

**Axial-vector mesons from  $\tau \rightarrow AP\nu_\tau$  decays**

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Axial-vector mesons  $a_1(1260)$ ,  $f_1(1285)$ ,  $h_1(1170)$ ,  $K_1(1270)$ , and  $K_1(1400)$  can be produced in semileptonic  $\tau \rightarrow AP\nu_\tau$  decays, where  $P$  stands for the pseudoscalar mesons  $\pi$  or  $K$ . We calculate the branching ratios based in a meson dominance model. The exclusive channels  $\tau^- \rightarrow a_1(1260)^- \pi^0 \nu$ ,  $\tau^- \rightarrow a_1(1260)^0 \pi^- \nu$ , and  $\tau^- \rightarrow h_1(1170) \pi^- \nu$  turn out to be of order  $\mathcal{O}(10^{-3})$ , the channel  $\tau^- \rightarrow f_1(1285) \pi^- \nu$  of order  $\mathcal{O}(10^{-4})$ , and channels  $\tau^- \rightarrow K_1(1270)^- \pi^0 \nu$ ,  $\tau^- \rightarrow K_1(1270)^0 \pi^- \nu$ ,  $\tau^- \rightarrow K_1(1400)^- \pi^0 \nu$ , and  $\tau^- \rightarrow K_1(1400)^0 \pi^- \nu$  of order  $\mathcal{O}(10^{-6})$ . These results indicate that the branching ratios could be measured in experiments.

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**I. INTRODUCTION**

Recently the *BABAR* Collaboration [1] performed measurements of the branching fractions of three-prong and five-prong  $\tau$  decay modes. Specifically, the branching fraction for the  $\tau^- \rightarrow f_1(1285) \pi^- \nu_\tau$  decay is measured,

$$\mathcal{B}(\tau^- \rightarrow f_1(1285) \pi^- \nu_\tau) = (3.86 \pm 0.25) \times 10^{-4}, \quad (1)$$

where the axial-vector meson  $f_1(1285)$  is detected using the  $f_1 \rightarrow 2\pi^+ 2\pi^-$  and  $f_1 \rightarrow \pi^+ \pi^- \eta$  modes. We have averaged in quadrature the two branching ratios and added the statistical and systematic errors reported by *BABAR* Collaboration [1]. This result supersedes a value reported by *BABAR* Collaboration [2]. Besides, the *CLEO* Collaboration [3] published a branching ratio of  $(5.8_{-1.3}^{+1.4} \pm 1.8) \times 10^{-4}$ , which is also based on the  $f_1 \rightarrow 2\pi^+ 2\pi^-$  mode.

The measurements of  $\tau \rightarrow AP\nu$  decays can give information about the hadronic matrix elements  $\langle AP | J_\mu | 0 \rangle$  at the moderate energy regime of  $\tau$  decays. This can be relevant in nonleptonic two-body  $D$  and  $B$  decays, where the annihilation contribution plays a relevant role in the branching ratios and  $CP$  asymmetries, see Refs. [4–7].

The  $\tau \rightarrow AP\nu$  decays can contribute to the  $\tau$  decay modes with four-pseudoscalar mesons in the final state. The exclusive  $\tau^- \rightarrow f_1(1285) \pi^- \nu$  decay contributes as an intermediate state in  $\tau$  decay involving five-pseudoscalar mesons. The former decays can be related to  $e^+ e^- \rightarrow (4\pi)^0$ . The relation between the  $\tau^- \rightarrow (4\pi)^- \nu$  decays and the electromagnetic annihilation  $e^+ e^- \rightarrow (4\pi)^0$  process appears because the components of the same current are present in both cases, i.e., the current  $\bar{d}\gamma_\mu u$  in  $\tau$  decay and  $1/\sqrt{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$  in the case of  $e^+ e^-$

annihilation. Thus, these two processes are related by isospin symmetry.

On the other hand, at the theoretical level, the production of p-wave mesons in semileptonic  $\tau$  decays has not been widely studied in the literature. In effect, Ref. [8] and this paper are the only works that have considered the production of orbitally excited mesons in  $\tau \rightarrow MP\nu_\tau$  channels, where  $M$  can be an axial-vector meson or a tensor meson. The exception has been Ref. [9], where the branching ratio for the channel  $\tau^- \rightarrow f_1(1285) \pi^- \nu$  estimated is  $2.91 \times 10^{-4}$ .

In this paper we estimate the branching ratios of the  $\tau \rightarrow AP\nu$  decays, where  $P$  is the pseudoscalar meson  $\pi$  or  $K$  and  $A$  is the axial-vector meson  $a_1(1260)$ ,  $f_1(1285)$ ,  $h_1(1170)$ ,  $K_1(1270)$ , or  $K_1(1400)$ . We have considered the kinematically allowed, the  $G$ -parity conserved, and the first-class currents decays. We use a meson dominance model, where the resonances that contribute are coming from the vector mesons  $\rho(770)$ ,  $\rho(1450)$ , and  $K^*(982)$  and the axial-vector meson  $a_1(1260)$ . The weak decay and strong coupling constants are determined from experimental data.

In our estimations, we find that the branching ratios for the exclusive channels  $\tau^- \rightarrow a_1(1260)^- \pi^0 \nu$ ,  $\tau^- \rightarrow a_1(1260)^0 \pi^- \nu$ , and  $\tau^- \rightarrow h_1(1170) \pi^- \nu$  are of order  $\mathcal{O}(10^{-3})$ . The channels  $\tau \rightarrow K_1(1270) \pi \nu$  and  $\tau \rightarrow K_1(1400) \pi \nu$  decays are of order  $\mathcal{O}(10^{-6})$ . Finally, the modes  $\tau \rightarrow a_1(1260) K \nu$  and  $\tau \rightarrow K_1(1270) K \nu$  are kinematically suppressed. Up to now, the  $\tau^- \rightarrow f_1(1285) \pi^- \nu$  decay of order  $\mathcal{O}(10^{-4})$  is the only mode that can be compared with experimental measurements.

The paper is organized as follows. In Sec. II, we describe the amplitude in the meson dominance model and give the expression to estimate the width decay. The calculation of the weak decay and strong coupling constants are given in Sec. III. In Sec. IV, we present the estimation of the branching ratios for the modes considered in this paper. And finally, we conclude in Sec. V.

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## II. AMPLITUDE AND WIDTH DECAY FOR SEMILEPTONIC $\tau \rightarrow AP\nu$ DECAYS

The decay amplitude for the  $\tau(p_\tau) \rightarrow A(p_A)P(p) \nu_\tau(p_\nu)$  process, where  $A$  denotes the axial-vector meson  $a_1(1260)$ ,  $f_1(1285)$ ,  $h_1(1170)$ ,  $K_1(1270)$ , or  $K_1(1400)$ , and  $P$  is the pseudoscalar meson  $\pi$  or  $K$ , can be written by

$$\mathcal{M}(\tau \rightarrow AP\nu_\tau) = \frac{G_F}{\sqrt{2}} V_{ui} \bar{u}(p_\tau) \gamma^\mu (1 - \gamma_5) u(p_\nu) \times \langle A(p_A)P(p) | J_\mu(0) | 0 \rangle, \quad (2)$$

where  $G_F$  is the Fermi constant,  $V_{ui}$  ( $i = d$  or  $s$ ) is the corresponding element of the Cabibbo-Kobayashi-Maskawa matrix, and  $J_\mu(0)$  is the  $(V - A)$  weak current.

The hadronic matrix element  $\langle A(p_A, \varepsilon)P(p) | J_\mu | 0 \rangle$  is parametrized by [10]

$$\begin{aligned} \langle A(p_A, \varepsilon)P(p) | J_\mu | 0 \rangle &= l(t) \epsilon_\mu + (\varepsilon \cdot p) [c_+(t)(p - p_A)_\mu + c_-(t)(p + p_A)_\mu] \\ &\quad + iq(t) \epsilon_{\mu\nu\rho\sigma} \varepsilon^\nu (p_A - p)^\rho (p_A + p)^\sigma, \end{aligned} \quad (3)$$

where the hadronic matrix element is expressed in terms of the form factors  $l(t)$ ,  $c_+(t)$ ,  $c_-(t)$ , and  $q(t)$ , which are Lorentz-invariant functions of squared momentum transfer  $t = (p_A + p)^2$ ; the polarization vector  $\epsilon_\mu$  describes the axial-vector meson  $A$ .

To estimate the form factors, we use the meson dominance model, i.e., the decay amplitude is given by the sum of intermediate meson resonance contributions,

$$\mathcal{M}(\tau \rightarrow AP\nu_\tau) = \sum_{R^*} \mathcal{M}(\tau \rightarrow R^* \nu_\tau \rightarrow AP\nu_\tau), \quad (4)$$

where  $R^*$  is the intermediate resonance, which has the right quantum numbers for the specific processes. The sum is extended over all possible resonance contributions; see Fig. 1.

The production of the resonance  $R^*$  can be classified as either first- or second-class current depending on the spin  $J$ , parity  $P$ , and  $G$ -parity of the resonance particle. In the Standard Model, the first-class current is considered to

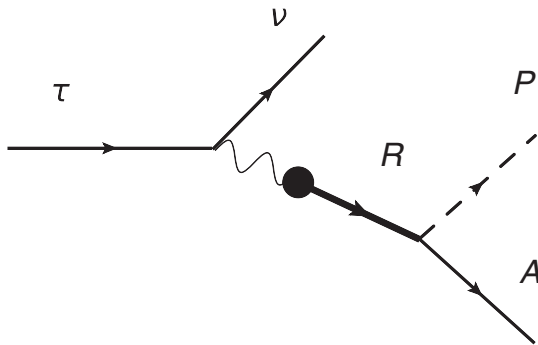


FIG. 1. Intermediate meson dominance resonance of  $\tau \rightarrow AP\nu$  decays.

dominate. The second-class currents are associated with a decay constant proportional to the mass difference between an up and down quark, and in the exact isospin limit symmetry they vanish [11]. The vector meson resonances  $\rho(770)$ ,  $\rho(1450)$ ,  $K^*(982)$  and the axial-vector meson  $a_1(1260)$  are produced by first-class currents, which are the contributions included in this work. The vertex  $R^* \rightarrow AP$  decay, which is produced by strong interaction, must conserve  $G$ -parity.

Comparing the hadronic current, see Eq. (3), with the amplitude build from the meson dominance model, which is calculated using Feynman rules, we determine expressions for the form factors  $q(t) = 0$  and  $c_-(t) = -c_+(t) = c(t)$ . Thus, the amplitude is expressed in terms of only two form factors  $l(t)$  and  $c(t)$ . These form factors can be expressed by means of the Breit-Wigner (BW) function,

$$\begin{aligned} l(t) &= -\frac{if_J g_{AJP}}{M_J} (p_A \cdot p_J) \text{BW}_J(t), \\ c(t) &= \frac{if_J g_{AJP}}{2M_J} \text{BW}_J(t), \end{aligned} \quad (5)$$

where the subindex  $J = V, A'$  stands for vector or axial-vector meson contributions, i.e.,  $V = \rho(770)$ ,  $\rho(1450)$  or  $K^*(982)$  and  $A' = a_1(1260)$ . The function  $\text{BW}_J(t)$  is defined by

$$\text{BW}_J(t) = \frac{M_J^2}{M_J^2 - t - i\sqrt{t}\Gamma_J(t)}, \quad (6)$$

the parameter  $M_J$  is the mass, and the function  $\Gamma_J(t)$  is the off-shell width decay of the intermediate meson resonant particle, see Refs. [12,13], where we established the relevance of considering the moment transfer dependency on the width decay of the resonance for semileptonic  $\tau$  decays. The function  $\text{BW}_J(t)$  appears from the Feynman rule propagator for the unstable charged vector or axial-vector mesons considered as intermediate virtual resonance, which is derived in general form in Ref. [14],

$$\begin{aligned} D_J^{\mu\nu}(q^2) &= \frac{1}{M_J^2 - q^2 - i\sqrt{q^2}\Gamma_J(q^2)} \\ &\quad \times \left[ -g^{\mu\nu} + \frac{q^\mu q^\nu}{M_J^2 - i\sqrt{q^2}\Gamma_J(q^2)} \right], \end{aligned} \quad (7)$$

where  $q^2 = t$ .

The unpolarized squared amplitude in terms of the form factors  $l(t)$  and  $c(t)$  is the following:

$$\begin{aligned} \sum_{\text{pol}} |\bar{\mathcal{M}}|^2 &= 2G_F^2 |V_{ui}|^2 [a_1(t, s) |l(t)|^2 + a_2(t, s) |c(t)|^2 \\ &\quad + a_3(t, s) \text{Re}[l^*(t)c(t)]], \end{aligned} \quad (8)$$

where the kinematic factors are defined by

$$\begin{aligned}
 a_1(t, s) &= \frac{1}{m_A^2} [(s - m_\tau^2)(s + t - m_\tau^2 - m^2) + m_A^2(m^2 + 2m_\tau^2 - 2t - s)], \\
 a_2(t, s) &= \frac{1}{m_A^2} [((m^2 - t)^2 - 2(m^2 + t)m_A^2 + m_A^4)((m^2 - s)m_A^2 + (s - m_\tau^2)(s + t - m_\tau^2 - m^2))], \\
 a_3(t, s) &= \frac{2}{m_A^2} [(m^2 - s)m_A^4 + (m^2 - t)(s - m_\tau^2)(m^2 + m_\tau^2 - s - t) + m_A^2(s^2 + m^2t + m_\tau^2t - m^4 - m^2m_\tau^2 - m_\tau^4)].
 \end{aligned} \tag{9}$$

The kinematics factors  $a_1(t, s)$ ,  $a_2(t, s)$ , and  $a_3(t, s)$  are functions of squared moments  $t = (p_A + p)^2$  and  $s = (p_\nu + p_A)^2$  and the masses of  $\tau$  lepton  $m_\tau$ , axial-vector meson  $m_A$ , and pseudo-scalar meson  $m$ .

The width decay for the channel  $\tau \rightarrow AP\nu_\tau$  is calculated integrating the unpolarized amplitude, Eq. (8), with respect to the variables  $t$  and  $s$  and the appropriate kinematic limits.

The vector meson resonant contribution is the only contribution for the channels studied in this paper, with the exception of the  $\tau^- \rightarrow f_1(1285)\pi^- \nu$  decay, where it has an axial-vector meson  $a_1(1260)$  as the only present contribution. Specifically, for the channels  $\tau \rightarrow a_1(1260)\pi\nu$ ,  $\tau \rightarrow h_1(1170)\pi\nu$ ,  $\tau \rightarrow K_1(1270)K\nu$ , and  $\tau \rightarrow K_1(1400)K\nu$ , we have the contributions of vector meson  $\rho(770)$  and  $\rho(1450)$  resonances. The strange vector meson  $K^*(982)$  contributes to the processes  $\tau \rightarrow a_1(1260)K\nu$ ,  $\tau \rightarrow K_1(1270)\pi\nu$ , and  $\tau \rightarrow K_1(1400)\pi\nu$ .

In studies of the  $\tau^- \rightarrow \pi^- \pi^0 \nu$  channel carried out by the Belle [15] and ALEPH [16] Collaborations, it was established that in order to correctly describe the spectral function, it is necessary to introduce two resonances—the vector mesons  $\rho(770)$  and  $\rho(1450)$ . We model the two contributions with a linear combination normalized by

$$\text{BW}_V(t) = \frac{\text{BW}_\rho(t) + \beta \text{BW}_{\rho'}(t)}{1 + \beta}, \tag{10}$$

where the parameter  $\beta$  is determined by a fit to the hadronic spectral function obtained by the experiments. In our work, we model in the same manner the contribution of the two resonances  $\rho(770)$  and  $\rho(1450)$  to the channels considered above. We use a conservative value of  $\beta = \pm(0.2 \pm 0.1)$ .

### III. WEAK DECAY AND STRONG COUPLING CONSTANTS

In order to obtain the weak decay and strong coupling constants that are required in this work, we make use of the available experimental information. However, for some channels, we use isospin  $SU(2)$  symmetries to relate them to the constants obtained by experimental data.

We use the experimental reported branching ratios in Ref. [17] to determine the strong coupling constants from the decays of strange axial-vector mesons  $K_1(1270) \rightarrow K\rho$ ,  $K_1(1270) \rightarrow K^*(982)\pi$ , and  $K_1(1400) \rightarrow K^*(982)\pi$  and from the  $h_1(1170) \rightarrow \rho\pi$  decay.

The strong coupling constant  $g_{\text{AVP}}$  (in  $\text{GeV}^{-1}$ ) is determined from the amplitude of probability for the  $A \rightarrow VP$  process, which is defined by

$$\begin{aligned}
 \mathcal{M}(A(p_A, \epsilon_A) \rightarrow V(p_V, \epsilon_V)P(p)) \\
 = g_{\text{AVP}} \epsilon_A^\mu \epsilon_V^\nu [(p_A \cdot p_V) g_{\mu\nu} - (p_A)_\nu (p_V)_\mu].
 \end{aligned} \tag{11}$$

The decay rate in the rest frame of the decaying axial-vector meson is

$$\begin{aligned}
 \Gamma(A \rightarrow VP) &= \frac{1}{3} \frac{g_{\text{AVP}}^2 \lambda^{1/2}(m_A^2, m_V^2, m^2)}{32\pi m_A^3} \\
 &\times [2m_A^2 m_V^2 + (m_A^2 + m_V^2 - m^2)^2],
 \end{aligned} \tag{12}$$

where  $\lambda(x, y, z)$  is the Kallen function, which is defined by  $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$ .

Using the measured width decay for the channels  $K_1(1270) \rightarrow K\rho$ ,  $K_1(1270) \rightarrow K^*\pi$ ,  $K_1(1400) \rightarrow K\rho$ ,  $K_1(1400) \rightarrow K^*\pi$ , and using the isospin  $SU(2)$  symmetry to relate with the charged modes, we obtain the following values in ( $\text{GeV}^{-1}$ ):  $g_{K_1(1270)^- K^0 \rho^-} = 5.04 \pm 0.67$ ,  $g_{K_1(1270)^0 K^- \rho^-} = 5.04 \pm 0.67$ ,  $g_{K_1(1270)^- K^* \pi^0} = 0.69 \pm 0.13$ ,  $g_{K_1(1270)^0 K^* \pi^-} = 0.97 \pm 0.19$ ,  $g_{K_1(1400)^- K^0 \rho^-} = 0.68 \pm 0.34$ ,  $g_{K_1(1400)^0 K^- \rho^-} = 0.68 \pm 0.34$ ,  $g_{K_1(1400)^- K^* \pi^0} = 1.95 \pm 0.10$ , and  $g_{K_1(1400)^0 K^* \pi^-} = 2.75 \pm 0.14$ . The error reported is due to the experimental uncertainty in the width decay.

In order to extract the strong coupling constant  $g_{h_1 \rho \pi}$ , we have assumed that only the mode  $h_1(1170) \rightarrow \rho\pi$  contributes to the total width decay of the axial-vector meson  $h_1(1170)$ . We obtain  $g_{h_1 \rho \pi^+} = 3.85 \pm 0.21 \text{ GeV}^{-1}$ . The uncertainty is determined by the experimental error in the width decay.

To obtain the strong coupling constants  $g_{a_1 \rho \pi}$  and  $g_{a_1 K^* K}$ , we use the branching ratios reported in Ref. [17] for the semileptonic  $\tau^- \rightarrow \pi^- \pi^+ \pi^- \nu$  and  $\tau^- \rightarrow K^- \pi^- K^+ \nu$  decays, respectively. We consider the approximation that the channel  $\tau^- \rightarrow \pi^- \pi^+ \pi^- \nu$  has the only contribution from  $\tau^- \rightarrow a_1^-(1260)\nu \rightarrow \rho^0 \pi^- \nu$ . Using the value for the weak decay constant  $f_{a_1}$  as is discussed below and the meson dominance model for the process  $\tau^- \rightarrow \rho^0 \pi^- \nu$ , we obtain  $g_{a_1 \rho \pi} = 5.14 \pm 2.00 \text{ GeV}^{-1}$ , where the principal error is coming from the width and weak decay constant of the axial-vector meson  $a_1(1260)$ . The channel  $\tau^- \rightarrow K^- \pi^- K^+ \nu$  has contributions due to vector and

axial-vector currents. There are different experimental determination of the axial-vector contribution which are inconsistent, see Refs. [18,19]. These authors consider the value  $(75 \pm 25)\%$  to account for the experimental discrepancy. Additionally, in estimates such as  $\approx 37\%$  [20] obtained by the resonance chiral perturbation theory, the axial-vector contribution is not dominant. In view of these considerations we make a conservative assessment for the axial-vector contribution of  $(65 \pm 25)\%$ . We approximate the channel  $\tau^- \rightarrow K^- \pi^- K^+ \nu$  to  $\tau^- \rightarrow K^{*0} K^- \nu$  with the only contribution coming in the axial-vector current from the axial-vector  $a_1(1260)$ . We obtain  $g_{a_1 K^* K} = 15.7 \pm 6.7 \text{ GeV}^{-1}$ , with the error is principally due to the measured axial-vector contribution and parameters associated with the meson  $a_1(1260)$ , the width, and the weak decay constant.

The strong coupling constant  $g_{f_1 a_1 \pi}$  can be obtained from the branching ratio of the channel  $f_1(1285) \rightarrow \rho^0 \pi^- \pi^+$  reported in Ref. [17]. This decay can be modeled by the contribution of the axial-vector meson  $a_1(1260)$ , given the sequence decays  $f_1(1285) \rightarrow a_1(1260) \pi^+ \rightarrow \rho^0 \pi^- \pi^+$ , in the following manner. Two vertexes are produced by the strong interactions—the first one  $f_1(1285) \rightarrow a_1(1260) \pi$ , and after that the meson  $a_1(1260)$  propagates and decays by the vertex  $a_1(1260) \rightarrow \rho \pi$ . Thus, with a value for the strong coupling constant  $g_{a_1 \rho \pi}$ , we obtain the other coupling  $g_{f_1 a_1 \pi} = 0.73 \pm 0.29 \text{ GeV}^{-1}$ , where the error is principally coming from the error in width decay of the intermediate axial-vector meson  $a_1(1260)$ , the strong coupling constant  $g_{a_1 \rho \pi}$ , and the branching ratio for the channel  $f_1(1285) \rightarrow \rho^0 \pi^- \pi^+$ .

We have estimated the weak decay constant of hadron mesons  $\rho(770)$  and  $K^*(982)$  from the experimental data on branching ratio from  $\tau^- \rightarrow h^- \nu_\tau$  decays, where  $h = \rho(770)$  or  $K^*(982)$ . We obtain  $f_\rho = 218 \pm 2$  and  $f_{K^*} = 210 \pm 5 \text{ MeV}$ . In the calculations above, the error in the weak decay constants is coming from the experimental error in branching ratios of  $\tau^- \rightarrow h^- \nu_\tau$  decays. The weak decay constant for the axial-vector meson  $a_1(1260)$  used is  $f_{a_1} = 165 \pm 34 \text{ MeV}$ , which is obtained using the value of the peak of the axial-vector meson  $a_1$  in the tau spectral distributions of the decays  $\tau^- \rightarrow \pi^- \pi^0 \pi^0 \nu_\tau$  and  $\tau^- \rightarrow \pi^- \pi^+ \pi^- \nu_\tau$  from ALEPH Collaboration [16]. See publicity accessible at the web site [21].

The necessary values for the relevant parameters such as width decays and masses of the particles involved in the channels calculated in this work have been taken from Ref. [17]. In particular, for the axial-vector meson  $a_1(1260)$ , Ref. [17] quotes 250 to 600 MeV for its width decay, which in our calculations we consider  $425 \pm 175 \text{ MeV}$ .

#### IV. BRANCHING RATIOS

The branching ratios for the exclusive  $\tau^- \rightarrow AP\nu_\tau$  decays estimated in this work are shown in Table I.

TABLE I. Branching ratios for  $\tau^- \rightarrow AP\nu$  decays.

AP mode	Branching ratio	Parameter $\beta$
$a_1(1260)^- \pi^0$	$(6.9 \pm 6.3) \times 10^{-3}$	$\beta = +0.2 \pm 0.1$
$a_1(1260)^- \pi^0$	$(6.1 \pm 5.9) \times 10^{-3}$	$\beta = -0.2 \pm 0.1$
$a_1(1260)^0 \pi^-$	$(6.8 \pm 6.1) \times 10^{-3}$	$\beta = +0.2 \pm 0.1$
$a_1(1260)^0 \pi^-$	$(5.9 \pm 5.7) \times 10^{-3}$	$\beta = -0.2 \pm 0.1$
$a_1(1260)^- K^0$	$(3.1 \pm 2.2) \times 10^{-7}$	
$a_1(1260)^0 K^-$	$(3.8 \pm 2.7) \times 10^{-7}$	
$f_1(1285) \pi^-$	$(1.3 \pm 1.2) \times 10^{-4}$	
$h_1(1170) \pi^-$	$(3.1 \pm 1.8) \times 10^{-3}$	$\beta = +0.2 \pm 0.1$
$h_1(1170) \pi^-$	$(2.7 \pm 2.2) \times 10^{-3}$	$\beta = -0.2 \pm 0.1$
$K_1(1270)^- \pi^0$	$(0.8 \pm 0.2) \times 10^{-6}$	
$K_1(1270)^0 \pi^-$	$(1.4 \pm 0.5) \times 10^{-6}$	
$K_1(1400)^- \pi^0$	$(1.1 \pm 0.1) \times 10^{-6}$	
$K_1(1400)^0 \pi^-$	$(2.1 \pm 0.2) \times 10^{-6}$	
$K_1(1270)^0 K^-$	$(13.0 \pm 8.9) \times 10^{-9}$	$\beta = +0.2 \pm 0.1$
$K_1(1270)^- K^0$	$(2.8 \pm 1.9) \times 10^{-9}$	$\beta = +0.2 \pm 0.1$

The largest branching ratios estimated in this work turn out to be of order  $\mathcal{O}(10^{-3})$ , corresponding to the exclusive modes  $\tau^- \rightarrow a_1(1260)^- \pi^0 \nu$ ,  $\tau^- \rightarrow a_1(1260)^0 \pi^- \nu$  and  $\tau^- \rightarrow h_1(1170) \pi^- \nu$ . In these modes we have the contribution of two vector meson resonances  $\rho(770)$  and  $\rho(1450)$ , which produce broad form factors  $l(t)$  and  $c(t)$  as a function of the momentum transferred  $t$ , consequently resulting in a large branching ratio estimation.

Up to now, the only measured branching ratio from the calculations in this work is for the channel  $\tau^- \rightarrow f_1(1285) \pi^- \nu$ . Our estimate for this channel is  $(1.3 \pm 1.2) \times 10^{-4}$ . We can compare with the recent result of BABAR Collaboration  $(3.86 \pm 0.25) \times 10^{-4}$  [1] and determine that our estimation differs by 2.1 sigma error with respect to the experimental result.

The channels with branching ratios of order  $\mathcal{O}(10^{-6})$  are  $\tau^- \rightarrow K_1(1270) \pi \nu$  and  $\tau^- \rightarrow K_1(1400) \pi \nu$  decays. They have only the contribution of the vector meson  $K^*(982)$  as intermediate resonance.

The exclusive channels  $\tau^- \rightarrow K_1(1400)^0 K^- \nu$  and  $\tau^- \rightarrow K_1(1400)^- K^0 \nu$  decays are not kinematically allowed. The modes  $\tau^- \rightarrow a_1(1260) K \nu$  and  $\tau^- \rightarrow K_1(1270) K \nu$  decays are of the order of  $\mathcal{O}(10^{-7})$  and  $\mathcal{O}(10^{-9})$ , respectively. This is due to the small phase space available for their decay; i.e., they are kinematically suppressed decays.

The channels  $\tau^- \rightarrow a_1(1260) \pi \nu$ ,  $\tau^- \rightarrow h_1(1170) \pi^- \nu$ ,  $\tau^- \rightarrow K_1(1270) \pi \nu$ , and  $\tau^- \rightarrow K_1(1400) \pi \nu$  decays, with resonance contributions coming from the vector mesons  $\rho(770)$  and  $\rho(1450)$ , have the largest error due to the error in the parameter  $\beta$ . In the other cases, where the meson resonance contribution is the strange vector meson  $K^*(982)$  or the axial-vector meson  $a_1(1260)$ , the principal error in the branching ratio estimate is due to the uncertainty in the strong coupling constant  $g_{AJ\nu}$ , where  $J$  is the resonance contribution  $K^*(982)$  or  $a_1(1260)$ .

In addition, there are two other sources of error in our estimations not considered. We use isospin  $SU(2)$  symmetry to relate the strong coupling constant to that obtained by experimental information. The error in isospin  $SU(2)$  symmetry can reach 5% in the estimation. The other source of theoretical uncertainty is coming from the contact term in the weak vertex of the hadronic current  $\langle AP|J_\mu(0)|0\rangle$ . However, we do not consider this contribution because it is associated with the continuum, i.e., there is no function  $BW_V(t)$ . The contact term contribution is not possible to estimate in a meson dominance model.

## V. CONCLUSIONS

We have estimated branching ratios of the  $\tau \rightarrow AP\nu$  decays. The axial-vector mesons  $A$  considered are  $a_1(1260)$ ,  $f_1(1285)$ ,  $h_1(1170)$ ,  $K_1(1270)$ , and  $K_1(1400)$ . The pseudoscalar mesons  $P$  are  $\pi$  and  $K$ . In total, 12 exclusive channels are kinematically allowed, first class-currents, and G-parity conserved.

To calculate the branching ratios we use a meson dominance model, where the form factors of the current  $\langle AP|J_\mu|0\rangle$  are calculated from the vector meson contributions  $\rho(770)$ ,  $\rho(1450)$ , and  $K^*(982)$  with the exception of the channel  $\tau^- \rightarrow f_1(1285)\pi^-\nu$ , where the contribution is due to the axial-vector meson  $a_1(1260)$ .

The channels  $\tau^- \rightarrow a_1(1260)^-\pi^0\nu$ ,  $\tau^- \rightarrow a_1(1260)^0\pi^-\nu$ , and  $\tau^- \rightarrow h_1(1170)\pi^-\nu$  decays are of order  $\mathcal{O}(10^{-3})$ . We can compare the estimated branching ratio  $\text{Br}(\tau^- \rightarrow f_1(1285)\pi^-\nu) = (1.3 \pm 1.2) \times 10^{-4}$  with the measured value  $(3.86 \pm 0.25) \times 10^{-4}$ , recently reported by *BABAR* Collaboration [1].

We observe a pattern in the branching ratios calculated in this work. The modes of order  $\mathcal{O}(10^{-7})$  and  $\mathcal{O}(10^{-9})$  are

kinematically suppressed. The channels of order  $\mathcal{O}(10^{-6})$  are Cabibbo suppressed and they have only the contribution of the vector meson  $K^*(982)$ . Finally, the channels with branching ratios of order  $\mathcal{O}(10^{-3})$  and the channel  $\tau^- \rightarrow f_1(1285)\pi^-\nu$  of order  $\mathcal{O}(10^{-4})$  are Cabibbo allowed and have two vector meson resonance contributions  $\rho(770)$  and  $\rho(1450)$ , with the exception of the  $\tau^- \rightarrow f_1(1285)\pi^-\nu$  where the resonance is the axial-vector meson  $a_1(1260)$ .

The processes studied in this work can contribute as intermediate states in branching ratios with four and five pseudoscalar mesons  $\pi$  and/or  $K$  in final states for the  $\tau$  decay modes.

Eventually, the branching ratios of order  $\mathcal{O}(10^{-3})$  for the channels calculated in this work, with the exception of the channel  $\tau^- \rightarrow f_1(1285)\pi^-\nu$  (which have been already measured), could be measured by four-prong of  $\tau$  decays, with the data sample of  $\tau$  lepton pairs accumulated by the B-factories *BABAR* and *Belle* experiments [22]. This will not be the case for the channels  $\tau^- \rightarrow K_1(1260)\pi\nu$  and  $\tau^- \rightarrow K_1(1400)\pi\nu$ , with branching ratios of order  $\mathcal{O}(10^{-6})$ , in view of the sensitivity reached in some channels involving strange mesons in final states; see Refs. [1,23]. Nevertheless, SuperB [24] and Belle-II [25,26] Collaborations have programs to produce  $\tau$  pairs and measure rare  $\tau$  decays of this order.

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