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Universal formula for the muon-induced neutron yield

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The experimental data on the yield of muon-induced neutrons for liquid scintillator, iron, and lead accumulated during 60 years of muon interaction underground study have been analyzed. A universal formula connecting the yield with muon energy loss in the matter and neutron production in hadronic and electromagnetic showers is presented.

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I. INTRODUCTION

In the last decade a renewed interest in the problem of the yield Y_n of muon-induced neutrons has become evident. This is due to both the increased requirements of the accuracy of background definition in underground experiments and the growth of computing resources. The yield dependence on both the mean muon energy $\bar{E}_\mu$ and atomic weight A of the medium has been investigated using the FLUKA and GEANT simulation packages and their versions [1–4]. Currently no expression exists for Y_n that binds together the muon energy deposition, the nuclear properties of the matter, and the neutron production processes in hadronic (h) and electromagnetic (em) showers generated by muons and developing in the matter. To calculate a yield value the approximate empirical laws $Y_n = p_E \times E_\mu^\alpha$ (for fixed A) or $Y_n = p_A \times A^\beta$ (for fixed E_μ) are used (E_μ is in GeV). The constants α, β are defined based on the results of calculations. Numerical fitting coefficients p_E, p_A are entered to get the agreement between calculations and a set of available experimental data.

The form of the dependence $Y_n(\bar{E}_\mu) = a\bar{E}_\mu^\alpha$ was proposed in Ref. [5]. As follows from results of the measurements in Refs. [6–9] and the calculations in Refs. [1,2,5,10,11], the α value is in the range of 0.7 to 0.9. The values of the exponents α and β represent the contributions of the neutron production channels.

Experimentally, the yield Y_n is given by

$$Y_n = \frac{N_n}{\bar{l}_\mu \rho}(\mathrm{n/g/cm^2}), \tag{1}$$

where N_n is the number of neutrons produced by a muon on the path length $\bar{l}_\mu$ in the matter with density ρ. The yield is connected with the medium properties and the characteristics of the reactions of neutron production by expression

$$Y_n = \frac{N_0\langle\nu\sigma\rangle}{A}, \tag{2}$$

where N_0 is Avogadro's number, $\langle\nu\sigma\rangle$ is a mean value of the product of the photonuclear μA-interaction cross section and neutron multiplicity ν, and A is the atomic weight. Equation (2) follows from the dependence of N_n on $\langle\nu\sigma\rangle$ and $\bar{l}_\mu$,

$$N_n = c_A\langle\nu\sigma\rangle l_\mu = \frac{\rho N_0}{A}\langle\nu\sigma\rangle = \frac{\langle\nu\sigma\rangle}{A}\rho l_\mu N_0, \tag{3}$$

where c_A [cm^{-3}] is a concentration of nuclei A.

II. EXPERIMENTAL DATA

Table I lists the measured yields Y_{LS} for liquid scintillator (LS), Fe (Y_{Fe}), and Pb (Y_{Pb}). The data are listed in the order of increasing of energy $\bar{E}_\mu$ to which authors have attributed their results. An error in determining the average muon energy $\bar{E}_\mu$ was only shown in Ref. [19]. To estimate the $\bar{E}_\mu$-value error in other experiments summarized in Table I we have used the expression $\delta\bar{E}_\mu = 2/\sqrt{\bar{E}_\mu}$. It covers both an uncertainty of the $\bar{E}_\mu$ calculation at different sets of parameters offered in Refs. [22–24] and deviations of $\bar{E}_\mu$ values from the $\bar{E}_\mu(H)$ dependence, which can be seen in Table I. The Monte Carlo calculations carried out recently [25,26] have resulted in a need to revise some Y_{LS} values. The majority of the LS data [6,8,11,14,15,18,21] was obtained using scintillator C_nH_{2n} ($n = 9.6$, $\rho = 0.78$ g/cm^3) [6,8,11,18,21] [the table includes the refined value $Y_{LS} = 4.1\times10^{-4}\,n/\mu/(\mathrm{g/cm^2})$ taken from Ref. [27]].

It should be noted that Ref. [3] incorrectly cites (Table IV in Ref. [3]) the results of Refs. [6,8,18], namely, out of 15 values taken from these works and included in their Table IV, seven values do not correspond to the published original data. The correct values of H, $\bar{E}_\mu$, and Y_{LS} are presented in Table I of this paper and also in Ref. [4].

The experiments in Refs. [6,18] detected the neutrons produced only in the counter LS; the results of Refs. [8,11,21] covered the neutrons generated in LS and iron of the setup structures.

The counters were located close to the mine ceiling of gypsum ($\bar{E}_\mu = 16.7$ GeV) or salt ($\bar{E}_\mu = 86$ GeV) in the

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TABLE I. Measured neutron yield.

		$Y_n \times 10^{-4}$, $n/\mu/(\mathrm{g/cm^2})$				
$\bar{E}_\mu$, GeV	H, m.w.e.	Y_{LS}	Y_{Fe}	Y_{Pb}	References	Year
10.0 ± 6.3[a]	20	$\cdots$	0.98 ± 0.01	2.43 ± 0.13	[12]	1954
10.0 ± 6.3[a]	60	$\cdots$	$\cdots$	4.8 ± 0.6	[13]	1970
11.0 ± 6.6[a]	40	$\cdots$	1.32 ± 0.30	4.03 ± 0.36	[9]	1971
13.0 ± 7.2	20	0.20 ± 0.07	$\cdots$	$\cdots$	[14]	1995
16.5 ± 8.1	32	0.36 ± 0.03	$\cdots$	$\cdots$	[15]	2000
16.7 ± 8.2	25	0.47 ± 0.05 0.36 ± 0.05[b]	$\cdots$	$\cdots$	[6]	1973
17.8 ± 8.4[a]	80	$\cdots$	1.69 ± 0.30	5.66 ± 0.36	[9]	1971
20 ± 9[a]	110	$\cdots$	$\cdots$	6.8 ± 0.9	[13]	1970
40 ± 12.6[a]	150	$\cdots$	3.31 ± 0.96	11.56 ± 1.1	[16]	1968
86 ± 18	316	1.21 ± 0.12 0.93 ± 0.12[b]	$\cdots$	$\cdots$	[6]	1973
110 ± 21[a]	800	$\cdots$	$\cdots$	17.5 ± 3.0	[17]	1970
125 ± 22	570	2.04 ± 0.24 1.57 ± 0.24[b]	$\cdots$	$\cdots$	[18]	1986
260 ± 8	2700	2.8 ± 0.3	$\cdots$	$\cdots$	[19]	2010
280 ± 33	4300	$\cdots$	$\cdots$	116 ± 44	[20]	1973
280 ± 33	3100	4.1 ± 0.5 3.3 ± 0.5[b]	16.4 ± 2.3	$\cdots$	[21]	2005
280 ± 33	3100	3.2 ± 0.2	19.0 ± 1.0	$\cdots$	[11]	2011
385 ± 39	5200	$5.3^{+0.95}_{-1.02}$ 4.1 ± 0.6[b]	20.3 ± 2.6	$\cdots$	[8]	1989

[a]Vertical flux.
[b]Corrected values.

experiment described in Ref. [6] and close to the mine ceiling of salt ($\bar{E}_\mu = 125$ GeV) in Ref. [18]. As a result of the Monte Carlo calculations in Ref. [25], it was obtained that the contribution of neutrons produced by shower particles in the standard rock around the detecting volume LS ($C_{12}H_{26}$) enlarges the measured yield Y_{LS} by $\sim 30\%$. Taking this fact into account and disregarding the small difference between the compositions of LS and rock in the experiments in Refs. [6,18] and the calculations in Ref. [25], we have obtained the corrected values of Y_{LS}, which are presented in Table I.

A similar correction is not suited for the results of Refs. [8,21,27] since in those experiments an inner detecting volume of the setups consisted of LS and iron in the same proportion as the peripheral one.

With the Liquid Scintillation Detector (LSD) and the Large Volume Detector (LVD), the yield was measured under various conditions: a) with inner counters crossed by a muon (LSD [8]), b) with all counters of the inner setup volume crossed by a muon (LVD [21,27]), and c) with inner counters fired by any trigger pulse, including the muon trigger (LVD [28]). In papers using the LSD and LVD data the yield was defined by the formula

$$Y_{\mathrm{LS}} = \frac{N^{\mathrm{det}} Q}{N_\mu \rho_{\mathrm{LS}} \bar{l}_{\mathrm{LS}} \eta}, \quad (4)$$

where $N^{\mathrm{det}} = N^{\mathrm{det}}_{\mathrm{LS}} + N^{\mathrm{det}}_{\mathrm{Fe}}$ is the number of detected neutrons, including those produced in LS (N_{LS}) and iron (N_{Fe}), while $N^{\mathrm{det}} = N_{\mathrm{LS}}\eta_{\mathrm{LS}} + N_{\mathrm{Fe}}\eta_{\mathrm{Fe}}$, where η_{LS}, η_{Fe} are respective detection efficiencies, Q is a fraction of neutrons produced in LS, and N_μ is number amount of muons. N^{det}, N_μ, and $\bar{l}_{\mathrm{LS}}$ have been determined directly in the experiment. The Q fraction was calculated with the assumption that $\eta = \eta_{\mathrm{LS}} = \eta_{\mathrm{Fe}}$. The values of $Q = 0.61$, 0.60, 0.85 and $\eta = 0.60$, 0.90, 0.60 have been used for cases a), b), and c), respectively. Case c) leads to the selection of neutrons at energies above 10 MeV and a significant reduction in Y_{LS} [21,29]. For this reason, the result of Ref. [28] is not included in the table.

The recent Monte Carlo calculations in Ref. [26] have shown that $\eta_{\mathrm{LS}} \neq \eta_{\mathrm{Fe}}$. This leads to the need to change the formula (4),

$$Y_{\mathrm{LS}} = \frac{N^{\mathrm{det}}}{N_\mu \rho_{\mathrm{LS}} \bar{l}_{\mathrm{LS}}} \times \frac{Q}{Q\eta_{\mathrm{LS}} + (1-Q)\eta_{\mathrm{Fe}}}. \quad (5)$$

Given the fraction Q the yield Y_{Fe} can also be defined,

$$Y_{\mathrm{Fe}} = \frac{N^{\mathrm{det}}}{N_\mu \rho_{\mathrm{Fe}} \bar{l}_{\mathrm{Fe}}} \times \frac{1-Q}{Q\eta_{\mathrm{LS}} + (1-Q)\eta_{\mathrm{Fe}}}. \quad (6)$$

New Q values were calculated in Ref. [30] for cases a) and b). The Q fraction depends on the ratios of the masses

$k_M = M_{\rm LS}/M_{\rm Fe}$, the surface areas $k_S = S_{\rm Fe}/S_{\rm LS}$ calculated per counter, the atomic weights $k_A = A_{\rm LS}/A_{\rm Fe}$, and the exponent β,

$$Q = \frac{k_A^\beta k_M k_S}{(1 + k_A^\beta k_M k_S)}. \tag{7}$$

For inner counters of the first LSD level, it was found that $Y_{\rm Fe} = (20.3 \pm 2.6) \times 10^{-4}$ and $\beta = 0.95 \pm 0.03$. These counters were detecting the neutrons from a 8-cm thick steel platform beneath the setup. The uncertainty of 3% in parameter β results from an uncertainty of 13% in determining $Y_{\rm Fe}$. Using the data of the inner LSD counters of the second level, the fraction $Q = 0.138$ has been determined for case a). The $Y_{\rm LS}$ value of $(4.1 \pm 0.6) \times 10^{-4}$ corresponds to this fraction at efficiencies $\eta_{\rm LS} = 0.45$, $\eta_{\rm Fe} = 0.10$, and $\beta = 0.95$. In the LVD experiment the value $Q = 0.18$ and the corresponding yields $Y_{\rm LS} = (3.3 \pm 0.5) \times 10^{-4}$, $Y_{\rm Fe} = (16.4 \pm 2.3) \times 10^{-4}$ were obtained under detection conditions b). Efficiencies $\eta_{\rm LS} = 0.75$, $\eta_{\rm Fe} = 0.65$ were taken from Ref. [26]. Thus, the $Y_{\rm LS}$ values from the reviewed papers exceed the corrected magnitudes by $\sim$30% (see Table I and Fig. 1).

The recent LVD results have been presented in Ref. [11]: $Y_{\rm LS} = (3.2 \pm 0.2) \times 10^{-4}$, $Y_{\rm Fe} = (19 \pm 1) \times 10^{-4}$. The yield values were obtained using data from counters without triggering pulses at the detection efficiencies $\eta_{\rm LS} = 0.0075$, $\eta_{\rm Fe} = 0.0107$. All the values $(Q, \eta, \bar{l})$ except for the starting number $N^{\rm det}$ were calculated by the Monte Carlo method.

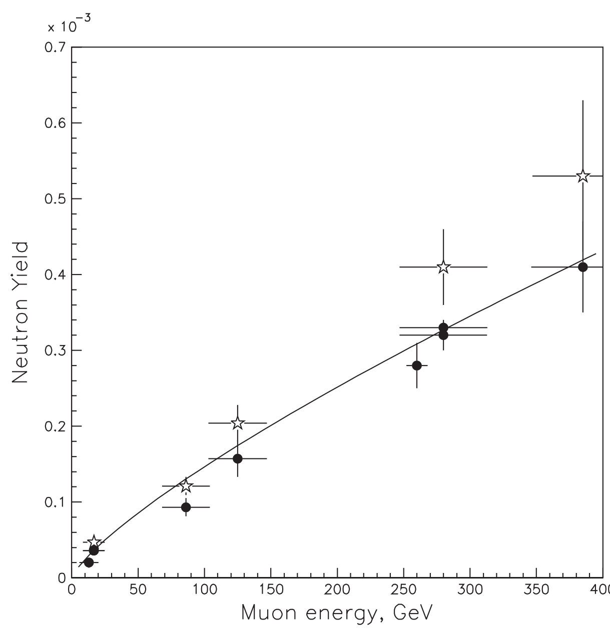

FIG. 1. Dependence of the neutron yield on muon energy for the scintillator. The curve is the function $Y_n = 4.03 \times 10^{-6} \bar{E}_\mu^{0.78}$ that fits the experimental points (filled circles); open stars are uncorrected data.

III. FORMULA FOR THE MUON-INDUCED NEUTRON YIELD

The table data including early measurements [12,13,16,17,20] with iron and lead were analyzed using the conventional approach: α and β are constants independent of $\bar{E}_\mu$ and A, respectively. Using the independence of α on $\bar{E}_\mu$, for any A we can reduce the yield $Y(\bar{E}_\mu)$ values to a certain arbitrarily chosen energy $\bar{E}_\mu^*$ and calculate the average value of $\langle Y(\bar{E}_\mu)\rangle$: $\langle Y_{\rm LS}\rangle = 0.34$, $\langle Y_{\rm Fe}\rangle = 1.70$, $\langle Y_{\rm Pb}\rangle = 6.33 \times 10^{-4} n/\mu/({\rm g/cm}^2)$ for $\bar{E}_\mu^* = 16.7$ GeV. The ratio $\langle Y_{\rm LS}\rangle/\langle Y_{\rm Fe}\rangle$ is consistent with $\beta = 0.95$, while $\langle Y_{\rm LS}\rangle/\langle Y_{\rm Pb}\rangle$ with $\beta = 0.97$ and $\langle Y_{\rm Fe}\rangle/\langle Y_{\rm Pb}\rangle$ with $\beta = 1.00$. The large β values in the last two cases are mostly associated with the excessive yield $Y_{\rm Pb} = 116 \times 10^{-4}$ in the experiment [20]. We assume $\beta = 0.95 \pm 0.03$ because this value is consistent with the result of the direct LSD measurement of $Y_{\rm Fe}$.

The table data presented in Fig. 2 can be described by the expression

$$Y_n(A, \bar{E}_\mu) = cA^\beta \bar{E}_\mu^\alpha, \tag{8}$$

where $\beta = 0.95$, and c is a constant. Using the independence of β with A and assuming $\beta = 0.95$, the $Y_{\rm Fe}(\bar{E}_\mu)$ and $Y_{\rm Pb}(\bar{E}_\mu)$ data sets can be reduced to the $Y_{\rm LS}(\bar{E}_\mu)$ set (Fig. 2, lower panel). By fitting the yield set of 24 values

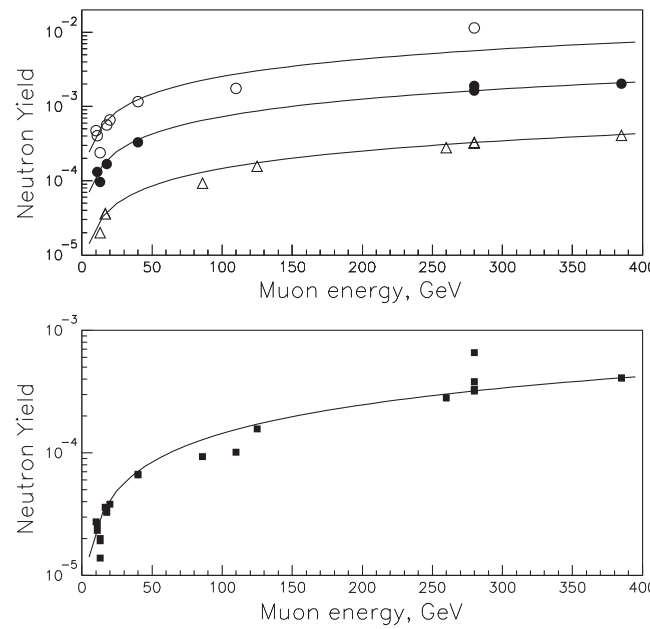

FIG. 2. Dependence of the neutron yield on A and $\bar{E}_\mu$. Upper panel: Experimental points for lead ($A = 207$, open circles), iron ($A = 56$, filled circles), and scintillator ($A = 10.3$, open triangles); the curves represent the function $Y = cA^\beta \bar{E}_\mu^\alpha$ at different values of A and $c = 4.4 \times 10^{-7}$, $\beta = 0.95$, $\alpha = 0.78$. Lower panel: Neutron yield for scintillator; the experimental data for iron and lead are reduced to scintillator, and the curve is the function $Y_{\rm LS} = 4.4 \times 10^{-7} 10.3^{0.95} \bar{E}_\mu^{0.78}$.

$Y_{\mathrm{LS}}(\bar{E}_\mu)$ by the expression $Y_{\mathrm{LS}} = c(10.3)^{0.95}\bar{E}_\mu^\alpha$ we get the best agreement with the data at $c = (4.4 \pm 0.3) \times 10^{-7}$ and $\alpha = 0.78 \pm 0.02$. The same values for c and α result from corrected LS data (Fig. 1, nine values).

The value of the constant c is close to the relative muon nuclear energy loss $b_h = 4.0 \times 10^{-7}$ $(\mathrm{g/cm^2})^{-1}$. Neutrons are produced mainly in h-showers. Therefore, taking into account additional neutron production in em-showers we can conclude that c is a relative muon energy loss for neutron production $c = b_n$. The b_h value does not depend on $\bar{E}_\mu$ and weakly depends on A: $b_h = 4.0 \times 10^{-7}$ for standard rock and 4.2×10^{-7} for water [22]. So, given the dominant role of h-showers in neutron production b_n is practically constant in a wide range of $\bar{E}_\mu$ and A. The values of the exponents α, β in Eq. (8) are determined by neutron production processes in showers: $Y_n \propto \bar{E}_\mu^{1.0}$ in em-showers [7] and $Y_n \propto \bar{E}_\mu^{0.75}$ in h-showers [8,10,18]. Therefore, the resultant values $\alpha = 0.78$, $\beta = 0.95$, and $b_n = 4.4 \times 10^{-7}$ obtained above are associated with the contributions of all neutron production processes, namely, the shower generation by muons and the neutron production in showers via πA, NA, and γA reactions.

The yield value is included in the formula for the neutron production rate $r_n = I_\mu(H)\rho_A Y_n(E_\mu, A)$ $(n/\mathrm{cm^3} c)$, where $I_\mu(H)$ $(\mu/\mathrm{cm^2} c)$ is the muon intensity at a depth H. Using this formula one can write the expression for the rate R_n of muon-induced neutrons in the detector and its shield consisting of different materials. The neutron rate for material A_i of volume v_i and mass m_i is given by

$$R_{ni} = v_i r_n = I_\mu(H)\rho_{A_i} v_i Y_{n_i} = I_\mu(H) m_i Y_{n_i}(n/c). \quad (9)$$

For all materials of the detector and the shield we have

$$R_n = I_\mu(H)\Sigma m_i Y_{n_i} = I_\mu(H) b_n E_\mu^\alpha \Sigma m_i A_i^\beta (n/c). \quad (10)$$

As follows from Eq. (8), the neutron yield is highly dependent on $\bar{E}_\mu(\propto \bar{E}_\mu^{0.78})$ and $A(\propto A^{0.95})$. So, its value for heavy material (Fe, Pb) can be used for the experimental determination of $\bar{E}_\mu$ at any overburden topography and rock composition. The accuracy of the procedure might not be worse than finding $\bar{E}_\mu$ by formulas in Refs. [22–24]. An approximation with constant parameters $b_n = 4.4 \times 10^{-7}$ $\mathrm{cm^2/g}$, $\alpha = 0.78$, $\beta = 0.95$ allows one to use the formula (8) to calculate the yield for any $\bar{E}_\mu$ and A in underground experiments. Since all nuclear effects produced by muons in the matter—including the production of radionuclides—are proportional to the neutron yield value the formula (8) is universal. However, the magnitudes of the parameters are determined by the contributions of nuclear and electromagnetic processes and therefore depend—albeit weakly—on $\bar{E}_\mu$ and A. Due to the increasing requirements of the accuracy of the background determination in underground experiments the study of the neutron yield is of crucial importance.

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