PHYSICAL REVIEW D **87,** 114014 (2013)

Large N_c deconfinement transition in the presence of a magnetic field

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(Received 9 August 2012; published 17 June 2013)

We investigate the effect of a homogeneous magnetic field on the thermal deconfinement transition of QCD in the large N_c limit. First we discuss how the critical temperature decreases due to the inclusion of $N_f \ll N_c$ flavors of massless quarks in comparison to the pure glue case. Then we study the equivalent correction in the presence of an external Abelian magnetic field. To leading order in N_f/N_c, the deconfinement critical temperature decreases with the magnetic field if the flavor contribution to the pressure behaves paramagnetically, with a sufficiently large magnetization as to overcome any possible magnetic effects in the string tension. Finally, we discuss the effects from a finite quark mass and its competition with magnetic effects.

DOI: 10.1103/PhysRevD.87.114014 PACS numbers: 25.75.Nq, 11.15.Pg, 12.38.Aw, 12.38.Mh

I. INTRODUCTION

The phase diagram of strong interactions in the presence of a classical, constant, and uniform magnetic background has been attracting increasing interest in the last few years. Strong (Abelian) magnetic fields not only provide another control parameter to probe the phase structure of QCD but are also currently generated in noncentral ultrarelativistic heavy ion collisions at the Brookhaven National Laboratory's Relativistic Heavy Ion Collider and at the CERN LHC. In fact, these fields are believed to be the largest ever produced since the electroweak phase transition in the early Universe, reaching values on the order of $B \sim 10^{19}$ Gauss ($eB \sim 6m_\pi^2$) and even much higher [1]. Furthermore, lattice Monte Carlo simulations are not constrained by the sign problem in this case and can produce a trustworthy $T \times eB$ phase diagram, among other results. Nevertheless, the mapping of this new phase diagram is still in its infancy and presents some conflicting pictures coming from different model calculations.

In this paper we study the behavior of the deconfining critical temperature T_c in the presence of a strong magnetic field in the large N_c limit of QCD. This provides a well-defined setup for a clean, semiquantitative description by essentially counting powers of N_f/N_c (with N_f being the number of quark flavors) when matching pressures for the confined and deconfined sectors. Our analysis suggests that the deconfinement temperature decreases with the magnetic field for small N_f/N_c, provided that the flavor contribution to the large N_c pressure is paramagnetic. We also discuss how the critical temperature for the pure glue theory decreases due to the leading order correction in N_f/N_c in the absence of a magnetic field.

All model calculations so far have suggested that sufficiently large magnetic fields, typically $eB \sim 10m_\pi^2$, could bring remarkable modifications in the QCD phase diagram, from shifting the chiral and the deconfinement phase transition lines [2–12] to transforming the vacuum into a superconducting medium via ρ-meson condensation [13]. In particular, most model descriptions have predicted either an increase or a flat behavior for the deconfinement critical line as eB is increased to very large values. Exceptions can be found in Ref. [2], where the critical temperature vanishes at a finite critical value of $eB_c \sim 25m_\pi^2$, featuring the disappearance of the confined phase at large magnetic fields, and in [3], where vacuum corrections are disregarded, and T_c diminishes with eB.

The first pioneering lattice simulations [14], still with large values for the pion mass, also suggested a very mild increase of the critical temperature with eB. However, recent lattice simulations with physical masses [15] have shown that the critical temperature for deconfinement actually falls as the magnetic field increases. However, instead of falling with a rate that will bring it to zero at a given critical value of eB, it falls less and less rapidly, tending to saturate at large values of B in agreement with what one would expect from the phenomenon of magnetic catalysis [16,17]. An exercise within the MIT bag model with the appropriate treatment of the subtleties of renormalization at finite B has shown remarkable qualitative agreement with these lattice findings with respect to the behavior of $T_c(eB)$; i.e., it decreases and saturates for very large fields [18]. To the best of our knowledge, even if known to be crude in numerical precision and missing the correct nature of the (crossover) transition, this is the only description to date that captures the correct qualitative behavior of the deconfining transition in a magnetic background.

Although a description of the deconfinement transition in the presence of an external magnetic field in terms of the MIT bag model is, of course, very simple, we believe it encodes an essential ingredient to provide a qualitative description of the behavior of $T_c \times eB$: confinement. The fact that the MIT bag model incorporates confinement (even if in its simplest fashion) seems to make it suitable to describe the behavior of T_c as a function of external parameters, as hinted by a previous successful description of the behavior of the critical temperature as a function of the pion mass and isospin chemical potential, as compared to lattice data, where chiral models failed even qualitatively [19,20]. This suggests that confinement dynamics may play a central role in guiding the functional behavior of T_c and points towards a large N_c description of the associated magnetic thermodynamics.

II. LARGE N_c THERMODYNAMICS

The large N_c limit provides a great opportunity to study several aspects of QCD [21–24]. Feynman diagrams are reorganized according to their dependence on N_c and, when $N_c \to \infty$, only planar diagrams are relevant. The theory is still asymptotically free, with a perturbative beta function defined in terms of the 't Hooft coupling $\lambda \equiv g^2 N_c$ and a renormalization group invariant energy scale Λ_{QCD} at which the associated coupling becomes strong. While confinement has not been proven in this limit, it is widely believed that in the vacuum the physical degrees of freedom are weakly interacting (since interactions go as $1/N_c$), colorless glueballs. N_f quark degrees of freedom in the fundamental representation can be added to this theory and the corresponding mesons are free when $N_c \to \infty$ while baryons become extremely heavy, $M_{\mathrm{baryon}} \sim N_c \Lambda_{\mathrm{QCD}}$ [23,24].

Lattice QCD calculations [25] show that the deconfinement phase transition of pure glue $SU(N_c)$ gauge theory becomes first order when $N_c \geq 3$ [26–29] with a critical temperature $\lim_{N_c\to\infty} T_c/\sqrt{\sigma_0} = 0.5949(17) + 0.458(18)/N_c^2$ [30], where $\sigma_0 \sim (440\ \mathrm{MeV})^2$ is the string tension of the large N_c pure glue theory. The thermodynamic properties of pure glue do not seem to change appreciably when $N_c \geq 3$ [31,32], which suggests that large N_c arguments may indeed capture the main physical mechanism behind the deconfinement phase transition of QCD (at least when N_c is sufficiently large).

The fact that $\lim_{N_c\to\infty} T_c/\sqrt{\sigma_0} \sim \mathcal{O}(N_c^0)$ and that the deconfinement phase transition is a strong first order transition can be readily understood using the following argument [28]. When $N_c \to \infty$ and $N_f = 0$, in the confined phase glueballs are very weakly interacting and, since they are colorless, they only contribute to the pressure at $\mathcal{O}(N_c^0)$. String breaking processes cannot occur when $N_f = 0$. Therefore, when $N_c \to \infty$ the only contribution to the pressure of the confined phase comes from the gluon condensate $\sim N_c^2 \Lambda_{\mathrm{QCD}}^4$, which we write in terms of the renormalization group invariant σ_0 as $P_{\mathrm{conf}} = c_0^4 N_c^2 \sigma_0^2$, where c_0 is a positive number of order 1. Moreover, it should be noticed that the entropy density in the confined phase vanishes.

On the other hand, asymptotic freedom implies that in the planar limit the gluon pressure is $P_{\mathrm{gluon}}(T) = N_c^2 T^4 c_{\mathrm{SB}}^4 f_{\mathrm{glue}}(T/\sqrt{\sigma_0})$, where c_{SB} is a positive constant determined from the Stefan-Boltzmann limit and $\lim_{T/\sqrt{\sigma_0}\to\infty} f_{\mathrm{glue}}(T/\sqrt{\sigma_0}) = 1$. The function f_{glue} depends implicitly on the 't Hooft coupling $\lambda(T)$ and, while its general form is not known when $T \sim \sqrt{\sigma_0}$, thermodynamical equilibrium imposes that it should be a monotonically increasing function of T that interpolates from 0 when $T \to 0$ to 1 for $T \to \infty$. Its form can be computed using perturbation theory at sufficiently high temperatures, where λ becomes very small [33]. If $N_f = 0$, since the pressure is always continuous at any phase transition, we see that there must be a deconfinement critical temperature defined by the condition $P_{\mathrm{glue}}(T_c^{(0)}/\sqrt{\sigma_0}) = P_{\mathrm{conf}}$ or, equivalently,

$$c_0^4 N_c^2 \sigma_0^2 = N_c^2 T_c^{(0)\,4} c_{\mathrm{SB}}^4 f_{\mathrm{glue}}(T_c^{(0)}/\sqrt{\sigma_0}), \tag{1}$$

which implies that the solution $T_c^{(0)}$ is a pure number of $\mathcal{O}(N_c^0)$ that in general cannot be computed perturbatively since it is obtained from the self-consistent equation

$$\frac{T_c^{(0)}}{\sqrt{\sigma_0}} f_{\mathrm{glue}}^{1/4}\left(\frac{T_c^{(0)}}{\sqrt{\sigma_0}}\right) = \frac{c_0}{c_{\mathrm{SB}}}. \tag{2}$$

Since f_{glue} increases monotonically with T, one obtains that $T_c^{(0)}$ must increase with c_0 (note that the critical temperature only vanishes if $c_0 \to 0$) [34]. Lattice calculations have shown that $T_c^{(0)}/\sqrt{\sigma_0} \sim 0.59$ [30]. The phase transition to a Z_{N_c} symmetric deconfined phase is then of first order when $N_c \to \infty$, $N_f = 0$, and the entropy density jumps from zero to a finite number of $\mathcal{O}(N_c^2)$ at $T_c^{(0)}$.

III. LEADING N_f/N_c CORRECTIONS

The first correction to this picture appears with the inclusion of N_f flavors of *massless* quarks. The previous Z_{N_c} symmetry is broken explicitly in the deconfined phase because of the presence of quarks. While the $U(N_f) \otimes U(N_f) \to U(N_f)_{\mathrm{vector}}$ pattern of (spontaneous) symmetry breaking leads to $N_f^2 - 1$ Goldstone bosons (the "pions"), their contribution to the pressure of the confined phase is of $\mathcal{O}(N_f^2 N_c^0)$, being negligible when $N_f \ll N_c$.

The presence of quark flavors, even in the massless limit, can lead to corrections of order $\sim N_f N_c$ to the vacuum pressure. In the double line notation [21], the addition of quark flavors leads to diagrams with boundaries, and it is possible to write down an infinite series of diagrams (each one with a power of λ) that can enter at that order due to production of quark-antiquark loops. Once quark loops appear in the theory, it is natural to assume that the value of the string tension decreases with the leading N_f/N_c correction with respect to the $N_f = 0$ value. This occurs because $q\bar{q}$ pairs can now be produced, which should decrease the linear confining potential experienced by infinitely massive probes in the fundamental representation (i.e., the heavy quark potential). Therefore, we assume that the string tension in the presence of the leading flavor correction is given by $\sigma/\sigma_0 = 1 - \alpha N_f/(2N_c)$, where α is *positive definite*. Given this expression for the string tension, the large N_c vacuum pressure becomes, in the presence of massless quarks,

$$P_{\rm conf} = c_0^4 N_c^2 \sigma_0^2 \left(1 - \alpha \frac{N_f}{N_c}\right). \tag{3}$$

When quarks are massive, there is another term of order $N_f N_c$ in the vacuum pressure given by the quark condensate contribution to the trace anomaly. We will discuss the massive quark case later; for now we keep the focus on the massless quark limit.

Once N_f flavors are included in the theory, the deconfined pressure also receives a contribution of order $N_c N_f$, which we denote here by $P_{\rm quark}(T)$. The most general expression for this quantity has the form $P_{\rm quark}(T) = N_c N_f T^4 c_{q\rm SB}^4 f_{\rm quark}(T/\sqrt{\sigma_0})$, where $c_{q\rm SB}$ is the corresponding positive dimensionless number computed in the Stefan-Boltzmann limit and $f_{\rm quark}$ is a monotonically increasing function of T such that $\lim_{T/\sqrt{\sigma_0}\to\infty} f_{\rm quark}(T) = 1$.

When $N_f/N_c \ll 1$ the explicit breaking of Z_{N_c} symmetry is small, slightly smoothing the phase transition into a very rapid crossover. The Polyakov loop below the transition is small, i.e., of order N_f/N_c. (This is why the contribution from a Polyakov loop potential to the pressure goes effectively as $\sim N_f^2$, i.e., a mesonlike contribution.) The balance equation that defines the critical temperature $T_c^{(1)}$ modified by the quark flavors is obtained by equating the pressures $P_{\rm conf} = P_{\rm glue}(T_c^{(1)}) + P_{\rm quark}(T_c^{(1)})$. Since $f_{\rm quark}$ is a monotonic function of T, one should expect that the critical temperature gets shifted towards smaller values. In fact, in the limit where $N_f/N_c \ll 1$ one finds the self-consistent equation

$$\frac{T_c^{(1)}}{\sqrt{\sigma_0}} = \frac{c_0}{c_{\rm SB} f_{\rm glue}^{1/4}\left(\frac{T_c^{(1)}}{\sqrt{\sigma_0}}\right)} \left[1 - \frac{\alpha}{4}\frac{N_f}{N_c} - \frac{1}{4}\frac{N_f}{N_c}\frac{c_{q\rm SB}^4 f_{\rm quark}\left(\frac{T_c^{(1)}}{\sqrt{\sigma_0}}\right)}{c_{\rm SB}^4 f_{\rm glue}\left(\frac{T_c^{(1)}}{\sqrt{\sigma_0}}\right)}\right]. \tag{4}$$

It is possible to obtain the effect of the leading order N_f/N_c correction on $T_c^{(1)}$ in terms of $T_c^{(0)}$. Keeping only the first correction in N_f/N_c, one may take $T_c^{(1)} \mapsto T_c^{(0)}$ inside the brackets in the equation above. Since the ratio $f_{\rm quark}/f_{\rm glue}$ is positive, one can define a new (still positive) constant given by

$$c_1(N_f) \equiv c_0 \left[1 - \frac{\alpha}{4}\frac{N_f}{N_c} - \frac{1}{4}\frac{N_f}{N_c}\frac{c_{q\rm SB}^4 f_{\rm quark}\left(\frac{T_c^{(0)}}{\sqrt{\sigma_0}}\right)}{c_{\rm SB}^4 f_{\rm glue}\left(\frac{T_c^{(0)}}{\sqrt{\sigma_0}}\right)}\right]. \tag{5}$$

Therefore, the self-consistent equation for $T_c^{(1)}$ actually has the same form as Eq. (2) and is given by

$$\frac{T_c^{(1)}}{\sqrt{\sigma_0}} f_{\rm glue}^{1/4}\left(\frac{T_c^{(1)}}{\sqrt{\sigma_0}}\right) = \frac{c_1(N_f)}{c_{\rm SB}}. \tag{6}$$

Thus, since $c_1(N_f) < c_0$ and $f_{\rm glue}$ is monotonically increasing with T, we see that the leading effect of N_f massless flavors in the large N_c limit is to decrease the critical temperature by a small amount of order N_f/N_c with respect to $T_c^{(0)}$. In other words, the addition of a small number of light quark flavors should decrease the value of the deconfinement critical temperature at large N_c. While the validity of any result obtained in the large N_c limit cannot be straightforwardly extended to the physical $N_c = N_f = 3$ case, it is reassuring to know that lattice QCD simulations [35–38] performed with $N_c = 3$ have found that light quark flavors decrease the deconfinement temperature.

IV. LARGE N_c BEHAVIOR OF $T_c \times (eB)$

The same line of argument used above can be employed to study what happens to the deconfinement critical temperature in the presence of an external magnetic field in the large N_c limit of QCD. Assuming that $N_f/N_c \ll 1$ and the quark mass $m_q = 0$, the magnetic field affects the confined pressure at order $N_f N_c$ via the effects of quark loops (higher order corrections were studied in [39–41]). Thus, we promote α to be a function of the magnetic field as follows: $\alpha \to \tilde{\alpha}(eB/\sigma_0)$. While we cannot say anything about the explicit magnetic field dependence of $\tilde{\alpha}$, since it depends on the nonperturbative QCD dynamics, we assume that $\tilde{\alpha}(eB/\sigma_0)$ is still positive definite. The confined pressure to leading order will, then, be

$$P_{\rm conf}(eB/\sigma_0) = c_0^4 N_c^2 \sigma_0^2 \left[1 - \tilde{\alpha}\left(\frac{eB}{\sigma_0}\right)\frac{N_{\rm pairs}(N_f)}{N_c}\right] \tag{7}$$

with $N_{\rm pairs}(N_f)/N_c \ll 1$ being the number of pairs of quark flavors with electric charges $\{(N_c - 1)/N_c, -1/N_c\}$ in units of the fundamental charge. Only the largest ($\sim N_c^0$) charge in each pair contributes to leading order in N_f/N_c.

In the deconfined phase, the N_c^2 contribution to the pressure is again $P_{\rm glue}(T) = N_c^2 T^4 c_{\rm SB}^4 f_{\rm glue}(T/\sqrt{\sigma_0})$ but the $N_f N_c$ flavor correction $P_{\rm quark}$ feels the effects of the magnetic field directly. In fact, the regularized contribution [42] of the massless quarks to the pressure is $P_{\rm quark}(T, eB) = N_c N_{\rm pairs}(N_f) T^4 c_{q\rm SB}^4 \tilde{f}_{\rm quark}(T/\sqrt{\sigma_0}, eB/T^2)$.

Notice that the function $\tilde{f}_{\rm quark}$ is positive definite and must increase monotonically with T for a fixed value of eB until it goes to 1 in the high temperature limit $T \gg \sqrt{\sigma_0}$, eB. Given our previous analysis for the case where $N_f \neq 0$ and $B = 0$, one should expect that the critical temperature as a function of the magnetic field, $T_c(eB)$, must decrease with respect to pure glue value $T_c^{(0)}$ by an amount of $\mathcal{O}(N_f/N_c)$.

This can be seen directly by equating the pressures at T_c,

$$c_0^4 N_c^2 \sigma_0^2 \left[1 - \tilde{\alpha}\left(\frac{eB}{\sigma_0}\right)\frac{N_{\rm pairs}(N_f)}{N_c}\right] = N_c^2 T_c^4 c_{\rm SB}^4 f_{\rm glue}\left(\frac{T_c}{\sqrt{\sigma_0}}\right) + N_c N_{\rm pairs}(N_f) T_c^4 c_{q\rm SB}^4 \tilde{f}_{\rm quark}\left(\frac{T_c}{\sqrt{\sigma_0}}, \frac{eB}{T_c^2}\right) \quad (8)$$

and noticing that, since the left-hand side of the equation above is fixed, the addition of the quark contribution on the right-hand side must lead to a decrease of the critical temperature by an amount of order N_f/N_c. In fact, the solution to the equation above for $T_c(eB)$, to leading order in N_f/N_c, is

$$\frac{T_c(eB)}{\sqrt{\sigma_0}} f_{\rm glue}^{1/4}\left(\frac{T_c(eB)}{\sqrt{\sigma_0}}\right) = \frac{c_2(N_{\rm pairs}, eB)}{c_{\rm SB}}, \quad (9)$$

where we defined

$$c_2(N_{\rm pairs}, eB) \equiv c_0\left[1 - \frac{1}{4}\tilde{\alpha}\left(\frac{eB}{\sigma_0}\right)\frac{N_{\rm pairs}(N_f)}{N_c}\right] \times \left[1 - \frac{1}{4}\frac{N_{\rm pairs}(N_f)}{N_c}\frac{c_{q\rm SB}^4 \tilde{f}_{\rm quark}\left(\frac{T_c^{(0)}}{\sqrt{\sigma_0}}, \frac{eB}{T_c^{(0)2}}\right)}{c_{\rm SB}^4 f_{\rm glue}\left(\frac{T_c^{(0)}}{\sqrt{\sigma_0}}\right)}\right]. \quad (10)$$

Since $c_2(N_{\rm pairs}, eB) < c_0$, the same arguments used before show that $T_c(eB)/T_c^{(0)} < 1$ by an amount $\sim N_f/N_c$. Therefore, one concludes that, in the presence of an external magnetic field, the deconfinement critical temperature decreases with respect to its value for pure glue in the large N_c limit of QCD. Whether $T_c(eB)$ is also lower than the critical temperature in the presence of N_f/N_c flavors of massless quarks at $B = 0$, $T_c^{(1)}$ requires that $c_2(N_{\rm pairs}, eB) < c_1$. This can be rewritten as a condition on the derivatives with respect to B of the quark pressure, i.e., the magnetization $M(T_c, eB)$, and of the modification of the string tension, $\partial_B\tilde{\alpha}$: $M(T_c, eB) > \max\{0, -c_{\rm SB}^4 f_{\rm glue}\partial_B\tilde{\alpha}\}$. This occurs if the flavor contribution behaves paramagnetically, with positive magnetization $M(T_c, eB)$ that is sufficiently large.

For a free gas implementation of the deconfined phase $f_{\rm glue} = 1$ and in the limit of strong magnetic fields $eB/T^2 \gg 1$, one finds that $\tilde{f}_{\rm quark} \sim eB/T_c^2$ [18]. Assuming that the magnetic effects on the string tension are negligible, we may set $\tilde{\alpha} = \alpha$. Thus, in this case the magnetic suppression of the deconfinement critical temperature goes like $eBN_{\rm pairs}/(N_c\sigma_0)$. In fact, this simple implementation in the limits of low and high magnetic fields provides a scenario in which the slope in $T_c(eB)$ decreases for large fields, as illustrated in Fig. 1.

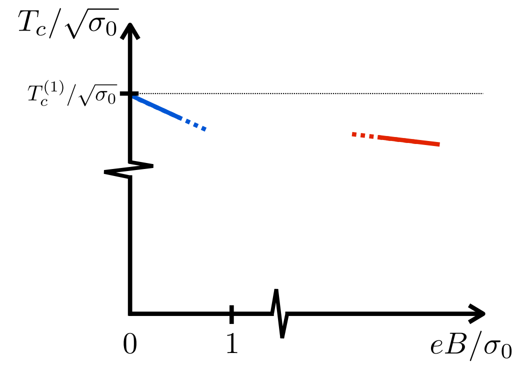

FIG. 1 (color online). Cartoon of the $T_c \times eB$ phase diagram in the large N_c limit, using the approximation of free deconfined quarks and gluons and the assumption that magnetic effects on the string tension are negligible, i.e., $\tilde{\alpha} = \alpha$.

An eventual saturation of T_c as a function of eB, as observed on the lattice [15] and in model calculations [18], cannot be obtained using the limits discussed in this paper in a general fashion. As mentioned above, the implications of large N_c estimates to the actual QCD phase diagram must be taken with great caution. The specific form of T_c as a function of eB depends on the nonperturbative functions $f_{\rm glue}$, $\tilde{f}_{\rm quark}$, and $\tilde{\alpha}$. In fact, in the large N_c limit, our results indicate that $T_c(eB)$ can only be a flat curve if $\tilde{f}_{\rm quark}$ and $\tilde{\alpha}$ are such that $M(T_c, eB)$ is positive but vanishes for large fields. In this scenario, a reasonable explanation for the nearly flat curve found in the $N_c = N_f = 3$ lattice study performed in Ref. [15] is a net cancellation effect that occurs for sufficiently large fields due to a magnetic field dependent contribution to the pressure below the phase transition (which in the physical case includes the dynamics of mesons).

V. QUARK MASS EFFECTS

When $m_q \neq 0$ the pressure of the confined phase is increased by the quark contribution to the vacuum trace anomaly, $N_c N_f m_q(-\langle\bar{q}q\rangle)$, where we used the fact that the quark condensate is negative. This is equivalent to a small positive shift of c_0 and, to leading order in N_f/N_c, the confined phase pressure when $eB = 0$ is $P_{\rm conf} = c_{m_q}^4 N_c^2 \sigma_0^2$, where

$$c_{m_q} = c_0\left(1 - \frac{\alpha_{m_q}}{4}\frac{N_f}{N_c} + \frac{1}{4}\frac{N_f}{N_c}\frac{m_q}{\sqrt{\sigma_0}}\frac{(-\langle\bar{q}q\rangle)}{c_0^4\sigma_0^{3/2}}\right). \quad (11)$$

Here α_{m_q} (assumed to be positive) includes possible quark mass effects on the α coefficient. In the deconfined phase only the quark pressure will be affected by the quark mass effects, decreasing e.g. in perturbation theory [33]. In a large temperature expansion, we may write $f_{\rm quark} \mapsto f_{\rm quark} - c_3 m_q^2/T^2$, where c_3 is positive. Therefore, the critical temperature computation in this massive case follows the same steps that led to Eqs. (5) and (6), with the substitution

$f_{\rm quark}|_{m_q=0} \mapsto f_{\rm quark}|_{m_q=0} - c_3 m_q^2/T^2 < f_{\rm quark}|_{m_q=0}$. As a consequence, $c_1(N_f, m_q) > c_1(N_f, m_q = 0)$ and $T_c^{(m_q)}$ is higher than its massless counterpart, $T_c^{(1)}$. Interestingly enough, however, the corrections to $f_{\rm quark}$ are respectively $\sim (m_q/T_c^{(0)})^2$, being extremely small for reasonable values of quark masses, $m_q \ll \sqrt{\sigma_0}$, $T_c^{(0)}$. Therefore, in this large N_c regime, we find that the critical temperature as a function of m_q is essentially flat. Similar behavior has been observed on the lattice for $SU(3)$ [43,44].

Of course, the explicit dependence of T_c with respect to the quark mass (or equivalently the pion mass) will also depend on the details of the functions $f_{\rm glue}$, $f_{\rm quark}$, α (which may acquire an extra dependence on the quark mass) as well as the quark condensate. In the study performed in [19,20] within an effective model implementation of the $N_c = 3$ and $N_f = 2$ deconfined phase, $T_c/\sqrt{\sigma_0}$ was found to be nearly constant with respect to variations in the pion mass.

In the presence of a magnetic field, the quark condensate and its influence on T_c are unaltered at this order in N_f/N_c, while the quark pressure receives magnetic contributions, becoming $\hat{f}_{\rm quark}(T/\sqrt{\sigma_0}, m_q/T, eB/T^2)$. Therefore, the critical temperature $T_c^{(2,m_q)}$ is the solution of Eq. (9) with c_2 replaced by

$$\frac{c_2(N_{\rm pairs}, eB, m_q)}{c_{m_q}(N_{\rm pairs}, eB, m_q)} = \left[1 - \frac{1}{4}\frac{N_{\rm pairs}(N_f)}{N_c}\frac{c_{q{\rm SB}}^4 \hat{f}_{\rm quark}\left(\frac{T_c^{(0)}}{\sqrt{\sigma_0}}, \frac{m_q}{T_c^{(0)}}, \frac{eB}{T_c^{(0)2}}\right)}{c_{\rm SB}^4 f_{\rm glue}\left(\frac{T_c^{(0)}}{\sqrt{\sigma_0}}\right)}\right] \tag{12}$$

where $c_{m_q}(N_{\rm pairs}, eB, m_q)$ is the corresponding generalization of c_{m_q} in Eq. (11) that takes into account magnetic field effects. In this more complicated scenario there will be a competition between mass and magnetic effects and it is hard to obtain even a qualitative estimate of the general behavior of the critical temperature as a function of eB. If, however, the term that is most sensitive to the magnetic field is $\hat{f}_{\rm quark}$, then if this term is paramagnetic the critical temperature would assume values that are lower than $T_c^{(m_q)}$ as one varies the magnetic field.

VI. FINAL COMMENTS

It would be interesting to extend the discussion about the magnetic effects on the deconfinement critical temperature to the Veneziano limit of QCD [45]. In this case, one could also study whether chiral symmetry restoration coincides with the deconfinement transition when $N_f, N_c \to \infty$ in the presence of an external magnetic field.

ACKNOWLEDGMENTS

E. S. F. and J. N. acknowledge the hospitality of Departamento de Física Teórica at UERJ, where most of this work has been conducted. The authors thank M. Chernodub, M. D'Elia, R. D. Pisarski, and I. Shovkovy for comments and discussions. This work was partially supported by CAPES, CNPq, FAPERJ, FAPESP and FUJB/UFRJ.

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