B decays into radially excited charmed mesons

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(Received 19 April 2013; published 10 June 2013)

It has been recently argued that some longstanding problems in semileptonic B decays can be solved provided the branching ratio for the $B \to D^{(\ast)}$ semileptonic decays are large enough. We have studied these decays in a constituent quark model, which has been successful in describing semileptonic and nonleptonic B decays into orbitally excited charmed mesons. Our results do not confirm the hypothesis of large branching ratios for the $B \to D^{(*)}$ semileptonic decays. In addition, we calculate the nonleptonic $B \to D^{\prime}\pi$ decays which can provide an independent test of the form factors involved in the $B \to D^{(\prime\prime)}$ reactions.

DOI: [10.1103/PhysRevD.87.114009](http://dx.doi.org/10.1103/PhysRevD.87.114009) PACS numbers: 12.39.Jh, 12.39.Pn, 13.20.He, 14.40.Lb

I. INTRODUCTION

The determination of exclusive branching fractions of $B \to X_c l^+ \nu_l$ decays is an essential part of the *B*-factory program to understand the dynamics of b-quark semileptonic decays and therefore to determine the relevant Cabibbo-Kobayashi-Maskawa matrix elements, $|V_{cb}|$ and $|V_{ub}|$, which are essential parameters of the Standard Model. The methods used to determine the decay rates as a function of $|V_{ab}|$ are very different for inclusive and exclusive decays, and the comparison of these complementary determinations provides an important check on the results.

Current measurements show a discrepancy between the inclusive rate $\mathcal{B}(B^+ \to X_c l^+ \nu_l)$ and the sum of the measured exclusive channels. At present, the disagreement between the inclusive rate $\mathcal{B}(B^+ \to X_c l^+ \nu) = (10.92 \pm 0.16)\%$ and the sum of the measured exclusive rates is

$$
\mathcal{B}(B^+ \to X_c l^+ \nu_l) - \mathcal{B}(B^+ \to D^{(*)} l^+ \nu_l) \n- \mathcal{B}(B^+ \to D^{(*)} \pi l^+ \nu_l) = (1.45 \pm 0.13)\%.
$$
 (1)

The authors of Ref. [[1\]](#page-4-0) argue that this 1.45% discrepancy could be solved, or at least eased, by an unexpectedly large B decay rate to the first radially excited D' and D'^* states $(B(B \to D^{1(*)} l^+ \nu_l) \sim \mathcal{O}(1\%)$). It is also stated in Ref. [\[1\]](#page-4-0) that a potentially large $\mathcal{B}(B^+ \to D^{(\prime\prime)} l^+ \nu_l)$ could help to solve the so-called $1/2$ vs $3/2$ puzzle. This puzzle refers to the discrepancy between heavy quark symmetry based model predictions for B decays into $1P D^{**}$ states and observations. Those calculations predict that these decays should have a substantially smaller rate to the $j_q^P = \frac{1}{2}^+$ doublet than to the $j_q^P = \frac{3}{2}^+$ doublet (see for instance Refs. [[2](#page-4-1),[3\]](#page-4-2)) whereas roughly equal branching ratios are found experimentally. A large $\mathcal{B}(B^+ \to D^{(*)} l^+ \nu_l)$ would result in an excess of the detected $B \to D_{1/2} \pi l \nu_l$ with respect to $B \to D_{3/2} \pi l \nu$ due to the fact that the $\Gamma(D' \to D_{1/2}\pi)$ is much larger than the $\Gamma(D' \to D_{3/2}\pi)$ because in the first case the pion is emitted in S-wave while in the $D_{3/2}$ one is emitted in P-wave.

Evidence for the $D^{(*)}$ radial excitations that would correspond to 2S quark model states have recently been found by the BABAR Collaboration [\[4\]](#page-4-3). Their results are summarized in Table [I](#page-1-0). The $D(2550)$ meson has only been seen in the decay mode $D^*\pi$ and its helicity-angle distribution turns out to be consistent with the prediction of a $2^{1}S_{0}$ state. The $D^{*}(2600)$ meson decays into $D\pi$ and $D^{*}\pi$ final states, and its helicity-angle distribution is consistent with the meson being a $J^p = 1⁻$ state. Moreover, its mass makes it the perfect candidate to be the spin partner of the $D(2550)$ meson.

Previous theoretical determinations of $\mathcal{B}(B^+ \rightarrow D^{(\ast)} l^+ \nu)$ include an earlier calculation within the ISGW2 model, which incorporates constraints imposed by heavy quark symmetry, where a value of 0.06% was obtained [\[5](#page-4-4)]. This is in agreement with the heavy quark effective theory calculation of Suzuki et al. [\[6](#page-4-5)] which obtained $\mathcal{B}(B^+ \to D^{(\ast)} l^+ \nu) = 0.05\%$. A much larger value is obtained in Ref. [\[7\]](#page-4-6). There the authors use a relativistic quark model finding that the calculated branching ratio increases from 0.29%, for infinite heavy quark mass, to 0.40% when $1/m_a$ corrections are taken into account. Although the value is still far from the \sim 1% needed to explain the experimental discrepancy, the size of the corrections suggests that a complete calculation may further approach the 1% result.

In this work we shall perform a full determination of $\mathcal{B}(B^+ \to D^{(\ast)} l^+ \nu)$ within the framework of the constituent quark model described in Ref. [[8](#page-4-7)]. The model has recently been applied to mesons containing heavy quarks obtaining a satisfactory description of many physical observables: spectra [[9,](#page-4-8)[10\]](#page-4-9), strong decays and reactions [[11](#page-4-10),[12](#page-4-11)], and semileptonic and nonleptonic B and B_s decays into orbitally excited charmed and charmed-strange mesons $[13,14]$ $[13,14]$. We will use the factorization approximation which gave a satisfactory explanation of the decays analyzed in Refs. [[13](#page-4-12),[14](#page-4-13)]. The form factors that parametrize the $\Gamma(B \to D'l\nu)$ decay also appear, evaluated at $q^2 = m_\pi^2$, in the nonleptonic decay $B \to D'\pi$ evaluated in factorization approximation. The latter decay, if experimentally

TABLE I. Mass and total decay width of the $D^{(\ast)}$ resonances as measured by the BABAR Collaboration [[4](#page-4-3)].

Resonance	Mass (MeV)	Width (MeV)
$D(2550)^{0}$	$2539.4 \pm 4.5 \pm 6.8$	$130 + 12 + 13$
$D^{*}(2600)^{0}$	$2608.7 \pm 2.4 \pm 2.5$	$93 + 6 + 13$

accessible, could be used to extract information on the form factors near $q^2 = 0$. In this work we also evaluate the $\bar{B}^0 \to D'^+ \pi^-$ branching ratio and its branching fraction relative to the $\bar{B}^0 \to D^+ \pi^-$ decay.

The paper is organized as follows: In Sec. [II](#page-1-1), we describe the constituent quark model predictions for the first radially excited S-wave states paying special attention to their strong decays. Section [III](#page-2-0) is dedicated to explaining the theoretical framework through which we calculate the $\mathcal{B}(B^+ \to D^{(\ast)}/{}^{\dagger} \nu)$ and $\mathcal{B}(\bar{B}^0 \to D^{/\dagger} \pi^-)$. Our results are shown in Sec. [IV.](#page-3-0) We summarize our conclusions in Sec. [V.](#page-4-14)

II. CONSTITUENT QUARK MODEL PREDICTIONS FOR 2S STATES

All the details on the constituent quark model we use have been described in Ref. [\[8](#page-4-7)] and we will only sketch here its main features. The model is based on the assumption that the constituent quark mass of the light quarks is due to the spontaneous chiral symmetry breaking of the QCD Lagrangian. To restore the original symmetry an interaction term, due to Goldstone-Boson exchanges, appears between light quarks. This interaction is added to the perturbative one-gluon exchange (OGE) and the nonperturbative confining interactions. In the heavy quark sector, chiral symmetry is explicitly broken and Goldstoneboson exchanges do not appear. Therefore, the corresponding potential for the system stems from the nonrelativistic reduction of the OGE interaction and the confinement component. Explicit formulas and model parameters used herein can be find in Refs. [[9](#page-4-8)[,13\]](#page-4-12).

Concerning the new discovered mesons, $D(2550)$ and $D^*(2600)$, we predict masses of 2.70 and 2.75 GeV which are larger than the experimental ones, 2.54 and 2.61 GeV. While masses are large, the goodness of the meson wave functions has been tested through their strong decays. Those were evaluated in Ref. [[11](#page-4-10)] in which a modification of the phenomenological ${}^{3}P_{0}$ decay model was proposed. The strength γ of the decay interaction is scale-dependent being a function of the reduced mass of the quarkantiquark pair of the decaying meson. In this way a satisfactory global description of strong decays of mesons which belong to charmed, charmed-strange, hidden charm and hidden bottom sectors was achieved [[11](#page-4-10)]. Using physical masses for the 2S states, we obtained the results shown in Tables [II](#page-1-2) and [III](#page-1-3).

The $D(2550)$ meson has been only seen in the decay mode $D^*\pi$, thus its possible spin-parity quantum numbers

TABLE II. Open-flavor strong decay widths, in MeV, and branchings, in %, of the $D(2550)$ meson with quantum numbers $nJ^P = 20^{-}$.

$D(2550)$ as $nJ^P = 20^-$				
Channel	$\Gamma_{^3P_{\alpha}}$			
$D^*\pi$	131.90	$\frac{{\cal B}_{^3P_{_0}}}{99.87}$		
$\frac{D_0^*\pi}{\textrm{Total}}$	0.18	0.13		
	132.07	100		

up to $J = 3$ are $J^P = 0^{-}$, 1^{+} , 2^{-} and 3^{+} . It is the lower in mass of the newly discovered mesons and within the possible assignments, the $0⁻$ is the most plausible. Assuming it is a 2S, $J^P = 0^-$ state, its total width, shown in Table [II,](#page-1-2) predicted by the ${}^{3}P_0$ model is in very good agreement with the experimental value given by the BABAR Collaboration.

The $D^*(2600)$ meson decays into $D\pi$ and $D^*\pi$ final states. Thus, its possible quantum numbers are $J^P = 1^-$, 2^+ and 3^- . The helicity-angle distribution of $D^*(2600)$ is found to be consistent with $J^P = 1⁻$. Moreover, its mass makes it the perfect candidate to be the spin partner of the $D(2550)$ meson. Again, the total decay width predicted by the ${}^{3}P_0$ model, assuming it is a 2S, $J^P = 1^-$ state, is in good agreement with the experimental data. Besides, we predict the ratio of branching fractions

$$
\frac{\mathcal{B}(D^*(2600)^0 \to D^+\pi^-)}{\mathcal{B}(D^*(2600)^0 \to D^{*+}\pi^-)} = 0.20\tag{2}
$$

in reasonable agreement with experiment, $0.32 \pm 0.02 \pm 0.02$ 0:09 [[4\]](#page-4-3).

In Tables [II](#page-1-2) and [III](#page-1-3) we also show the contribution of each channel to the total decay width of the $D^{(*)}$ states. In both cases we obtain that the decay widths of these mesons into 1P states are much smaller than the ones into 1S states. This is in contrast to Ref. [\[1](#page-4-0)] where the authors find plausible that the $D^{(\ast)}$ decay rates to 1S and 1P charmed states may be comparable. That was used in Ref. [\[1\]](#page-4-0) as a

TABLE III. Open-flavor strong decay widths, in MeV, and branchings, in %, of the $D^*(2600)$ meson with quantum numbers $nJ^P = 21^{-}$.

$D^*(2600)$ as $nJ^P = 21^-$				
Channel	$\Gamma_{^3P_{_0}}$	$\mathcal{B}_{^3P_{\scriptscriptstyle 0}}$		
$D\pi$	10.84	11.19		
$D^*\pi$	54.10	55.83		
$D\eta$	11.86	12.24		
$D_s K$	8.73	9.01		
$D^*\eta$	9.65	9.95		
$D_1\pi$	0.28	0.29		
$D_1'\pi$	1.44	1.49		
$D_2^*\pi$	0.01	0.00		
Total	96.91	100		

possible explanation of the so-called $1/2$ vs $3/2$ puzzle. However, we studied this issue within our model in Ref. $[13]$ $[13]$ $[13]$ where we observed similar B semileptonic decay rates into the two $j_q^P = 1/2^+$ and $j_q^P = 3/2^+$ doublets, being our results for the different channels in agreement with experiment.

III. THE $B^+ \to D^{(\ast)} l^+ \nu_l$ AND $B^0 \to D^{(\ast)} \pi$ DECAY WIDTH

The presence of the heavy quark in the initial and final meson states in these decays considerably simplifies their theoretical description. Let us start our analysis in the infinitely heavy quark limit, $m_Q \rightarrow \infty$. In this limit the heavy quark symmetry arises. This leads to a considerable reduction of the number of independent form factors which are necessary for the description of heavy-to-heavy semileptonic decays. For example, in this limit only one form factor is necessary for the semileptonic B decay to S-wave D mesons. It is important to note that the heavy quark symmetry requires that matrix elements between a B meson and an excited D meson should vanish at zero recoil as a result of the orthogonality of the wave functions.

As the D' and D'^* states have 0^- and 1^- quantum numbers, the hadronic matrix elements for the semileptonic transition can be parametrized in terms of form factors as [[15](#page-4-15)]

$$
\frac{\langle D'(p')|\bar{\Psi}_c(0)\gamma^{\mu}(1-\gamma_5)\Psi_b(0)|B(p)\rangle}{\sqrt{m_B m_{D'}}}
$$
\n= $h_+(w)(v+v')^{\mu} + h_-(w)(v-v')^{\mu}$,
\n
$$
\frac{\langle D'^*(p')|\bar{\Psi}_c(0)\gamma^{\mu}(1-\gamma_5)\Psi_b(0)|B(p)\rangle}{\sqrt{m_B m_{D'^*}}}
$$
\n= $h_V(w)\epsilon^{\mu\nu\alpha\beta}\epsilon_{\nu}^*v_{\alpha}'v_{\beta} - i[-h_{A_1}(w)(w+1)\epsilon^{*\mu} + h_{A_2}(w)(\epsilon^* \cdot v)v'^{\mu} + h_{A_3}(w)(\epsilon^* \cdot v)v'^{\mu}],$ \n(3)

where $v(v')$ is the four velocity of the $B(D^{(*)})$ meson, $\epsilon^{0123} = -1$, ϵ^{μ} is a polarization vector of the final vector charmed meson and the form factors h_i are dimensionless functions of the product of four-velocities $w = v \cdot v'$. The differential $d\Gamma/dw$ decay widths are given by [\[15\]](#page-4-15)

$$
\frac{d\Gamma}{dw}(B^+ \to D'l^+ \nu_l)
$$
\n
$$
= \frac{G_F^2 |V_{cb}|^2 m_B^5}{48\pi^3} (w^2 - 1)^{3/2} r^3 (1 + r)^2 G^2(w),
$$
\n
$$
\frac{d\Gamma}{dw}(B^+ \to D'^* l^+ \nu_l)
$$
\n
$$
= \frac{G_F^2 |V_{cb}|^2 m_B^5}{48\pi^3} (w^2 - 1)^{3/2} (w + 1)^2 r^{*3} (1 - r^*)^2
$$
\n
$$
\times \left[1 + \frac{4w}{w + 1} \frac{1 - 2wr^* + r^{*2}}{(1 - r^*)^2}\right] F^2(w), \tag{4}
$$

where G_F is the Fermi decay constant, $|V_{cb}|$ the modulus of the Cabibbo-Kobayashi-Maskawa matrix element for a $b \to c$ transition and $r^{(*)} = m_{D^{(*)}}/m_B$. As for $G(w)$ and $F(w)$, they are given in term of the form factors as

$$
G(w) = h_{+}(w) - \frac{1-r}{1+r}h_{-}(w),
$$

\n
$$
F^{2}(w) = \left\{2(1 - 2wr^{*} + r^{*2})\left[h_{A_{1}}^{2}(w) + \frac{w-1}{w+1}h_{V}^{2}(w)\right] + \left[(1 - r^{*})h_{A_{1}}(w) + (w-1)(h_{A_{1}}(w) - h_{A_{3}}(w) - r^{*}h_{A_{2}}(w))\right]^{2}\right\}
$$

\n
$$
\times \left\{(1 - r^{*})^{2} + \frac{4w}{w+1}(1 - 2wr^{*} + r^{*2})\right\}^{-1}.
$$
 (5)

Similarly to the semileptonic $B^+ \to D^{(*)}l^+\nu_l$ case we have that, in the limit of very large heavy quark masses, heavy quark symmetry predicts that all form factors are given in terms of just one Isgur-Wise function $\xi(w)$. One has in that limit

$$
h_{+}(w) = h_{V}(w) = h_{A_{1}}(w) = h_{A_{3}}(w) = \xi(w), \quad (6)
$$

while

$$
h_{-}(w) = h_{A_{2}}(w) = 0. \tag{7}
$$

From these results one obtains in that limit

$$
G(w) = F(w) = \xi(w). \tag{8}
$$

Heavy quark symmetry at the point of zero recoil (w = 1) implies $\xi(1) = 0$ in the $B \to D^{(*)}$ case since the radial parts of the wave functions for the $D^{(\ast)}$ and B mesons are orthogonal in the infinitely heavy quark mass limit. Thus, the value of $\xi(w)$ near zero recoil comes entirely from corrections beyond that limit. This is different from the $B \to D^{(*)}$ case where $\xi(1) = 1$ in that limit.

As mentioned in Ref. [[1](#page-4-0)], the nonleptonic $\bar{B}^0 \to D'^+ \pi^$ decay could also give valuable information on $F(w)$ and $G(w)$, as in factorization approximation the width of this process is related to the form factors involved in the semileptonic decay. Factorization approximation has been proven to be correct for $B \to D\pi$ in the infinite heavy quark mass limit [\[16\]](#page-4-16), and we expect it should also work for decays B into $D'\pi$.

Following Ref. [\[17\]](#page-4-17), we calculate the ratio $\mathcal{B}(\bar{B}^0 \to D^{\prime +} \pi^-)/\mathcal{B}(\bar{B}^0 \to D^+ \pi^-)$, which in factorization approximation is given by

$$
\frac{\mathcal{B}(\bar{B}^0 \to D^{+\pi^-})}{\mathcal{B}(\bar{B}^0 \to D^{+\pi^-})} = \left(\frac{m_B^2 - m_{D'}^2}{m_B^2 - m_D^2}\right)^2 \left(\frac{\lambda(m_B^2, m_{D'}^2, m_\pi^2)}{\lambda(m_B^2, m_D^2, m_\pi^2)}\right)^{1/2} \left|\frac{f_+^{B \to D'}(0)}{f_+^{B \to D}(0)}\right|^2, \tag{9}
$$

where $\lambda(a, b, c) = (a + b - c)^2 - 4ab$, and the $f_+(q^2)$ form factor is related with h_{\pm} via

$$
f_{+}(q^{2}) = \frac{1}{2} \left(\sqrt{\frac{m_{D'}}{m_{B}}} + \sqrt{\frac{m_{B}}{m_{D'}}} \right) h_{+}(w)
$$

$$
+ \frac{1}{2} \left(\sqrt{\frac{m_{D'}}{m_{B}}} - \sqrt{\frac{m_{B}}{m_{D'}}} \right) h_{-}(w). \tag{10}
$$

Note q^2 and w are related through $q^2 = m_B^2 + m_{D^{(i)}}^2$ $2m_B m_{D^{(i)}} w$. If the $\mathcal{B}(\bar{B}^0 \to D'^+ \pi^-)$ branching ratio is measured experimentally, it could be used to extract information on $f_+^{B\to D'}(0)$.

IV. RESULTS

In the calculation we use physical masses for the $D^{(*)}$ mesons. In Fig. [1](#page-3-1) we show the different form factors evaluated in our model for the $B \to D^{(*)}$ transitions. For the sake of comparison we also show in a different panel the corresponding ones for the $B \to D^{(*)}$ transitions. We see that, even for the actual heavy quark masses the relations, in Eqs. ([6](#page-2-1)) and ([7](#page-2-2)) are approximately satisfied for the $B \to D^{(*)}$ case over the whole w range. Deviations are expected due to the finite heavy quark masses and the difference between m_b and m_c . For the $B \to D^{(*)}$ decays

FIG. 1 (color online). Upper panel: Form factors for the $B^+ \rightarrow D^{(*)} l^+ \nu_l$ transition evaluated in our model. Lower panel: The same for the $B^+ \to D^{(\ast)} l^+ \nu_l$ transition.

FIG. 2. $G(w)$ and $F(w)$ factors for the $B^+ \rightarrow D'l^+ \nu_l$ and $B^+ \rightarrow D'^* l^+ \nu_l$ transitions. We show with error bars the estimates for maximum recoil obtained in Ref. [[1](#page-4-0)] adapting the light cone sum rule calculation of Ref. [\[18\]](#page-4-18). We also show the $1/m_O$ corrected $G(1)$ and $F(1)$ values obtained in Ref. [[7](#page-4-6)] using the relativistic quark model.

those differences are magnified by the fact that in the infinite heavy quark mass limit we have $\xi(1) = 0$ in this case.

The $F(w)$ and $G(w)$ factors are depicted in Fig. [2.](#page-3-2) Our results at maximum recoil, $F(w_{\text{max}}) = 0.2$ and $G(w_{\text{max}}) =$ 0:1, are compatible with the estimates in Ref. [\[1](#page-4-0)]

$$
F(w_{\text{max}}) = 0.25 \pm 0.15, \quad G(w_{\text{max}}) = 0.15 \pm 0.1 \quad (11)
$$

obtained adapting the light cone sum rule calculation of Ref. [\[18\]](#page-4-18).

Integrating the differential decay width we obtain $\mathcal{B}(B^+ \to D'I^+ \nu_l) = (0.012 \pm 0.006)\%$ and $\mathcal{B}(B^+ \to$ $D^{i*}l^+\nu_l$ = (0.097 ± 0.015)%, where theoretical uncertainties have been estimated varying the different model parameters within 10% of their central values. For the case of total strong decay widths calculated in Tables [II](#page-1-2) and [III](#page-1-3), we have seen that this 10% variation in the parameters induces changes which are smaller than the experimental error bars. For the sum of the two semileptonic branching ratios, we thus obtain $\mathcal{B}(B^+ \to D^{(\ast)} l^+ \nu_l) =$ $(0.109 \pm 0.016)\%$. Although this branching ratio is larger than those found by Refs. $[5,6]$ $[5,6]$ $[5,6]$ it is still a factor of ten smaller than the expectation in Ref. [[1](#page-4-0)]. By looking at the individual ratios we also find that $\mathcal{B}(B^+ \to D'l^+ \nu_l)$ is smaller than $\mathcal{B}(B^+ \to D^* l^+ \nu_l)$, in agreement with Refs. [\[5](#page-4-4)[,6](#page-4-5)].

Our results for $G(w)$ and $F(w)$ at zero recoil differ from the values obtained in Ref. [\[7\]](#page-4-6) using the relativistic quark model. The discrepancy, clearly visible in Fig. [2,](#page-3-2) explains why our branching ratios are much smaller than the ones in Ref. [[7](#page-4-6)]. The change is larger for $F(1)$ which also explains why $\mathcal{B}(B^+ \to D'l^+ \nu_l) > \mathcal{B}(B^+ \to D'^* l^+ \nu_l)$ in Ref. [[7\]](#page-4-6).

Concerning the nonleptonic decay our theoretical result, obtained within the factorization approximation, is

B DECAYS INTO RADIALLY EXCITED CHARMED MESONS PHYSICAL REVIEW D 87, 114009 (2013)

 \Rightarrow \Rightarrow \Rightarrow \Rightarrow

$$
\frac{\mathcal{B}(\bar{B}^0 \to D'^+ \pi^-)}{\mathcal{B}(\bar{B}^0 \to D^+ \pi^-)} = 0.011 \pm 0.004,\tag{12}
$$

which combined with the experimental value for $\mathcal{B}(\bar{B}^0 \to D^+ \pi^-) = (2.68 \pm 0.13) \times 10^{-3}$ gives for $\mathcal{B}(\bar{B}^0 \to D'^+ \pi^-) \sim 3 \times 10^{-5}$, sufficiently large to be measured experimentally.

V. CONCLUSIONS

We have performed a theoretical calculation of the branching ratio for the $B^+ \to D^{(*)} l^+ \nu_l$ decays within the framework of the constituent quark model described in Ref. [\[8](#page-4-7)].

We find branching ratios much smaller than expectations in Ref. [[1](#page-4-0)]. However, the results are sensitive to the amount of orthogonality between the B and $D^{(\ast)}$ wave functions and the latter depend on the quark model used. In order to check that sensitivity, we have performed an estimation of the theoretical uncertainties in our calculation, finding individual branching ratios can change at most by 50%. Our results would not then confirm that these contributions can explain the difference between the inclusive $\mathcal{B}(B^+ \to X_c l^+ \nu)$ rate and the various exclusive channels.

Concerning the $1/2$ vs $3/2$ puzzle there is no need for a large branching ratio into the $D^{(*)}$ states to solve it. In fact, it was already shown in Ref. [[13](#page-4-12)] that our model predicts similar B semileptonic decay rates into the two $j_q^P = 1/2^+$ and $j_q^P = 3/2^+$ doublets, being our results for the different channels in agreement with experiment. To us, this apparent puzzle appears only when one works in the infinite heavy quark mass limit and neglects corrections on the inverse of the heavy quark masses.

We have also evaluated, in factorization approximation, the nonleptonic $\mathcal{B}(\bar{B}^0 \to D'^+ \pi^-)$ branching fraction. The latter reaction may give additional information on the size of the form factors involved in the semileptonic decay $[1,17]$ $[1,17]$ provided it can be measured in B-factories or at LHCb in the near future.

ACKNOWLEDGMENTS

This work has been partially funded by Ministerio de Ciencia y Tecnología under Contracts No. FPA2010-21750-C02-02 and No. FIS2011-28853-C02-02, by the European Community-Research Infrastructure Integrating Activity 'Study of Strongly Interacting Matter' (HadronPhysics3 Grant No. 283286) by the Spanish Ingenio-Consolider 2010 Program CPAN (CSD2007- 00042) and by U.S. Department of Energy, Office of Nuclear Physics, Contract No. DE-AC02-06CH11357.

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