Anti-D3 Branes: Singular to the bitter end

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We study the full backreaction of anti-D3 branes smeared over the tip of the deformed conifold. Requiring the 5-form flux and warp factor at the tip to be that of anti-D3 branes, we find a simple power-counting argument showing that if the 3-form fluxes have no IR singularity, they will be necessarily imaginary anti-self-dual. Hence the only solution with anti-D3 branes at the tip of the conifold that is regular in the IR and the UV is the anti-Klebanov-Strassler solution, and there is no regular solution whose D3-charge is negative in the IR and positive in the UV.

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I. INTRODUCTION

A nonzero positive cosmological constant appears to be the most plausible cause for the observed accelerated expansion of our universe, and thus, in order to be a candidate for a theory of everything, string theory must contain low-energy de Sitter (dS) space solutions. On the other hand, the generic low-energy compactifications of string theory on six-dimensional manifolds with flux produces very large numbers of anti–de Sitter (AdS) vacua, but does not produce classical dS solutions with a cosmological constant smaller than the compactification scale.

To obtain phenomenologically relevant dS solutions one needs to uplift the negative cosmological constant of AdS to a positive one, without disturbing the delicate balance needed to keep the compact dimensions stable, and the only known mechanism for doing this is to place objects with D-brane charge opposite to that of the background (like anti-D3 branes [1]) in regions of high redshift (or high warp factor) of the latter. This ensures that the contribution of the antibranes to the cosmological constant can be parametrically small, and implies that the many AdS low-energy flux compactifications can be uplifted to dS vacua, and hence string theory has a landscape of dS low-energy vacua.

The best-studied model for a highly warped region of a flux compactification is the so-called Klebanov-Strassler warped deformed conifold (KS) solution [2], and anti-D3 branes placed in this solution have been argued to be metastable [3] and are the key ingredient in the Kachru-Kallosh-Linde-Trivedi mechanism for uplifting AdS vacua and producing a de Sitter landscape [1]. The suitability of anti-D3 branes in KS throats for describing metastable vacua and for uplifting AdS to dS vacua has been recently put into question by the perturbative investigation of the backreaction of these antibranes [4,5], which found that near the antibranes the solution develops an unphysical-looking singularity, and hence anti-D3 branes in KS may not give an asymptotically decaying small deformation of this solution.

The stakes raised by this investigation are very high. If the singularity is not an artifact of perturbation theory (as suggested by Ref. [6]), and if moreover it cannot be resolved in string theory, this implies that all solutions with anti-D3 branes in backgrounds with D3 charge dissolved in fluxes are unphysical. This would invalidate the Kachru-Kallosh-Linde-Trivedi mechanism for uplifting AdS vacua to dS ones, and imply that string theory does not have a landscape of vacua with a small positive cosmological constant.

The purpose of this paper is to demonstrate that there exists *no* fully backreacted singularity-free solution describing smeared anti-D3 branes in a warped deformed conifold background with positive D3 charge dissolved in fluxes. Furthermore, the only fully backreacted *regular* solution with anti-D3 branes in the infrared has anti-D3 charge dissolved in fluxes, and hence it is just the supersymmetric KS solution with a different charge orientation (which we will refer to as *anti-KS*). This was first conjectured in Ref. [4] (based on an analogy with the brane-bending calculation of Ref. [7]) and our results confirm this conjecture. The setup is shown in Fig. 1.

To make such a statement one may naively try to construct the fully backreacted anti-D3 solution by solving analytically or numerically the underlying eight nonlinear coupled second-order differential equations [8], but this is not necessary. We believe there exist at least *three* ways to demonstrate that imposing regularity near the anti-D3-branes cannot give a solution with positive D3 charge at infinity, and in this paper we present the three proofs:

(1) We solve by brute force the equations in a Taylor expansion around the infrared. Setting to zero all the coefficients that give a singular metric and 3-form fluxes, we found that the full solution up to order τ^{10} (where τ is the radial coordinate away from the KS tip) has three independent parameters all of which, as we will show below, are singular in the

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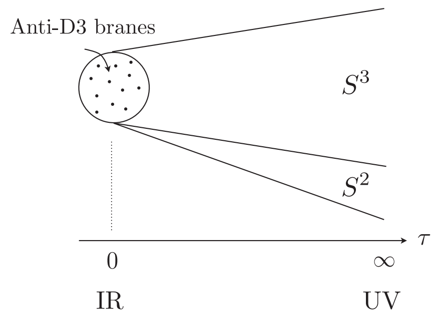

FIG. 1. The deformed conifold with anti-D3-branes smeared over the three-sphere at the tip ($\tau = 0$). This setup is a concrete model of a warped throat in flux compactification, of the kind used to uplift AdS vacua to dS ones.

ultraviolet. The only *regular* solution is hence the Bogomol'nyi-Prasad-Sommerfield (BPS) anti-KS solution with anti-D3-branes.

(2) We explore the boundary conditions for the fields and their derivatives in the infrared, and show that if one imposes singularity-free boundary conditions the right-hand sides of some of the equations are zero at all orders in perturbation theory. The remaining equations only have UV-singular solutions, and the only possible regular solution is the supersymmetric one.

(3) A more elegant way to prove that there is no regular solution whose D3-brane charge changes sign from IR to UV is to find a topological argument similar to that of Ref. [9]. We present an argument along these lines. This argument may be generalizable to the case of *localized* anti-D3-branes.

II. THE SETUP

As argued in Ref. [4], the ansatz for the solution describing smeared D3- and anti-D3-branes in the KS solution is [8]

$$\begin{aligned} ds_{10}^2 &= e^{2A+2p-x}ds_{1,3}^2 + e^{-6p-x}(d\tau^2 + g_5^2) \\ &\quad + e^{x+y}(g_1^2+g_2^2) + e^{x-y}(g_3^2+g_4^2), \\ H_3 &= \frac{1}{2}(k-f)g_5 \wedge (g_1\wedge g_3 + g_2 \wedge g_4) \\ &\quad + d\tau \wedge (\dot f g_1 \wedge g_2 + \dot k g_3 \wedge g_4), \end{aligned} \tag{1}$$

$$\begin{aligned} F_3 &= F g_1\wedge g_2\wedge g_5 + (2P-F)g_3\wedge g_4\wedge g_5 \\ &\quad + \dot F d\tau \wedge (g_1\wedge g_3 + g_2\wedge g_4), \\ F_5 &= \mathcal{F}_5 + *\mathcal{F}_5, \qquad \mathcal{F}_5 = K g_1\wedge g_2\wedge g_3\wedge g_4\wedge g_5, \end{aligned} \tag{2}$$

with

$$K = -\frac{\pi}{4}Q + (2P-F)f + kF, \tag{3}$$

where all the functions depend only on the radial variable τ and the angular forms g_i are defined in Ref. [2]. The constant P is proportional to the 5-brane flux of the KS solution and Q is the number of (anti-)D3-branes.

In order to handle the second-order equations of motion for the scalars of the Papadopoulos-Tseytlin ansatz, we find it crucial to define particular combinations of fields, inspired by the Giddings-Kachru-Polchinski [10] notations. The warp factor $e^{4A+4p-2x}$ and the 5-form flux $K\mathrm{vol}_5$ are combined into scalar modes $\xi_1^\pm$, defined as

$$\xi_1^\pm = -e^{4(p+A)}\Big(\dot x - 2\dot p - 2\dot A \mp \frac{1}{2}e^{-2x}K\Big). \tag{4}$$

These modes have a clear physical interpretation: they parametrize the force on probe D3- and anti-D3-branes in a given solution,

$$F_{D3} = -2e^{-2x}\xi_1^+, \qquad F_{\overline{D3}} = -2e^{-2x}\xi_1^-. \tag{5}$$

We also introduce imaginary self-dual (ISD) and imaginary anti-self-dual (IASD) 3-form fluxes,

$$G_\pm = (\star_6 \pm i)G_3, \tag{6}$$

with $\star_6$ the six-dimensional Hodge star and $G_3 = F_3 + ie^{-\phi}H_3$. The scalar components of $G_\pm$ will be called $\xi_f^\pm$, $\xi_k^\pm$ and $\xi_F^\pm$. This notation follows from the fact that these modes are the conjugate momenta to the fields f, k and F in Eq. (2), in the reduced one-dimensional system that describes the dynamics of the eight scalar functions [seven in Eqs. (1) and (2) plus the dilaton ϕ].

Supersymmetry imposes either that $G_- = F_{D3} = 0$ or $G_+ = F_{\overline{D3}} = 0$, depending on which supersymmetries are preserved. We will refer to the solutions with ISD and IASD fluxes as KS and *anti-KS* respectively. With this notation the KS solution has $\xi_a^+ = 0$, $a = 1, f, k, F$, while for the anti-KS solution $\xi_a^- = 0$. A crucial fact is that the equations of motion for the scalars $\xi_a^\pm$ are just first-order ordinary differential equations. For the ξ_a^- modes we find

$$\begin{aligned} &\dot\xi_1^- + Ke^{-2x}\xi_1^- \\ &\quad = 4e^{2x-4(p+A)}\Big[e^{\phi+2y}(\xi_f^-)^2 + e^{\phi-2y}(\xi_k^-)^2 + \frac{1}{2}e^{-\phi}(\xi_F^-)^2\Big] \end{aligned} \tag{7}$$

and

$$\begin{aligned} \dot\xi_f^- &= \frac{1}{2}e^{-2x}(2P-F)\xi_1^- + \frac{1}{2}e^{-\phi}\xi_F^-, \\ \dot\xi_k^- &= \frac{1}{2}e^{-2x}F\xi_1^- - \frac{1}{2}e^{-\phi}\xi_F^-, \\ \dot\xi_F^- &= \frac{1}{2}e^{-2x}(k-f)\xi_1^- + e^{\phi}(e^{2y}\xi_f^- - e^{-2y}\xi_k^-). \end{aligned} \tag{8}$$

Remarkably, these are the only equations that we will need in this paper. One can define additional scalars $\xi_b^\pm$ which are the conjugate momenta for the four additional modes x, y, p, ϕ, in such a way that the BPS KS solution with Q

mobile D3-branes has all the ξ^+ modes equal to zero. The eight integration constants of the BPS system $\xi^+ = 0$ are fixed as follows: 1. The zero-energy condition of the effective Lagrangian fixes the τ-redefinition gauge freedom and is automatically solved when $\xi_a = 0$, but the constant shift $\tau \longrightarrow \tau + \tau_0$ still remains unfixed, and so τ_0 appears as a "trivial" integration constant. 2. The conifold deformation parameter ϵ and the constant dilaton e^{ϕ_0} give two other free parameters. 3. An additional parameter renders the conifold metric singular in the IR [11] and has to be discarded. 4. The three equations for the flux functions f, k and F appear to have three free parameters [12]. One gives singular BPS fluxes in the IR, the second gives a (0, 3) complex 3-form $G_3 \equiv F_3 + ie^{-\phi}H_3$ that is singular in the UV, and the third corresponds to a B-field gauge transformation $(f, k) \longrightarrow (f + c, k + c)$ that can be absorbed in the redefinition of Q. 5. The warp function $h \equiv e^{-4(p+A)+2x}$ can only be determined up to a constant, which is fixed requiring that h vanishes at infinity.

To summarize, the KS solution with Q mobile D3-branes and the free parameters ϵ and e^{ϕ_0} is the only (IR and UV) regular solution with $\xi_a^+ = 0$, where by IR-regular we denote a solution whose only singularities are those coming from D-branes.

III. THE BOUNDARY CONDITIONS FOR ANTI-D3-BRANES

The main goal of this paper is to show that there is no IR-regular solution with smeared anti-D3-branes ($Q < 0$, hence $K > 0$) at the tip of the conifold and with KS asymptotics ($K < 0$) in the UV. Starting with a singularity-free anti-brane solution in the IR, one necessarily ends up with an anti-KS solution in the UV. Moreover, we will prove that the *only* regular solution with $|Q|$ anti-D3-branes is the anti-KS flip of the solution with Q mobile branes that we reviewed above.

To obtain IR-regular solutions we require the following:

(i) The six-dimensional conifold metric has the tip structure of the KS solution: the 2-sphere shrinks smoothly at $\tau = 0$ and the 3-sphere has finite size.

(ii) The warp factor comes from $|Q|$ anti-D3-branes smeared on the 3-sphere, and hence goes like $h \sim |Q|/\tau$. As a result the Taylor expansions of the functions x, p, A and y start with the same logarithmic *and* constant terms as in the KS solution with mobile branes and can differ only by linear (and higher) terms. The constant term in A cannot be fixed by the regularity condition, since it corresponds to the conifold deformation parameter ϵ.

(iii) There is no singularity in the 3-form fluxes; their energy densities, H_3^2 and F_3^2, do not diverge at $\tau = 0$. Hence, the Taylor expansions of the functions f, k and F start from τ^3, τ and τ^2 terms respectively, exactly like in the KS background. To be more precise, in a solution with branes at the tip the functions f, k and F can also start with noninteger powers ($\tau^{9/4}$, $\tau^{1/4}$ and $\tau^{5/4}$), but one can show that the logarithmic terms in the metric imply that the IR expansion of the solution only has integer powers of τ.

(iv) The dilaton is finite at $\tau = 0$.

It is important to stress that we *do not* impose any kind of anti-KS IR boundary conditions for the 3-form fluxes, and *a priori* the 3-form can be either ISD or IASD (or have both components). On the other hand, we do require the singularities in the warp factor and the 5-form flux to correspond to objects that exist in string theory.

These observations are helpful to determine the possible leading-order behaviors of the ξ_a^+'s and ξ_a^- (for our argument we mostly need the latter). Let us denote by n_a the lowest *possible* leading orders of the fields ξ_a^-. For small τ, the metric regularity conditions imply that the functions e^{2x} and $e^{4(p+A)}$ go like τ and τ^2 respectively. From the explicit definitions of the ξ_a^- modes we find

$$(n_1, n_f, n_k, n_F) = (2, 1, 3, 2). \tag{9}$$

IV. THE IR OBSTRUCTION

Our goal is to show that when solving the equations of motion (7) and (8) for ξ_1^-, ξ_f^-, ξ_k^- and ξ_F^- in the IR (small τ) and imposing the IR regularity conditions, one finds only trivial solutions for these functions. This essentially means that the IASD conditions $\xi_f^- = \xi_k^- = \xi_F^- = 0$ will be satisfied all the way to the UV and not only at $\tau = 0$. To prove this, a simple counting argument is sufficient, as we will now prove.

Let us assume that ξ_F^- and ξ_1^- start from τ^n and τ^{n+l} for some $n \geq 2$. We treat the two possibilities separately:

(1) $l > -1$. Recalling that $e^y \approx \frac{\tau}{2} + \ldots$, we can see from a simple power analysis that the ξ_1^- term is subleading both in the ξ_k^- and ξ_F^- equations in Eq. (8). In the latter equation the ξ_f^- term is also subleading. We arrive at the set of two simple equations near $\tau = 0$: $\dot{\xi}_k^- \approx -\frac{1}{2}e^{-\phi_0}\xi_F^-$ and $e^{-\phi_0}\dot{\xi}_F^- \approx -4\tau^{-2}\xi_k^-$. They have only two solutions, $\xi_F^- \sim \tau^{-2}$ and $\xi_F^- \sim \tau$ and both fall short of the regularity conditions (9). Remarkably, in showing that the system has no regular solution we have not used Eq. (7).

(2) $l \leq -1$. Now the right-hand side of Eq. (7) is certainly negligible with respect to the left-hand side. This means that we have $\dot{\xi}_1^- + \tau^{-1} \cdot \xi_1^- \approx 0$ for small τ leading to the singular solution $\xi_1^- \sim \tau^{-1}$.

For noninteger powers, the argument above can be straightforwardly extended, and the two regimes of parameters corresponding to those above are $l > -1/4$ and $l \leq -1/4$.

To conclude, we see that regularity in the IR implies that the functions ξ_1^-, ξ_f^-, ξ_k^- and ξ_F^- vanish identically.

Consequently, the solution will remain IASD for any value of τ, and the force on probe anti-D3-branes will remain identically zero.

A second way to see that a solution with negative D3 charge in the infrared remains anti-KS all the way to the UV is to solve directly the second-order equations for the scalar in the Papadopoulos-Tseytlin ansatz in a power expansion around the origin. Upon eliminating all the singular modes, we find that to order τ^{10} the space of solutions is parametrized by *three* constants. None of these constants breaks the IASD condition, which confirms the results of the previous section.

One can also use the fact that ξ_1^-, ξ_f^-, ξ_k^-, ξ_F^- are necessarily zero to identify the three IR modes we find, and to show that they actually correspond to UV-singular solutions:

(1) Plugging $\xi_{1,f,k,F}^- = 0$ into the remaining equations of motion, it is easy to show that there exists an IR-regular but UV-divergent one-parameter family of solutions.

(2) A second mode is the (3, 0)-form solution of the superpotential $\xi_a^- = 0$ equations, which breaks supersymmetry and diverges in the UV (see Ref. [12]).

(3) A third "superpotential mode" is related to the shift of the warp function and, following our previous discussion, has to be excluded.

Summarizing, we see that the only solution with smeared *anti*-D3-branes at the KS tip that is regular both in the UV and in the IR is the *anti*-KS solution. Stated differently, the only way to obtain a sensible supersymmetry-breaking solution corresponding to the backreaction of smeared anti-D3's is to allow for IR singularities in the energy densities of the 3-form fluxes.

V. THE GLOBAL OBSTRUCTION

We can also present a "global" argument why the functions ξ_1^-, ξ_f^-, ξ_k^- and ξ_F^- have to vanish in a regular solution, without focusing on their Taylor expansions. The proof for the remaining four functions proceeds precisely as above.

Our key observation is that the flux functions $f(\tau)$, $k(\tau)$ and $F(\tau)$ appear only in Eqs. (7) and (8). None of the remaining $\dot{\xi}_a^-$ equations has any flux function in it. Next, the equations in Eq. (8) might be derived from the following *reduced* Lagrangian:

$$\mathcal{L}_{\text{fluxes}} = 4e^{2x-4(p+A)+\phi} \times \left[e^{2y}(\xi_f^-)^2 + e^{-2y}(\xi_k^-)^2 + \frac{1}{2}e^{-2\phi}(\xi_F^-)^2 \right] + e^{-4(p+A)}(\xi_1^-)^2. \tag{10}$$

We treat $\mathcal{L}_{\text{fluxes}}$ as the effective Lagrangian *only* for the fields $f(\tau)$, $k(\tau)$ and $F(\tau)$ with the remaining five fields being free but subject to the proper boundary conditions ensuring the IR regularity. This means that the first three terms in Eq. (10) are kinetic terms, while the last one is a potential term. Recall also that the ξ^-'s are first order in the derivatives of ϕ's and so the Lagrangian is of the second order, precisely as it should be.

This Lagrangian has a remarkable property: it is strictly non-negative and vanishes only for ξ_1^-, ξ_f^-, ξ_k^-, $\xi_F^- = 0$. In other words, the *global* minimum of Eq. (10) corresponds to the IASD solution. The only way to arrive at a different solution, which describes only a *local* minimum of the action, is to impose boundary conditions (either in the IR or in the UV) that are at odds with the IASD solution $\xi_{f,k,F,1}^- = 0$.

The regularity requirement, however, constrains all three flux functions and their conjugate momenta $\xi_{f,k,F}^-$ in the IR. Indeed, we saw that both (f, k, F) and $(\xi_f^-, \xi_k^-, \xi_F^-)$ have to vanish at $\tau = 0$ for a regular solution. Similarly $\xi_1^- = 0$ in the IR, and thus the IR boundary conditions following solely from the regularity are consistent with the "trivial" IASD solution. We conclude again that requiring regularity forces upon us the anti-KS solution.

Note, however, that the equations derived from Eq. (10) are singular in the IR, and so our arguments should be taken very cautiously. At the same time, this approach may prove efficient for the localized anti-D3-branes, where one cannot use the Taylor expansion argument.

VI. CONCLUSIONS

We presented a detailed analysis of the nonlinear backreaction of smeared anti-D3-branes on the KS geometry. In the near-brane (IR) region we impose boundary conditions coming from the presence of a smeared source: singular warp factor and commensurate 5-form flux, while in the UV region we require the absence of highly divergent modes. We showed that with these assumptions there is a unique solution of the equations of motion, namely the supersymmetric anti-KS solution.

We have thus proven that any supersymmetry-breaking solution associated to the backreaction of smeared anti-D3-branes on the KS geometry has singularities in the IR region not directly associated with the anti-D3-branes themselves. Moreover, these singularities appear in the 3-form fluxes, confirming the linearized analysis of Ref. [4].

A singularity in a supergravity solution does not necessarily mean that this solution should be automatically discarded. However, physically acceptable singularities are those which are resolved in the full string theory (for example, the singularity in the supergravity solution for a brane is resolved by the open strings on the brane, while other singularities are resolved by brane polarization [13,14] or geometric transitions). Here it is not at all clear that there is any mechanism capable of explaining the present singularities. If there is no such mechanism, then the singularity in the supergravity background is telling us that there is no (meta)stable anti-D3-brane solution, and the whole system of branes with charge opposite to that of the background is unstable. This would invalidate the antibrane AdS-to-dS uplifting mechanism, and therefore most of the string theory de Sitter landscape.

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