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Chiral condensate in hadronic matter

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The finite temperature chiral condensate for 2 + 1 quark flavors is considered in the framework of the hadron resonance gas model. This requires some dynamical information, for which two models are employed: one based on the quark structure of hadrons combined with the Nambu–Jona-Lasinio approach to chiral symmetry breaking, and one originating from gauge/gravity duality. Using these insights, hadronic sigma terms are discussed in the context of recent first-principles results following from lattice QCD and chiral perturbation theory. For the condensate, in generic agreement with lattice data it is found that chiral symmetry restoration in the strange quark sector takes place at higher temperatures than in the light quark sector. The importance of this result for a recently proposed dynamical model of hadronic freeze-out in heavy ion collisions is outlined.

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I. INTRODUCTION

Spontaneous chiral symmetry breaking is, apart from color confinement, the most important physical aspect of strong interactions. The fact that one observes mass splittings of chiral partners in the hadron spectrum and that pions have properties attributed to Goldstone bosons strongly suggests that chiral symmetry is spontaneously broken in the vacuum. These, and other theoretical arguments [1], imply that in the vacuum there exists a chiral condensate giving rise to an expectation value of the bilinear fermionic operator $\bar{\psi}\psi$. Dynamical details of this phenomenon, which is inherently nonperturbative in nature, are part of the long-standing problem of strong interactions, but in the course of time different model mechanisms for all its different aspects have been developed.

As temperature and/or baryon density is increased, thermal hadron excitations, because of their quark substructure, will affect the vacuum condensate, causing its melting and eventually vanishing at the transition line to the chirally symmetric phase. Microscopic quantification of this phenomenon comes from first-principles lattice QCD (lQCD) simulations and confirms the intuitive predictions [2].

To get physical insight into this effect for low temperatures (and densities) one can use the hadron resonance gas (HRG) model [3], which was previously successfully applied to give a physical interpretation of lQCD data [2,4] as well as a description of the abundances of particles produced in heavy ion collisions at very different center-of-mass energies [5,6] in terms of freeze-out parameters. The assumption underlying this approach is that for conditions below the QCD transition line, the system is composed of noninteracting hadronic degrees of freedom, and so the partition function is that of an ideal mixture of free quantum gases. To have a reliable physical description of the system, one needs to take into account all hadron resonances with masses up to ~2 GeV.

To calculate the condensate in this framework it is necessary to know the dependence of hadron masses on the current quark masses. This, apart from the Goldstone boson octet, is not straightforward to determine and either requires some assumptions about the underlying dynamics or is the result of a phenomenological fit. Approaches to this which can be regarded as based on first principles are chiral perturbation theory (ChPT) [7], lattice QCD simulations [8] and Dyson-Schwinger equations (DSE) [9,10]. They provide a consistent picture of hadrons with a reliable account of the quark mass dependence. However, in the ChPT framework there are still large uncertainties concerning, for example, the nucleon strange sigma term [11], for which, when different orders of approximation are considered, even the sign is not clear [12].

This article explores the consequences of various hadron mass formulas proposed recently and compares them with the results mentioned above.

One set of mass formulas which was used in the analysis reported here comes from a new model based on quark counting and is a generalization of what was proposed by Leupold [13] a few years ago. In Leupold's scheme, hadron masses were assumed to be linear in the current quark masses. This approach was used (and generalized) in Ref. [14]. In the present work a further step is taken: it is assumed that the dependence of hadron masses on the current quark mass arises solely due to the dependence of constituent masses of valence quarks. The response of these constituent masses to the change in the current quark

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mass is determined based on the Nambu—Jona-Lasinio (NJL) model [15–17]. In this way a fairly good description of the hadronic sigma terms is obtained. The only flaw is that the sea quark contributions are neglected entirely, which, for example, leads to the vanishing of the nucleon strange sigma term.

The second approach considered in this paper is the use of baryon mass formulas which were obtained in a large-N [18] holographic model of QCD due to Sakai and Sugimoto [19]. The last ten years have witnessed a lot of progress coming from gauge/gravity duality, allowing for valuable insights into the dynamics of strongly coupled gauge theories. Recent developments have made it possible to study in a quasianalytical way theories which have very realistic properties. The spectrum of mesons and chiral symmetry breaking in the chiral limit was studied in Ref. [19], and static baryon properties (such as masses, magnetic moments or charge radii) [20,21] were found to be in qualitative agreement with experiment. Also, form factors [21] agree quite well with the data. Further progress was made with the extension to finite current quark masses [22] (see Ref. [23] for an alternative construction), where for example Gell-Mann-Oakes-Renner relations for the pseudoscalar octet have been demonstrated. The impact of finite current quark masses on the spectrum of nucleon octet and delta decuplet baryons has been considered in the two-flavor case [24] and for $2 + 1$ flavors [25] with nontrivial results. The leading-order corrections are proportional to the squares of Goldstone boson masses and determined in a similar way to the leading order of ChPT [12]. The results are in good agreement with the data and other theoretical expectations in the light quark sector, while the contribution of the strange quark is overestimated by the model. It is very likely that going beyond the leading order in the expansion in powers of the current quark masses will give more reasonable results (as is the case in ChPT). Also, at leading order, vector mesons were argued not to receive mass corrections from finite current quark mass [24]. The mass formulas obtained in Refs [24,25] make it possible to estimate sigma terms, including those for the nucleon octet and delta decuplet. This is then used to calculate the chiral condensate in the framework of the HRG model, and the results are compared with calculations based on chiral perturbation theory.

In the context of DSE studies [9,10], sigma terms for the two light quark flavors have been considered. In addition to the nucleon and delta baryons, vector mesons were included. Due to the $\rho - \pi\pi$ and $\rho - \pi - \omega$ couplings, one gets a sigma term of the ρ meson. The ω meson has no pion loop dressing, and therefore only $\omega - \rho\pi$ coupling remains. Since in the DSE approach strange sigma terms were not included yet, we do not use these results as a base for calculating the chiral condensates of interest. We will only use it as a reference point to other calculations.

The importance of the hadronic contribution to the melting of the chiral condensate was appreciated in a model for the freeze-out stage of heavy ion collisions, where it was related to the Mott-Anderson delocalization of hadrons [14]. The model is based on assumptions for hadron-hadron interactions and on the evolution of the matter formed in heavy ion collisions. The main point is that freeze-out phenomena are assumed to take place in the hadronic phase and are entirely attributed to the hadron dynamics. In general, each hadron is assigned a medium dependent radius $r_h(T, \mu_B)$, which is then related in a universal way to the chiral condensate. Hadron-hadron reactions are described by the Povh-Hüfner law [26], and in consequence the cross section is determined by the medium dependence of the condensate. As the temperature decreases, the mean time between the interactions is getting larger, since it is inversely proportional to the reaction cross section and hadron density (in the *relaxation time approximation*). At some point the reaction rate becomes smaller than the rate of expansion, and reactions between hadrons stop changing the final composition. The freeze-out parameters μ_B^f and T^f are determined by the equality of both time scales. However, in Ref. [14] only the light quark condensate was considered, so one of the possible improvements of the model is to include also the strange sector. This is one of the motivations for the present studies.

The organization of the paper is as follows: in Sec. II the generic theoretical setup is described and the relevant quantities used in further calculations are defined. This section also reviews some thermodynamic quantities computed in the HRG model and highlights their very good agreement with lattice computations. These considerations do not require any detailed assumptions about hadron dynamics. On the other hand, the computation of the chiral condensate strongly depends on hadron mass formulas expressed in terms of current quark masses, as discussed in Sec. III. This dependence is captured by the hadronic sigma terms. In Sec. IV we describe our baseline, which comes from results obtained within ChPT as the low-energy effective theory of QCD. In Sec. V, hadron mass formulas based on their constituent quark structure are presented and contrasted with a previously established parametric dependence [27,28] and with first-principles results. Section VI contains novel results following from the Sakai-Sugimoto holographic model together with a discussion in the light of lowest-order ChPT results. Section VII contains conclusions, some discussion and possible open directions.

II. HADRON RESONANCE GAS MODEL

The hadron resonance gas model implements the idea [3] that QCD thermodynamics in the hadronic phase can be described as a multicomponent ideal hadron gas. For very low temperatures the dominant degrees of freedom are pions and kaons, and due to the Goldstone theorem their interactions are weak. Therefore, in the first approximation they can be considered as free particles. As the temperature and/or density is increased, contributions from heavier hadrons become important. Because strong interactions are of finite range in the thermodynamical limit of infinite volume $V \rightarrow \infty$, the grand canonical potential[1] can be expressed as a sum of contributions from free hadrons [29]:

$$\Omega(T, \{\mu_i\}) = \Omega_0 + \Omega_{\rm HRG}(T, \{\mu_i\}). \quad (1)$$

In the above formula, Ω_0 is the vacuum part, whose detailed form is irrelevant for the following considerations, and the medium-dependent part contains contributions from mesons and baryons:

$$\Omega_{\rm HRG}(T, \{\mu_i\}) = \Omega_{\rm M}(T, \{\mu_i\}) + \Omega_{\rm B}(T, \{\mu_i\}). \quad (2)$$

Here $\{\mu_i\}$ is the set of chemical potentials corresponding to conserved charges such as baryon number B, electric charge Q, isospin I_3 and strangeness S. The free meson contribution reads

$$\Omega_{\rm M}(T, \{\mu_i\}) = \sum_M d_M \int \frac{d^3k}{(2\pi)^3} T \ln(1 - z_M e^{-\beta E_M}), \quad (3)$$

while the free baryon contribution is

$$\Omega_{\rm B}(T, \{\mu_i\}) = -\sum_B d_B \int \frac{d^3k}{(2\pi)^3} T \ln(1 + z_B e^{-\beta E_B}), \quad (4)$$

where $E_i = \sqrt{k^2 + m_i^2}$, and d_B and d_M count the degeneracy of hadrons. Fugacities are defined by

$$z_j = \exp\left(\beta \sum_a X^a \mu_{x^a}\right), \quad (5)$$

where the index a runs over all conserved charges in the system, X^a is the corresponding charge and $\beta = 1/T$ is the inverse temperature. Although inclusion of chemical potentials is straightforward in the HRG approach, for much of this paper they are all set to zero. The residual repulsive interactions can be taken into account, e.g., by the van der Waals excluded volume corrections [30].

All thermodynamic quantities, such as equations of state for pressure and energy density, as well as material properties such as the speed of sound, can be obtained from the grand canonical thermodynamical potential $\Omega(T, \{\mu_i\})$. In the following, let us discuss in more detail the case of vanishing chemical potentials. The pressure is given by

$$p = -\Omega(T, \{\mu_i = 0\}), \quad (6)$$

and the energy density is

$$\varepsilon = \frac{\partial[\beta\Omega(T, \{\mu_i = 0\})]}{\partial\beta}. \quad (7)$$

Figure 1 shows the equations of state as obtained for the HRG and compares it with recent lQCD simulations [31] normalized to T^4, the Stefan-Boltzmann behavior of a massless ideal gas. There is clearly an excellent agreement

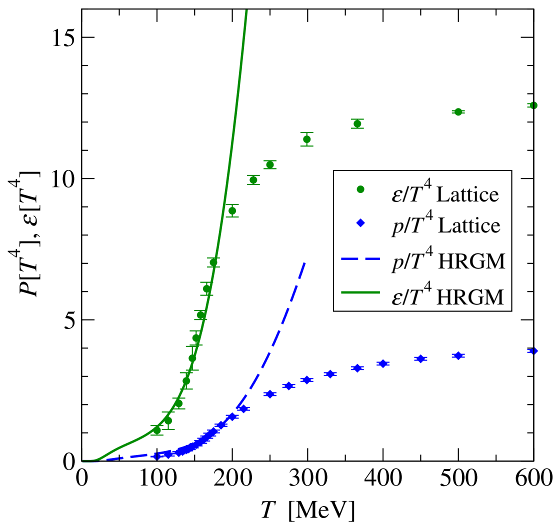

FIG. 1 (color online). Energy density and pressure for the HRG compared to lQCD data [31]. The upper limit for the mass of hadrons included in the calculation is $m_{\rm max} = 2$ GeV.

for temperatures up to ~170 MeV, which means that the dominant effect in that range of temperatures comes from the excitation of hadronic degrees of freedom rather than from their interactions. This is a very well-known effect [2,4]. Agreement for higher temperatures can be obtained when medium modifications of hadronic states are taken into account. For example, in Ref. [32] it was demonstrated that the inclusion of state-dependent hadronic width $\Gamma_h(T)$ taken on the inverse collision time of the Mott-Anderson freeze-out [14] and the proper introduction of quark-gluon degrees of freedom based on the Polyakov-loop extended Nambu–Jona-Lasinio (PNJL) model nicely reproduces lattice QCD data in the whole temperature range.

The velocity of sound is given by

$$c_s^2 = \frac{dp}{d\varepsilon} = \frac{\varepsilon + p}{T}\left(\frac{d\varepsilon}{dT}\right)^{-1}, \quad (8)$$

where the second equality holds only for zero chemical potentials. Its temperature dependence is shown in Fig. 2 for the HRG model compared to lQCD data [31].

Qualitatively, in the HRG model, the increase for low temperatures is related to the appearance of a large number of light degrees of freedom. When heavier hadrons are excited, they contribute considerably to the energy density but almost nothing to the pressure, which leads to the characteristic dip. For high temperatures, because the number of states included is finite, there is an approximately constant behavior approaching the massless gas limit $c_s^2 = 1/3$ only for very high temperatures. On the other hand, for lQCD the dip is an indicator of the crossover transition. For a first-order transition, the sound velocity should be strictly zero. For high temperatures, lattice data approach the massless limit, which is consistent with the interpretation of deconfinement to a massless quark-gluon medium.

[1]Since only homogeneous systems are considered here, the symbol Ω denotes the grand canonical potential as usually defined in statistical mechanics divided by the volume.

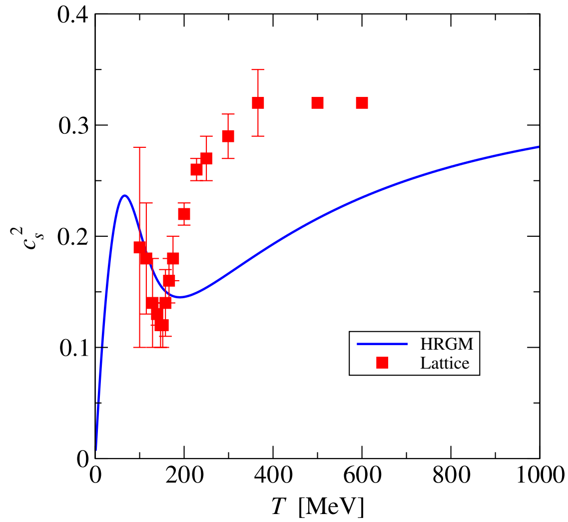

FIG. 2 (color online). Squared sound velocity for HRG compared to lQCD data [31]. The upper limit for the mass of hadrons included in the calculation is $m_{\max} = 2$ GeV.

The importance of the speed of sound for the phenomenology of heavy ion collisions was noticed, e.g., by Florkowski *et al.* [33,34] in the context of the HBT puzzle.

III. CHIRAL CONDENSATE AND SIGMA TERMS

Using the standard formula for the chiral condensate

$$\langle \bar{q}q \rangle = \frac{\partial \Omega(T, \{\mu_i\})}{\partial m_0}, \tag{9}$$

one obtains the quark-antiquark condensate in the light and strange flavor sectors, respectively:

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 + \frac{\partial \Omega_{\mathrm{HRG}}(T, \{\mu_i\})}{\partial m_q}, \tag{10}$$

$$\langle \bar{s}s \rangle = \langle \bar{s}s \rangle_0 + \frac{\partial \Omega_{\mathrm{HRG}}(T, \{\mu_i\})}{\partial m_s}. \tag{11}$$

The derivatives are taken with respect to the current quark masses and lead to the generic formulas

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 + \sum_M \frac{\sigma_q^M}{m_q} n_M(T, \{\mu_i\}) + \sum_B \frac{\sigma_q^B}{m_q} n_B(T, \{\mu_i\}), \tag{12}$$

$$\langle \bar{s}s \rangle = \langle \bar{s}s \rangle_0 + \sum_M \frac{\sigma_s^M}{m_s} n_M(T, \{\mu_i\}) + \sum_B \frac{\sigma_s^B}{m_s} n_B(T, \{\mu_i\}), \tag{13}$$

where the scalar densities of mesons and baryons have been introduced as

$$n_M(T, \{\mu_i\}) = \frac{d_M}{2\pi^2} \int_0^\infty dk k^2 \frac{m_M}{E_M} \frac{1}{z_M^{-1} e^{\beta E_M} - 1}, \tag{14}$$

$$n_B(T, \{\mu_i\}) = \frac{d_B}{2\pi^2} \int_0^\infty dk k^2 \frac{m_B}{E_B} \frac{1}{z_B^{-1} e^{\beta E_B} + 1}, \tag{15}$$

and the response of hadron masses to changes in the current quark mass of flavor $f = u, d, s, \ldots, q_{N_f}$ is captured by the hadron sigma terms

$$\sigma_f^h = m_f \frac{\partial m_h}{\partial m_f}. \tag{16}$$

Thus, for every hadron state, there are different sigma terms related to contributions from quark flavors constituting the hadron.

The above formulas are valid for the noninteracting gas. In the light flavor sector, effects of meson-meson and pion-nucleon interactions as described by ChPT were implemented in Ref. [35].

In the following, isospin symmetry is assumed, setting the light quark masses $m_q = m_u = m_d \approx 5.5$ MeV and the light quark condensate $\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0 = \langle \bar{q}q \rangle_0 = (-240)^3$ MeV^3. Analysis based on lQCD and QCD sum rules together with the low-energy theorem for the correlation functions allows one to estimate the ratio of strange to light quark condensates to be 0.8 ± 0.3 [36]. (Other estimates give 0.75 ± 0.12 [37], but note also recent explicit lattice calculations [38].) One can understand this hierarchy of condensates using the spectral representation of the expectation value for the quark of current mass m_f [39,40],

$$\langle \bar{q}_f q_f \rangle = -2m_f \int_0^\infty d\lambda \frac{\rho(\lambda)}{\lambda^2 + m_f^2}, \tag{17}$$

and noting that the spectral integral is increasingly suppressed with the higher current quark mass, thus lowering the value of the quark condensate. Furthermore, the characteristic length scale related to the quark-antiquark condensate can be taken as $1/m_f$, which is smaller for greater masses. This implies that the medium effect—expressed as screening length—will affect heavier quark condensates at higher temperatures. This can also be understood as arising from the fact that the contribution to the strange quark condensate—and its melting—comes from strange hadrons, which are fewer in number than hadrons containing light quarks.

Another important quantity, an approximate order parameter for the deconfinement phase transition, is the Polyakov loop. It is very well studied in lQCD, and recently it has been addressed within the HRG framework [41]. Good agreement with the lattice data was found in the temperature range 150 MeV $< T <$ 190 MeV.

IV. HADRON MASSES IN CHIRAL PERTURBATION THEORY

As explained above, the finite-temperature behavior of chiral condensates in the HRG approach is determined by the sigma terms, which express the dependence of hadron masses on the current quark masses. A very important approach to this problem is provided by chiral perturbation theory. This approach is most effective in the pseudoscalar sector, since in the limit of vanishing quark masses these

TABLE I. Sigma terms for the lowest-lying baryons in the leading-order ChPT in MeV.

	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω^-
σ_q	36.2	19.9	10.8	27.8	−1.08	15.6	3.31	−8.96
σ_s	162	438	591	317	789	523	729	935

states are massless Goldstone bosons of spontaneously broken chiral symmetry. The importance of the chiral perturbation theory results for the sequel is twofold. Firstly, in the following section they are used to compute sigma terms for the pseudo-Goldstone bosons—the model introduced there is used for the remaining hadronic states. Secondly, it is a natural point of reference for calculations carried out in Sec. VI, where a detailed comparison with the holographic approach is described.

In the case of the Goldstone boson octet, the relevant mass formula is the Gell-Mann-Oakes-Renner (GMOR) relation, which takes the form [42]

$$f_\pi^2 m_\pi^2\left(1-\kappa\frac{m_\pi^2}{f_\pi^2}\right) = -\langle\bar{q}q\rangle_0(m_u+m_d), \quad (18)$$

$$f_K^2 m_K^2\left(1-\kappa\frac{m_K^2}{f_\pi^2}\right) = -\frac{\langle\bar{q}q\rangle_0+\langle\bar{s}s\rangle_0}{2}(m_q+m_s). \quad (19)$$

These formulas include next-to-leading-order corrections expressed in terms of the parameter $\kappa = 0.021 \pm 0.008$ [36]. If one assumes $\langle\bar{s}s\rangle_0 = 0.8\langle\bar{q}q\rangle_0$, $f_\pi = 92.4$ MeV, $f_K = 113$ MeV (which gives $f_K/f_\pi \approx 1.22$ [43]) and $m_s = 138$ MeV, then one finds $(m_q+m_s)/m_s \approx 1.040$ as compared to the lattice choice, $(m_q+m_s)/m_s \approx 1.036$ [2].

Taking the derivative of the above equations with respect to the light quark masses, one finds

$$\frac{\partial m_\pi^2}{\partial m_q} = -\frac{\langle\bar{q}q\rangle_0}{f_\pi^2(1-2\kappa\frac{m_\pi^2}{f_\pi^2})} \approx -\frac{\langle\bar{q}q\rangle_0}{f_\pi^2}\left(1+2\kappa\frac{m_\pi^2}{f_\pi^2}\right), \quad (20)$$

and similarly, for the derivatives of the kaon mass with respect to m_q (and m_s),

$$\frac{\partial m_K^2}{\partial m_{q,s}} = -\frac{\langle\bar{q}q\rangle_0+\langle\bar{s}s\rangle_0}{2f_K^2\left(1-2\kappa\frac{m_K^2}{f_\pi^2}\right)} \approx -\frac{\langle\bar{q}q\rangle_0+\langle\bar{s}s\rangle_0}{2f_K^2}\left(1+2\kappa\frac{m_K^2}{f_\pi^2}\right). \quad (21)$$

In ChPT, mass formulas for the ground-state baryons can be also computed in the $N_f = 2$ [44] and $N_f = 2+1$ cases [12]. At lowest order, the shift due to the finite current quark mass is proportional to the square of the Goldstone boson mass [12]. The lowest-order contributions to the baryon masses read

$$M_B = M_B^0 - \sum_{\phi=\pi,K}\xi_{B,\phi}m_\phi^2, \quad (22)$$

where $\xi_{B,\phi}$ are expressed in terms of the parameters of the low-energy Lagrangian. Sigma terms following from this formula are listed in Table I. In Sec. VI, these results will compared with the holographic mass formulas.

Higher-order contributions are given by the loop corrections to baryon self-energies and are evaluated using different regularization schemes. All of them are carefully compared to the recent lattice data. It is interesting to note that the strange nucleon sigma term turns out to be negative at next-to-leading order (NLO), $\sigma_s^N \approx -4$ MeV [12], which means that the nucleon mass decreases if the strange quark mass is raised. On the other hand, different ChPT studies give at $\mathrm{N^3LO}$ $\sigma_s^N \approx 130$ MeV [11], which means that higher-order corrections can be relevant and the existing answers must be regarded as somewhat tentative. First-principles lattice simulations with $N_f = 2+1$ dynamical quark flavors give $\sigma_s^N = 49 \pm 25$ MeV [45].

In this context, an interesting quantity to look at is the strangeness content of the nucleon, which was estimated in the form [7]

$$y = \frac{2\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u+\bar{d}d|p\rangle} = \frac{m_\pi^2}{\sigma_{\pi N}}\left(m_K^2-\frac{1}{2}m_\pi^2\right)^{-1} m_s\frac{\partial m_N}{\partial m_s} \approx 0.21, \quad (23)$$

where $|p\rangle$ is a nucleon state of momentum p. This is similar to the famous Wroblewski factor [46] introduced in heavy ion and pp collisions for quantifying strangeness production. For the $SU(3)$ symmetric case, $y = 1$; while when there are no strange quark pairs, $y = 0$. The second equality of Eq. (23) is quite generic and relies only on the Hellman-Feynman theorem and tree-level GMOR relations. Its importance lies in the fact that making some statement about the nucleon strange sigma term is in fact equivalent to making a statement about the strangeness contribution to the nucleon.

V. CONSTITUENT QUARK PICTURE

The model described in this section is based on the valence quark structure of hadrons and is a nontrivial generalization of the formulas used in Ref. [14] for the light quark condensate (following earlier work by Leupold [13]). This model is also compared with a another approach, which gives a parametric dependence of hadron masses on the pion mass [27,28] and was previously used for the calculation of both light and strange quark condensates [47]. In these two models, mass formulas are in principle given for all the hadron states, and so the sums over mesons (M) and baryons (B) which appear in the

HRG model take into account all states up to mass ~2 GeV.

The scenario introduced here assumes that baryon and meson masses scale as

$$m_B = (3 - N_s)M_q + N_s M_s + \kappa_B, \quad (24)$$

$$m_M = (2 - N_s)M_q + N_s M_s + \kappa_M. \quad (25)$$

Equation (24) is used for all baryonic states, while Eq. (25) is used for all mesons except pions and kaons, for which the GMOR relations of the previous section are employed. The quark masses in mass formulas (24) and (25) are the dynamical (constituent) ones and are denoted by M_q for the light quarks and by M_s for the strange quark. The parameter N_s measures the strangeness content of the hadron, and the quantities κ_B, κ_M depend on the state, but not on the current quark masses. For the open strange hadrons, N_s is simply the number of strange (antistrange) quarks. For hidden strange mesons—such as, for example, the η or h_1 state—it is modified by the squared modulus of the coefficient of the $\bar{s}s$ contribution to the meson wave function. There are two possible wave function assignments related to the flavor singlet $\psi_0 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)$ and flavor octet $\psi_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$ wave functions for the hidden strange mesons. The strangeness-counting parameters $N_s^{(0)} = 2/3$ for the singlet and $N_s^{(8)} = 4/3$ for the octet have been adopted.

It is easy to see that the baryon octet Gell-Mann-Okubo relation $3M_\Lambda + M_\Sigma = 2(M_N + M_\Xi)$ is translated into a constraint on the state-dependent contributions: $3\kappa_\Lambda + \kappa_\Sigma = 2(\kappa_N + \kappa_\Xi)$.

Two further simplifying assumptions are made: excited states are assumed to have the same flavor structure in their wave functions as their respective ground states, and any possible mixing between octet and singlet states (such as $\eta - \eta'$ mixing) is neglected.

The dynamical (constituent) quark masses M_q and M_s appearing in Eqs. (24) and (25) are a way of partially accounting for the dynamics of strong interactions. For the purposes of computing the condensates, only the dependence of these constituent masses on the current quark masses is relevant. This dependence is taken from the NJL model, where the dynamically generated mass changes by $\Delta M_q = 12.5$ MeV as the quark mass is turned on from zero in the chiral limit to $m_q = 5.5$ MeV [48]. This gives the nucleon sigma term $\sigma_N = 37.5$ MeV. For the strange quark mass, the value of the dynamical quark mass is $M_s = 587.4$ MeV for $m_s = 140.7$ MeV, which gives $\Delta M_s = 227.4$ MeV [49]. This valence quark counting implies that the strange contribution to the nucleon is zero, which is an approximation hard to control. For the Λ baryon, which has one strange quark, the same arguments as above lead to the estimate $\sigma_s^\Lambda = 252.9$ MeV. The resulting sigma terms are shown in Figs. 3 and 4.

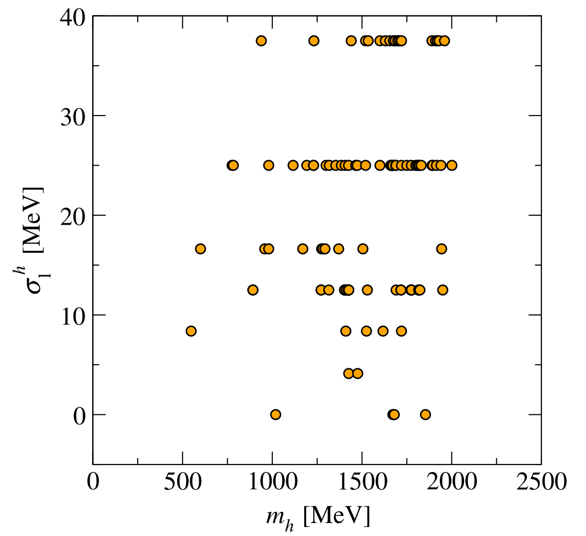

FIG. 3 (color online). Light sigma terms calculated with the constituent quark picture.

In principle, the scaling of Eqs. (24) and (25) will be corrected by various effects such as contributions of the sea quarks, which one would expect to give a logarithmic correction $\ln(m_q/\Lambda_{QCD})^2$ at the one-loop order.

At this point it is interesting to consider another way to quantify the dependence of hadron masses on the explicit breaking of chiral symmetry, i.e., on the pion mass squared. Let us define the quantities A_h by

$$\frac{\partial m_h}{\partial m_\pi^2} = \frac{A_h}{m_h}. \quad (26)$$

The rationale for doing this is that a parametrization of this type was used in the past [27,28,47], taking A_h to be a constant for all hadrons heavier than the pion and the kaon.

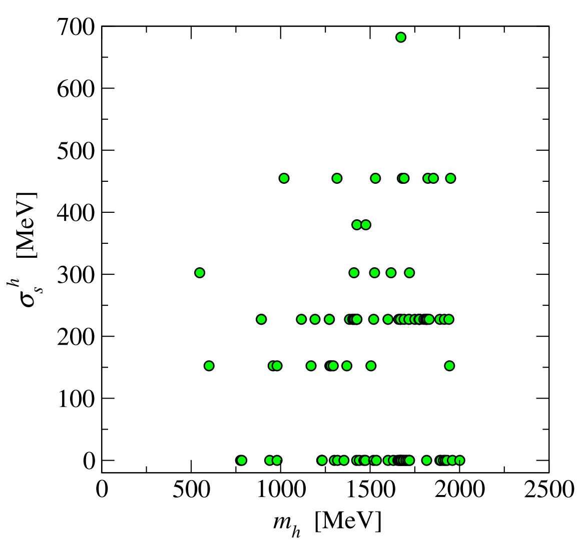

FIG. 4 (color online). Strange sigma terms calculated with the constituent quark picture.

The value of this constant was estimated to be ≈ 0.9–1.2 on the basis of fits to data from lQCD simulations (performed at unphysical values of quark masses). One may ask how strongly the quantities A_h defined by Eq. (26) depend on h in the model under consideration.

The state-dependent coefficient A_h can be used to replace the sigma terms in the condensate formulas [Eqs. (13) and (14)], according to (assuming $\kappa = 0$)

$$\sigma_q^h = \frac{m_\pi^2}{m_h} A_h \tag{27}$$

for the light quark sigma terms. Below we will also use this formula to translate sigma terms calculated within the CQP to estimate A_h. Using the GMOR relation [Eq. (18)], one can write for the light quark condensate in a HRG the compact expression

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 \left(1 - \frac{A_{\rm av} n_{\rm tot}}{m_{\rm red} f_\pi^2} \right), \tag{28}$$

where the averaged A_h coefficient is introduced as

$$A_{\rm av} = \frac{\sum_{h=\{M\},\{B\}} A_h n_h / m_h}{\sum_{h=\{M\},\{B\}} n_h / m_h}, \tag{29}$$

while

$$m_{\rm red} = \left[\frac{\sum_{h=\{M\},\{B\}} n_h / m_h}{\sum_{h=\{M\},\{B\}} n_h} \right]^{-1} \tag{30}$$

is the weighted reduced mass and $n_{\rm tot} = \sum_{h=\{M\},\{B\}} n_h$ the total scalar density of hadrons. Note that $A_{\rm av}$ and $m_{\rm red}$ are temperature dependent.

Equation (28) provides a compact expression for the modification of the light quark condensate in a HRG medium. Since $n_{\rm tot}$ and $m_{\rm red}$ are model-independent characteristics of the HRG, the evaluation of the medium dependence requires solely the determination of $A_{\rm av}$ for a given model.

The reduced mass defined in Eq. (30) is analogous to the reduced mass $\mu_{\rm red}$ used in many particle systems. The latter obeys two inequalities, $m_{\rm lightest}/n \le \mu_{\rm red} \le m_{\rm lightest}$, where $m_{\rm lightest}$ is the lightest mass in the system of n particles. Those inequalities have a direct analogy in our case and read $m_\pi \le m_{\rm red} \le m_\pi n_{\rm tot}(T)/n_\pi(T)$, where the pion is the lightest hadron and $n_\pi(T)$ is the scalar density of the pion. Figure 5 shows the temperature dependence of the scalar densities for pions, kaons, and for all hadrons included in the calculation.

We exemplify this for the simple quark-counting model with the mass formulas of Eqs. (24) and (25), for which we have already given the sigma terms. The corresponding values of the A_h coefficient as a function of hadron mass are shown in Fig. 6. For this model, the averaged value [Eq. (29)] comes out to be temperature dependent, and its behavior is shown in Fig. 7 for three different upper limits of the mass spectrum of included hadrons.

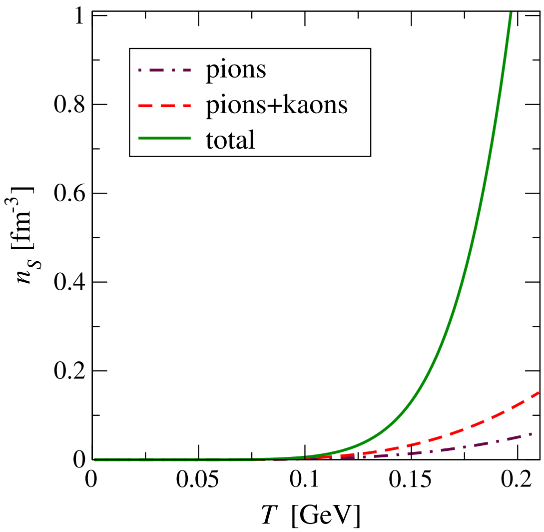

FIG. 5 (color online). Scalar densities defining the hadron contribution to the melting of the chiral condensate.

The straight line structures of Fig. 6 reflect the fact that different hadrons admit different flavor structures and the assumption that excited states have the same structure as their respective ground states.

Figure 8 shows chiral condensates calculated with the two mass formulas described above. What is apparent is that in the quark-counting scenario there is a more pronounced difference between the light and the strange condensates. In Ref. [47] it was found that for the parametric mass formulas [27,28] at the temperature where the light condensate vanishes, the strange condensate is ≈0.4 of its vacuum value. The temperature where the light condensate vanishes is about $T \approx 178$ MeV. In contrast, for the quark-counting scheme used here, this ratio is $\langle \bar{s}s \rangle / \langle \bar{s}s \rangle_0 \approx 0.83$.

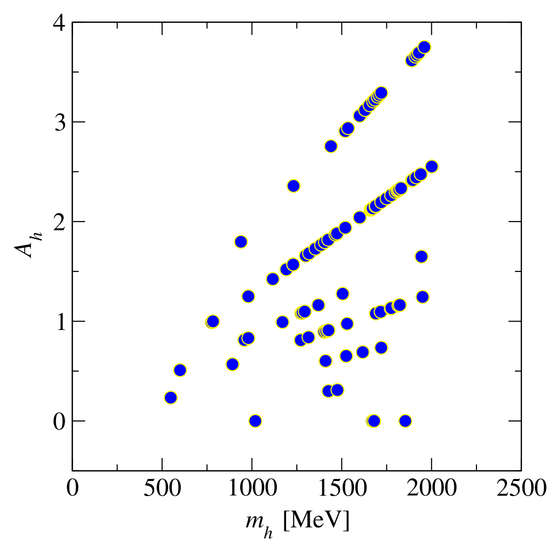

FIG. 6 (color online). Values of the A_h coefficient for hadrons of different mass from Eq. (27) as evaluated with the generalized quark-counting formula.

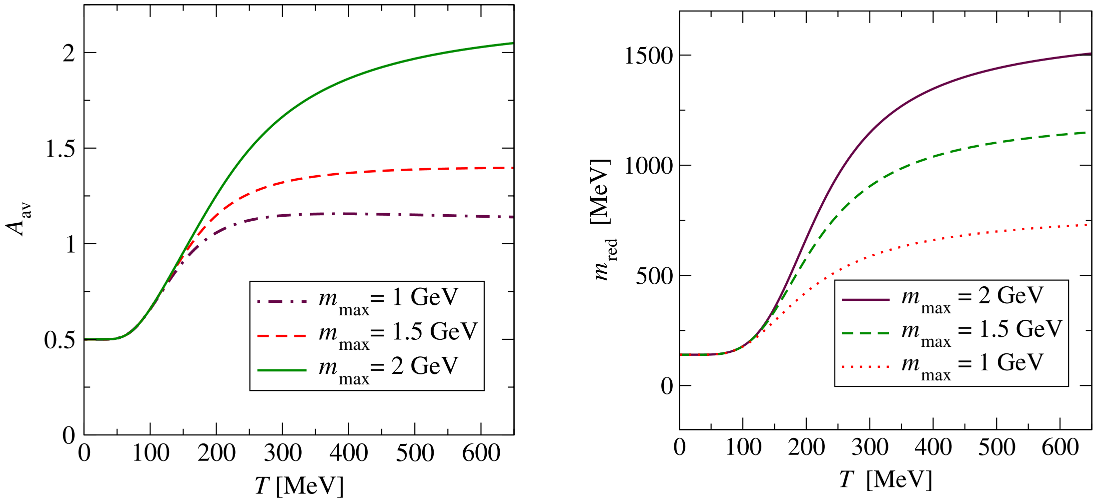

FIG. 7 (color online). HRG model results. Left panel: Temperature dependence of the quantity A_{av} defined by Eq. (29). Right panel: Temperature dependence of the weighted reduced mass m_{red} defined by Eq. (30). The parameter m_{max} denotes the upper limit for the mass of hadrons included in the calculation.

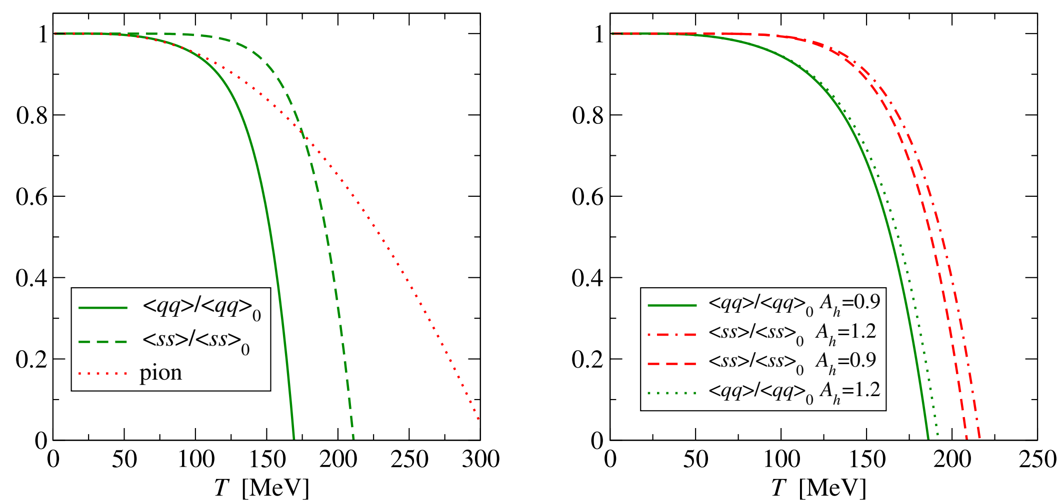

FIG. 8 (color online). HRG model results. Left panel: Quark-counting-mass-formulas-based result for the light quark condensate (green solid line) and the strange quark condensate (green dashed line). Condensate based only on the pion gas contribution (red dotted line). Right panel: Chiral condensate with parametric dependence of hadron masses [27,28,47].

The temperature where the light quark vanishes is $T \approx 168$ MeV. This difference comes from the fact that taking into account sea quark effects diminishes the difference between contributions from strange and nonstrange hadrons. For example, nucleons would contribute to the strange condensate, and hadrons composed only of (anti)strange quarks would contribute to the light quark condensate. This effect is captured by the parametric mass dependence.

To compare with the lattice results of the Wuppertal-Budapest group [2], the quantity

$$\Delta_{q,s}(T) = \frac{\langle \bar{q}q \rangle - \frac{m_q}{m_s}\langle \bar{s}s \rangle}{\langle \bar{q}q \rangle_0 - \frac{m_q}{m_s}\langle \bar{s}s \rangle_0} \tag{31}$$

is considered. The reason to define this quantity on the lattice is purely technical: in this form it eliminates a quadratic singularity at a nonzero value of the quark mass m_q/a^2 (where a is the lattice spacing), and the ratio eliminates multiplicative ambiguities in the definition of condensates. The lattice results for $\Delta_{q,s}(T)$ are calculated for lattices with temporal extent $N_t = 6$, 10, 12 and 16, and an extrapolation to the continuum limit has been given in Ref. [2], to which we compare our models. Physically this quantity is sensitive to chiral symmetry restoration: it is normalized to unity in vacuum and vanishes with the vanishing of the condensates as temperature grows. Figure 9 shows a comparison of the lattice data to the HRG results with the CQP mass formulas. There is overall agreement up

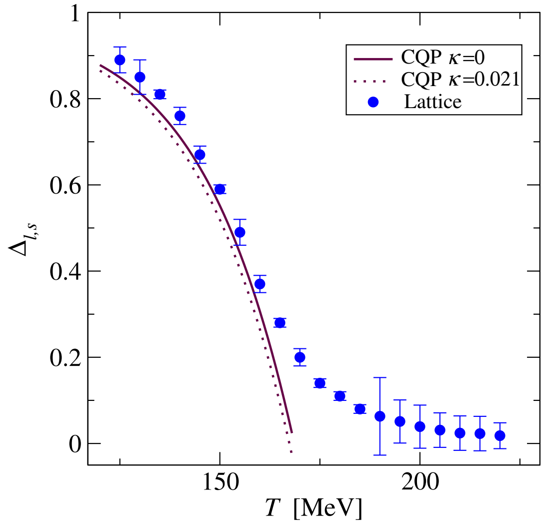

FIG. 9 (color online). Comparison of the HRG results for the temperature dependence of the chiral condensate from the constituent quark picture (CQP) to lQCD results from the Wuppertal-Budapest collaboration [2] (blue dots with error bars).

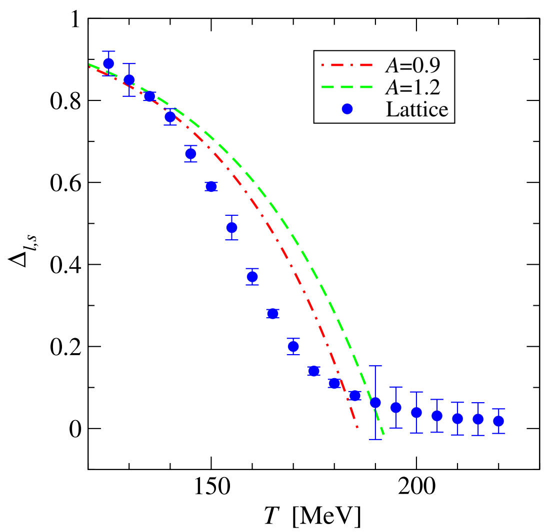

FIG. 10 (color online). Comparison of the HRG results using the parametric mass formula (red and green lines) to the lattice Wuppertal-Budapest results [2] as in Fig. 9 (blue dots with error bars).

to temperatures ≈ 155 MeV, which is the critical temperature from the lattice data. The effects of the NLO corrections on the contribution of the pseudo-Goldstone bosons (κ corrections) to the condensate are minor. To compare, in Fig. 10 the HRG results are shown together with those for the parametric mass formulas. There is good agreement only for temperatures up to $\sim$140 MeV.

VI. HOLOGRAPHIC MASS FORMULAS

The second model considered in this paper is the holographic model of Sakai and Sugimoto [19]. This model is based on a D-brane construction in string theory and assumes both large N and large 't Hooft coupling g^2N. Even though this model is neither supersymmetric nor conformal, the approximations used are sufficiently under control to justify the serious effort that has gone into exploring its phenomenology. Even though in its original formulation the model did not allow for nonzero quark masses, it leads to a large number of quantitative predictions which agree very well with experiment despite the model having just two parameters [50]. The inclusion of explicit chiral symmetry breaking by nonvanishing quark masses was studied[2] in subsequent work [22,51]. The resulting hadron mass shifts were calculated for the case of two flavors in Ref. [24] and for three flavors in Ref. [25]. The latter reference provides hadron mass formulas which were used in the present study. The results reported here include only the nucleon octet and delta decuplet states in the sums over hadrons, since mass formulas have only been calculated for these states.

In the quasi-Goldstone boson sector, the holographic model leads to the GMOR formula [22]. Although in the holographic model the GMOR relations were only obtained in the leading order $m_\pi^2 = 2c/f_\pi^2(m_u + m_d)$ and $m_K^2 = 2c/f_\pi^2(m_u + m_s)$ [22], in the following Eqs. (18) and (19) will be used, which include an estimate of higher-order corrections parameterized in terms of the constant κ.

For the baryon sector, the results are as follows. In the case of two quark flavors, the formula for the nucleon octet and delta decuplet reads [24]

$$\delta M_B = c m_\pi^2, \tag{32}$$

where $c = 4.1\ \text{GeV}^{-1}$. The leading-order chiral perturbation theory result is of exactly the same form, with $c = 3.6\ \text{GeV}^{-1}$ [44] (and references therein). For the choice of parameters made in this paper, this mass shift gives $\delta M = 80.36$ MeV, and the resulting sigma term is

$$\sigma = -c m_q \frac{\langle \bar{q} q \rangle_0}{f_\pi^2}\left(1 + 2\kappa \frac{m_\pi^2}{f_\pi^2}\right) = 36.86\ \text{MeV}. \tag{33}$$

Note that the above results are state independent.

The estimated pion-nucleon sigma term in chiral perturbation theory changes from $\sigma_{\pi N} \approx 59 \pm 19$ MeV at NLO [12] to $\sigma_{\pi N} = 43 \pm 7$ MeV at N^{3}LO, already quite close to what was obtained above (within error bars). There is no essential difference for this sigma term when one includes the strange quark, which is why one can compare this with the $2 + 1$ flavor results. In the chiral limit, the nucleon octet mass was found [7] to be $M_0 = 767$ MeV, which gives $\delta M = 171$ MeV from the physical proton mass.

For the two-flavor DSE studies [10], the nucleon and delta sigma terms were found to be $\sigma_N \simeq 60$ MeV and $\sigma_\Delta \simeq 50$ MeV. This is within the reasonable limits defined

[2]For an alternative approach, see Ref. [23].

by various model approaches but will turn out to be a little closer to the holographic results of the 2 + 1 flavor case.

In the three-flavor case [25], the nucleon octet mass formula reads

$$\delta M_N = \frac{1}{3} c_8 (a_0 m_{K^0}^2 + a_K m_{K^\pm}^2 + a_\pi m_{\pi^\pm}^2), \tag{34}$$

and for the delta decuplet

$$\delta M_\Delta = \frac{1}{3} c_{10} (a_0 m_{K^0}^2 + a_K m_{K^\pm}^2 + a_\pi m_{\pi^\pm}^2), \tag{35}$$

where $c_8 = 7.9\ \mathrm{GeV}^{-1}$, $c_{10} = 9.5\ \mathrm{GeV}^{-1}$ and the a coefficients are given in Tables II and III. Using Eqs. (20) and (22), one can calculate the derivatives of baryon masses

$$\frac{\partial(\delta M_B)}{\partial m_u} = \frac{1}{3} c_\# \left[a_K \frac{\langle \bar{q} q \rangle_0 + \langle \bar{s} s \rangle_0}{2 f_K^2} \left(1 + 2\kappa \frac{m_K^2}{f_\pi^2} \right) + a_\pi \frac{\langle \bar{q} q \rangle_0}{2 f_\pi^2} \left(1 + 2\kappa \frac{m_\pi^2}{f_\pi^2} \right) \right], \tag{36}$$

TABLE II. Coefficients in the nucleon mass formula.

8	P	N	Λ	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
a_0	3/5	4/5	9/10	3/5	11/10	8/5	4/5	8/5
a_K	4/5	3/5	9/10	8/5	11/10	3/5	8/5	4/5
a_π	8/5	8/5	6/5	4/5	4/5	4/5	3/5	3/5

TABLE III. Coefficients in the delta mass term.

10	Δ^{++}	Δ^+	Δ^0	Δ^-	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^-
a_0	1/2	3/4	1	5/4	3/4	1	5/4	1	5/4	5/4
a_K	5/4	1	3/4	1/2	5/4	1	3/4	5/4	1	5/4
a_π	5/4	5/4	5/4	5/4	1	1	1	3/4	3/4	1/2

$$\frac{\partial(\delta M_B)}{\partial m_d} = \frac{1}{3} c_\# \left[a_0 \frac{\langle \bar{q} q \rangle_0 + \langle \bar{s} s \rangle_0}{2 f_K^2} \left(1 + 2\kappa \frac{m_K^2}{f_\pi^2} \right) + a_\pi \frac{\langle \bar{q} q \rangle_0}{2 f_\pi^2} \left(1 + 2\kappa \frac{m_\pi^2}{f_\pi^2} \right) \right], \tag{37}$$

$$\frac{\partial(\delta M_B)}{\partial m_s} = \frac{1}{3} c_\# (a_0 + a_K) \frac{\langle \bar{q} q \rangle_0 + \langle \bar{s} s \rangle_0}{2 f_K^2} \left(1 + 2\kappa \frac{m_K^2}{f_\pi^2} \right), \tag{38}$$

where $B = N, \Delta$ and $\# = 8, 10$.

The resulting hadronic sigma terms are presented in Tables IV and V.

In the holographic setup, the strange nucleon sigma term is significantly overestimated, indicating that higher-order corrections are needed. For ChPT, the leading-order tree-level result is expressed in terms of five low-energy constants and gives a reasonably good evaluation of the nucleon strange sigma term. For the purpose of comparison, the results for the leading-order ChPT sigma terms are shown in Table I. The strange sigma term for the nucleon, $\sigma_s^N \approx 162$ MeV, is a bit large but still reasonable. It should be noted that when compared to the NLO results from Ref. [12] even the sign of the sigma terms can change, meaning that higher-order corrections cannot be ignored. It is to be expected that including higher-order corrections in the holographic approach should cure the problem of overestimating the strangeness contribution as it does in the case of ChPT.

Figure 11 presents the result for the chiral condensates obtained with holographic and NLO ChPT mass formulas where only the nucleon octet and delta decuplet baryons are included (apart from the quasi-Goldstone bosons). In the holographic case, due to the overestimated sigma terms, the difference between strange and light condensates is diminished. For the same reason, the too-small number of states included in the strange sector is compensated.

TABLE IV. Sigma terms for the nucleon octet in MeV.

8	P	N	Λ	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
σ_u	50.6	47.4	42.7	44.3	36.4	28.4	39.6	26.9
σ_d	47.4	50.6	42.7	28.4	36.4	44.3	26.9	39.6
σ_s	1190	1270	1070	714	913	1110	674	993

TABLE V. Sigma terms for the delta decuplet in MeV.

10	Δ^{++}	Δ^+	Δ^0	Δ^-	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^-
σ_u	49.4	45.4	41.5	37.5	43.5	39.5	35.6	37.6	33.6	31.6
σ_d	37.5	41.5	45.4	49.4	35.6	39.5	43.5	33.6	37.6	31.6
σ_s	941	1040	1140	1240	892	992	1090	843	942	794

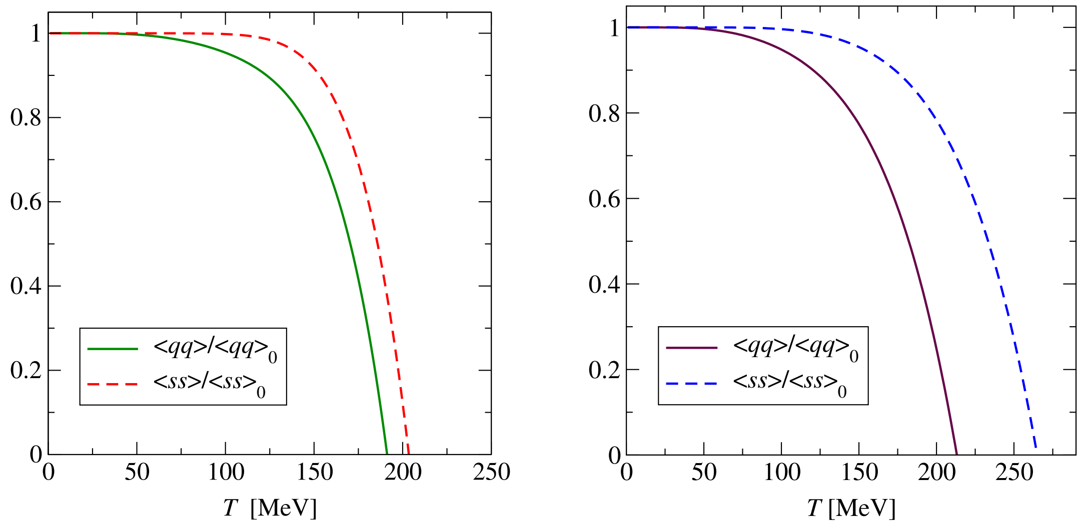

FIG. 11 (color online). Temperature dependence of strange and light condensates using holographic mass formulas (left panel) and NLO ChPT mass formulas [12] (right panel). For details, see the text.

As is clear from the above discussion, one important extension of the existing calculations in the holographic model would be to calculate higher-order corrections in the current quark masses. One motivation for it was already mentioned: this would improve the resulting sigma terms, especially in the strange sector. A second, more formal, motivation is that in ChPT, for $N_f = 2$, the second-order correction to the proton mass has the universal form [44]

$$M_N^{(3)} = M_0 + 4c_1 m_\pi^2 + \frac{3g_A^3}{32\pi f_\pi^2} m_\pi^3, \tag{39}$$

where g_A is the axial coupling. If one adopts the usual scaling of parameters with N, then one gets $g_A \sim N$ and $f_\pi^2 \sim N$, so that the subleading contribution would scale like $\sim N^2$, which would dominate the leading order result $M_0 \sim N$. On the other hand, if one follows recent argumentation [52] that $g_A \sim N^0 = 1$, then NLO contributions would be of the order $\sim 1/N$. It would be interesting to check if in the Sakai-Sugimoto model this universality also holds.

VII. CONCLUSIONS

This paper was devoted to a discussion of the finite-temperature behavior of the chiral condensate within the HRG framework, exploring different microscopic descriptions of the dependence of hadron masses on the current quark mass. In particular, a constituent quark scheme and holographic mass formulas have been used. It was also studied how the results are affected by including different numbers of states in the sums over resonances. It turns out that with a sensible choice of mass formulas and including hadron states with masses, up to ~ 2 GeV generic agreement with recent lattice results is obtained. This is yet another confirmation of the well-known fact that for low temperatures the HRG model gives a satisfactory physical interpretation of lQCD data. Chiral symmetry restoration in the strange sector was seen to take place at higher temperatures than in the light quark sector [53,54], which is related both to the lower number of strange hadrons contributing to the condensate as well as to the response of hadron masses to changes in the current strange quark mass.

A generalization of the quark-counting approach of Refs. [13,14] was proposed, and it was shown that the mass relations where only valence quarks of the hadron are taken into account already lead to a behavior of the condensate which is close to what is seen in the full lattice data. In this scheme, dynamically generated (constituent) quark masses are considered, and their dependence on the current quark mass is quantified in the framework of the NJL model. This step takes into account part of the nonperturbative QCD dynamics. The sea quark contributions are neglected, resulting in a vanishing strange sigma term for the nucleon and a vanishing light quark contribution for the Ω^- baryon. This is somewhat in the spirit of the large-N expansion where quark loops are suppressed.

Along with this, a careful analysis of the hidden strange mesons has been performed based on the flavor symmetry structure of the mesons. This affects the simple quark-counting rules used by Refs. [13,14], taking into account neglected effects which overestimated the light quark condensate and underestimated the strange quark condensate.

Another new aspect considered in this paper concerns the sigma terms and the condensate following from the mass formulas of the holographic model of QCD due to Sakai and Sugimoto [19]. These formulas take on a form similar to the tree-level ChPT results with strange sigma terms overestimated due to the inaccuracy of the approximation for the relatively large value of m_s. Since those shifts were only calculated for the nucleon octet and delta decuplet baryons, the computation of the condensate is incomplete. This also shows the importance of heavier hadrons for temperatures near the QCD transition temperature.

The results obtained here are of great importance in the context of hadron production under extreme conditions in heavy ion collisions. Recently, it has been conjectured that the behavior of the chiral condensate determines the collision rates of hadrons and thus may provide a microscopic approach to the chemical freeze-out of hadron species [14]. This approach, however, has yet been considered only in the light quark sector. Including the strange quark condensate in that analysis could advance the understanding of strangeness production in heavy ion collision experiments.

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