Blackness of the cosmic microwave background spectrum as a probe of the distance-duality relation

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A violation of the reciprocity relation, which induces a violation of the distance-duality relation, reflects itself in a change in the normalization of the cosmic microwave spectrum in such a way that its spectrum is grey. We show that existing observational constraints imply that the reciprocity relation cannot be violated by more than 0.01% between decoupling and today. We compare this effect to other sources of violation of the distance-duality relations which induce spectral distortion of the cosmic microwave background spectrum.

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I. INTRODUCTION

In the standard cosmological model [1,2], in which the universe is described by a spatially homogeneous and isotropic geometry of the Friedmann-Lemaître family, the luminosity distance D_L and angular diameter distance D_A are related by the distance-duality relation,

$$D_L(z) = (1+z)^2 D_A(z), \tag{1}$$

with a coupling β and mass scale M. where z is the redshift. This relation is actually far more general [3,4]. It can be shown (see Ref. [5] for a demonstration) that it holds in any spacetime in which (i) the reciprocity relation holds and (ii) the number of photons is conserved.

The *reciprocity relation* connects the area distances up and down the past light cone and is a relation between the source angular distance, r_s, and the observer area distance, r_o. The former is defined by considering a bundle of null geodesics diverging from the source and which subtends a solid angle $d\Omega_s^2$ at the source. This bundle has a cross section $d^2 S_o$ at the observer and the source angular distance is defined by the relation

$$d^2 S_o = r_s^2 d\Omega_s^2. \tag{2}$$

Similarly, the observer area distance is defined by considering a reciprocal null geodesic bundle converging at the observer by

$$d^2 S_s = r_o^2 d\Omega_o^2. \tag{3}$$

As long as photons propagate along null geodesics and the geodesic deviation equation holds then these two distances are related by the reciprocity relation [5]

$$r_s = (1+z) r_o, \tag{4}$$

regardless of the metric and matter content of the spacetime. While r_o is related to the angular distance, the solid angle $d\Omega_s^2$ cannot be measured so that r_s is not an observable quantity. If one further assumes that the number of photons is conserved, the source angular distance is related to the luminosity distance by $D_L = (1+z) r_s$, which leads to the distance-duality relation (1), where $D_A(z) = r_o$.

II. VARYING THE DISTANCE-DUALITY RELATION

Violations of the distance-duality relation (1) can arise from (i) a violation of the reciprocity relation, which can occur in the case where photons do not follow (unique) null geodesics (e.g., in theories involving torsion and/or nonmetricity or birefringence [6]) or (ii) from the non-conservation of photons, which occurs, e.g., when photons are coupled to axions [7] or to gravitons in an external magnetic field [8], to Kaluza-Klein modes associated with extra dimensions [9], or to a chameleon field [10–12]. Theories involving the conversion of photons → X violate the distance-duality relation in that the flux of photons arriving from standard candles (such as type Ia supernovae) may be altered based on the theory and environment that the photons travel through between the source and detector. By contrast, distance measures from standard rulers (such as baryon acoustic oscillations) are unmodified in such theories, as the important measure is the angle subtended on the sky, and not the actual flux of the object itself. The fact that such a violation of the distance-duality relation can account for a dimming of the supernovae luminosity [13,14]—since, e.g., in the case of photon-axion mixing the luminosity distance has to be rescaled as $D_L/\sqrt{1-P_{\gamma-a}(r)}$ with $P_{\gamma-a}$ being the probability for a photon to oscillate in an axion after having propagated over

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a distance r—has motivated the design of many tests of this relation [15–19] using independent measurements of D_L and D_A [15,17] based on the Sunyaev-Zel'dovich effect and X-ray measurements [16,18,19]. Furthermore, observations of birefringence (a frequency-independent rotation of linear polarization vectors over cosmological distances) may be linked to scalar-field models of dark energy [20], as the scalar field couples to photons via an interaction term

$$\frac{\beta}{2}\frac{\phi}{M}F_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}. \tag{5}$$

with a coupling β and mass scale M.

These tests play an important role in understanding the physics behind the acceleration of cosmic expansion [21]. Defining the deviation from Eq. (1) as

$$\eta(z) = (1+z)^2\frac{D_A(z)}{D_L(z)}, \tag{6}$$

so the distance-duality relation holds iff $\eta(z) = 1$, it was concluded [16] that $\eta = 0.91 \pm 0.04$ up to $z \simeq 0.8$.[1] In particular, this sets strong constraints on photon-axion oscillation models [15,16]. It also allows us to prove [15] that the dimming of the supernovae did not result from absorption by a grey-dust model [24].

It has also been pointed out [25–27] in the particular case of photon-axion mixing that the oscillation probability depends on the frequency of the photon [7], so that a spectral distortion of the cosmic microwave background spectrum was expected (see below).

The cosmic microwave background (CMB) radiation is considered to enjoy one of the most precise black-body spectra ever produced, that is, it has the Planck form

$$I_{\rm BB}(\nu, T) = \nu^3 f(\nu/T),\, f(\nu/T) := \frac{2h}{c^2}\frac{1}{e^{h\nu/kT}-1}, \tag{7}$$

where $I_{\rm BB}(\nu, T)$ is the energy per unit time (or the power) radiated per unit area of emitting surface in the normal direction per unit solid angle per unit frequency by a black body at temperature T at emission (h is the Planck constant, c the speed of light in a vacuum, and k the Boltzmann constant.) We note that the normalization factor $\frac{2h}{c^2}$ incorporated into the definition of $f(\nu/T)$ is crucial to the black-body nature of the spectrum, coming directly from quantum mechanics.

The accuracy of the CMB black-body spectrum sets [28–30] constraints on various spectral distortions parametrized as the effective chemical potential μ, the Compton parameter y, and the free-free distortion parameter $Y_{\rm ff}$ of the order

$$|\mu| < 19\times 10^{-6}, \quad |y| < 9\times 10^{-5}, \quad |Y_{\rm ff}| < 15\times 10^{-6}. \tag{8}$$

This sets stringent constraints on violations of the distance-duality relation induced by the nonconservation of photons, as, e.g., in the case of photon-axion mixing.

III. CHANGES IN THE CMB SPECTRUM

Our goal is to investigate the effect of a violation of the distance-duality related to a violation of the reciprocity relation on the spectrum of CMB photons. We assume that Eq. (4) is modified to

$$r_{\rm s}^2 = (1+z)^2 r_{\rm o}^2\beta(z), \tag{9}$$

where $\beta(0) = 1$. $\beta(z)$ is a function depending on the particular physical mechanism responsible for the violation of the reciprocity relation; that relation holds precisely iff $\beta(z) = 1$.

First, the integrated flux of radiation F received from an isotropically emitting source of intrinsic luminosity L is the amount of radiation received by the detector per unit area per unit time, and is given by

$$F = \frac{L}{4\pi}\frac{1}{r_{\rm s}^2(1+z)^2}, \tag{10}$$

where the factor 4π arises from the integral of $\mathrm{d}\Omega_{\rm s}^2$ over the whole sky [5]. The specific flux F_ν received from the source, i.e., the flux per unit frequency range, is given by

$$F_\nu \mathrm{d}\nu = \frac{L}{4\pi}\frac{\mathcal{I}[\nu(1+z)]\mathrm{d}\nu}{r_{\rm s}^2(1+z)}, \tag{11}$$

where $\mathcal{I}(\nu)$ is the source spectrum. Note that here ν is the frequency measured by the observer, which corresponds to a frequency $(1+z)\nu$ at emission.

What is actually measured from an extended source by a detector is the specific intensity I_ν in a solid angle $\mathrm{d}\Omega_{\rm o}^2$ in each direction of observation,

$$I_\nu \mathrm{d}\nu \equiv \frac{F_\nu \mathrm{d}\nu}{\mathrm{d}\Omega_{\rm o}^2}, \tag{12}$$

and reduces, using Eqs. (11), (2), and (3), to

$$I_\nu \mathrm{d}\nu = I_{\rm s}\left(\frac{r_{\rm o}}{r_{\rm s}}\right)^2\frac{\mathcal{I}[\nu(1+z)]\mathrm{d}\nu}{(1+z)}, \tag{13}$$

where $I_{\rm s} := L/4\pi\mathrm{d}^2 S_{\rm s}$ is the source surface brightness in that direction. It follows from Eq. (9) that

$$I_\nu \mathrm{d}\nu = I_{\rm s}\frac{\mathcal{I}[\nu(1+z)]\mathrm{d}\nu}{(1+z)^3\beta(z)}. \tag{14}$$

This expression is completely general (and does not assume any specific geometry for the spacetime) and thus holds in any curved or flat spacetime. In the laboratory, it simply means that the intensity of the radiation is independent of the distance from the source, as long as the source

[1] A deviation of $\eta(z)$ away from unity in optical wavelengths was studied in Refs. [22,23], where Ref. [22] put strong constraints on any deviation from cosmic transparency in optical wavelengths, with $\eta(z) = 1/(1+z)^\epsilon$ and $\epsilon = -0.04^{+0.08}_{-0.07}$ at 2σ.

has no relative motion compared to the detector ($z = 0$). In cosmology, the relation depends only on redshift and is thus achromatic (so that the spectrum is redshifted but not distorted) and is independent of area distance.

In the case of the cosmic microwave background, the photons are coupled to the electrons and baryons by Thomson scattering up to recombination [1,2] (see Ref. [31] for a description of the physical origins of the different spectral distortions). Because the collision term entering the Boltzmann equation has a very weak dependence on the energy of the photon, the CMB spectrum enjoys a Planck spectrum (7). Deviations from the Planck spectrum induced by nonlinear dynamics [32,33] are negligible, as recalled in Eq. (7); see also Refs. [34–36].

Then, denoting the emission temperature by T_e, we have that

$$I_s \mathcal{I}[\nu(1+z)] = \nu^3(1+z)^3 f\left[\frac{\nu(1+z)}{T_e}\right]. \tag{15}$$

Using Eq. (14) and the relation $\nu \propto (1+z)^{-1}$, which follows from the definition of redshift (which is purely a time dilation effect, so this relation is quite independent of area distances), we finally get

$$I_\nu = \frac{\nu^3}{\beta(z)} f\left[\frac{\nu(1+z)}{T_e}\right] \tag{16}$$

after simplifying by a factor $(1+z)^3$.

As long as $\beta(z) = 1$, the redshift prefactor combines with the factor ν^3 in Eq. (15) so that the initial Planck spectrum with temperature T_e remains a Planck spectrum,

$$I_\nu = \nu^3 f\left[\frac{\nu}{T(z)}\right], \tag{17}$$

with a redshifted temperature

$$T(z) = \frac{T_e}{1+z}. \tag{18}$$

If $\beta(z) \neq 1$, then the spectrum has the form

$$I_\nu = \beta^{-1}(z) I_{\rm BB}[\nu, T(z)]. \tag{19}$$

We conclude that if the reciprocity relation is violated, the black-body spectrum is observed as a grey-body spectrum. However, there is no spectral distortion, so such an achromatic effect can be confused with calibration errors.

Note that the grey-body factor does not impact Wien's displacement law, that the wavelength $\lambda_{\rm max}$ at which the intensity of the radiation is maximum obeys

$$\lambda_{\rm max} = b/T, \tag{20}$$

where Wien's displacement constant is $b = 2.8977721 \times 10^3$ Km. This corresponds to a frequency $\nu_{\rm max} = 58.8(T/1\ {\rm K})$ GHz. The Stefan-Boltzmann law states that the power emitted per unit area of the surface of a black body is proportional to the fourth power of its absolute temperature; $j_* = \sigma T^4$, where j_* is the total power radiated per unit area and $\sigma = 5.67 \times 10^8\ {\rm m^2\,K^4}$ is the Stefan-Boltzmann constant. For a grey body, this is modified to

$$j_* = \beta^{-1}(z)\sigma T^4 \tag{21}$$

because all intensities get changed by this same factor. Hence, the Wien temperature

$$T_{\rm W} := b/\lambda_{\rm max} \tag{22}$$

and the Stefan-Boltzmann temperature

$$T_{\rm SB} := \{j_* \beta(z)/\sigma\}^{1/4} \tag{23}$$

are different iff $\beta(z) \neq 1$. The first is independent of z, while the second is not.

In summary, a violation of the distance-duality relation (1) has an imprint on the CMB spectrum. Two possible origins of such a violation are a violation of the reciprocity relation, or nonconservation of the number of photons.

Generically, the evolution of the distribution function f of the CMB is discussed in terms of the Boltzmann equation, $\mathcal{L}[f] = C[f]$. In a Friedmann-Lemaître universe, the distribution function is, by symmetry, a function of the energy E and cosmic time t so that the Liouville term reduces to $\mathcal{L}[f] = E\partial_t f - HE^2\partial_E f$. If the collision term does not depend on energy, the integration of the Boltzmann equation over energy gives the following evolution equation for the photon number density (see, e.g., Refs. [1,37]):

$$\dot{n} + 3Hn = \tilde{C}. \tag{24}$$

In general, the collision term depends on energy, as for example in the case of axion-photon mixing, so the effect of photon nonconservation is expected to be chromatic. It follows that the general form of the observed spectrum can be parametrized as

$$I(\nu, T) = \Phi_0(\nu, z) I_{\rm BB}[\nu, T(z)], \tag{25}$$

where

$$\Phi_0(\nu, z) := \beta^{-1}(z)\eta(\nu, z). \tag{26}$$

The factor β is related to the violation of the reciprocity relation as shown above and the factor $\eta(\nu, z)$ to the nonconservation of photons. In Fig. 1 we compare the factor $\Phi_0(\nu, z)$ for photon-axion mixing, a violation of the reciprocity relation, and a μ-type spectral distortion. It demonstrates that each physical phenomenon impacts a different part of the CMB spectrum.

Concerning the possibility of a violation of the reciprocity relation, the COBE-FIRAS experiment [28,29] showed that the CMB photons have a black body spectrum within $3.4 \times 10^{-8}\ {\rm ergs\,cm^{-2}\,s^{-1}\,sr^{-1}\,cm}$ over the frequency range from 2 to 20 ${\rm cm^{-1}}$. More importantly, concerning our work it showed that the deviations are less than 0.03% of the peak brightness with an rms value of 0.01%. This means that the normalization of the spectrum can be

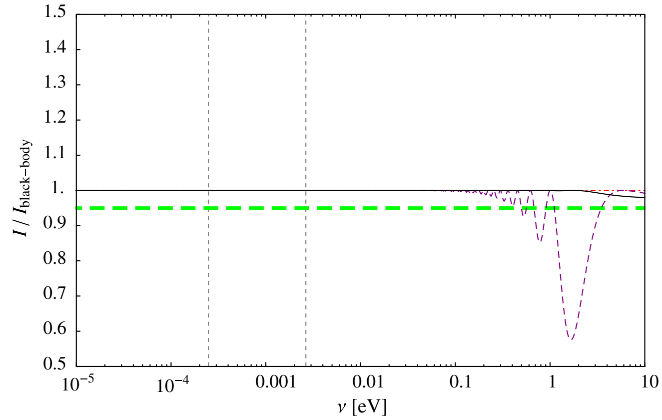

FIG. 1 (color online). Comparison of the function $\Phi_0(\nu, z)$ defined in Eq. (26) at redshift $z = 0$ for a μ-type spectral distortion (red dot-dashed), a violation of the reciprocity theorem leading to a grey body (green thick dashed) and photon-axion mixing (black solid, blue dotted, and purple dashed) with different physical parameters for the distribution of the intergalactic magnetic field, calculated with the results of Ref. [14]. The vertical dashed lines correspond to the frequency range covered by the FIRAS instrument. Because of the smallness of any spectral distortion on the CMB, only the solid black curve corresponds to realistic parameters. The unphysical CMB spectral distortions include the assumption of a 20 nG intergalactic magnetic field for distortions from an axion-photon coupling and a deviation towards a grey-body spectrum at the level of $\sim$1 part in 20 (compared to $\sim$1 part in 100 from previous works and improved to $\sim$1 part in 10^4 in this work).

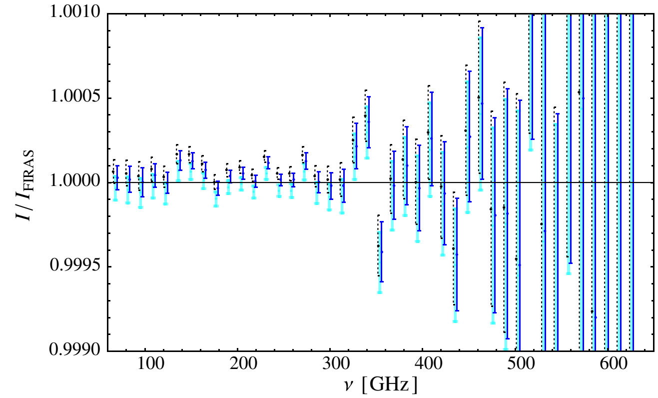

FIG. 2 (color online). The ratio of the spectral radiance from a 2.725 K grey-body spectrum [Eq. (19)] to that measured by FIRAS for $\beta^{-1}(z_{\text{LSS}}) - 1 = 10^{-4}$ (black dotted), $10^{-4.2}$ (blue solid), and 0 (cyan thick). Error bars are the 1σ uncertainty from the FIRAS data.

considered accurate at this level so that it indicates a constraint of the order $|\beta^{-1}(z_{\text{LSS}}) - 1| < 10^{-4}$, with $z_{\text{LSS}} \sim 1100$ for the redshift of the last scattering surface.

To check the order of magnitude of this effect, we focus on the CMB monopole spectrum [29] and compare to a 2.725 K grey-body spectrum for several values of $\beta^{-1}(z_{\text{LSS}})$. The ratio of the expected CMB monopole signal (19) to the measured CMB monopole (publicly available at Ref. [38]) is depicted in Fig. 2. Error bars designate the 1σ uncertainty of the FIRAS data. From Fig. 2 we see that any violation of the reciprocity relation must be less than one part in 10^4 at the surface of last scattering,

$$|\beta^{-1}(z_{\text{LSS}}) - 1| < 10^{-4}. \tag{27}$$

Including the spectral distortions of the CMB monopole using the constraints listed in Eq. (8) (see also Refs. [29,39]) increases the deviation from unity of the ratio I/I_{FIRAS} plotted in Fig. 2, but does not significantly change any constraint on $\beta^{-1}(z_{\text{LSS}})$.

In this article we have shown that a violation of the reciprocity relation is associated with a deviation from blackness and that the COBE/FIRAS data sets the constraints $|\beta^{-1}(z_{\text{LSS}}) - 1| < 10^{-4}$. The second factor $\eta(\nu, z_{\text{LSS}})$ induces a spectral distortion and can be constrained independently; see, e.g., Refs. [26,27] for an example of the case of photon-axion mixing. To compare to previous constraints [16], we use the fact that they were based on bolometric observations so that they concerned the parameter $\eta(z)$ defined in Eq. (6), which is related to parameters introduced in this article by

$$\eta(z) = \beta^{-1}(z)\frac{\int \eta(\nu, z) I_{\text{BB}}[\nu, T(z)]\mathrm{d}\nu}{\int I_{\text{BB}}[\nu, T(z)]\mathrm{d}\nu}. \tag{28}$$

We recall that $\eta = 0.91 \pm 0.04$ up to $z \simeq 0.8$ [16]. Note that Eq. (27) gives much tighter limits over a much longer range of redshifts.

For completeness, we shall also mention that bounds have been set on the relation between CMB temperature and redshift, assuming a form $T = T_0(1+z)^{1-\gamma}$. The index γ may be constrained through measurements of the Sunyaev-Zeldovich effect for redshifts $z < 1$ and fine structure excitations in quasar spectroscopy for redshifts $z > 1$, specifically $\gamma = -0.004 \pm 0.016$ up to a redshift of $z \sim 3$ [40]. A nonvanishing γ has been argued to appear in models with decaying dark energy [41–43], but it has been argued to have an unphysical ansatz [31]. Such constraints on the temperature are not easily related to ours since these analyses assume a Planck spectrum. However, when the spectrum is no longer a Planck spectrum the different notions of temperatures differ. We have already noted the difference between the Wien and Stefan-Boltzmann temperatures. The latter is also related to the bolometric temperature in the case of spectral distortions. Note also that while the brightness and the bolometric temperatures agree at the background and first-order level, it has been shown [44] that the nonlinear dynamics sources a y-type spectral distortion and this would affect the brightness and thus both the brightness and the bolometric temperatures. In such a case [45] it was proposed to use occupation number temperature defined as the temperature of a black body which would have the same number density of photons as the actual distribution (to be contrasted with the bolometric temperature which is the temperature of the black body which carries the same energy density as the actual distribution). From an observational perspective, some projects such as PIXIE [46] plan to improve the constraints on the spectral distortions of the CMB. While they claim to reach an upper bound of 10^{-8} on both the μ and y parameters, the case of a grey-body distortion and the level of the constraint on the parameter β is not evaluated.

IV. CONCLUSION

We have shown that CMB spectral observations allow one to test the distance-duality relation. We have emphasized the difference between the imprint induced by the nonconservation of photons, usually chromatic, and a violation of the reciprocity relation, which is achromatic. In the latter case we have shown that the CMB spectrum is a grey spectrum, with the same shape as the CMB power spectrum, up to a normalization factor. The FIRAS/COBE data allowed us to set constraints of the order of 0.01% on the relative deviation from the reciprocity relation for the CMB.

As a final remark, we note that the limits above are for radiation coming from the surface of last scattering at $z = 1100$. However, it is likely that any effect violating the distance-duality relation would be cumulative, and hence proportional to distance. While it probably implies stronger constraints for closer sources, no robust and model-independent bound can be derived. Note however that our constraint improves those at low redshift by at least two orders of magnitude.

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