Dark radiation and the CMB bispectrum

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Non-Gaussianities in the cosmic microwave background maps arising from correlations between lensing and time variations of the gravitational potential (the so-called integrated Sachs-Wolfe effect) are one of the most important contaminants to the determination of the primordial inflationary bispectrum and may bias its determination. The presence of an extra dark radiation component, as suggested by some recent osmic microwave background measurements from the South Pole Telescope, could bias the expected value of the local bispectrum. In this paper we investigate the impact of dark radiation on the local bispectrum. As a by-product we also quantify the additional information on the dark radiation component that could come from a future precise measurement of the lensing-integrated-Sachs-Wolfe bispectrum.

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I. INTRODUCTION

A precise measurement of the bispectrum of the cosmic microwave background (CMB) anisotropies is expected to be provided in the very near future from satellite experiments as Planck (see e.g., [1] and references therein).

While a measurement of a nonzero value of the CMB bispectrum could indicate the presence of small non-Gaussianities produced in the very early universe and could give valuable information on the inflationary process ([2]), the expected amplitude of this signal is strongly model dependent. There is indeed no guarantee that a significant and measurable primordial non-Gaussian signal was produced during inflation.

On the other hand the cosmological model of structure formation predicts that a certain degree of non-Gaussianity *must* be produced at much later epochs from interactions of the CMB photons with the local universe. In particular, lensing from the dark matter fluctuations and the time variation of the gravitational potential in a low matter density universe [the so-called "late" integrated Sachs-Wolfe (ISW) effect], will both produce new and correlated anisotropies on the CMB radiation. This correlation appears as a clear signal in the CMB bispectrum and has been already discussed by several authors (see e.g., [3–13]). Contrary to the primordial inflationary signal the lensing-ISW (LISW hereafter) bispectrum is a standard expectation of the standard model and is independent from the inflationary modelling.

In the past years, several studies have forecasted the ability of future experiments to detect the LISW bispectrum. The Planck satellite experiment is expected to detect it at the level of 4–5 standard deviations ([5,7,9]) opening the possibility not only to further constrain cosmological parameters but also to test modified gravity scenarios (see [14,15]).

All current cosmological data are in very good agreement with the expectations of the standard Lambda cold dark matter model. However, very recently, a tension seems to emerge with the number of relativistic degrees of freedom at recombination, $N_{\rm eff}$ (see e.g., [16] and references therein). In the standard scenario, assuming three relativistic neutrinos, this parameter is expected to be consistent with the value $N_{\rm eff} = 3.04$. Recent analysis of the damping tail of the CMB anisotropies from the South Pole Telescope, are however in better agreement, especially when combined with measurements of the Hubble constant from the Hubble space telescope satellite and results from galaxy surveys, with a larger value of $N_{\rm eff}$ at more than 2 standard deviations (see [17]). The situation is rather puzzling since another recent CMB experiment, the Atacama Cosmology Telescope, reported a value of $N_{\rm eff}$ consistent with the standard value. However this experiment also reported a value for the lensing signal larger than the expected value at about 2 standard deviations [18]. Parametrizing the lensing amplitude with the lensing parameter A_L as introduced in [19], the ACT Collaboration found $A_L = 1.70 \pm 0.38$ at 68% C.L.

In any case, it is clearly timely to assess the implications of a different value for $N_{\rm eff}$ on the LISW bispectrum. There are two reasons to do this: first, it is important to discuss what kind of information a measurement of LISW bispectrum could bring in the determination of the value of $N_{\rm eff}$. Secondly, since the LISW signal is an important contaminant in the determination of the primordial inflationary bispectrum, it is useful to understand the possible bias that a different value for $N_{\rm eff}$ could introduce.

The paper is organized as follows. In Sec. II we review the LISW bispectrum. In Sec. III we describe the analysis method and in Secs. IV and V we present our results. We conclude in Sec. VI.

II. THE LISW BISPECTRUM

CMB lensing is correlated with the CMB anisotropies that originate from the ISW effect. The anisotropy induced by the ISW effect is indeed given by

$$\frac{\delta T}{T}(\hat{\mathbf{n}})|_{\mathrm{ISW}} = \int d\chi(\Phi - \Psi)_{,\tau}(\hat{\mathbf{n}}, \chi), \tag{1}$$

where $\hat{\mathbf{n}}$ is the direction of the line of sight, ψ is the Newtonian potential, ϕ is the perturbation induced in spatial curvature, τ is the conformal time and χ is the comoving distance (see e.g., [15]). Temperature fluctuations of the CMB due to the ISW effect can be expanded in spherical harmonics

$$\frac{\delta T}{T}(\hat{\mathbf{n}})|_{\mathrm{ISW}} = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m}^{\mathrm{ISW}} Y_{\ell m}(\hat{\mathbf{n}}). \tag{2}$$

On the other hand, the paths of CMB photons are deflected by the gravitational lensing induced by the fluctuations of matter density while traveling from the recombination to the observer

$$\delta\tilde{T}(\hat{\mathbf{n}}) = \delta T(\hat{\mathbf{n}} + \partial\phi) \simeq \delta T(\hat{\mathbf{n}}) + [(\partial\phi)\cdot(\partial\delta T)](\hat{\mathbf{n}}), \tag{3}$$

where the lensing potential is defined as

$$\phi(\hat{\mathbf{n}}) = -\int_0^{\chi_*} d\chi \frac{\chi_* - \chi}{\chi_*\chi}(\Phi - \Psi)(\hat{\mathbf{n}}, \chi). \tag{4}$$

From Eqs. (1) and (3), we can see that Weyl potential $(\Phi - \Psi)$ sources both ISW and weak lensing, so the long-wavelength mode from ISW couples with the short-wavelength mode from weak lensing.

Going to harmonic space, the theoretical angular averaged CMB bispectrum is

$$\begin{aligned} B_{\ell_1\ell_2\ell_3} &= \sum_{m_1m_2m_3}\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}\langle \tilde{a}_{\ell_1m_1}\tilde{a}_{\ell_2m_2}\tilde{a}_{\ell_3m_3}\rangle \\ &= f_{\ell_1\ell_2\ell_3} C_{\ell_2}^{T\phi} C_{\ell_3}^{TT} + 5\,\mathrm{perm.}, \end{aligned} \tag{5}$$

where $\langle\cdots\rangle$ is the ensemble average, C_ℓ^{TT} is the usual temperature power spectrum and $C_\ell^{T\phi} = \langle \phi_{lm}^* a_{lm}^{\mathrm{ISW}}\rangle$ is the cross temperature-lensing angular power spectrum (see e.g., [5–7,20]) and where the coefficient $f_{\ell_1\ell_2\ell_3}$ is given by

$$f_{\ell_1\ell_2\ell_3} = \left(\frac{-\ell_1(\ell_1+1) + \ell_2(\ell_2+1) + \ell_3(\ell_3+1)}{2}\right)\Upsilon_{\ell_1\ell_2\ell_3}, \tag{6}$$

with

$$\Upsilon_{\ell_1\ell_2\ell_3} \equiv \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}}\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

As we can see clearly, if $C_\ell^{T\phi} = 0$ the bispectrum signal is zero, i.e., thanks to the correlations between lensing and ISW the bispectrum that give $C_\ell^{T\phi} \neq 0$ of the CMB anisotropies is different from zero even if the original anisotropies are expected to perfectly gaussian.

A reduced bispectrum can be derived from

$$b_{\ell_1\ell_2\ell_3} = \frac{\Upsilon_{l_1l_2l_3}}{B_{\ell_1\ell_2\ell_3}}. \tag{8}$$

In what follows we are interested in checking the sensitivity of the CMB bispectrum from neutrino physics. We therefore consider three neutrino parameters: the number of relativistic degrees of freedom N_{eff}, the total neutrino mass Σm_ν, and the neutrino perturbation viscosity c_{vis}. In the case of three, massless, neutrinos these parameters are $N_{\mathrm{eff}} = 3.046$, $\Sigma m_\nu = 0$, $c_{\mathrm{vis}}^2 = 1/3$.

In Figs. 1 and 2 we plot different theoretical predictions for $C_\ell^{T\phi}$ and the reduced bispectrum $b_{\ell_1\ell_2\ell_3}$ in function of

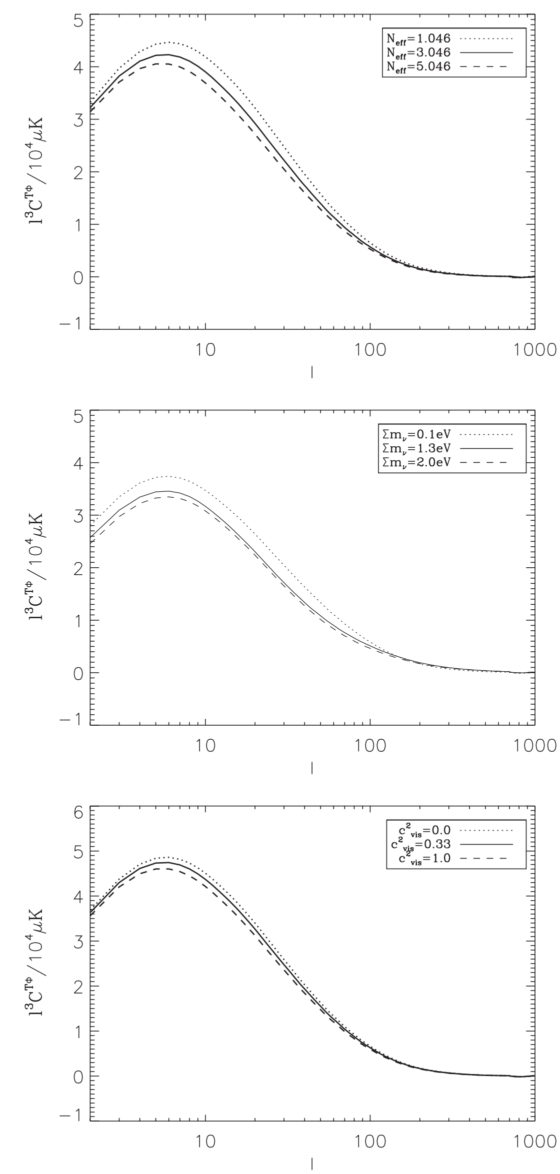

FIG. 1. Theoretical predictions for $C_\ell^{T\phi}$ for different neutrino parameters: the relativistic degrees of freedom N_{eff} (top panel), the total neutrino mass Σm_ν (center panel), and the viscosity sound speed c_{vis}^2 (bottom panel).

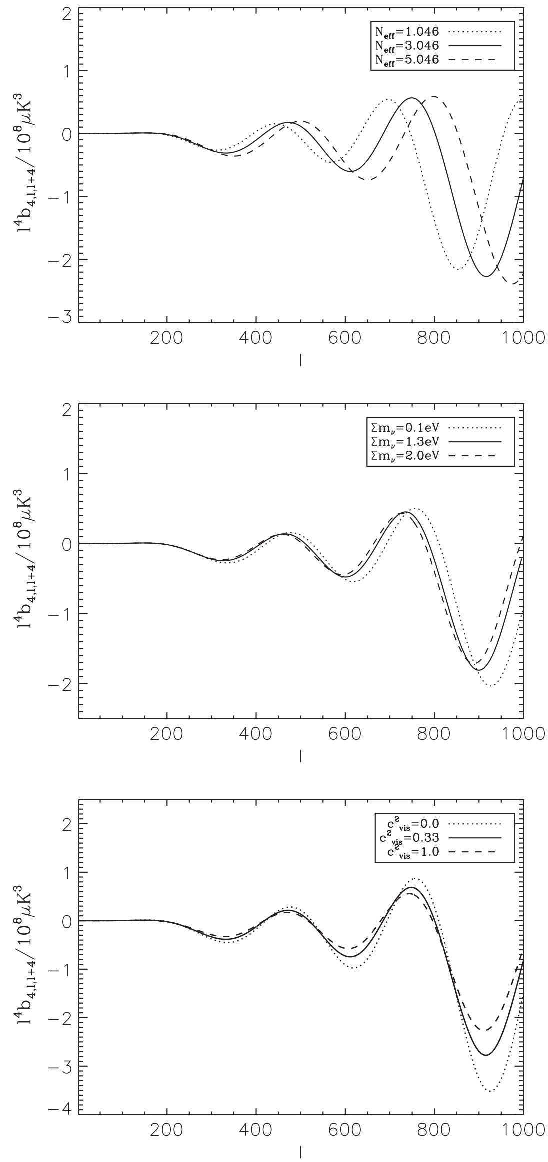

FIG. 2. Theoretical predictions for the reduced bispectrum for different neutrino parameters: the relativistic degrees of freedom N_{eff} (top panel), the total neutrino mass Σm_ν (center panel), and the viscosity sound speed c^2_{vis} (bottom panel).

these parameters. As it can be noticed, the LISW reduced bispectrum is mildly sensitive to changes in Σm_ν and c_{vis} while there are larger differences in the variation of N_{eff}.

This different dependence can be explained as follows. As we can see from Eq. (1) the bispectrum signal is given by the contribution of three terms: a geometrical factor arising from the Wigner 3J symbol selection rules, the power spectrum C_l^{TT} evaluated at the last scattering surface and the cross-correlation spectrum $C_l^{T\phi}$. The first term is responsible for the high frequency oscillations in the total (not reduced) bispectrum slice shape. The third term produces a smooth reprojection of the acoustic peaks over the angular scales, determining a low frequency modulation of the bispectrum signal. As a result, modifications affecting

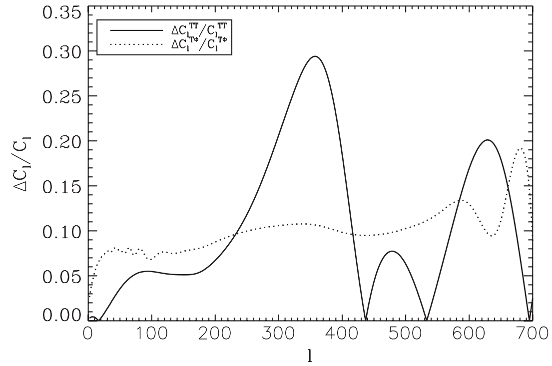

FIG. 3. Percentage difference between a model with $N_{\mathrm{eff}} = 3.046$ and a model with $N_{\mathrm{eff}} = 5.046$ for the C_l^{TT} and the $C_l^{T\phi}$. As we can see the percentage variation in the spectra is of the same order, around $\sim 10\%$.

the primordial power spectrum via different choices of fiducial models must also affect the bispectrum shape. In particular, an increase in radiation density (i.e., an increase in N_{eff}) changes the Hubble rate at decoupling, it reduces the size of the acoustic horizon and shifts the acoustic peaks towards small angular scales. This effect is clearly seen also in the bispectrum. In Fig. 3 we plot the percentage difference between a model with $N_{\mathrm{eff}} = 3.046$ and a model with $N_{\mathrm{eff}} = 5.046$ for the C_l^{TT} and the $C_l^{T\phi}$. As we can see the percentage variation in the spectra is of the same order, around $\sim 10\%$. This indicates that the bispectrum signal is affected by a change in N_{eff} not only from of a change in the temperature spectrum but also from a change in the $C_l^{T\phi}$ term, due mainly to a variation in the matter clustering that affects CMB lensing.

The neutrino mass, on the contrary, mainly affects the lensing spectrum while leaves the primary anisotropy spectrum (for $\Sigma m_\nu < 2$ eV) as practically unaffected. The variation in the bispectrum are therefore less pronounced. The viscosity parameter c_{vis} produces just mild variations both in the primary anisotropy spectrum and in the lensing spectrum. We can therefore expect that the LISW bispectrum will be more powerful in constraining N_{eff} than the absolute neutrino mass scale or the viscosity parameter c_{vis}.

III. FUTURE CONSTRAINTS FROM CMB: METHOD

We shall now estimate the potential of upcoming LISW bispectrum measurements from CMB Planck-like experiments to constrain the neutrino parameters. We perform an analysis following the same method presented in [14] and also adopted in [15]. Namely, we assume a fiducial model with parameters given by the WMAP 7-year data best fit [21] in the case of three, active, neutrinos and quantify how well the future LISW data could discriminate any deviation in the neutrino parameters.

Assuming that the bispectrum is well approximated by Gaussian variables, we can forecast the constraints on cosmological parameters building a simple χ^2 function (see, for example, the same procedure adopted in [5,7–9,14,15,22,23]):

$$\chi_b^2 = \sum_{\ell_1,\ell_2,\ell_3=2}^{l_{\max}} f_{\rm sky}\left[\frac{B^{\rm th}_{\ell_1\ell_2\ell_3} - B^{\rm fid}_{\ell_1\ell_2\ell_3}}{\sigma_{\ell_1\ell_2\ell_3}}\right]^2, \tag{9}$$

where $B^{\rm fid}_{\ell_1\ell_2\ell_3}$ is the fiducial temperature bispectrum; $B^{\rm th}_{\ell_1\ell_2\ell_3}$ is the bispectrum with nonstandard neutrino parameters. The sum is over all possible combinations of ℓ_1, ℓ_2, ℓ_3 with ($\ell_1 < \ell_2 < \ell_3$), $\ell_1 + \ell_2 + \ell_3$ even and we set $\ell_{\max} = 1000$, which roughly corresponds to the maximum multipole sensibility for Planck-like experiments, (since at higher multipoles the contamination from foreground point sources starts to be dominant).

The uncertainty $\sigma_{\ell_1\ell_2\ell_3}$ is given by [24]

$$(\sigma_{\ell_1\ell_2\ell_3})^2 = \bar{C}^{TT}_{\ell_1}\bar{C}^{TT}_{\ell_2}\bar{C}^{TT}_{\ell_3}, \tag{10}$$

where the $\bar{C}^{TT}_{\ell}$ are defined by

$$\bar{C}^{TT}_{\ell} = C^{TT}_{\ell} + \mathcal{N}_{\ell}, \tag{11}$$

and $\mathcal{N}_{\ell}$ is the experimental noise given by

$$\mathcal{N}_{\ell} = w^{-1}\mathcal{B}_{\ell}^{-2}, \tag{12}$$

with

$$w \equiv (\sigma_{\rm pix}\theta_{\rm pix})^{-2}, \qquad \mathcal{B}^2_{\ell} \approx e^{-\ell(\ell+1)/\ell_s^2}, \tag{13}$$

where we assume that the experimental beam profile $\mathcal{B}$ is Gaussian with width $\ell_s \equiv \sqrt{8\ln 2}\,\theta^{-1}_{f\rm whm}$. We have adopted $f_{\rm sky} = 0.65$, a resolution $\theta_{f\rm whm} = 8'$, a sensitivity $\sigma_{\rm pix} = 2.0 \times 10^{-6}\ \mu$K and a noise power parameter $w^{-1} = 0.022 \times 10^{-15}\ \mu\mathrm{K}^2$ as roughly expected for the 150 GHz frequency channel of the Planck experiment (see [25]).

Once the χ^2 function is computed, we can build a likelihood from the bispectrum data given by:

$$\mathcal{L}_b = \exp\left(-\frac{\chi_b^2}{2}\right). \tag{14}$$

Since we are modeling the (primordial plus ISW) spectrum as a Gaussian variable, we are effectively neglecting any inflationary non-Gaussian signal; furthermore, we ignore contributions to the bispectrum from the lensing-Sunyaev-Zel'dovich correlation. Both signals could anyway be removed exploiting their different angular dependence (see e.g., [26]).

IV. RESULTS AND CONSTRAINTS

In Fig. 4 we present the likelihood distribution functions when a single neutrino parameter is let to vary. In case of massless neutrinos, the fiducial model is taken as the WMAP7 best fit model with baryon density $\omega_b = 0.02258$, cold dark matter density $\omega_{\rm cdm} = 0.1109$, Hubble parameter $h = 0.71$, and the standard neutrino parameters ($N_{\rm eff} = 3.046$, $c^2_{\rm vis} = 1/3$). In case of massive neutrinos the fiducial model is taken with baryon density $\omega_b = 0.02219$, cold dark matter density $\omega_{\rm cdm} = 0.1122$, Hubble parameter $h = 0.65$, a neutrino density of $\omega_\nu = 0.014$ (corresponding to a neutrino mass of $m_\nu = 1.3$ eV), and standard neutrino parameters ($N_{\rm eff} = 3.046$, $c^2_{\rm vis} = 1/3$).

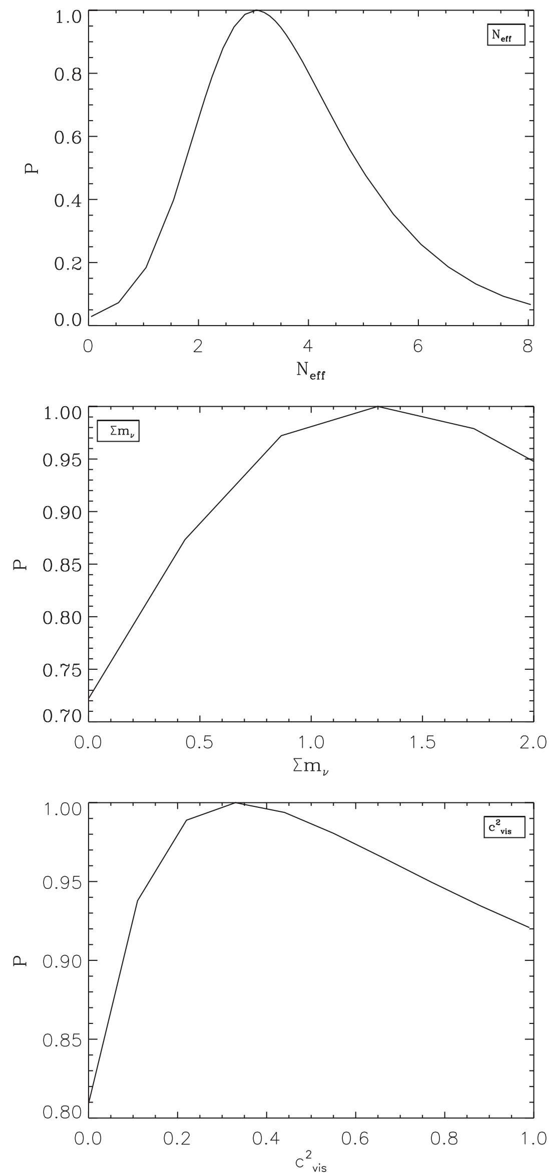

FIG. 4. Likelihood functions for different neutrino parameters from the bispectrum data.

As we can see from the likelihoods functions, even if the very optimistic case of complete knowledge of all cosmological parameters, the bispectrum can provide an interesting constraint only on the neutrino effective number $N_{\rm eff}$, with $2.0 < N_{\rm eff} < 4.6$ at 68% C.L. The viscosity sound speed is practically left as unconstrained. The neutrino

TABLE I. Fisher errors σ_{fnl} and $\sigma_{\rm lens}$ on the amplitudes of the corresponding bispectrum templates, the correlation between the two bispectrum shapes and the systematic error, i.e., the bias, on $f_{\rm NL}$ if the CMB lensing contribution is neglected. $\sigma_{fnl}^{\rm marge}$ is the Fisher error on $f_{\rm NL}$ if the amplitude of the lensing contribution is marginalized over.

Model	σ_{fnl}	$\sigma_{\rm lens}$	Correlation	Bias on $f_{\rm NL}$	$\sigma_{fnl}^{\rm marge}$
$N_{\rm eff}^{\rm rel} = 3.046 \sum m_\nu = 0$	4.33	0.18	0.24	9.7	4.47
$N_{\rm eff}^{\rm rel} = 0.046 \sum m_\nu = 0$	4.40	0.16	0.24	12.5	4.54
$N_{\rm eff}^{\rm rel} = 5.046 \sum m_\nu = 0$	4.30	0.19	0.25	9.3	4.44
$N_{\rm eff}^{\rm rel} = 0.046$ $N_{\rm eff}^{\rm mass} = 3 \sum m_\nu = 1$ eV	4.17	0.22	0.23	7.5	4.29
$N_{\rm eff}^{\rm rel} = 0.046$ $N_{\rm eff}^{\rm mass} = 4 \sum m_\nu = 2$ eV	4.13	0.24	0.24	7.1	4.26
$N_{\rm eff}^{\rm rel} = 3.046 \sum m_\nu = 0$ $A_L = 1.7$	4.35	0.19	0.26	9.63	4.51

mass is also very weakly constrained as well. These constraints have been obtained under the optimistic assumption of neglecting correlations with other parameters. However we expect strong correlations, for example, between $N_{\rm eff}$ and the Hubble constant H_0 since they both change the expansion rate at decoupling and affect in a similar way the size of the acoustic horizon and the angular displacement of the acoustic peaks.

We have therefore performed an analysis letting the Hubble constant to vary with a Gaussian prior of $H_0 = 71 \pm 5$ that is conservative considering the current bounds on this parameter. We have found that also in this case the bispectrum can provide useful constraints with $1.8 < N_{\rm eff} < 4.7$ at 68% C.L.

V. BIAS ON $f_{\rm NL}$

An important aspect is to evaluate the bias produced by a wrong assumption in the neutrino parameters in constraining the primordial $f_{\rm NL}$ arising during inflation.

We remind that the optimal estimator for $f_{\rm NL}$ in case of small levels of non-Gaussianity is given by [6]

$$\langle \hat{f}_{\rm NL} \rangle_{\rm lens} = \frac{F_0(B^{\rm lens}, B^{\rm prim})}{F_0(B^{\rm prim}, B^{\rm prim})}, \tag{15}$$

where F_0 is the Fisher Matrix for bispectra with expected null $f_{\rm NL}$ signal, $B^{\rm lens}$ is the lensing bispectrum and $B^{\rm prim}$ is the primordial bispectrum. $F_0 = F_0(B^a, B^b)$ (where a and b refer to *lens* and *prim*) is given by

$$F_0(B^a, B^b) = \frac{1}{6} \sum_{l_1 l_2 l_3} (B^a_{l_1 l_2 l_3})^* (\tilde{\bar{C}}^{TT}_{l_1} \tilde{\bar{C}}^{TT}_{l_2} \tilde{\bar{C}}^{TT}_{l_3})^{-1} B^b_{l_1 l_2 l_3}, \tag{16}$$

where $\tilde{\bar{C}}^{TT}_\ell$ is the lensed power spectrum $\bar{C}^{TT}_\ell = C^{TT}_\ell + \mathcal{N}_\ell$ that includes noise.

In Table I we report, for several choices of neutrino parameters, the Fisher errors σ_{fnl} and $\sigma_{\rm lens}$ on the amplitudes of the corresponding bispectrum templates, the correlation between the two bispectrum shapes and the systematic error, i.e., the bias, on $f_{\rm NL}$ if the CMB lensing contribution is neglected. $\sigma_{fnl}^{\rm marge}$ is the Fisher error on $f_{\rm NL}$ if the amplitude of the lensing contribution is marginalized over. We assume that the signal is cosmic variance limited up to $\ell \sim 2000$.

As we can see, there is a non-negligible variation in the reported values. It is therefore important to consider the possibility of nonstandard neutrino background when removing the ISW-lensing contribution. Otherwise the determination of the primordial $f_{\rm NL}$ value could be substantially biased.

Finally, we have also considered variation in the lensing amplitude parameter A_L since the ACT experiment recently reported deviations in this parameter respect to the standard value at about ~68% C.L. with $A_L = 1.7 \pm 0.38$. We see that assuming the best fit value obtained by ACT of $A_L = 1.7$ does not change the bias in a significant way.

VI. CONCLUSIONS

In this brief paper we have investigated the cosmological utility of the CMB bispectrum in determining some neutrino properties. We have found that while a measurement of the CMB bispectrum can provide only very weak constraints on the neutrino mass and on the viscosity speed, it can achieve interesting constraints on the effective number of relativistic degrees of freedom $N_{\rm eff}$. These constraints are significant weaker with respect to those that could be achieved from the temperature spectrum (around $\Delta N_{\rm eff} \sim 0.4$) but clearly provide an useful cross-check of the theory.

We have also investigated the bias introduced by current uncertainties in neutrino parameters in the determination of the primordial $f_{\rm NL}$ parameter that could arise in some inflationary model. We have found that the bias varies in a significant way between model with different neutrino parameters. It will be therefore important to take this aspect in to account in future non-Gaussianity searches based on CMB high resolution maps as those expected from the Planck satellite experiment.

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