine-tuned if the dominant contributions to Δ_{EW} arise from $\Sigma_u^u(\tilde{t}_i)$. In Fig. 4(f), we plot Δ_{EW} vs m_A . We see that low Δ_{EW} can be found over the entire range of $m_A \sim 0.15$ –1.5 TeV, so this parameter is not so relevant toward achieving low EWFT.

Next, to gain a sense of the sparticle mass ranges expected from RNS, we plot Δ_{EW} vs selected sparticle masses. First, since $m_0 \sim 2$ –8 TeV for $\Delta_{EW} \lesssim 30$, we

expect first- and second-generation squark and slepton masses also within this range (which is for the most part inaccessible in LHC SUSY searches). Next, in Fig. 5(a), we show Δ_{EW} vs $m_{\tilde{g}}$. We find that requiring $\Delta_{EW} \lesssim 30$ requires $m_{\tilde{g}} \sim 1$ –4 TeV. The lower portion of this range should be accessible to LHC14 searches, while the upper part lies beyond any LHC luminosity upgrade [55].

In Fig. 5(b), we show Δ_{EW} vs the lighter top-squark mass $m_{\tilde{t}_1}$. Here, we see that $\Delta_{EW} \lesssim 30$ allows $m_{\tilde{t}_1} \sim 0.5$ –2.5 TeV range. This is well above the range expected in generic NS models [9,10], where $m_{\tilde{t}_{1,2}}$ has been advocated to lie below about 600 GeV. In Fig. 5(c), we show Δ_{EW} vs $m_{\tilde{t}_2}$. Here, we find that $m_{\tilde{t}_2}$ can range up to ~ 6 TeV for $\Delta_{EW} \lesssim 30$. Such high values of $m_{\tilde{t}_2}$ are helpful to increase radiative corrections to the light Higgs mass m_h into the 125 GeV range. However, such heavy top squarks lie far beyond any conceivable LHC reach. In Fig. 5(d), we show Δ_{EW} vs $m_{\tilde{b}_1}$. Here, we see $m_{\tilde{b}_1} \sim 0.8$ –6 TeV, which again allows for far heavier bottom squarks than previous NS models, where $m_{\tilde{t}_{1,2}}$ and $m_{\tilde{b}_1}$ all were suggested to be $\lesssim 600$ GeV.

In Fig. 6(a), we show Δ_{EW} vs $m_{\tilde{W}_1}$. For RNS models, $m_{\tilde{W}_1} \simeq m_{\tilde{Z}_{1,2}} \sim |\mu|$, i.e., the chargino's mass is roughly equal to that of the two lighter neutralinos. Since \tilde{W}_1 is mainly Higgsino-like near the lower edge of the envelope of points, the distribution follows a similar pattern as for

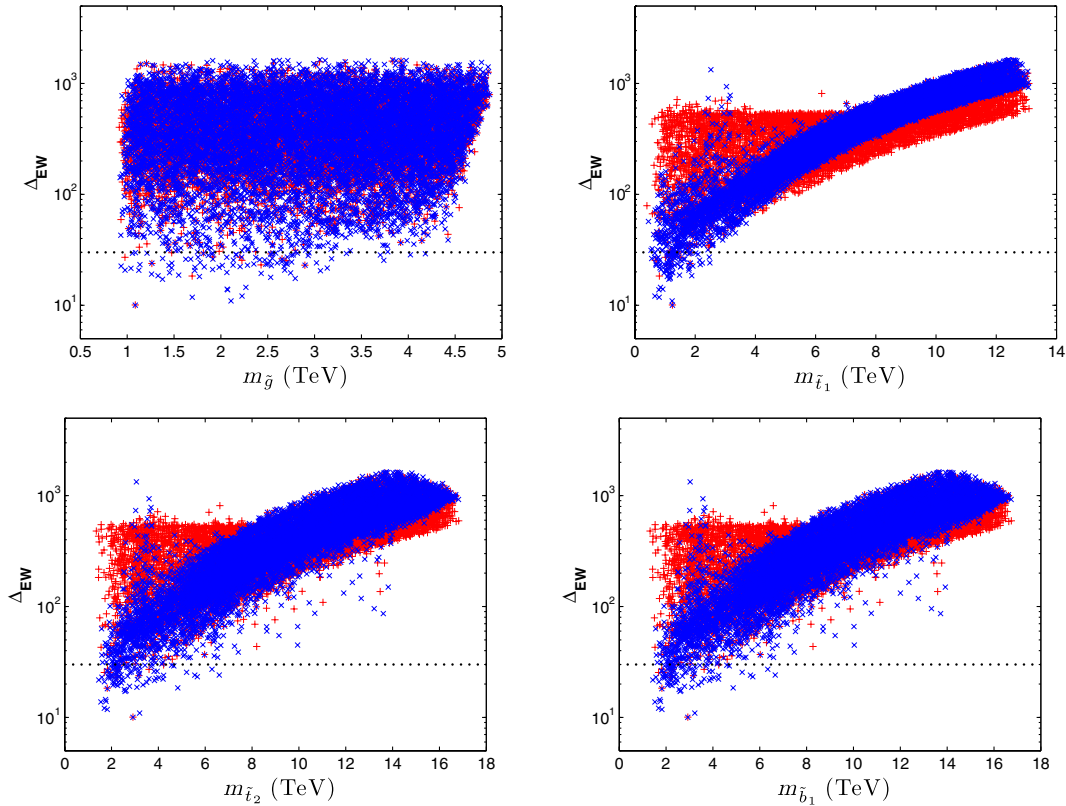


FIG. 5 (color online). The value of Δ_{EW} vs gluino and third-generation squark masses from a scan over NUHM2 parameter space. As in Fig. 4, the red pluses denote the distributions from the complete scan, whereas the blue crosses depict the results for the dedicated low μ scan. The line at $\Delta_{EW} = 30$ is to guide the eye.

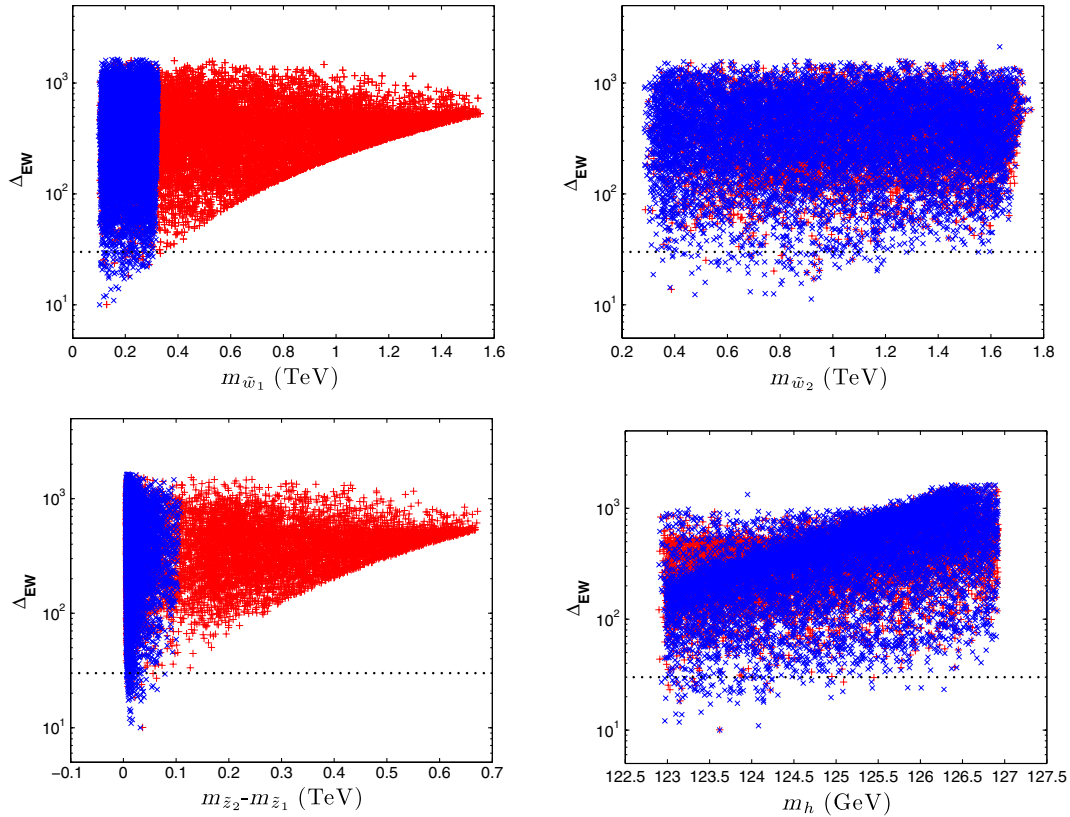


FIG. 6 (color online). The value of Δ_{EW} vs electroweakino and Higgs boson masses from a scan over NUHM2 parameter space. As in Fig. 4, the red pluses denote the distributions from the complete scan, whereas the blue crosses depict the results for the dedicated low μ scan. The line at $\Delta_{EW} = 30$ is to guide the eye.

the Δ_{EW} vs μ plot in Fig. 4. We see that for $\Delta_{EW} \lesssim 20$, $m_{\tilde{w}_1} \lesssim 250$ GeV. Thus, a linear collider operating with $\sqrt{s} > 2m_{\tilde{w}_1}$ will *directly probe* the lowest (and hence most lucrative) values of Δ_{EW} if the relatively soft visible daughters of the chargino can be distinguished over two-photon backgrounds [56]. In this sense, it has been emphasized that for models of natural SUSY, a linear e^+e^- collider would be a Higgsino factory in addition to a Higgs factory [11,14,20]. In Fig. 6(b), we show Δ_{EW} vs $m_{\tilde{w}_2}$. In the RNS model, the \tilde{W}_2 is nearly pure winolike, and its mass can range between ~ 0.3 – 1.2 TeV for $\Delta_{EW} \lesssim 30$. Since RNS as presented here includes gaugino mass unification, then typically $\tilde{Z}_{1,2}$ are Higgsino-like, \tilde{Z}_3 is binolike, and \tilde{Z}_4 is winolike. Since the $SU(2)$ gauge coupling g is rather large, we expect significant rates for $\tilde{W}_2^\pm \tilde{Z}_4$ production at the LHC, at least for the lower portion of the range of $m_{\tilde{w}_2}$. In Fig. 6(c), we show the $m_{\tilde{z}_2} - m_{\tilde{z}_1}$ mass difference in RNS vs Δ_{EW} . For most points with $\Delta_{EW} \lesssim 30$, we find that $m_{\tilde{z}_2} - m_{\tilde{z}_1} \lesssim 10$ – 20 GeV. Some points with $\Delta_{EW} \sim 30$ – 40 have a mass difference as large as 100 GeV; these points arise from sampling the lower portion of the $m_{1/2}$ range, which gives rise to gaugino masses comparable in magnitude to $|\mu|$ so that the lighter electroweakinos are actually gaugino-Higgsino mixtures. For the more likely small mass gap case, the lighter

neutralinos are dominantly Higgsino-like and decay via $\tilde{Z}_2 \rightarrow \tilde{Z}_1 f \bar{f}$ (here f denotes SM fermions) through the virtual Z . Then decays into opposite-sign same-flavor (OS/SF) isolated dileptons should occur at $\sim 3\%$ for each charged lepton species. The presence of low invariant mass OS/SF isolated dileptons from boosted \tilde{Z}_2 produced in gluino or gaugino cascade decay events could then be a distinctive signature of RNS at the LHC. For NUHM2 models with larger values of Δ_{EW} falling outside the RNS low EWFT requirement, $m_{\tilde{z}_2}$ can be greater than $m_{\tilde{z}_1} + M_Z$ or $m_{\tilde{z}_1} + m_h$ so that two-body decays of \tilde{Z}_2 are then allowed. Finally, in Fig. 6(d), we show Δ_{EW} vs m_h . Here, we see the lower $m_h \sim 123$ – 124 GeV values are just slightly preferred by EWFT over the higher range, although values of m_h as high as ~ 126.5 GeV occur for $\Delta_{EW} = 30$.

While our methodology allows one to find a low value of μ^2 for any value of m_0 and $m_{1/2}$, this by itself does not guarantee a small value of Δ_{EW} . In addition, the GUT scale value of $m_{H_u}^2$ has to be adjusted very precisely to obtain low EWFT, which could be viewed as a different sort of fine-tuning, that only a very narrow range of $m_{H_u}^2(M_{GUT})$ values will yield $-m_{H_u}^2 \sim M_Z^2$ at the weak scale.³ This can be seen

³From the perspective introduced in Sec. I, we would look for an underlying model where $m_{H_u}^2$ is thus determined.

TABLE I. An illustration of the sensitivity of the EWFT fine-tuning measure Δ_{EW} to $m_{H_u}^2(M_{\text{GUT}})$. For case A the NUHM2 parameters are $m_0 = 2.5$ TeV, $m_{1/2} = 400$ GeV, $\tan\beta = 10$, and $m_A = 1$ TeV, while for case B we have $m_0 = 4$ TeV, $m_{1/2} = 1$ TeV, $\tan\beta = 15$, and $m_A = 2$ TeV. For both cases, we take $A_0 = -1.6m_0$. The numbers in the table are in GeV units.

Case A			Case B		
$m_{H_u}^2(M_{\text{GUT}})$	μ	Δ_{EW}	$m_{H_u}^2(M_{\text{GUT}})$	μ	Δ_{EW}
1.03×10^7	150	9.04	2.73×10^7	150	15.4
1.02×10^7	250	18.8	2.72×10^7	250	24.1
1.00×10^7	400	42.4	2.70×10^7	400	49.5

from Table I where we plot the value of $m_{H_u}^2(M_{\text{GUT}})$ which is needed to generate small μ solutions for two different cases of NUHM2 model parameters. Optimistically speaking, we would view this as essentially determining the GUT scale value of $m_{H_u}^2/m_0^2$ to be very nearly 1.65 (case A) or 1.71 (case B). It is gratifying to see that the GUT scale values of all scalar mass parameters have no hierarchy as expected in models of gravity-mediated SUSY breaking where all scalar masses might be expected to be comparable at the high scale.

To be more general, we show in Fig. 7 a scatter plot of Δ_{EW} vs the GUT scale ratio $m_{H_u}^2/m_0^2$ from our scan over NUHM2 models. We find that for points with $\Delta_{\text{EW}} \lesssim 30$, then $m_{H_u}(M_{\text{GUT}}) \sim (1-2)m_0$.

B. RNS from the NUHM1 model?

Up to this point, we have focused on RNS from the NUHM2 model. However, it is of interest to see if low EWFT is also possible within the NUHM1 [32] framework in which H_u and H_d have equal GUT scale mass

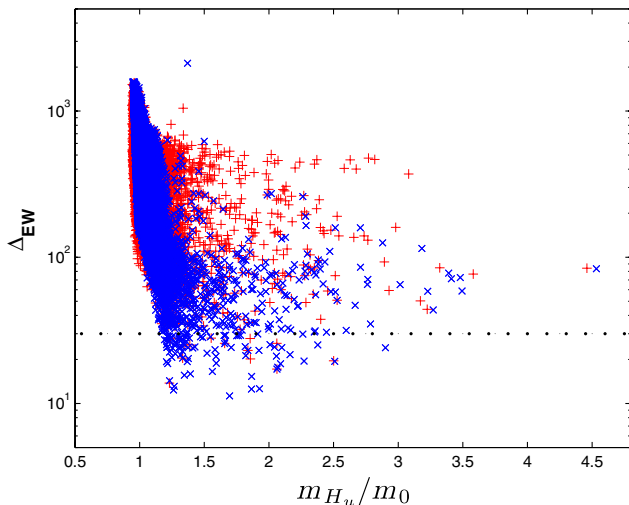


FIG. 7 (color online). The value of Δ_{EW} vs $m_{H_u}^2/m_0^2(M_{\text{GUT}})$ from the scan over the NUHM2 parameter space. As before, the red pluses are for the scan over the entire range of μ while the blue crosses are for the dedicated scan with μ limited to the 100–300 GeV range. The line at $\Delta_{\text{EW}} = 30$ is to guide the eye.

parameters; i.e., $m_{H_u}^2(M_{\text{GUT}}) = m_{H_d}^2(M_{\text{GUT}}) \equiv m_\phi^2$. As mentioned above, m_ϕ^2 is then adjusted to $m_{H_u}^2(M_{\text{SUSY}}) \sim M_Z^2$. For brevity, we confine our investigation to model lines where we fix m_0 , $m_{1/2}$, and A_0 to be the same as for the RNS2 model point but where we vary m_ϕ , the common GUT scale Higgs mass parameter, and $\tan\beta$. In Fig. 8, we show the value of (a) μ , (b) m_A , and (c) Δ_{EW} vs m_ϕ for $\tan\beta = 8.85$ (the RNS2 value), 25, 40, and 50. We see that the various curves in Fig. 8(a) are quite close (except at very large values where they dive down). This is essentially because the top-Yukawa coupling that dominantly affects $m_{H_u}^2$ [remember that $m_{H_u}^2(M_{\text{SUSY}})$ determines μ] hardly varies with $\tan\beta$; the small differences arise from the (subdominant) effects of bottom-Yukawa couplings. In contrast, the m_A values in Fig. 8(b) reduce considerably as $\tan\beta$ increases. We can understand this if we remember that the bottom-Yukawa coupling—which increases with $\tan\beta$ —drives $m_{H_d}^2$ to low values, thus reducing $m_A^2 \approx m_{H_d}^2 + \mu^2$ for larger $\tan\beta$ values. Turning to Fig. 8(c), we see that for this model line with $\tan\beta = 8.55$ (uppermost curve), Δ_{EW} reduces with increasing m_ϕ as in the μ curve in Fig. 8(a) until the kink at which it starts increasing. We have checked that the kink occurs when μ^2 becomes so low that the $m_{H_d}^2$ term becomes larger than all other terms in Eq. (1.1). For larger values of $\tan\beta$, the $m_{H_d}^2$ contribution is suppressed, resulting in smaller values of Δ_{EW} . However, in none of the cases shown does Δ_{EW} drop below ~ 80 . It may be a useful exercise to scan the NUHM1 parameter space to see just how small the EWFT can be when all LHC constraints are satisfied.

III. RNS FROM THE NUHM3 (SPLIT GENERATION) MODEL

In this section, we investigate if any advantage can be gained for RNS models if we allow for a splitting between scalars of the third generation and those of the first/second generations. We adopt the parameter set

$$m_0(1, 2), m_0(3), m_{1/2}, A_0, \tan\beta, \mu, m_A \quad (\text{NUHM3}), \quad (3.1)$$

where $m_0(3)$ is the GUT scale third-generation soft SUSY breaking mass parameter and $m_0(1, 2)$ is the corresponding (common) parameter for the first/second generation.

We search again for RNS solutions from the split generation parameter space by implementing a random scan over the parameters:

$$\begin{aligned} m_0(3): & 0-20 \text{ TeV}, & m_0(1, 2): & m_0(3) - 30 \text{ TeV}, \\ m_{1/2}: & 0.3-2 \text{ TeV}, & -3 < A_0/m_0 < & 3, \end{aligned} \quad (3.2)$$

$$\mu: 0.1-1.5 \text{ TeV}, \quad m_A: 0.15-1.5 \text{ TeV}, \quad \tan\beta: 3-60.$$

We implement the same LHC sparticle mass and $m_h = 125 \pm 2$ GeV constraints as before.

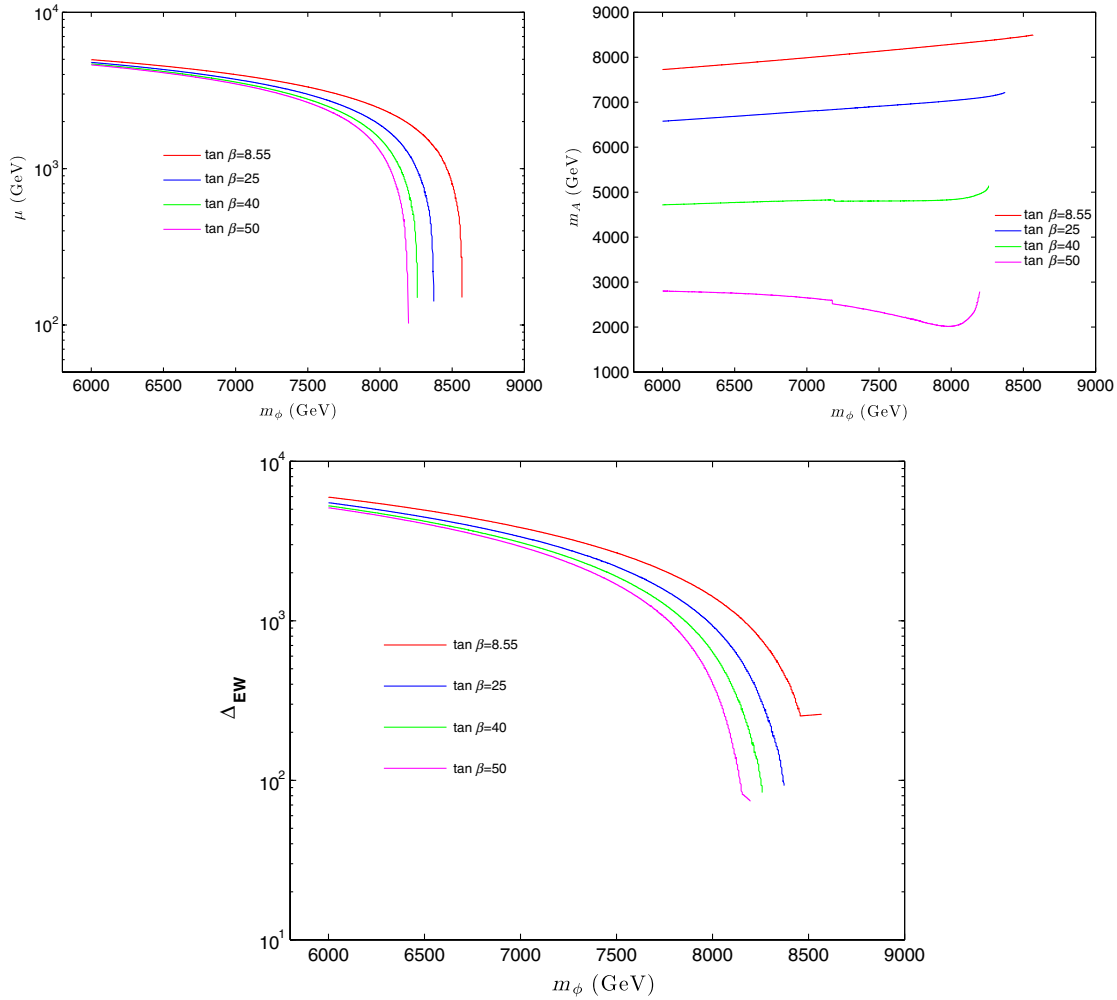


FIG. 8 (color online). Plot of (a) μ , (b) m_A , and (c) Δ_{EW} in the 1-parameter NUHM1 model vs m_ϕ for RNS2 model parameters $m_0 = 7025$ GeV, $m_{1/2} = 568.3$ GeV, $A_0 = -11426.6$ GeV, and for several values of $\tan\beta$. In all the frames the order of the lines is that of increasing $\tan\beta$, with $\tan\beta = 8.55$ on the top and $\tan\beta = 50$ on the bottom.

In Fig. 9, we show Δ_{EW} vs $m_0(3)$ and also vs $m_0(1, 2)$. The results for Δ_{EW} vs other parameters are very similar to Fig. 4 so we do not repeat them here. From Fig. 9(a), we see that RNS solutions with $\Delta_{EW} \lesssim 30$ can be found for $m_0(3)$

values ranging between 1 and 8 TeV, similar to results found in Fig. 4 for the NUHM2 model. It is interesting to note that the smallest values of Δ_{EW} in the figure are no smaller than for the NUHM2 model. The gap at small

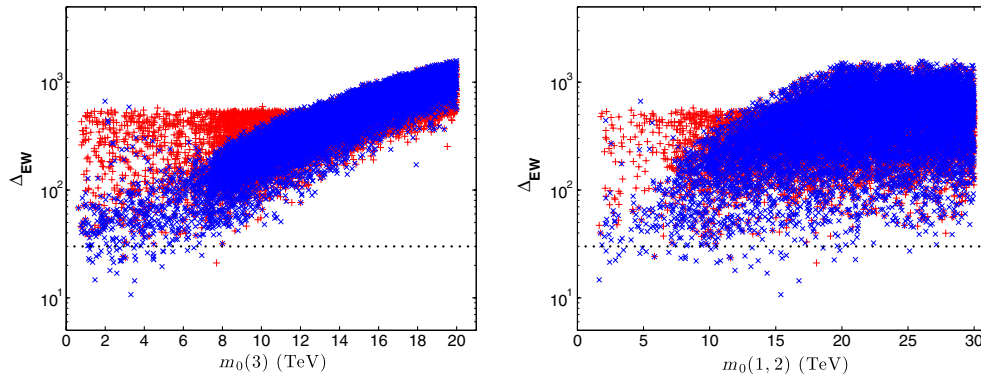


FIG. 9 (color online). The value of Δ_{EW} vs $m_0(3)$ and $m_0(1, 2)$ from a scan over the NUHM3 model with split first/second and third generations. As in Fig. 4, the red pluses denote the distributions from the complete scan, whereas the blue crosses depict the results for the dedicated low μ scan. The line at $\Delta_{EW} = 30$ is to guide the eye.

values of $m_0(3)$ is an artifact of the upper limit on $m_{1/2}$ in our scan: for small values of $m_0(3)$, the lighter t squark is often driven to tachyonic masses by two-loop contributions of heavy first-/second-generation squarks. We have checked that with larger values of $m_{1/2}$ in the scan, solutions fill in the entire gap. Again, even though the GUT scale value of $m_0(3)$ is in the multi-TeV regime, the \tilde{t}_2 and especially \tilde{t}_1 physical masses are considerably lower—in the few TeV regime—due to radiative effects from RGE running and also large mixing.

The key advantage of the NUHM3 model is seen in Fig. 9(b), where we plot Δ_{EW} vs $m_0(1, 2)$. In this case, we see that GUT scale first-/second-generation scalar masses can easily range between 1 and 30 TeV while still maintaining low Δ_{EW} . The solutions with $m_0(1, 2)$ in the multi-TeV region will also produce first-/second-generation squark and slepton masses which are comparable to $m_0(1, 2)$. This allows for a much more robust solution to the SUSY flavor/ CP problems. It also ameliorates the cosmological gravitino problem if $m_{3/2} \sim m_0(1, 2)$ as is expected in simple models of gravity mediation.

We do not show plots of Δ_{EW} vs sparticle masses since these are very similar to results shown in Figs. 5 and 6 except for the fact that NUHM3 scans allow for much heavier first-/second-generation squark and slepton masses in the 10–30 TeV range, whereas in the NUHM2 model, the squarks and sleptons are typically constrained to be below 8 TeV due to the imposed relation $m_0(3) = m_0(1, 2)$.

IV. RARE B DECAY CONSTRAINTS ON RNS

A. $BF(b \rightarrow s\gamma)$

The combination of several measurements of the $b \rightarrow s\gamma$ decay rate finds that $BF(b \rightarrow s\gamma) = (3.55 \pm 0.26) \times 10^{-4}$ [57]. This is slightly higher than the SM prediction [58] of $BF^{\text{SM}}(b \rightarrow s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$. SUSY contributions to the $b \rightarrow s\gamma$ decay rate come mainly from chargino-stop loops and the W -charged Higgs loops and so are large when these particles are light and when $\tan\beta$ is large [59]. Thus, in generic natural

SUSY where $m_{\tilde{t}_{1,2}, \tilde{b}_1} \lesssim 500$ GeV, one finds generally large deviations from the SM value for $BF(b \rightarrow s\gamma)$, as shown in Ref. [11]. In contrast, in radiative natural SUSY where third-generation squarks are in the TeV range, SUSY contributions to $BF(b \rightarrow s\gamma)$ are more suppressed. The situation is shown in Fig. 10(a) along with the measured central value (green solid line) and errors. The red points all have $\Delta_{EW} < 30$ and qualify as RNS points. We see the bulk of RNS points are consistent with the measured $BF(b \rightarrow s\gamma)$, although there are outliers.

B. $B_s \rightarrow \mu^+ \mu^-$

Recently, the LHCb Collaboration has discovered an excess over the background for the decay $B_s \rightarrow \mu^+ \mu^-$ [41]. They find a branching fraction of $BF(B_s \rightarrow \mu^+ \mu^-) = 3.2^{+1.5}_{-1.2} \times 10^{-9}$ in accord with the SM prediction of $(3.2 \pm 0.2) \times 10^{-9}$ [60]. In supersymmetric models, this flavor-changing decay occurs through pseudoscalar Higgs A exchange [61], and the contribution to the branching fraction from SUSY is proportional to $\frac{(\tan\beta)^6}{m_A^4}$. We show the value of $BF(B_s \rightarrow \mu^+ \mu^-)$ from RNS in Fig. 10(b). The decay is most constraining at large $\tan\beta \sim 50$ as occurs in Yukawa-unified models [62] and low m_A . In the case of RNS with lower $\tan\beta$ and heavier m_A , the constraint is less important. The bulk of the RNS points in Fig. 10(b) fall well within the newly measured error bands, although there are some outlier red points, mainly at larger values of the branching fraction.

C. $(g - 2)_\mu$

In addition, the well-known $(g - 2)_\mu$ anomaly has been reported as a roughly 3σ deviation from the SM value: $\Delta a_\mu = (28.7 \pm 8.0) \times 10^{-10}$ [63]. In RNS, since the $\tilde{\mu}_{1,2}$ and $\tilde{\nu}_\mu$ masses are expected to be in the multi-TeV range, only a tiny nonstandard contribution to the $(g - 2)_\mu$ anomaly is expected, and alternative explanations for this anomaly would have to be sought.

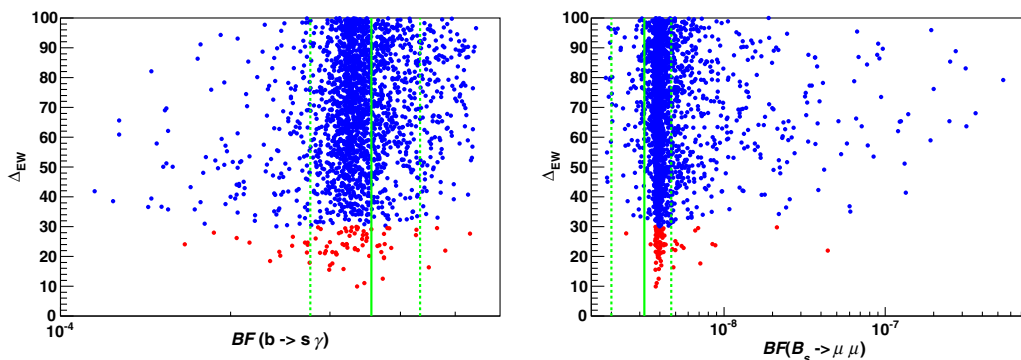


FIG. 10 (color online). The values of Δ_{EW} vs (a) $BF(b \rightarrow s\gamma)$ and (b) $BF(B_s \rightarrow \mu^+ \mu^-)$. The vertical lines represent the experimental measurements with uncertainties.

V. RNS AT LHC

Here, we list a few of the possibilities for an LHC search for radiative natural SUSY. A thorough study of signal and background simulations will be presented in an upcoming study [64].

The hallmark feature of radiative natural SUSY models is the presence of light Higgsino states \tilde{W}_1 and $\tilde{Z}_{1,2}$ with masses $\sim |\mu| \sim 100\text{--}300$ GeV and usually a small mass gap $m_{\tilde{W}_1} - m_{\tilde{Z}_1}$ and $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ of order 10–30 GeV with a possible exception of low $m_{1/2}$ and larger μ where there can be substantial gaugino-Higgsino mixing.

One possibility for RNS at LHC is to search for clean trilepton events from $\tilde{W}_1\tilde{Z}_2$ production followed by $\tilde{W}_1 \rightarrow \tilde{Z}_1\ell\nu_\ell$ and $\tilde{Z}_2 \rightarrow \tilde{Z}_1\ell^+\ell^-$ decays where $\ell = e$ or μ . This signal has been investigated in Ref. [14]. There, the $p_T(\ell)$ values were typically found to be quite low in the 5–15 GeV range making detection difficult. The small mass difference between the parent and daughter neutralino will also mean that the invariant mass of the opposite-sign/same-flavor dilepton pair will be small, making it more challenging to separate it from SM origins. Nevertheless, this reaction certainly motivates an LHC search for clean trilepton states with very soft lepton p_T values, as low as is experimentally feasible.

While first-/second-generation squarks are expected to be in the multi-TeV range, the value of $m_{\tilde{g}}$ is expected to be $\sim 1\text{--}5$ TeV. The lower portion of this mass range $m_{\tilde{g}} \sim 1\text{--}2$ TeV should be accessible to LHC searches for gluino pair production $pp \rightarrow \tilde{g}\tilde{g}$. For RNS models, since $m_{\tilde{t}_{1,2}} \ll m_{\tilde{q}}$, then gluino three-body decays to third-generation particles typically dominate: $\tilde{g} \rightarrow t\tilde{W}_1$ or $t\tilde{Z}_i$. Thus, we would expect $\tilde{g}\tilde{g}$ events to contain up to four b jets and 2–4 reconstructable top quarks. A small fraction of events would contain $\tilde{Z}_2 \rightarrow \tilde{Z}_1\ell^+\ell^-$, where $m(\ell^+\ell^-)$ is bounded by $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \sim 10\text{--}20$ GeV. Normally the leptons from \tilde{Z}_2 decay would be rather soft, but in the case of large boosts from the gluino cascade decay, the opposite-sign/same-flavor pair would be highly collimated in the opening angle. We expect LHC14 with 100 fb^{-1} to be able to probe $m_{\tilde{g}} \sim 1\text{--}2$ TeV via \tilde{g} cascade decays using analyses similar to those used for gluino searches in mSUGRA when m_0 is very large [55].

A novel search for RNS at the LHC is to look for pair production of the *heavier* gaugino states \tilde{W}_2 and \tilde{Z}_3 and \tilde{Z}_4 . Wino pair production occurs via the large $SU(2)$ gauge couplings and leads to large rates for $\tilde{W}_2\tilde{Z}_4$ and $\tilde{W}_2\tilde{W}_2$ processes. The decays $\tilde{W}_2 \rightarrow W\tilde{Z}_{1,2}$, $Z\tilde{W}_1$ and $\tilde{Z}_4 \rightarrow W\tilde{W}_1$, $Z\tilde{Z}_{1,2}$ occur with significant branching fractions and yield a variety of diboson final states that include spectacular $W^\pm W^\pm$ and WZ [65] plus E_T^{miss} events with soft debris from the decays of Higgsinos. Events with light Higgs bosons instead of gauge bosons in the final state are also possible [66].

As with the mSUGRA model, a wide range of RNS signatures for LHC can be found by exploring the m_0 vs

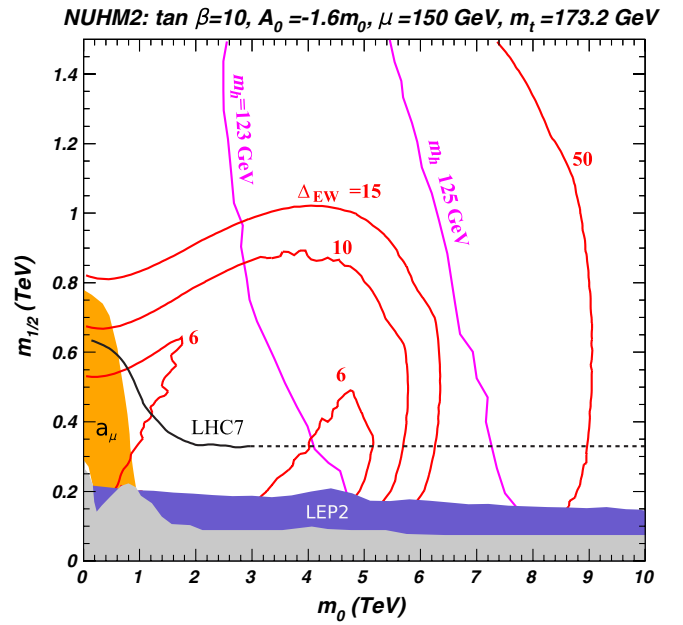


FIG. 11 (color online). Contours of Δ_{EW} (red curves) and m_h (purple curves) in the m_0 vs $m_{1/2}$ plane of the RNS model with $A_0 = -1.6m_0$, $\tan\beta = 10$, $\mu = 150$ GeV, and $m_A = 1$ TeV.

$m_{1/2}$ plane. This plane will look quite different from the mSUGRA case since now we will require small $\mu \sim 100\text{--}200$ GeV in accord with EWFT and also $A_0 \sim -1.6m_0$ in accord with $m_h = 125$ GeV and low EWFT. We show the plane in Fig. 11 for $\mu = 150$ GeV with $A_0 = -1.6m_0$, $\tan\beta = 10$, and $m_A = 1$ TeV. Here, we plot contours $m_h = 123$ and 125 GeV and also contours of $\Delta_{EW} = 6, 10, 15,$ and 50 . Almost the entire plane has low $\Delta_{EW} < 50$, with $5.5 \leq \Delta_{EW} \leq 10$ in the lower left portion. In addition, the right-hand portion of the plane has $m_h \geq 123\text{--}125$ GeV. The purple-shaded region marked LEP2 has $m_{\tilde{W}_1} < 103.5$ GeV in violation of LEP2 limits on chargino pair production. We also show recent LHC constraints from gluino/squark searches within the mSUGRA model as the black contour.⁴ We extrapolate those constraints to much higher m_0 values via the dashed black contour. Over the entire plane, $m_{\tilde{W}_1} \sim m_{\tilde{Z}_{1,2}} \sim \mu = 150$ GeV, so there would always be light Higgsino pair production at the LHC. The region with $m_{1/2} \leq 0.6$ TeV yields $m_{\tilde{g}} \leq 2$ TeV and should be accessible to future gluino pair production searches. Signals from wino pair production may also be observable at the LHC, and perhaps even at LHC8, if the heavier chargino is sufficiently light.

⁴Strictly speaking, these are the constraints obtained in the mSUGRA model for $A_0 = 0$ from the nonobservation of signals from gluino and first-generation squark production. Since the masses of these sparticles depend mostly on m_0 and $m_{1/2}$, it is reasonable to suppose these also apply to the plane in Fig. 11.

VI. RNS AT ILC

Since the main feature of RNS models is the presence of light Higgsinos \tilde{W}_1 , \tilde{Z}_1 , and \tilde{Z}_2 , we expect excellent prospects for testing RNS at a linear e^+e^- collider. Pair production for charged Higgsinos via the reaction $e^+e^- \rightarrow \tilde{W}_1^+ \tilde{W}_1^-$ would yield soft but observable decay products from $\tilde{W}_1 \rightarrow \tilde{Z}_1 f \bar{f}'$ decay, where f and f' are SM fermions. These decay products should be easily detectable in the clean environment of e^+e^- colliders and moreover should be acollinear in the transverse plane as opposed to two-photon backgrounds $\gamma\gamma \rightarrow f\bar{f}$ where visible decay products tend to come out back-to-back. The entire m_0 vs $m_{1/2}$ plane shown in Fig. 11 will be accessible to an ILC with $\sqrt{s} \gtrsim 2m_{\tilde{W}_1} \approx 2|\mu|$.

Moreover, the cross section for the reaction $e^+e^- \rightarrow \tilde{Z}_1 \tilde{Z}_2$ should also be large and provide corroborative evidence. These cross sections will have a distinctive shape vs beam polarization as shown in Ref. [14], which should be indicative of Higgsino pair production. We note here that since $E_{\text{CM}} \gtrsim 2m_{\tilde{W}_1} \sim 2|\mu|$, and $\Delta_{\text{EW}} \sim \mu^2/(M_Z^2/2)$, then an e^+e^- collider with a center-of-momentum (CM) energy E_{CM} directly probes

$$\Delta_{\text{EW}} \sim E_{\text{CM}}^2/(2M_Z^2), \quad (6.1)$$

so that even a low energy e^+e^- linear collider would probe the most lucrative regions of RNS parameter space (that portion with lowest Δ_{EW}) and, as E_{CM} increases, would discover natural SUSY or increasingly exclude it.

VII. SEARCH FOR HIGGSINO-LIKE WIMPS FROM RNS

One of the distinctive features of natural SUSY models is that the lightest MSSM particle is a Higgsino-like neutralino \tilde{Z}_1 . If R parity is conserved, then the \tilde{Z}_1 may make up all or at least a portion of the dark matter in the Universe. Higgsinos with mass $m_{\tilde{Z}_1} > m_W, M_Z$ have high annihilation rates into vector boson pairs. Thus, if they are present in thermal equilibrium in the early Universe, then the Higgsino relic density may be computed approximately as

$$\Omega_{\tilde{Z}_1}^{\text{th}} h^2 = \frac{s_0}{\rho_c/h^2} \left(\frac{45}{\pi g_*} \right)^{1/2} \frac{x_f}{m_{\text{Pl}}} \frac{1}{\langle \sigma v \rangle}, \quad (7.1)$$

where s_0 is the entropy density of the Universe at the present time, ρ_c is the critical closure density, h is the scaled Hubble constant, g_* is the number of relativistic degrees of freedom at freeze-out, $x_f \sim 25$ is the scaled freeze-out temperature, m_{Pl} is the Planck mass, and $\langle \sigma v \rangle$ is the thermally averaged neutralino annihilation cross section times relative velocity. Higgsino-like WIMPs couple with gauge strength to vector bosons so that $\langle \sigma v \rangle$ is large and the relic density is suppressed.

We evaluate the relic density of Higgsinos using Isatools [67] from our scan over NUHM2 parameters as in Sec. II and show Δ_{EW} vs $\Omega_{\tilde{Z}_1} h^2$ in Fig. 12(a). Points

with $\Delta_{\text{EW}} < 30$ indicative of RNS are shown in red while the more highly fine-tuned points are in blue. The vertical green line shows the WMAP-measured value of the dark matter density. We see that the majority of points with $\Delta_{\text{EW}} < 30$ have $\Omega_{\tilde{Z}_1} h^2 \sim 0.005\text{--}0.05$, i.e., well below the measured abundance. Several points have $\Omega_{\tilde{Z}_1} h^2 > 0.12$; these points arise from cases where $\mu \sim M_1$ where the neutralino is of the mixed bino-Higgsino variety.

There exists a variety of nonstandard cosmologies with features which make them more attractive than the standard WIMP-only dark matter scenario. For instance, in stringy models with moduli fields at the 10–100 TeV scale, the moduli may decay after big bang nucleosynthesis into SM particles, thus diluting all relics present. Alternatively, if moduli decay to SUSY particles which cascade into the lightest SUSY particle, then the neutralino abundance may be enhanced [68].

Another possibility arises from SUSY models where the strong CP problem is solved by the Peccei-Quinn (PQ) mechanism [69] with its concomitant axion a . In the SUSY case, the axion superfield \hat{a} also contains an R -even spin-0 saxion s and an R -odd spin- $\frac{1}{2}$ axino \tilde{a} . In the case where \tilde{Z}_1 is lightest SUSY particle, then the dark matter would be a mixture of two particles: the axion and the neutralino. Axinos which are produced thermally at high reheat temperature T_R in the early Universe would cascade decay to neutralinos at a decay temperature $T_D < T_f$, causing a neutralino *reannihilation* which provides a much higher abundance of WIMPs than expected in a WIMP-only picture. In addition, saxions can be produced both thermally and via coherent oscillations and may decay to both SUSY and SM particles; the former case enhances the neutralino abundance while the latter case dilutes any relics present at the time of decay. Calculations of the neutralino abundance in the PQ-augmented MSSM depend on the various PQ parameters along with T_R and the SUSY particle spectrum and have been presented in Refs. [70–72]. In the case of models with a standard underabundance of neutralinos, the neutralino abundance is almost always *enhanced* beyond its standard value $\Omega_{\tilde{Z}_1} h^2$. If this scenario is applied to the case of RNS, then we may most likely expect an enhanced Higgsino-like WIMP abundance beyond its standard value. In this scenario, axions will also be produced via coherent oscillations at temperature around the QCD phase transition. Thus, the Higgsinos could make up either a small or a large fraction of the relic dark matter, with axions comprising the remainder. The important point here is that it is very difficult to suppress the Higgsino abundance below its standard thermal value which is shown in Fig. 12(a). Thus, we would expect relic Higgsinos to be present in the Universe today, but with an abundance which is suppressed by between 1 and 15 from the measured value. This opens up the opportunity to detect relic Higgsinos, albeit while these would only constitute a fraction of the measured dark matter abundance. At the same time, there is also the possibility to detect relic axions.

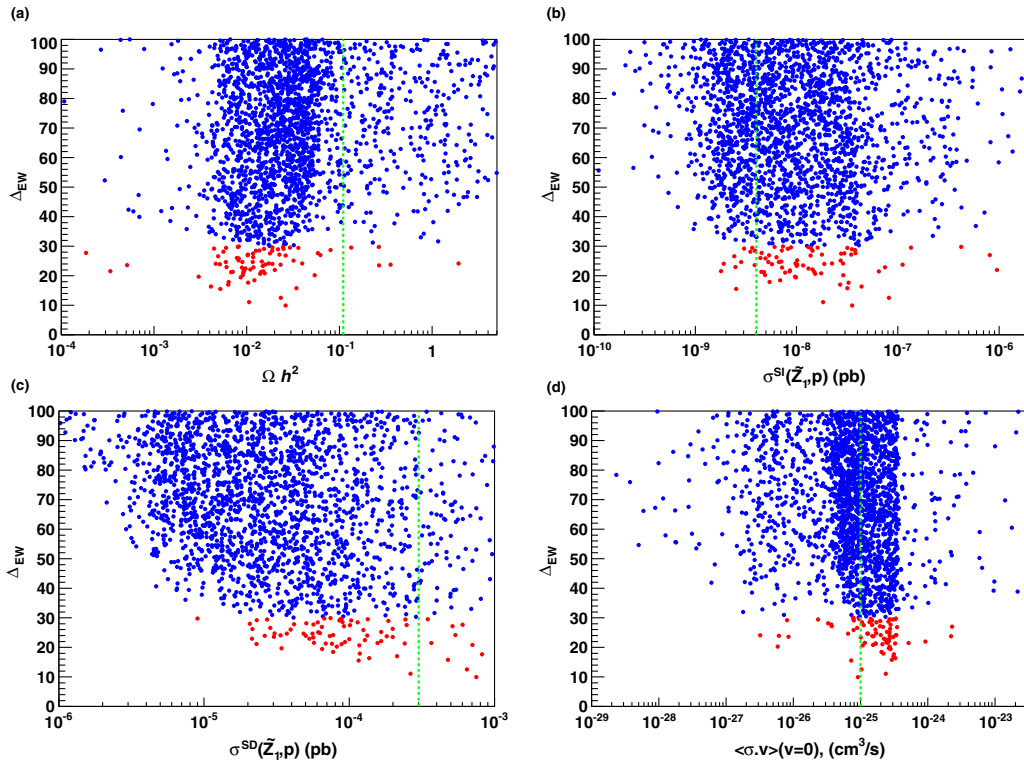


FIG. 12 (color online). The value of Δ_{EW} vs neutralino relic density and direct and indirect WIMP detection rates. Vertical green lines denote upper experimental limits obtained assuming that the WIMP saturates the observed density of dark matter. The predictions in the last three frames need to be rescaled by a factor $\Omega_{\tilde{Z}_1} h^2 / 0.11$ if the neutralinos make up only part of the dark matter.

With a view toward detecting relic Higgsinos from RNS, we show in Fig. 12(b) the value of Δ_{EW} vs the spin-independent neutralino-proton scattering cross section $\sigma^{SI}(\tilde{Z}_1 p)$ in pb from Isatools [73]. The red points with $\Delta_{EW} < 30$ occur with $\sigma^{SI}(\tilde{Z}_1 p) \sim 10^{-9} - 10^{-7}$ pb. For comparison, we show via the green vertical line the Xe-100 limit from 225 live days [74] for $m_{\tilde{Z}_1} \sim 150$ GeV. Naively, many of the RNS points would be excluded if Higgsinos comprised the entire dark matter density. However, in the mixed axion/Higgsino dark matter scenario, the expected local abundance of WIMPs can be scaled down by factors of 1–15 typically. Even with this rescaling of the expected local abundance, we still expect relic Higgsinos to be within detection range of near-future WIMP detectors.

In Fig. 12(c), we show the spin-dependent neutralino-proton cross section $\sigma^{SD}(\tilde{Z}_1 p)$ in pb. The bulk of RNS points with $\Delta_{EW} < 30$ populate the region with $\sigma^{SD}(\tilde{Z}_1 p) \sim 2 \times 10^{-5} - 10^{-3}$ pb. The IceCube neutrino detector at the South Pole is sensitive to the detection of neutrinos arising from Higgsino annihilation in the core of the Sun. The expected detection rate depends on the Sun's ability to sweep up neutralinos via $\tilde{Z}_1 p$ collisions, which depends mainly on $\sigma^{SD}(\tilde{Z}_1 p)$. For reference, we also show the current IceCube Weinberg-Wilczek limit [75] at $\sigma^{SD}(\tilde{Z}_1 p) \sim 3 \times 10^{-4}$ pb. This limit depends on the assumption that WIMPs comprise the entire dark matter abundance and would need to be rescaled for a mixed axion/Higgsino cosmology.

Figure 12(d) shows the thermally averaged neutralino annihilation cross section times relative velocity, evaluated as $v \rightarrow 0$. This quantity enters linearly into indirect searches for neutralino annihilation in the cosmos into γ s or e^+ , \bar{p} or \bar{D} . For the case of RNS, the bulk of points with $\Delta_{EW} < 30$ inhabit the region around $\langle \sigma v \rangle|_{v \rightarrow 0} \sim 10^{-25}$ cm^3/sec . The vertical green line shows the upper limit on the annihilation cross section times velocity for very nonrelativistic dark matter in dwarf spheroidal satellite galaxies of the Milky Way annihilating to W boson pairs obtained by the Fermi collaboration [76], assuming a ~ 150 GeV WIMP. Models with a larger annihilation cross section would have led to a flux of gamma rays not detected by the experiment, assuming a Navarro-Frenk-White profile [77] for each dwarf galaxy in the analysis. We see that the Fermi bound might exclude the bulk of points assuming Higgsinos saturate the dark matter density. This bound changes rather slowly with the WIMP mass, being just a factor of 2 weaker for a WIMP mass of 300 GeV. Further searches and improvements by the Fermi-LAT Collaboration and/or the impending AMS results should provide more stringent probes of the RNS model.

VIII. SUMMARY AND CONCLUSIONS

Models of natural supersymmetry reconcile the lack of a SUSY signal at LHC with the principle of electroweak

naturalness. Natural SUSY models are characterized by light Higgsinos of mass $\sim 100\text{--}300$ GeV, three light third-generation squarks with mass less than about 500 GeV, and gluinos of mass less than about 1.5 TeV. First-/second-generation squarks may be much heavier—in the multi-TeV regime—thus avoiding LHC searches and providing at least a partial decoupling solution to the SUSY flavor and CP problems. Attractive as they are, generic NS models based on the MSSM are at odds with the recent discovery of a light Higgs scalar at ~ 125 GeV, which requires TeV-scale top squarks along with large top-squark mixing.

We presented here an improved natural SUSY model dubbed radiative natural SUSY [20]. RNS is a SUSY model based on the MSSM, which may be valid all the way up to the GUT scale. Thus, it maintains the desirable features of gauge coupling unification and radiative electroweak symmetry breaking while avoiding the introduction of extra possibly destabilizing gauge singlets or other forms of exotic matter. The main features of the RNS model include 1) a low value of superpotential Higgsino mass $|\mu| \sim 100\text{--}300$ GeV and 2) a weak scale value of $-m_{H_u}^2 \sim M_{\tilde{Z}}^2$; both these qualities are required to fulfill electroweak naturalness at the tree level. The term $m_{H_u}^2$ is driven to low values radiatively by the same mechanism leading to radiatively broken electroweak symmetry breaking and depends on a large top-quark Yukawa coupling. We proceed further by evaluating EWFT at the 1-loop level. In this case, top-squark masses enter the computation of Δ_{EW} and are also driven radiatively to few-TeV values. By allowing for large top-squark mixing ($|A_0| \sim (1\text{--}2)m_0$), top-squark contributions to EWFT are suppressed at the same time as the light Higgs boson mass is uplifted; thus, the model reconciles electroweak fine-tuning with $m_h \simeq 125$ GeV all in the context of the MSSM valid up to the GUT scale.

RNS may be realized in the two-parameter nonuniversal Higgs models NUHM2. In this case, low EWFT with $\Delta_{EW} \lesssim 30$ can be attained for model parameters which lead to a distinctive mass spectrum:

- (i) light Higgsino-like \tilde{W}_1 and $\tilde{Z}_{1,2}$ with mass $\sim 100\text{--}300$ GeV;
- (ii) gluinos with mass $m_{\tilde{g}} \sim 1\text{--}4$ TeV;
- (iii) heavier top squarks than generic NS models: $m_{\tilde{t}_1} \sim 1\text{--}2$ TeV and $m_{\tilde{t}_2} \sim 2\text{--}5$ TeV;
- (iv) first-/second-generation squarks and sleptons with mass $m_{\tilde{q},\tilde{\ell}} \sim 1\text{--}8$ TeV; the $m_{\tilde{\ell}}$ range can be pushed up to 20–30 TeV if nonuniversality of generations with $m_0(1, 2) > m_0(3)$ is allowed.

The RNS model with the above spectra also fulfills limits from rare B -decay measurements, which can be an Achilles heel for generic NS models with much lighter third-generation squarks.

The RNS model can be tested at the LHC for $m_{1/2}$ in the lower portion of its range whereupon gluino pair

production and/or gaugino pair production may lead to observable signals. Many of the associated SUSY events will contain light Higgsinos arising from cascade decays with soft decay products which may be observable. The case of OS/SF dileptons with mass $\lesssim 10\text{--}20$ GeV would signal the presence of $\tilde{Z}_2 \rightarrow \tilde{Z}_1 \ell^+ \ell^-$ decay.

Linear e^+e^- colliders would likely provide the definitive test of RNS models since pair production of charged Higgsinos should be easily observable, and the lowest energy machines will scrutinize the most lucrative parameter choices with the lowest values of Δ_{EW} . For RNS, an ILC-type machine would be a Higgsino factory in addition to a Higgs factory.

In RNS, we also expect the presence of Higgsino-like WIMPs which have large rates for direct and indirect WIMP detection. Since Higgsinos are thermally underproduced, we expect them to constitute only a portion of the measured dark matter abundance, with perhaps axions comprising the remainder. Detectability via WIMP searches will depend on the Higgsino fraction of the dark matter.

The many elegant features presented above impel us to regard RNS as the possible new paradigm SUSY model. Its consequences for detection at colliders and at dark matter detectors merits a high level of scrutiny.

ACKNOWLEDGMENTS

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APPENDIX: RADIATIVE CORRECTIONS TO THE HIGGS POTENTIAL MINIMIZATION CONDITIONS

The Higgs portion of the scalar potential in the MSSM is given by

$$V_{\text{Higgs}} = V_{\text{tree}} + \Delta V, \quad (\text{A1})$$

where the tree level portion for the neutral Higgs sector is given by

$$\begin{aligned} V_{\text{tree}} = & (m_{H_u}^2 + \mu^2)|h_u^0|^2 \\ & + (m_{H_d}^2 + \mu^2)|h_d^0|^2 - B\mu(h_u^0 h_d^0 + \text{H.c.}) \\ & + \frac{1}{8}(g^2 + g'^2)(|h_u^0|^2 - |h_d^0|^2)^2 \end{aligned} \quad (\text{A2})$$

and the radiative corrections (in the effective potential approximation and using the \overline{DR} regularization scheme, as appropriate for SUSY models) by

$$\Delta V = \sum_i \frac{(-1)^{2s_i}}{64\pi^2} (2s_i + 1) c_i m_i^4 \left[\log\left(\frac{m_i^2}{Q^2}\right) - \frac{3}{2} \right], \quad (\text{A3})$$

where the sum over i runs over all fields that couple to Higgs fields, m_i^2 are the *Higgs-field-dependent* mass

squared values, and $c_i = c_{\text{col}}c_{\text{cha}}$, with $c_{\text{col}} = 3(1)$ for colored (uncolored) particles and $c_{\text{cha}} = 2(1)$ for charged (neutral) particles, and s_i is their spin quantum number.

Minimization of the scalar potential allows one to compute the gauge boson masses in terms of the Higgs field vacuum expectation values v_u and v_d and leads to the well-known conditions that

$$B\mu v_d = (m_{H_u}^2 + \mu^2 - g_Z^2(v_d^2 - v_u^2))v_u + \Sigma_u \quad (\text{A4})$$

$$B\mu v_u = (m_{H_d}^2 + \mu^2 + g_Z^2(v_d^2 - v_u^2))v_d + \Sigma_d, \quad (\text{A5})$$

where

$$\Sigma_{u,d} = \left. \frac{\partial \Delta V}{\partial h_{u,d}} \right|_{\min} \quad (\text{A6})$$

and $h_{u,d}^0 = (h_{u,dR}^0 + ih_{u,dI}^0)/\sqrt{2}$, $g_Z^2 = (g^2 + g'^2)/8$. By $SU(2)$ invariance, the scalar potential V depends on the scalar fields as [6] $V(h_u^\dagger h_u, h_d^\dagger h_d, h_u h_d + \text{c.c.})$; then we have

$$\Sigma_u = \Sigma_u^u v_u + \Sigma_u^d v_d, \quad (\text{A7})$$

$$\Sigma_d = \Sigma_d^u v_u + \Sigma_d^d v_d \quad \text{and} \quad (\text{A8})$$

$$\Sigma_u^u = \Sigma_u^d, \quad (\text{A9})$$

where

$$\Sigma_u^u = \left. \frac{\partial \Delta V}{\partial |h_u|^2} \right|_{\min}, \quad (\text{A10})$$

$$\Sigma_d^d = \left. \frac{\partial \Delta V}{\partial |h_d|^2} \right|_{\min} \quad \text{and} \quad (\text{A11})$$

$$\Sigma_u^d = \left. \frac{\partial \Delta V}{\partial (h_u h_d + \text{c.c.})} \right|_{\min}. \quad (\text{A12})$$

In this case, the minimization conditions may be expressed as

$$M_Z^2/2 = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2, \quad (\text{A13})$$

$$B\mu = ((m_{H_u}^2 + \mu^2 + \Sigma_u^u) + (m_{H_d}^2 + \mu^2 + \Sigma_d^d)) \times \sin\beta \cos\beta + \Sigma_u^d. \quad (\text{A14})$$

The advantage of writing the minimization conditions in terms of Σ_u^u and Σ_d^d and Σ_{ud} is that the corrections to $m_{H_u}^2$, $m_{H_d}^2$, and $B\mu$ are neatly separated so that Σ_u^d terms do not appear in Eq. (A13) and so do not contribute to the fine-tuning calculation.

The contributions of the various Σ s can be written as

$$\begin{aligned} \Sigma_u^u &= \sum_i \frac{1}{32\pi^2} (-1)^{2s_i} (2s_i + 1) c_i \left. \frac{\partial m_i^2}{\partial |h_u|^2} \right|_{\min} F(m_i^2), \\ \Sigma_d^d &= \sum_i \frac{1}{32\pi^2} (-1)^{2s_i} (2s_i + 1) c_i \left. \frac{\partial m_i^2}{\partial |h_d|^2} \right|_{\min} F(m_i^2), \\ \Sigma_u^d &= \sum_i \frac{1}{32\pi^2} (-1)^{2s_i} (2s_i + 1) c_i \\ &\quad \times \left. \frac{\partial m_i^2}{\partial (h_u h_d + \text{c.c.})} \right|_{\min} F(m_i^2) = \Sigma_d^u, \end{aligned} \quad (\text{A15})$$

where

$$F(m^2) = m^2 \left(\log \frac{m^2}{Q^2} - 1 \right), \quad (\text{A16})$$

with the optimized scale choice $Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$.

For the top-squark contributions, we find

$$\begin{aligned} \Sigma_u^u(\tilde{t}_{1,2}) &= \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \\ &\quad \times \left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_W)\Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right] \end{aligned} \quad (\text{A17})$$

$$\Sigma_d^d(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \left[g_Z^2 \mp \frac{f_t^2 \mu^2 + 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_W)\Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right], \quad (\text{A18})$$

where $\Delta_t = (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)/2 + M_Z^2 \cos 2\beta(\frac{1}{4} - \frac{2}{3}x_W)$ and $x_W \equiv \sin^2\theta_W$. In the denominator of Eqs. (A17) and (A18), the tree-level expressions of $m_{\tilde{t}_{1,2}}^2$ should be used.

For b -squark contributions, we have

$$\begin{aligned} \Sigma_u^u(\tilde{b}_{1,2}) &= \frac{3}{16\pi^2} F(m_{\tilde{b}_{1,2}}^2) \\ &\quad \times \left[g_Z^2 \mp \frac{f_b^2 \mu^2 - 8g_Z^2(\frac{1}{4} - \frac{1}{3}x_W)\Delta_b}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2} \right] \\ \Sigma_d^d(\tilde{b}_{1,2}) &= \frac{3}{16\pi^2} F(m_{\tilde{b}_{1,2}}^2) \\ &\quad \times \left[f_b^2 - g_Z^2 \mp \frac{f_b^2 A_b^2 - 8g_Z^2(\frac{1}{4} - \frac{1}{3}x_W)\Delta_b}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2} \right], \end{aligned} \quad (\text{A19})$$

where $\Delta_b = (m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2)/2 - M_Z^2 \cos 2\beta(\frac{1}{4} - \frac{1}{3}x_W)$. The expressions for $\Sigma_u^u(\tilde{\tau}_{1,2})$ and $\Sigma_d^d(\tilde{\tau}_{1,2})$ are similar to $\Sigma_u^u(\tilde{b}_{1,2})$ and $\Sigma_d^d(\tilde{b}_{1,2})$ but with $b \rightarrow \tau$, $c_{\text{col}} = 1$ and $(\frac{1}{4} - \frac{1}{3}x_W) \rightarrow (\frac{1}{4} - x_W)$.

For first-/second-generation sfermion contributions, we find

$$\Sigma_{u,d}^{u,d}(\tilde{f}_{L,R}) = \frac{c_{\text{col}}}{16\pi^2} F(m_{\tilde{f}_{L,R}}^2) (4g_Z^2(T_3 - Qx_W), \quad (\text{A20})$$

where T_3 is the weak isospin and Q is the electric charge assignment (taking care to flip the sign of Q for R sfermions). For instance,

$$\Sigma_u^u(\tilde{u}_L) = \frac{3}{16\pi^2} F(m_{\tilde{u}_L}^2) \left(f_u^2 - 4g_Z^2 \left(\frac{1}{2} - \frac{2}{3} x_W \right) \right) \quad (\text{A21})$$

$$\Sigma_u^u(\tilde{u}_R) = \frac{3}{16\pi^2} F(m_{\tilde{u}_R}^2) \left(f_u^2 - 4g_Z^2 \left(\frac{2}{3} x_W \right) \right) \quad (\text{A22})$$

$$\Sigma_d^d(\tilde{u}_L) = \frac{3}{16\pi^2} F(m_{\tilde{u}_L}^2) \left(4g_Z^2 \left(\frac{1}{2} - \frac{2}{3} x_W \right) \right) \quad (\text{A23})$$

$$\Sigma_d^d(\tilde{u}_R) = \frac{3}{16\pi^2} F(m_{\tilde{u}_R}^2) \left(4g_Z^2 \left(\frac{2}{3} x_W \right) \right). \quad (\text{A24})$$

These contributions, arising from electroweak D -term contributions to masses, cancel out separately for squarks and sleptons in the limit of mass degeneracy due to the fact that weak isospins and electric charges (or weak hypercharges) sum to zero in each generation. For this reason, we sum these contributions before taking the maximum contribution to the fine-tuning measure Δ_{EW} .

For chargino contributions, we find

$$\Sigma_u^u(\tilde{W}_{1,2}^\pm) = \frac{-g^2}{16\pi^2} F(m_{\tilde{W}_{1,2}}^2) \times \left(1 \mp \frac{M_2^2 + \mu^2 - 2m_W^2 \cos 2\beta}{m_{\tilde{W}_2}^2 - m_{\tilde{W}_1}^2} \right) \quad (\text{A25})$$

$$\Sigma_d^d(\tilde{W}_{1,2}^\pm) = \frac{-g^2}{16\pi^2} F(m_{\tilde{W}_{1,2}}^2) \times \left(1 \mp \frac{M_2^2 + \mu^2 + 2m_W^2 \cos 2\beta}{m_{\tilde{W}_2}^2 - m_{\tilde{W}_1}^2} \right). \quad (\text{A26})$$

For contributions from neutralinos, we find⁵

$$\Sigma_u^u(\tilde{Z}_i) = \frac{1}{16\pi^2} \frac{F(m_{\tilde{Z}_i}^2)}{D(\tilde{Z}_i)} \times [K(\tilde{Z}_i) - 2(g^2 + g'^2)\mu^2 M_Z^2 \cos^2 \beta (m_{\tilde{Z}_i}^2 - m_{\tilde{\gamma}}^2)], \quad (\text{A27})$$

$$\Sigma_d^d(\tilde{Z}_i) = \frac{1}{16\pi^2} \frac{F(m_{\tilde{Z}_i}^2)}{D(\tilde{Z}_i)} \times [K(\tilde{Z}_i) - 2(g^2 + g'^2)\mu^2 M_Z^2 \sin^2 \beta (m_{\tilde{Z}_i}^2 - m_{\tilde{\gamma}}^2)], \quad (\text{A28})$$

⁵Unlike the case of other contributions where it is easy to explicitly find the eigenvalues of the Higgs-field-dependent squared mass matrices, this is not possible for the neutralino. To evaluate the derivatives of the eigenvalues of the *squared* neutralino mass matrix that appear in Eq. (A15), we use the technique introduced in Ref. [39] and elaborated on further in Ref. [78].

where

$$K(\tilde{Z}_i) = -m_{\tilde{Z}_i}^6 (g^2 + g'^2) + m_{\tilde{Z}_i}^4 [g^2(M_1^2 + \mu^2) + g'^2(M_2^2 + \mu^2) + (g^2 + g'^2)M_Z^2] - m_{\tilde{Z}_i}^2 [\mu^2(g^2 M_1^2 + g'^2 M_2^2) + (g^2 + g'^2)M_Z^2 m_{\tilde{\gamma}}^2], \quad (\text{A29})$$

$D(\tilde{Z}_i) = \prod_{j \neq i} (m_{\tilde{Z}_i}^2 - m_{\tilde{Z}_j}^2)$, and $m_{\tilde{\gamma}} = M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W$. Our neutralino corrections differ in form as well as numerically from those in the literature where these were calculated using the neutralino mass (not mass squared) matrix [39,40].

For weak bosons, we find

$$\Sigma_u^u(W^\pm) = \Sigma_d^d(W^\pm) = \frac{3g^2}{32\pi^2} F(m_W^2) \quad (\text{A30})$$

$$\Sigma_u^u(Z^0) = \Sigma_d^d(Z^0) = \frac{3g^2}{64\pi^2 \cos^2 \theta_W} F(M_Z^2). \quad (\text{A31})$$

For Higgs bosons, we find

$$\Sigma_u^u(h, H) = \frac{g_Z^2}{16\pi^2} F(m_{h,H}^2) \times \left(1 \mp \frac{M_Z^2 + m_A^2 (1 + 4 \cos 2\beta + 2 \cos^2 2\beta)}{m_H^2 - m_h^2} \right), \quad (\text{A32})$$

$$\Sigma_d^d(h, H) = \frac{g_Z^2}{16\pi^2} F(m_{h,H}^2) \times \left(1 \mp \frac{M_Z^2 + m_A^2 (1 - 4 \cos 2\beta + 2 \cos^2 2\beta)}{m_H^2 - m_h^2} \right) \quad (\text{A33})$$

and

$$\Sigma_u^u(H^\pm) = \Sigma_d^d(H^\pm) = \frac{g^2}{32\pi^2} F(m_{H^\pm}^2). \quad (\text{A34})$$

For SM fermions t , b , and τ , we find

$$\Sigma_u^u(t) = -\frac{3f_t^2}{8\pi^2} F(m_t^2), \quad (\text{A35})$$

$$\Sigma_d^d(t) = 0 \quad (\text{A36})$$

$$\Sigma_u^u(b) = 0, \quad (\text{A37})$$

$$\Sigma_d^d(b) = -\frac{3f_b^2}{8\pi^2} F(m_b^2) \quad (\text{A38})$$

$$\Sigma_u^u(\tau) = 0, \quad (\text{A39})$$

$$\Sigma_d^d(\tau) = -\frac{f_\tau^2}{8\pi^2} F(m_\tau^2). \quad (\text{A40})$$

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