Equivalent neutrinos, light WIMPs, and the chimera of dark radiation

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According to conventional wisdom, in the standard model (SM) of particle physics and cosmology the "effective number of neutrinos" measured in the late Universe is $N_{\rm eff} = 3$ (more precisely, 3.046). For extensions of the standard model allowing for the presence of ΔN_ν "equivalent neutrinos" (or "dark radiation"), it is generally the case that $N_{\rm eff} > 3$. These canonical results are reconsidered, demonstrating that a measurement of $N_{\rm eff} > 3$ can be consistent with $\Delta N_\nu = 0$ ("dark radiation without dark radiation"). Conversely, a measurement consistent with $N_{\rm eff} = 3$ is not inconsistent with the presence of dark radiation ($\Delta N_\nu > 0$). In particular, if there is a light weakly interacting massive particle (WIMP) that annihilates to photons after the SM neutrinos have decoupled, the photons are heated beyond their usual heating from $e^\pm$ annihilation, reducing the late time ratio of neutrino and photon temperatures (and number densities), leading to $N_{\rm eff} < 3$. This opens the window for one or more equivalent neutrinos, including "sterile neutrinos," to be consistent with $N_{\rm eff} = 3$. By reducing the neutrino number density in the present Universe, this also allows for more massive neutrinos, relaxing the current constraints on the sum of the neutrino masses. In contrast, if the light WIMP couples only to the SM neutrinos and not to the photons and $e^\pm$ pairs, its late time annihilation heats the neutrinos but not the photons, resulting in $N_{\rm eff} > 3$ even in the absence of equivalent neutrinos or dark radiation. A measurement of $N_{\rm eff} > 3$ is no guarantee of the presence of equivalent neutrinos or dark radiation. In the presence of a light WIMP and/or equivalent neutrinos, there are degeneracies among the light WIMP mass and its nature (fermion or boson, as well as its couplings to neutrinos and photons), the number and nature (fermion or boson) of the equivalent neutrinos, and their decoupling temperature (the strength of their interactions with the SM particles). As the analysis here reveals, there's more to a measurement of $N_{\rm eff}$ than meets the eye.

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I. INTRODUCTION

For the standard model (SM) of particle physics and cosmology at late times in the early Universe, after the $e^\pm$ pairs have annihilated, the only massless or extremely relativistic particles remaining are the photons and the three SM neutrinos. In the SM the neutrinos decouple prior to $e^\pm$ annihilation so that only the photons are heated when the pairs annihilate. The strength of the SM weak interactions determines the neutrino decoupling temperature which, in turn, fixes the relative contributions of the photons and neutrinos to the late time, early Universe (radiation dominated) energy density. This relative contribution of neutrinos, measured by the "effective number of neutrinos" is $N_{\rm eff} = 3$ under the assumptions of the standard models of particle physics and cosmology. Many years ago, stimulated by the desire to test the prediction of asymptotic freedom limiting the number of particle physics families [1,2] and by the discovery of the third family of leptons, along with its neutrino [3], which led to $N_{\rm eff}$ increasing from 2 to 3, Steigman *et al.* [4] explored the consequences for big bang nucleosynthesis (BBN) of additional, "equivalent neutrinos" (see, also, the earlier related work of Hoyle and Tayler [5], Peebles [6], and Shvartsman [7]). Ever since, it has been a goal of a broad array of cosmological observations, from those of the abundances of the light elements produced during BBN to studies of the cosmic microwave background (CMB) radiation and of large scale structure (LSS), to measure $N_{\rm eff}$. In recent years both BBN and the CMB/LSS have favored values of $N_{\rm eff} > 3$ [8–12], hinting at the presence of equivalent neutrinos or "dark radiation." In anticipation that the results from the Planck experiment [13] will provide the most accurate determination of $N_{\rm eff}$ to date, it is timely to revisit the theoretical predictions for models beyond the SM containing equivalent neutrinos and WIMPs, weakly interacting massive particles that are candidates for the dark matter in the Universe. In the course of the analysis presented here, several degeneracies are noted in the determination of $N_{\rm eff}$ that will render the interpretation of any precision measurement of $N_{\rm eff}$ more problematic, and more interesting.

In Sec. II the standard model analysis is reviewed, allowing the neutrino decoupling temperature ($T_{\nu d}$) to be a free parameter, revealing how $N_{\rm eff}$ depends on its value. In the process, very small differences with the canonical, textbook results are revealed. With this as background, in Sec. III the analysis is extended to allow for equivalent neutrinos (ξ). It is noted here that $N_{\rm eff}$ now depends on the number (ΔN_ν) and nature (fermion or boson) of the

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equivalent neutrinos as well as on their decoupling temperature (how strongly they couple to the SM particles). Sterile neutrinos are equivalent neutrinos (Majorana fermions) that decouple along with the SM neutrinos ($T_{\xi d} = T_{\nu d}$), but more general equivalent neutrinos may decouple before or after the SM neutrinos ($T_{\xi d} \neq T_{\nu d}$), affecting both $N_{\rm eff}$ and the connection between the sum of the neutrino masses and their contribution to the present Universe mass density (for neutrinos with nonzero mass). This analysis is further extended in Sec. IV to allow for the presence of light WIMPs (χ) whose annihilation occurs around or after the time when the SM neutrinos decouple. The light WIMP annihilation can heat the photons beyond the usual heating from $e^{\pm}$ annihilation, reducing the relative contribution of the neutrinos to the energy density, leading to $N_{\rm eff} < 3$. In this case the degeneracies discussed above are expanded to include the nature (fermion or boson) of the WIMP and its mass (m_χ). Many more possibilities now emerge allowing, for example, for one or even two sterile neutrinos ($\Delta N_\nu = 1, 2$) even if observations should find $N_{\rm eff} \lesssim 3.5$. It is shown that an observation of $N_{\rm eff} = 3$ would not exclude the presence of equivalent neutrinos or dark radiation. The tables are turned in Sec. V, where it is assumed that the light WIMP couples only to the SM neutrinos and not to the photons or $e^{\pm}$ pairs. In this case the SM neutrinos are heated by WIMP annihilation, increasing their relative contribution to the early Universe energy density, resulting in $N_{\rm eff} > 3$ even in the absence of equivalent neutrinos or dark radiation. The results are reviewed and summarized in Sec. VI.

II. STANDARD MODEL NEUTRINOS

To set the stage for the subsequent discussion, in this section it is assumed that there are no light WIMPs (e.g., with $m_\chi \lesssim 20$ MeV) or "extra" neutrinos (e.g., sterile neutrinos) or other relativistic particles (equivalent neutrinos), only the standard model particles including the three SM neutrinos. However, the neutrino decoupling temperature, $T_{\nu d}$, assumed to be the same for all three flavors, is allowed to be a free parameter. Allowing $T_{\nu d}$ to vary is equivalent to imagining that the weak interactions are weaker, or stronger, than the SM weak interactions. Of course, the strength of the weak interactions and $T_{\nu d}$ are determined by laboratory and accelerator experiments ($T_{\nu d} \approx 2$–3 MeV [14–16]) and $T_{\nu d}$ is not really a free parameter. However, it is interesting and informative to ask, "How does allowing the neutrino decoupling temperature to vary change the well known, canonical SM neutrino results?". To facilitate comparison with the usual SM results, the neutrinos are assumed to decouple instantaneously, when $T_\gamma = T_{\nu d}$. For the analysis here, the instantaneous decoupling approximation, typically accurate to ~2% or better, replaces a coupled set of integro-differential equations which need to be solved numerically (see, e.g., [15,17]), with algebraic equations that follow from entropy conservation.

Prior to neutrino decoupling, for $T_\gamma \geq T_{\nu d}$, $T_\nu = T_\gamma$. After neutrino decoupling, for $T_\gamma < T_{\nu d}$, $T_\nu \leq T_\gamma$ as a consequence of the heating of the photons relative to the decoupled neutrinos. After $e^{\pm}$ annihilation, when $T_\gamma = T_{\gamma 0}$, where $T_{\gamma 0} \ll \min\{m_e, T_{\nu d}\}$, entropy conservation permits the calculation of the "frozen out" ratio of neutrino and photon temperatures (and number densities). The result is

$$\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \frac{g_\gamma}{g_s(T_{\nu d}) - 3g_\nu} = \frac{2}{g_s(T_{\nu d}) - 21/4}, \tag{1}$$

where $g_s = g_s(T)$ is defined by the ratio of the total entropy density to the entropy density contributed by photons alone,

$$s_{\rm tot}/s_\gamma \equiv g_s/g_\gamma = g_s/2, \tag{2}$$

and the entropy density at temperature T is defined by

$$s \equiv \frac{\rho + p}{T}, \tag{3}$$

where ρ is the energy density and p is the pressure. As a result,

$$\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \frac{11}{2g_s(T_{\nu d}) - 10.5}. \tag{4}$$

In the canonical, textbook analysis it is assumed that the neutrinos decouple instantaneously, and that at neutrino decoupling only the photons, the $e^{\pm}$ pairs, and the three SM neutrinos contribute to g_s. It is a further, unstated assumption that at neutrino decoupling the $e^{\pm}$ pairs are essentially massless so that $(s_e/s_\gamma)_{T_{\nu d}} = 7/4$. That is, it is assumed that $g_s(T_{\nu d}) = 43/4 = 10.75$, resulting in $(T_\nu/T_\gamma)_0^3 = 4/11$.

A. The effective number of neutrinos: $N_{\rm eff}$

At late times in the early Universe (e.g., after neutrino decoupling but prior to the epoch of matter—radiation equality and prior to any of the SM neutrinos becoming nonrelativistic), the only relativistic SM particles present are the photons and the three SM neutrinos. As a result, the total energy density (or, the "radiation" (R) energy density) is

$$\rho_{\rm R} = \rho_\gamma + 3\rho_\nu, \tag{5}$$

where ρ_ν is the contribution from one SM neutrino. After neutrino decoupling and $e^{\pm}$ annihilation,

$$\frac{\rho_\nu}{\rho_\gamma} = \left(\frac{\rho_\nu}{\rho_\gamma}\right)_0 = \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)_0^4. \tag{6}$$

If $\rho_{\nu 0}^0$ is defined to be the value of $\rho_{\nu 0}$ assuming that $(T_\nu/T_\gamma)_0^3 = 4/11$, then

$$\left(\frac{\rho_\nu}{\rho_\nu^0}\right)_0 = \left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3}, \tag{7}$$

and

$$\left(\frac{\rho_R}{\rho_\gamma}\right)_0 = 1 + 3\left(\frac{7}{8}\right)\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3}, \tag{8}$$

or, normalizing the difference between ρ_R and ρ_γ to ρ_ν^0,

$$\left(\frac{\rho_R - \rho_\gamma}{\rho_\nu^0}\right)_0 = 3\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3}. \tag{9}$$

This can be generalized from the three SM neutrinos to allow for $N_{\rm eff}$ "effective neutrinos." The effective number of neutrinos, $N_{\rm eff}$, here a function of the neutrino decoupling temperature, is defined by

$$N_{\rm eff}(T_{\nu d}) \equiv \left(\frac{\rho_R - \rho_\gamma}{\rho_\nu^0}\right)_0 = 3\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3}. \tag{10}$$

For the SM, assuming instantaneous neutrino decoupling and $g_e(T_{\nu d}) = 7/4$ (e.g., massless electrons), $N_{\rm eff} = 3$, corresponding to the three SM neutrinos. Allowing for the fact that the SM neutrinos don't decouple instantaneously, which enables them to share some of the energy released by $e^\pm$ annihilation, results in a small ($\sim$1.5%) increase, $N_{\rm eff} = 3 \rightarrow 3.046$ [17].

Since the late time, frozen-out ratio of neutrino and photon temperatures depends on the neutrino decoupling temperature, it is informative to allow $T_{\nu d}$ to vary and to explore the dependence of $N_{\rm eff}$ on $T_{\nu d}$. The relation between $N_{\rm eff}$ and the neutrino decoupling temperature is shown in Fig. 1. For very high neutrino decoupling temperatures (very weak, weak interactions) the neutrinos are diluted relative to the photons when the latter are heated relative to the decoupled neutrinos by the annihilations and/or decays of all the SM particles. As a result, for $T_{\nu d} \gg m_t$, $g_s \rightarrow 427/4$ and $N_{\rm eff} \rightarrow 0.06$. In the opposite limit of very strong, weak interactions, if the SM neutrinos were to remain coupled through the epoch of $e^\pm$ annihilation ($T_{\nu d} \ll m_e$), sharing the energy released along with the photons, $T_{\nu 0} \rightarrow T_{\gamma 0}$ and $N_{\rm eff} \rightarrow 3(11/4)^{4/3} = 11.56$.

It should be noted that the assumption that $s_e/s_\gamma = 7/4$ when $T_\gamma = T_{\nu d}$, while quite accurate, is not perfect since for all finite temperatures, $s_e/s_\gamma < 7/4$. Indeed, $s_e/s_\gamma \rightarrow 7/4$ only in the limit $m_e/T_{\nu d} \rightarrow 0$, and while $m_e/T_{\nu d} \approx 0.26$ is small, this ratio is not $\ll 1$. For $T_{\nu d} = 2$ MeV, $s_e/s_\gamma = 6.95/4$ and $g_s(T_{\nu d}) = 42.9/4 = 10.73$. Figure 2 is a zoomed version of Fig. 1, showing that for the neutrino decoupling temperature adopted here, $T_{\nu d} = 2$ MeV [14–16], assuming instantaneous neutrino decoupling, $N_{\rm eff} = 3.018$ (if, instead $T_{\nu d} = 3$ MeV were adopted, $N_{\rm eff} = 3.012$). Indeed, as may be seen from Fig. 2, the canonical, textbook value of $N_{\rm eff} = 3$ is actually only achieved (in the instantaneous decoupling approximation) for $T_{\nu d} \approx 8$ MeV. Although this correction ($g_s(T_{\nu d}) < 10.75$, $N_{\rm eff} = 3.018$) is small, it is comparable to (within $\sim$40% of) the detailed corrections [17] accounting, mainly, for noninstantaneous neutrino decoupling. Indeed,

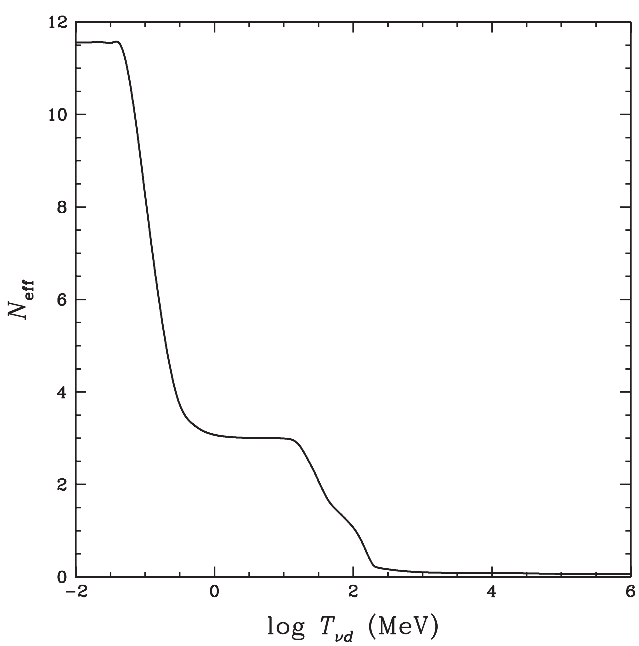

FIG. 1. The effective number of neutrinos, $N_{\rm eff}$, as a function of $T_{\nu d}$.

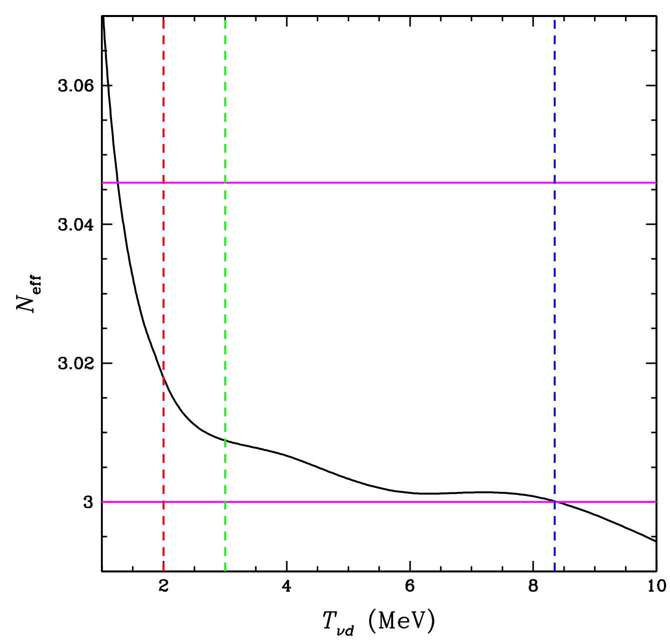

FIG. 2 (color online). A zoomed-in version of Fig. 1 for $1 \leq T_{\nu d} \leq 10$ MeV (for a linear temperature scale). Notice that $N_{\rm eff} = 3$ (lower horizontal, purple line) when $T_{\nu d} \approx 8.3$ MeV (dashed, vertical blue line). For $T_{\nu d} = 2$ MeV, $N_{\rm eff} = 3.018$ (dashed, vertical red line), while for $T_{\nu d} = 3$ MeV, $N_{\rm eff} = 3.012$ (dashed, vertical green line). In the instantaneous decoupling approximation, $N_{\rm eff} = 3.046$ (upper horizontal, purple line) when $T_{\nu d} \approx 1.3$ MeV.

as Fig. 1 shows, the longer the neutrinos remain coupled (the stronger the weak interaction), the more they are heated when the $e^{\pm}$ pairs annihilate, and the larger is the resulting value of $N_{\rm eff}$.

B. Neutrino decoupling and the neutrino mass constraint

Allowing the neutrino decoupling temperature to vary also has consequences for the CBM/LSS constraint on the sum of the neutrino masses. Since at least two of the three SM neutrinos have nonzero masses which are large enough so they are nonrelativistic in the present Universe, the neutrino contribution to the present Universe mass density is $\rho_{\nu 0} = \Sigma m_\nu n_{\nu 0}$, where Σm_ν is the sum of the three SM neutrino masses and $n_{\nu 0}$ is the present number density of one species of the SM neutrinos ($n_{\nu 0} \propto T^3_{\nu 0}$). In the present Universe, the "frozen-out" ratio of the neutrino (each flavor) and photon number densities is

$$\left(\frac{n_\nu}{n_\gamma}\right)_0 = \frac{3}{4}\left(\frac{T_\nu}{T_\gamma}\right)^3_0 = \frac{3}{11}\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)^3_0\right]. \qquad (11)$$

The canonical, instantaneous decoupling result for the SM neutrinos assumes that $(T_\nu/T_\gamma)^3_0 = 4/11$ which, when combined with the number density of CMB photons ($n_{\gamma 0}$) and the critical mass density, leads to a relation between the sum of the SM neutrino masses and $\Omega_\nu h^2$, $\Sigma m_\nu = 94.12\Omega_\nu h^2$ eV. While the detailed calculation

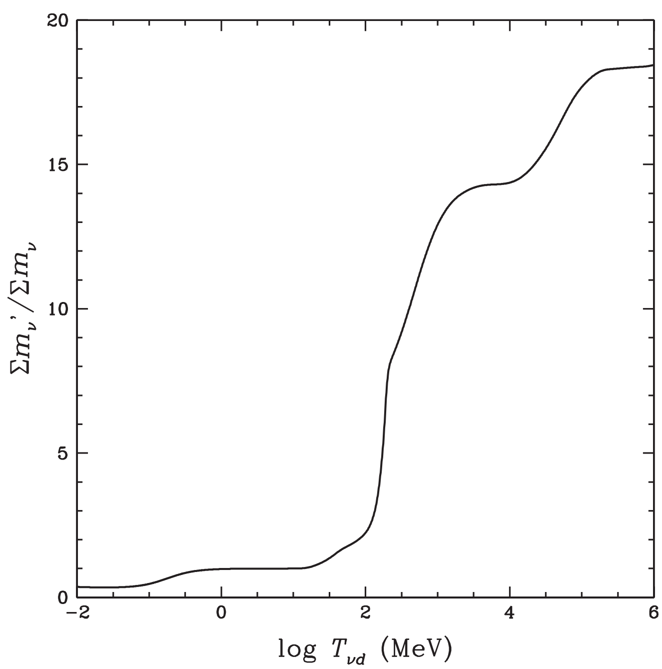

FIG. 3. The ratio of the sum of the neutrino masses to its canonical value (assuming instantaneous neutrino decoupling and $(T_\nu/T_\gamma)^3_0 = 4/11$), $\Sigma m'_\nu/\Sigma m_\nu$, as a function of the neutrino decoupling temperature, $T_{\nu d}$. If the upper bound to Σm_ν were 1 eV, the curve would show the upper bound to the sum of the SM neutrino masses ($\Sigma m'_\nu$), in eV.

of Mangano *et al.* [17], relaxing the instantaneous decoupling approximation, modifies this result to $\Sigma m_\nu = 93.14\Omega_\nu h^2$ eV, for consistency with the instantaneous decoupling analysis here, this small difference will be ignored. Here, as $T_{\nu d}$ varies from $\gg 2$ MeV to $\ll 2$ MeV, $(T_\nu/T_\gamma)^3_0$ increases from $\ll 4/11$ to 1 ($(n_\nu/n_\gamma)_0$ increases from $\ll 3/11$ to 3/4), modifying the constraint on the sum of the neutrino masses. If $\Sigma m'_\nu$ is defined to be the sum of the neutrino masses when $T_{\nu d}$ is allowed to vary and Σm_ν is the SM quantity, assuming $(n_\nu/n_\gamma)_0 = 3/11$, then their ratio is a function of the neutrino decoupling temperature,

$$\frac{\Sigma m'_\nu}{\Sigma m_\nu} = \frac{3}{11}\left(\frac{n_\gamma}{n_\nu}\right)_0 = \frac{4}{11}\left(\frac{T_\gamma}{T_\nu}\right)^3_0 = \frac{2g_s(T_{\nu d}) - 10.5}{11}. \qquad (12)$$

Figure 3 shows $\Sigma m'_\nu/\Sigma m_\nu$ as a function of $T_{\nu d}$. For example, if the current CMB and LSS upper bound to the sum of the neutrino masses were $\Sigma m_\nu \lesssim 1$ eV, then the vertical axis in Fig. 3 would be the upper bound to $\Sigma m'_\nu$ in eV. As $T_{\nu d}$ increases from $\ll m_e$ to $\gg m_t$, $\Sigma m'_\nu/\Sigma m_\nu$ increases by a factor of ~50, from ~0.36 to ~18.5.

III. EQUIVALENT NEUTRINOS

With the discussion in Sec. II as prelude, the analysis in this section allows for the presence of particles in addition to those provided by the content of the SM. Along with the SM particles, consider ΔN_ν additional particles, "equivalent neutrinos" ξ, chosen to be very light ($\Sigma m_\xi \lesssim 10$ eV), or massless, Majorana fermions. The assumption of a Majorana fermion is for simplicity so that aside from the strength of its coupling to the SM particles, ξ is just like a SM neutrino. It is important to note that ΔN_ν, a measure of the number of "extra" neutrinos, is not restricted to be an integer. In general, ΔN_ν has discrete values that depend on the nature of the equivalent neutrino and on how many of them are being considered. For fermionic equivalent neutrinos ΔN_ν is an integer, while for bosons ΔN_ν is an integer multiple of 4/7. For example, $\Delta N_\nu = 2$ for two sterile (Majorana) neutrinos or one Dirac neutrino, while $\Delta N_\nu = 3$ for three right-handed neutrinos, and $\Delta N_\nu = 4/7$ for a scalar. In the context of the discussion here, "sterile neutrino" is the special case of an equivalent neutrino that decouples along with the SM neutrinos ($T_{\xi d} = T_{\nu d}$). The restriction to very light particles is to ensure that the equivalent neutrinos are extremely relativistic when they decouple ($T_{\xi d} \gg m_\xi$).

In contrast to the analysis in Sec. II, here the SM neutrino decoupling temperature is fixed at $T_{\nu d} = 2$ MeV, chosen for consistency with most analyses in the literature [14–16]. This choice can be modified straightforwardly, e.g., $T_{\nu d} = 3$ MeV, or even for a choice of one decoupling temperature for ν_e (e.g., $T_{\nu e d} = 2$ MeV) and a different one for ν_μ and ν_τ (e.g., $T_{\nu\mu d} = T_{\nu\tau d} = 3$ MeV). The quantitative results for all three choices are very nearly the same. In contrast to the analysis in Sec. II where $T_{\nu d}$ was allowed to vary, here the free parameter is the equivalent neutrino decoupling temperature, $T_{\xi d}$, assumed to be the same for all (if there are more than one) equivalent neutrinos. If the equivalent neutrinos are more weakly coupled than are the SM neutrinos, they decouple earlier ($T_{\xi d} > T_{\nu d}$), when $g_s(T_{\xi d}) > g_s(T_{\nu d})$, sharing less of the heating of the SM neutrinos, resulting in $(T_\xi/T_\nu)_0 < 1$. On the other hand, if the equivalent neutrinos are more strongly coupled than the SM neutrinos so that $T_{\xi d} < T_{\nu d}$, they remain in equilibrium with the photons and other SM particles to later times, in particular sharing more of the energy or entropy released by the annihilation of the $e^\pm$ pairs. This leads to $(T_\xi/T_\nu)_0 > 1$, along with an increase in $(T_\nu/T_\gamma)_0$ from its SM value since the photons now have to share the $e^\pm$ annihilation energy with the equivalent neutrinos and are cooler than they would be in the absence of the more strongly coupled equivalent neutrinos. In this case both $(T_\xi/T_\gamma)_0^3$ and $(T_\nu/T_\gamma)_0^3 > 4/11$, so that $N_{\mathrm{eff},\nu} > 3$ and $N_{\mathrm{eff},\xi} > \Delta N_\nu$, resulting in $N_{\mathrm{eff}} > 3 + \Delta N_\nu$.

At late times in the early Universe, after the $e^\pm$ pairs have annihilated, the only particles contributing to the radiation energy density are the photons, the SM neutrinos, and any equivalent neutrinos. At these times, for $T_\gamma \to T_{\gamma 0} \ll m_e$, the radiation energy density, normalized to the energy density in photons alone is

$$\left(\frac{\rho_{\mathrm{R}}}{\rho_\gamma}\right)_0 = 1 + \frac{7}{8}\left[3\left(\frac{T_\nu}{T_\gamma}\right)_0^4 + \Delta N_\nu\left(\frac{T_\xi}{T_\gamma}\right)_0^4\right] = 1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)_0^4\left[3 + \Delta N_\nu\left(\frac{T_\xi}{T_\nu}\right)_0^4\right]. \tag{13}$$

Recall that the canonical, textbook result is that $(T_\nu/T_\gamma)_0^3 = 4/11$, so that the above result may be written as

$$\left(\frac{\rho_{\mathrm{R}}}{\rho_\gamma}\right)_0 \equiv 1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\mathrm{eff}}, \tag{14}$$

where the effective number of neutrinos is now a function of both ΔN_ν and $T_{\xi d}$,

$$N_{\mathrm{eff}} \equiv \left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3}\left[3 + \Delta N_\nu\left(\frac{T_\xi}{T_\nu}\right)_0^4\right] = 3\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3}\left[1 + \frac{\Delta N_\nu}{3}\left(\frac{T_\xi}{T_\nu}\right)_0^4\right]. \tag{15}$$

For the canonical result, if the equivalent neutrinos decouple along with the SM neutrinos (e.g., sterile neutrinos) so that $T_{\xi 0} = T_{\nu 0}$, $N_{\mathrm{eff}} = 3 + \Delta N_\nu$. However, aside from the very small correction $3 \to 3.018$, N_{eff} generally depends on the combination $\Delta N_\nu^* \equiv \Delta N_\nu (T_\xi/T_\nu)_0^4$, which is a function of the equivalent neutrino decoupling temperature $T_{\xi d}$. There are two interesting regimes, depending on whether $T_{\xi d} \ge T_{\nu d}$ or $T_{\xi d} \le T_{\nu d}$.

A. Weaker than weak equivalent neutrinos: $T_{\xi d} \ge T_{\nu d}$

First consider equivalent neutrinos that are more weakly interacting than the SM neutrinos so they decouple before the SM neutrinos, at $T_{\xi d} \ge T_{\nu d}$. In this case, in the early Universe before neutrino decoupling, when $T_\gamma \ge T_{\nu d}$, $T_\nu = T_\gamma$, while $T_\xi \le T_\gamma$. Early decoupling of any extra, equivalent neutrinos dilutes their contribution to the total energy density, possibly allowing them to avoid the cosmological constraints on N_{eff} (see, e.g., [18]). Entropy conservation enables us to find the ratio of the ξ to neutrino (and/or photon) temperatures at neutrino decoupling when $T_\gamma = T_{\nu d}$,

$$\left(\frac{T_\xi}{T_\nu}\right)_{T_{\nu d}}^3 = \left(\frac{T_\xi}{T_\gamma}\right)_{T_{\nu d}}^3 = \frac{g_s(T_{\nu d})}{g_s(T_{\xi d})}. \tag{16}$$

The cube of the equivalent neutrino to SM neutrino temperature ratio at the SM neutrino decoupling decreases from $(T_\xi/T_\nu)_{T_{\nu d}}^3 = 1$ when $T_{\xi d} = T_{\nu d}$ (e.g., for "sterile" neutrinos), down to $(T_\xi/T_\nu)_{T_{\nu d}}^3 = 0.10$ when $T_{\xi d} \gg m_t$, corresponding to $(T_\nu/T_\gamma)_{T_{\nu d}}^4 \approx 0.05$.

As the Universe continues to expand and cool, for $T_\gamma < T_{\nu d}$, the ratio of the equivalent neutrino to SM neutrino temperatures is preserved so that for $T_\gamma \to T_{\gamma 0}$,

$$\left(\frac{T_\xi}{T_\nu}\right)_0^3 = \left(\frac{T_\xi}{T_\nu}\right)_{T_{\nu d}}^3 = \frac{10.73}{g_s(T_{\xi d})} \le 1. \tag{17}$$

As a result of $e^\pm$ annihilation the photons are heated relative to both the decoupled SM neutrinos and the equivalent neutrinos, which decoupled earlier. In this regime ($T_{\xi d} \ge T_{\nu d}$) where both the SM and equivalent neutrinos are decoupled at $e^\pm$ annihilation, the heating is exactly the same as described in Sec. II so that,

$$\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \frac{g_\gamma}{g_s(T_{\nu d}) - 3g_\nu} = \frac{2}{10.73 - 5.25} = 0.365 \approx 1.004\left(\frac{4}{11}\right). \tag{18}$$

As a result,

$$\left(\frac{T_\xi}{T_\gamma}\right)_0^3 = \left(\frac{T_\nu}{T_\gamma}\right)_0^3\left[\frac{g_s(T_{\nu d})}{g_s(T_{\xi d})}\right] = 0.365\left[\frac{g_s(T_{\nu d})}{g_s(T_{\xi d})}\right] \approx \frac{3.92}{g_s(T_{\xi d})}. \tag{19}$$

For the case considered here, the SM neutrinos supplemented by ΔN_ν equivalent neutrinos which are more weakly coupled to the SM particles than the SM neutrinos,

$$N_{\mathrm{eff}} = N_{\mathrm{eff},\nu} + N_{\mathrm{eff},\xi} = 3\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3} + \Delta N_\nu\left[\frac{11}{4}\left(\frac{T_\xi}{T_\gamma}\right)_0^3\right]^{4/3}, \tag{20}$$

or,

$$N_{\rm eff} = \left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3}(3+\Delta N_\nu^*) = 3.018\left(1+\frac{\Delta N_\nu^*}{3}\right), \tag{21}$$

where,

$$\Delta N_\nu^* = \Delta N_\nu\left(\frac{T_\xi}{T_\nu}\right)_0^4 = \Delta N_\nu\left(\frac{g_s(T_{\nu d})}{g_s(T_{\xi d})}\right)^{4/3} = \Delta N_\nu\left(\frac{10.73}{g_s(T_{\xi d})}\right)^{4/3}. \tag{22}$$

For one equivalent neutrino, e.g., a Majorana fermion ($\Delta N_\nu = 1$), as $T_{\xi d}$ decreases from $T_{\xi d} \gtrsim m_t \gg T_{\nu d}$ ($g_s(T_{\xi d}) \to 106.75$) to $T_{\xi d} = T_{\nu d}$ ($g_s(T_{\xi d}) \to g_s(T_{\nu d}) = 10.73$), the effective number of neutrinos increases from $N_{\rm eff} = 3.065$ to $N_{\rm eff} = 4.024$ ($N_{\rm eff} = 3.018(4/3)$). These results can be generalized to other choices for the nature and the number of equivalent neutrinos.

For the special case of sterile neutrinos that decouple along with the SM neutrinos, the results of the Mangano *et al.* analysis [17], relaxing the assumption of instantaneous decoupling, may be appropriate. If so, then for one (two) sterile neutrinos ($\Delta N_\nu = 1(2)$), $N_{\rm eff} \to 3.046(1 + \Delta N_\nu/3) = 4.06(5.08)$. Since equivalent neutrinos that decouple before the SM neutrinos will not benefit from the additional heating resulting from relaxing the instantaneous decoupling assumption, $N_{\rm eff} = 3.046 + \Delta N_\nu$ is perhaps more appropriate for them. However, it is highly unlikely that such small differences will be tested in the foreseeable future.

Since a CMB/LSS constraint on or measurement of $N_{\rm eff}$ results in a constraint on ΔN_ν^*, which is a function of the equivalent neutrino decoupling temperature, for a fixed value of $N_{\rm eff}$ there is a degeneracy between ΔN_ν and $T_{\xi d}$. As an example of this additional freedom, consider the case of three right-handed neutrinos ($\Delta N_\nu = 3$) [18] which decouple at $T_{\xi d} \approx 180$ MeV, when $g_s(T_{\xi d}) \approx 29.6$. This corresponds to $N_{\rm eff} = 3.80$, consistent with the WMAP 9 year plus SPT results supplemented by information from LSS (e.g., BAO) and measurements of H_0 [19–21]. In contrast, if the same three equivalent neutrinos were to decouple much earlier at $T_{\xi d} \approx 1.5$ GeV, when $g_s(T_{\xi d}) \approx 79.3$, this would correspond to $N_{\rm eff} = 3.23$, in excellent agreement with the WMAP 9 year plus ACT results [20,22]. A measurement of $N_{\rm eff} < 4$ is not evidence for the absence of one, or even more, equivalent neutrinos.

B. Stronger than weak equivalent neutrinos: $T_{\xi d} < T_{\nu d}$

While it would seem difficult to have hidden from experimental scrutiny equivalent neutrinos that are more strongly coupled to the SM particles than are the SM neutrinos, for completeness this possibility is explored here. For more strongly coupled equivalent neutrinos, as $T_{\xi d}$ decreases below $T_{\nu d}$, the equivalent neutrino shares along with the photons some of the energy/entropy released by the $e^\pm$ annihilations. However, the decoupled SM neutrinos which have already "frozen out" prior to ξ decoupling are unheated. In this regime, when $T_\gamma = T_{\nu d}$, $T_\gamma = T_\xi = T_\nu$, while for photon temperatures in the range, $T_{\xi d} \le T_\gamma < T_{\nu d}$, $T_\gamma = T_\xi \ge T_\nu$. As the temperature decreases further, from $T_\gamma = T_\xi = T_{\nu d}$ until ξ decoupling when $T_\gamma = T_{\xi d}$, the photons and equivalent neutrinos are heated relative to the decoupled neutrinos. Entropy conservation permits the evaluation of the ratio of neutrino and photon and neutrino and equivalent neutrino temperatures when the equivalent neutrino finally decouples,

$$\left(\frac{T_\nu}{T_\gamma}\right)_{T_{\xi d}}^3 = \left(\frac{T_\nu}{T_\xi}\right)_{T_{\xi d}}^3 = \frac{2g_s(T_{\xi d})-7}{2g_s(T_{\nu d})-7} = \frac{2g_s(T_{\xi d})-7}{14.45}, \tag{23}$$

or, in terms of the normalized entropy density in the $e^\pm$ pairs, $\phi_e(x) \equiv s_e(x)/s_e(0)$, where $x \equiv m_e/T$ and $x_{\xi d} \equiv m_e/T_{\xi d}$,

$$\left(\frac{T_\nu}{T_\gamma}\right)_{T_{\xi d}}^3 = \frac{15+14\phi_e(x_{\xi d})}{15+14\phi_e(x_{\nu d})} = \frac{15+14\phi_e(x_{\xi d})}{28.90}. \tag{24}$$

Notice that as x increases from $x \ll 1$ (extremely relativistic) to $x \gg 1$ (extremely nonrelativistic), ϕ_e decreases from 1 to 0. Since for the considerations here $T_{\xi d}$ decreases from 2 MeV to $\ll m_e$, in this regime ϕ_e decreases from 0.993 to 0 and $(T_\nu/T_\gamma)^3_{T_{\xi d}}$ decreases from 1 to $15/28.90 = 0.519$. In this regime, the photons are less heated than when $T_{\xi d} \ge T_{\nu d}$.

As the Universe continues to expand and cool after the ξ have decoupled ($T_\gamma < T_{\xi d}$), the annihilation of any remaining $e^\pm$ pairs heats the photons relative to the decoupled neutrinos and the now decoupled equivalent neutrinos ($T_\gamma \ge T_\xi \ge T_\nu$) whose temperature ratio remains fixed (i.e., $(T_\xi/T_\nu)_0 = (T_\xi/T_\nu)_{T_{\xi d}}$). Entropy conservation in this regime then predicts the frozen-out ($T_\gamma \to T_{\gamma 0} \ll m_e$) ratio of the equivalent neutrino and photon temperatures,

$$\left(\frac{T_\xi}{T_\gamma}\right)_0^3 = \frac{4}{4+7\phi_e(x_{\xi d})}. \tag{25}$$

For $T_{\xi d} = T_{\nu d}$, $(T_\xi/T_\gamma)_0^3 = 0.365$. As $T_{\xi d}$ decreases below m_e, $\phi_e \to 0$ so that $(T_\xi/T_\gamma)_0^3 \to 1$ (the equivalent neutrino shares along with the photons all the energy or entropy released by $e^\pm$ annihilation). Since the SM neutrinos have already frozen out,

$$\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \left(\frac{T_\nu}{T_\xi}\right)_{T_{\xi d}}^3\left(\frac{T_\xi}{T_\gamma}\right)_0^3 = \left[\frac{15+14\phi_e(x_{\xi d})}{15+14\phi_e(x_{\nu d})}\right]\left[\frac{4}{4+7\phi_e(x_{\xi d})}\right]. \tag{26}$$

As already noted, in this case ($T_{\xi d} < T_{\nu d}$) the SM neutrinos are warmer relative to the photons, than for equivalent neutrinos which decouple before the SM neutrinos because now the photons have to share the $e^\pm$ energy or entropy with the equivalent neutrinos. For $T_{\xi d} = T_{\nu d} = 2$ MeV, $(T_\xi/T_\gamma)_0^3 = (T_\nu/T_\gamma)_0^3 = 0.365$. In contrast, in the limit that $T_{\xi d} \ll m_e$, $(T_\nu/T_\gamma)_0^3 \to 15/28.90 = 0.519$, while $(T_\xi/T_\gamma)_0^3 \to 1$.

As before when $T_{\xi d} > T_{\nu d}$, there are two contributions to $N_{\rm eff}$, from the SM neutrinos ($N_{{\rm eff},\nu}$) and from the ΔN_ν equivalent neutrinos ($N_{{\rm eff},\xi}$),

$$N_{\rm eff} = 3\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3} + \Delta N_\nu\left[\frac{11}{4}\left(\frac{T_\xi}{T_\gamma}\right)_0^3\right]^{4/3}$$
$$= 3\left(\frac{11}{4+7\phi_e(x_{\xi d})}\right)^{4/3}\left[\left(\frac{15+14\phi_e(x_{\xi d})}{15+14\phi_e(x_{\nu d})}\right)^{4/3} + \frac{\Delta N_\nu}{3}\right]. \tag{27}$$

In the limit where $T_{\xi d} = T_{\nu d}$ (e.g., for sterile neutrinos), $N_{\rm eff} = 3.018(1 + \Delta N_\nu/3)$, while in the limit of strongly coupled equivalent neutrinos ($T_{\xi d} \ll m_e$), $N_{{\rm eff},\nu} = 3\times((11/4)(0.519))^{4/3} = 4.82$ and $N_{{\rm eff},\xi} = (11/4)^{4/3}\Delta N_\nu = 3.85\Delta N_\nu$, so that $N_{\rm eff} = 4.82 + 3.85\Delta N_\nu$; for $\Delta N_\nu = 1$, $N_{\rm eff} = 8.67$. The results for $N_{\rm eff}$ as a function of $T_{\xi d}$ for $\Delta N_\nu = 1$ are shown in Fig. 4, where the contributions to $N_{\rm eff}$ from the SM neutrinos and the equivalent neutrino are shown separately. For $\Delta N_\nu = 1$, as the equivalent neutrino decoupling temperature decreases from $T_{\xi d} \gg m_t$ to $T_{\xi d} = T_{\nu d}$, $N_{\rm eff}$ increases from 3.07 to 4.02. As the equivalent neutrino decoupling temperature decreases further, from $T_{\xi d} = T_{\nu d}$ to $T_{\xi d} \ll m_e$, $N_{\rm eff}$ increases to 8.67, even though $\Delta N_\nu = 1$. A measurement of $N_{\rm eff} > 4$ could be consistent with the presence of only one equivalent neutrino ($\Delta N_\nu = 1$). Note that for a scalar equivalent neutrino, $\Delta N_\nu = 4/7$. As may be seen in Fig. 5, in this case in the limit $T_{\xi d} \ll m_e$, $N_{\rm eff} \to 7.02$.

C. Equivalent neutrinos and the neutrino mass constraint

After $e^\pm$ annihilation is complete, the present-day ratio of the number densities of the SM neutrinos and the equivalent neutrinos to that of the CMB photons is fixed. For each SM neutrino flavor and for each equivalent neutrino (assuming Majorana fermions),

$$\left(\frac{n_\nu}{n_\gamma}\right)_0 = \frac{3}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3; \qquad \left(\frac{n_\xi}{n_\gamma}\right)_0 = \frac{3}{4}\left(\frac{T_\xi}{T_\gamma}\right)_0^3. \tag{28}$$

The present Universe energy densities in massive (non-massless) SM neutrinos and in equivalent neutrinos are,

$$\rho_{\nu 0} = \Sigma m_\nu n_{\nu 0} \quad \text{and} \quad \rho_{\xi 0} = \Sigma m_\xi n_{\xi 0}. \tag{29}$$

As before in Sec. II B, if the results for the sum of the neutrino masses ($\Sigma m'_\nu$) and the sum of the masses of the equivalent neutrinos (if there is more than one, they are assumed to decouple at the same time) Σm_ξ, are compared

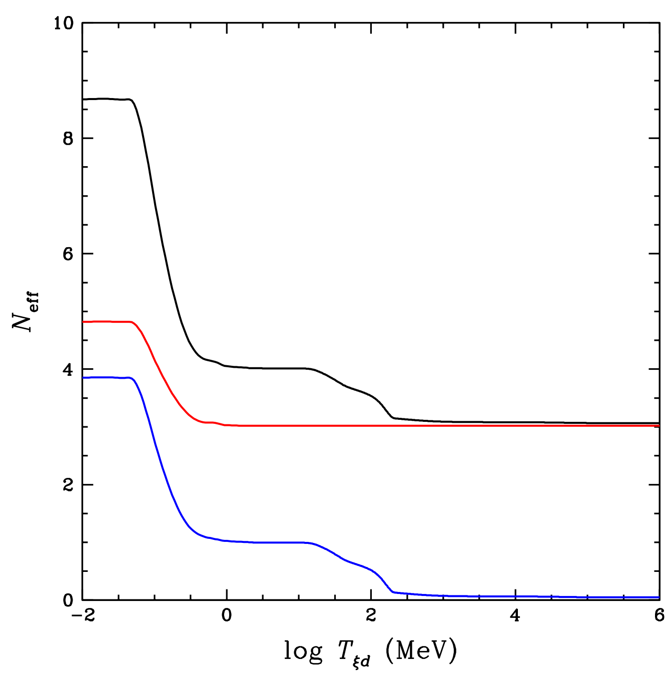

FIG. 4 (color online). Analogous to Fig. 1, $N_{\rm eff}$ is shown as a function of the equivalent neutrino decoupling temperature, $T_{\xi d}$, for one equivalent neutrino, $\Delta N_\nu = 1$ (upper, black curve). The lower, blue curve is the contribution to $N_{\rm eff}$ from the equivalent neutrino and the intermediate, red curve is the contribution to $N_{\rm eff}$ from the three SM neutrinos.

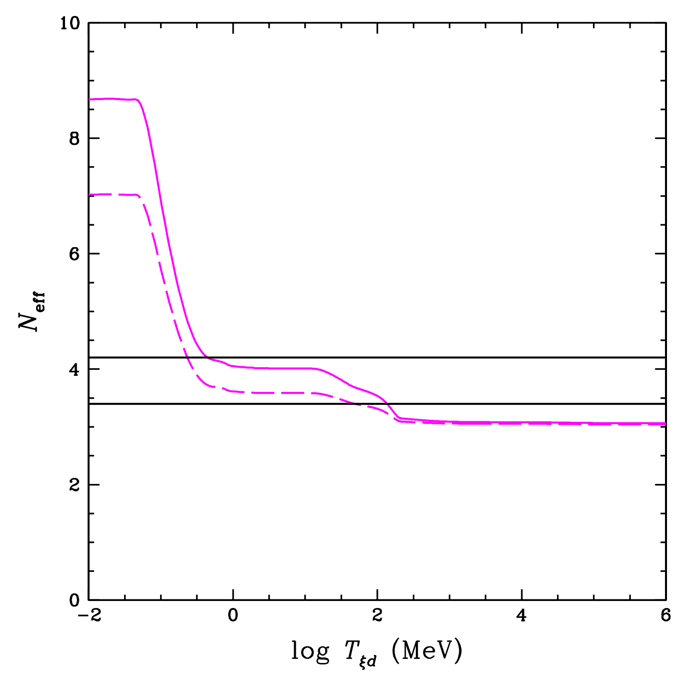

FIG. 5 (color online). $N_{\rm eff}$ is shown as a function of the equivalent neutrino decoupling temperature, $T_{\xi d}$, for one equivalent neutrino, $\Delta N_\nu = 1$, a Majorana fermion (solid curve; the upper, black curve in Fig. 4). The long-dashed curve shows $N_{\rm eff}$ for a scalar equivalent neutrino, $\Delta N_\nu = 4/7$. The horizontal band is the $\pm 1\sigma$ region allowed by WMAP9 [19].

to those for SM neutrinos which decouple instantaneously at $T_{\nu d} = 2$ MeV (in which case $\Sigma m_\nu = 94.12\Omega_\nu h^2$ eV),

$$\frac{\Sigma m'_\nu}{\Sigma m_\nu} = \frac{4}{11}\left(\frac{T_\gamma}{T_\nu}\right)^3_0, \qquad \frac{\Sigma m_\xi}{\Sigma m_\nu} = \frac{4}{11}\left(\frac{T_\gamma}{T_\xi}\right)^3_0. \tag{30}$$

Comparing with the results of the previous section, these results may be also written as

$$\frac{\Sigma m'_\nu}{\Sigma m_\nu} = \left(\frac{3}{N_{\mathrm{eff},\nu}}\right)^{3/4}, \qquad \frac{\Sigma m_\xi}{\Sigma m_\nu} = \left(\frac{1}{N_{\mathrm{eff},\xi}}\right)^{3/4}. \tag{31}$$

Since a constraint on the current energy density in hot, dark matter (Ω_{HDM}) leads to a constraint on the sum of the SM and equivalent neutrino masses, in the presence of equivalent neutrinos, this neutrino mass constraint is modified,

$$m \equiv \Sigma m_\xi + \Sigma m'_\nu \le 94.12\Omega_{\mathrm{HDM}} h^2 \left(\frac{\Sigma m'_\nu}{\Sigma m_\nu} + \frac{\Sigma m_\xi}{\Sigma m_\nu}\right) \text{ eV}$$
$$= 94.12\Omega_{\mathrm{HDM}} h^2 \left[\left(\frac{3}{N_{\mathrm{eff},\nu}}\right)^{3/4} + \left(\frac{1}{N_{\mathrm{eff},\xi}}\right)^{3/4}\right] \text{ eV}. \tag{32}$$

These results for the SM neutrino and equivalent neutrino masses as well as for their sum are shown in Fig. 6 as a function of the equivalent neutrino decoupling temperature. For example, if observations should find

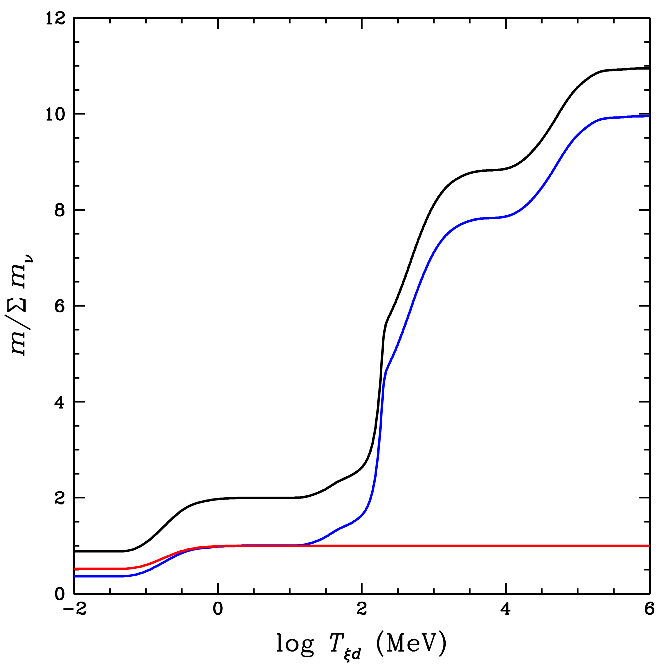

FIG. 6 (color online). Analogous to Fig. 3, $m \equiv \Sigma m_\xi + \Sigma m'_\nu$, normalized to Σm_ν ($\Sigma m_\nu \equiv 94.12\Omega_\nu h^2$ eV), is shown as a function of the equivalent neutrino decoupling temperature, $T_{\xi d}$ (upper, black curve). The intermediate, blue curve is for $\Sigma m_\xi/\Sigma m_\nu$ and the lower, red curve is for $\Sigma m'_\nu/\Sigma m_\nu$. If the upper bound to Σm_ν were 1 eV, the curves would be the upper bounds to the sums of SM neutrino masses ($\Sigma m'_\nu$), the equivalent neutrino masses (Σm_ξ), and their sum (m), in eV.

$94.12\Omega_{\mathrm{HDM}} h^2 = 1$, corresponding to $\Sigma m_\nu \le 1$ eV, then the vertical scale in Fig. 6 is the upper bound to the sum of the SM neutrino and equivalent neutrino masses, in eV. Notice that for very weakly coupled equivalent neutrinos, any CMB/LSS constraint on the sum of the neutrino masses is relaxed by ∼ an order of magnitude (in this limit, $\Sigma m'_\nu \approx \Sigma m_\nu$, while $\Sigma m_\xi \approx 10\Sigma m_\nu$). However, for sterile neutrinos ($T_{\xi d} = T_{\nu d} = 2$ MeV), the mass constraint is relaxed by only a factor of two (see Fig. 6).

IV. LIGHT OR VERY LIGHT WIMPS: RESCUING STERILE NEUTRINOS

In this section the effect on N_{eff} of the presence of a WIMP, sufficiently light so that its late time annihilation heats the photons beyond the usual heating from $e^\pm$ annihilation, is investigated. While the dark matter candidates (χ) supplied by most supersymmetric models tend to be very massive, $m_\chi \gtrsim$ tens or hundreds of GeV, in recent years there has also been interest in the light ($m_e \lesssim m_\chi \lesssim$ tens of MeV) or very light ($m_\chi \lesssim m_e$) WIMPs [23–34] considered here. The discussion in this section has some overlap with earlier work of Kolb *et al.* [23] and of Serpico and Raffelt [24], and with the recent analyses of Ho and Scherrer [35,36]. Assume, initially, that there are no equivalent neutrinos ($\Delta N_\nu = 0$), but there is a light WIMP, χ, a Majorana fermion (to be generalized later to a WIMP that is a Dirac fermion or a scalar boson, and to $\Delta N_\nu \ne 0$). The annihilation of a WIMP more massive than ∼20 MeV occurs prior to the decoupling of the SM neutrinos, heating them along with the photons and the $e^\pm$ pairs present at that time, preserving the standard results discussed in § II. Note that it is essential here to assume that the light WIMP couples to photons and $e^\pm$ pairs but does not couple to the SM neutrinos since through such coupling the neutrinos could be kept in equilibrium with the photons, leading to $(T_\nu/T_\gamma)_0 \to 1$ and $N_{\mathrm{eff}} \to 3(11/4)^{4/3} = 11.56$ (see Sec. II). This assumption will be reversed in the next section where WIMPs that couple only to the SM neutrinos are considered. In the presence of "massive" light WIMPs ($m_\chi \gtrsim 20$ MeV), there is no change from the standard result that for $\Delta N_\nu = 0$, $N_{\mathrm{eff}} = 3.018$ (or, 3.046 [17]). However, the late time annihilation of sufficiently light WIMPs ($m_\chi \lesssim 12$ MeV $\approx 6T_{\nu d}$) will further heat the photons relative to the now decoupled neutrinos, resulting in photons that are hotter than the SM neutrinos in the absence of the light WIMP. This dilutes the contribution of the SM neutrinos to the early Universe energy density, leading to the surprising result that, even in the presence of the three SM neutrinos, $N_{\mathrm{eff}} < 3$. This opens the door for $\Delta N_\nu > 0$ to be consistent with a measurement of $N_{\mathrm{eff}} = 3$.

As before, the late time ratio of neutrino to photon temperatures, $(T_\nu/T_\gamma)_0$, may be evaluated by comparing the entropy in a comoving volume at $T_\gamma = T_{\nu d}$ with the same quantity evaluated at $T_\gamma = T_{\gamma 0} \ll m_e(m_\chi)$. At late times,

$$\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \frac{2}{2 + \frac{7}{2}\phi_{ed} + \frac{7}{4}\phi_{\chi d}}, \tag{33}$$

where $\phi(x) \equiv s(x)/s(0)$ (for fermions, the same for Majorana and Dirac fermions) and ϕ_{ed} is evaluated at $x_{ed} = m_e/T_{\nu d,}$ while $\phi_{\chi d}$ is evaluated at $x_{\chi d} = m_\chi/T_{\nu d}$. It is usually assumed that $\phi_{ed} = 1$ but, as seen above in Sec. II, for $T_{\nu d} = 2$ MeV, $\phi_{ed} = 0.993$. For consistency, this latter value is adopted here (along with the assumption of instantaneous decoupling) resulting in

$$\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \frac{4}{10.95 + \frac{7}{2}\phi_{\chi d}}, \qquad \frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \frac{11}{10.95 + 3.5\phi_{\chi d}}. \tag{34}$$

As a result,

$$N_{\rm eff} \equiv 3\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3} = 3\left[\frac{11}{10.95 + 3.5\phi_{\chi d}}\right]^{4/3} \le 3.018. \tag{35}$$

In the presence of a light WIMP, $N_{\rm eff}$ is a function of the light WIMP mass through the dependence of $\phi_{\chi d}$ on $x_{\chi d} = m_\chi/T_{\nu d}$. In the limit of "high" light WIMP masses, $m_\chi \gg T_{\nu d}$, $\phi_{\chi d} \to 0$ and, $N_{\rm eff} \to 3.018$, recovering the SM result. However, in the opposite limit, for very light WIMPs with $m_\chi \ll m_e \lesssim T_{\nu d}/4$, $\phi_{\chi d} \to 1$ and, $N_{\rm eff} \to 2.085$ [35]. The evolution of $N_{\rm eff}$ with m_χ is shown by the intermediate, black curve in Fig. 7.

This result is for a WIMP that is a Majorana fermion. It is straightforward to generalize this result to a WIMP that is a Dirac fermion, or for bosons [35,36], by rewriting the entropy conservation equation [Eq. (34)] as

$$\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \frac{2}{2 + \frac{7}{2}\phi_{ed} + \tilde{g}_\chi\phi_{\chi d}}, \tag{36}$$

where $\tilde{g}_\chi = 7/4$ for a Majorana WIMP, $7/2$ for a Dirac WIMP, and 1 for a scalar WIMP; a vector boson WIMP would have $\tilde{g}_\chi = 3$. However, note that for bosons, the quantity $\phi = s(x)/s(0)$, which has been derived for the Majorana and Dirac WIMPs using the Fermi-Dirac distribution, must be replaced with the corresponding function evaluated using the Bose-Einstein distribution. The results for these different choices are shown in Fig. 7. In the limit of "high" WIMP mass, $m_\chi \gtrsim 12$ MeV, all these cases approach $N_{\rm eff} \approx 3.02$, but they differ for very light WIMPs with $m_\chi \lesssim 1$ MeV. While $N_{\rm eff} \to 2.09$ for a Majorana WIMP, for a Dirac WIMP, $N_{\rm eff} \to 1.56$, and for a scalar WIMP, $N_{\rm eff} \to 2.41$. As may be seen in Fig. 7, the transition from the "standard" value of $N_{\rm eff} \approx 3$ in the absence of extra equivalent neutrinos or dark radiation, to the asymptotic values of $N_{\rm eff} < 3$ occurs over a relatively small range in the light WIMP mass, $2 \lesssim m_\chi \lesssim 12$ MeV.

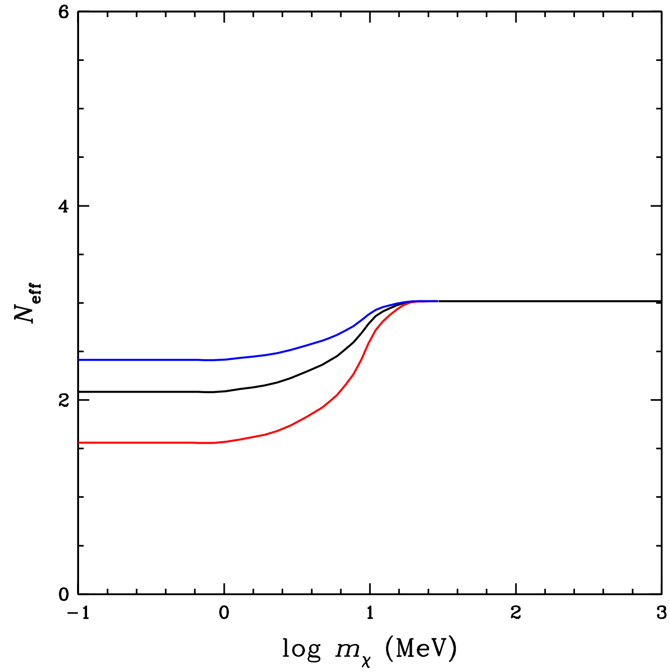

FIG. 7 (color online). The effective number of neutrinos as a function of the WIMP mass for a Majorana WIMP (intermediate, black curve), a Dirac WIMP (lower, red curve), and a scalar WIMP (upper, blue curve).

In the absence of "dark radiation" or equivalent neutrinos ($\Delta N_\nu = 0$), the presence of a sufficiently light WIMP allows the effective number of neutrinos to take on any value from $N_{\rm eff} \approx 1.56$ to $N_{\rm eff} \approx 3.02$, depending on the nature of the WIMP and its mass.

A. Light WIMP and sterile neutrinos: Degeneracy between m_χ and ΔN_ν

To explore how $N_{\rm eff}$ changes in the presence of both a light WIMP (χ) and equivalent neutrinos (ξ), allow for ΔN_ν equivalent neutrinos that, for simplicity, all decouple at the same temperature, $T_{\xi d}$. If $N^0_{\rm eff} \equiv N_{\rm eff}(\Delta N_\nu = 0)$ [see Eq. (35)] and $N_{\rm eff} \equiv N_{\rm eff}(\Delta N_\nu \neq 0)$, then

$$N_{\rm eff} = \left[3 + \Delta N_\nu\left(\frac{T_\xi}{T_\nu}\right)_0^4\right]\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]^{4/3} = N^0_{\rm eff}\left[1 + \frac{\Delta N_\nu}{3}\left(\frac{T_\xi}{T_\nu}\right)_0^4\right] = N^0_{\rm eff}\left(1 + \frac{\Delta N^*_\nu}{3}\right). \tag{37}$$

Suppose there are one or even two sterile neutrinos, so that $(T_\xi/T_\nu)_0 = 1$ and $N_{\rm eff} = N^0_{\rm eff}(1 + \Delta N_\nu/3)$. Depending on the nature of the WIMP and its mass it is possible to account for any value of $N_{\rm eff}$ in the range $2.08 \lesssim N_{\rm eff} \lesssim 4.02$ (for one sterile neutrino) or $2.60 \lesssim N_{\rm eff} \lesssim 5.03$ (for two sterile neutrinos). Since in the presence of sterile neutrinos the effective number of neutrinos depends on the WIMP mass and its nature, along with the number of sterile neutrinos, $N_{\rm eff} = N_{\rm eff}(m_\chi, \Delta N_\nu)$, there is a degeneracy between the number of sterile neutrinos and the WIMP mass (and its nature). The same observationally determined value of the effective number of neutrinos can be achieved with different combinations of the light WIMP mass and the number of sterile neutrinos. This degeneracy is illustrated in Fig. 8 for a Majorana fermion WIMP. As seen in Fig. 8, for one sterile neutrino ($\Delta N_\nu = 1$), $N_{\rm eff} = 4N^0_{\rm eff}/3$; for two sterile neutrinos ($\Delta N_\nu = 2$), $N_{\rm eff} = 5N^0_{\rm eff}/3$. For example, as shown in Fig. 8, for a sufficiently low mass Majorana fermion WIMP, $m_\chi \lesssim 1$ MeV, $N^0_{\rm eff} \to 2.09$, so that for one (two) sterile neutrino(s), $N_{\rm eff} = 2.78(3.48)$, consistent with current CMB/LSS constraints [19–22]. A CMB/LSS determination of $N_{\rm eff} \approx 3$ does not, by itself, exclude the possibility of one sterile neutrino. Indeed, the current CMB/LSS data appear to favor one, or possibly two, sterile neutrinos.

B. Light WIMP and equivalent neutrinos: Degeneracy among m_χ, ΔN_ν, and $T_{\xi d}$

Current pre-Planck constraints on $N_{\rm eff}$ from WMAP9, ACT3, and SPT, supplemented by LSS data from BAO and measurements of $H_{0,}$ are consistent with values for the effective number of neutrinos in the range, $3 \lesssim N_{\rm eff} \lesssim 4$ [19–22]. Values of $N_{\rm eff}$ in this range can be achieved by different combinations of $N^0_{\rm eff}(m_\chi)$ and $\Delta N^*_\nu \equiv \Delta N_\nu (T_\xi/T_\nu)^4_0$. If the restriction to sterile neutrinos is relaxed so that $(T_\xi/T_\nu)_0 \neq 1$, it is ΔN^*_ν and the WIMP mass that are degenerate, $N_{\rm eff}(m_\chi, \Delta N^*_\nu) = N^0_{\rm eff}(m_\chi)(1 + \Delta N^*_\nu/3)$. For example, if the equivalent neutrinos decouple prior to the decoupling of the SM neutrinos so that $(T_\xi/T_\nu)_0 \leq 1$, the contribution of the equivalent neutrinos to $N_{\rm eff}$ is diluted, $\Delta N^*_\nu \leq \Delta N_\nu$. ΔN^*_ν is shown as a function of the WIMP mass in Fig. 9 for three different choices of $N_{\rm eff} = 3.0$, 3.5, 4.0, demonstrating that depending on the WIMP mass, these values of $N_{\rm eff}$ are consistent with ΔN^*_ν in the range $0 \leq \Delta N^*_\nu \lesssim 2.8$. As an illustrative example, reconsider the case of three right-handed neutrinos [18] (see Sec. III A), so that $\Delta N_\nu = 3$ and $\Delta N^*_\nu = 3(T_\xi/T_\nu)^4_0$. For $m_\chi \lesssim 1$ MeV, $N_{\rm eff} = 3$ requires $\Delta N^*_\nu \approx 1.3$, or $(T_\xi/T_\nu)^4_0 \approx 0.4$, which is achieved for $T_{\xi d} \approx 120$ MeV $\approx 60 T_{\nu d}$ [18]. A determination of $N_{\rm eff} = 3$ does not exclude three, right-handed neutrinos. This is illustrated in Fig. 10, where $N_{\rm eff} = 3$ is adopted and ΔN^*_ν is shown as a function of the WIMP mass (as in Fig. 9) for Majorana and Dirac fermion WIMPs as well as for a scalar WIMP. As may be seen in Fig. 10, for $m_\chi \lesssim 1$ MeV, $\Delta N^*_\nu > 0$ even though $N_{\rm eff} = 3$. The absence of evidence for equivalent neutrinos ($N_{\rm eff} = 3$) is not evidence for the absence of equivalent neutrinos.

C. SM and sterile neutrino masses in the presence of a light WIMP

As has been noted in Secs. II B and III C above, the constraint on the sum of the neutrino masses is modified if

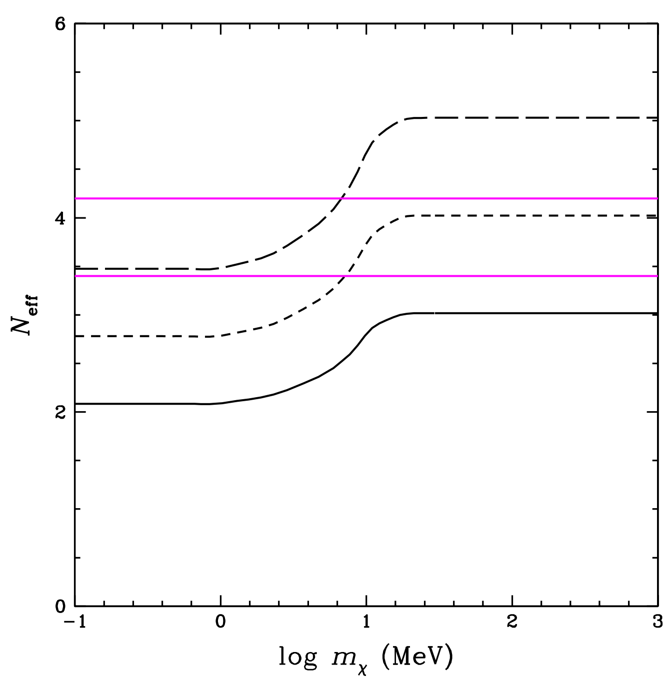

FIG. 8 (color online). The effective number of neutrinos, $N_{\rm eff}$, as a function of the WIMP mass for a Majorana WIMP. The solid curve is for the case of no sterile neutrinos. The short-dashed curve is for one sterile neutrino. The long dashed curve is for two sterile neutrinos. The horizontal (purple) lines show the $\pm 1\sigma$ band allowed by the WMAP 9 year data [19].

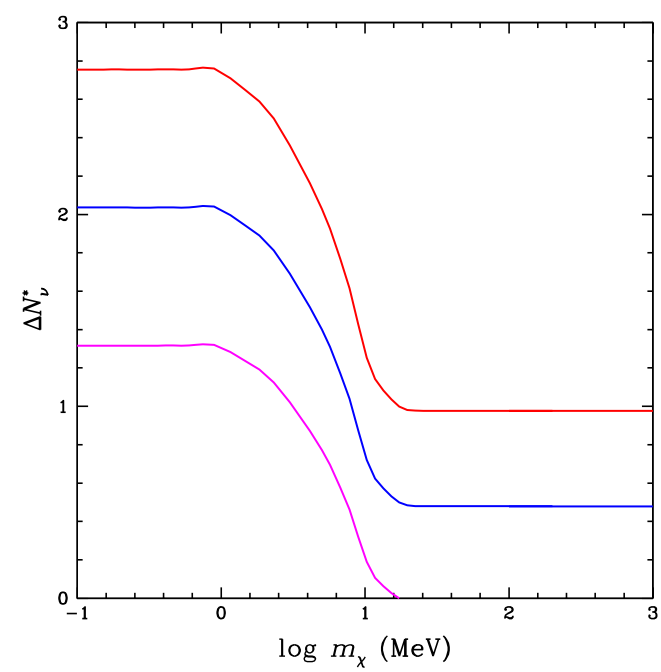

FIG. 9 (color online). The effective number of equivalent neutrinos, ΔN^*_ν (see the text), as a function of the WIMP mass, m_χ, for a Majorana WIMP, consistent with an observationally determined value of $N_{\rm eff}$. The lower, purple curve is for $N_{\rm eff} = 3.0$, the intermediate, blue curve is for $N_{\rm eff} = 3.5$, the upper, red curve is for $N_{\rm eff} = 4.0$.

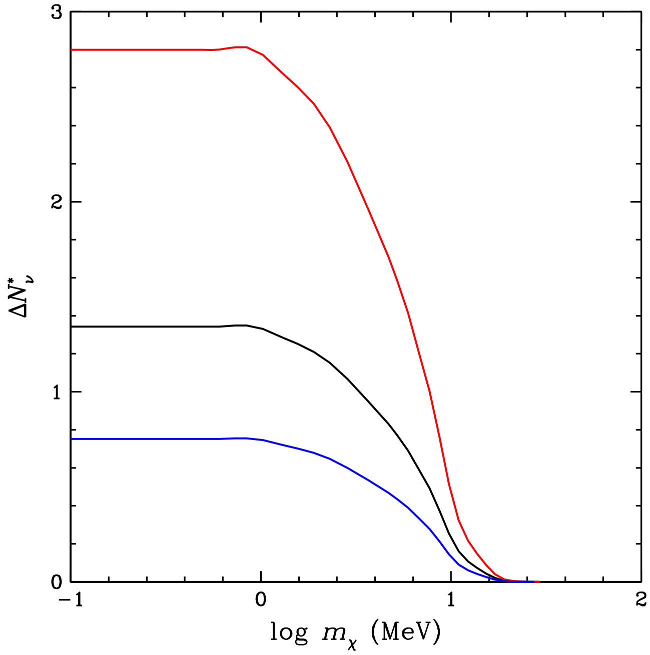

FIG. 10 (color online). The analog of Fig. 9 for the choice of $N_{\rm eff} = 3$. The intermediate, black curve is for a Majorana fermion WIMP, the upper, red curve is for a Dirac fermion WIMP, and the lower, blue curve is for a scalar boson WIMP (see Fig. 7).

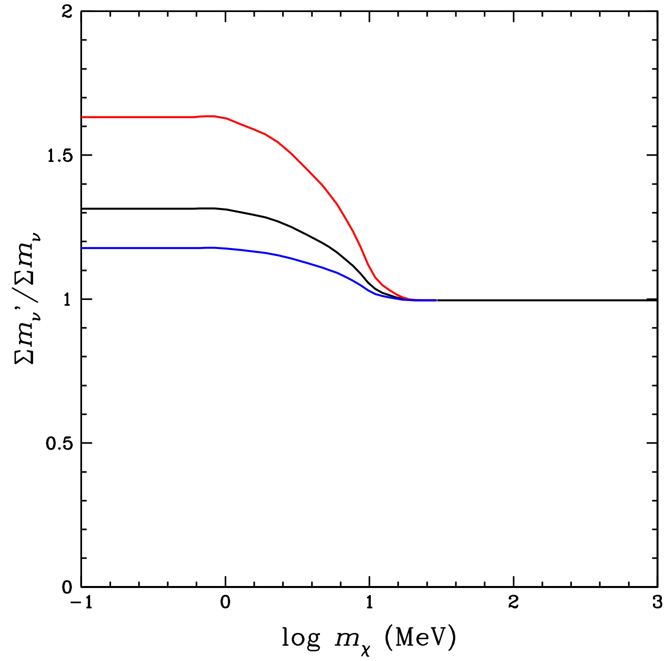

FIG. 11 (color online). The ratio of the sum of the SM plus sterile neutrino masses to its canonical value, $\Sigma m'_\nu/\Sigma m_\nu$, assuming instantaneous neutrino decoupling and $(T_\nu/T_\gamma)_0^3 = 4/11$, as a function of the WIMP mass for a Majorana WIMP (intermediate, black curve), a Dirac WIMP (upper, red curve), and a scalar WIMP (lower, blue curve). If the upper bound to Σm_ν were 1 eV, the curves would be the upper bounds to the sum of the SM plus sterile neutrino masses, in eV.

the late time ($T_\gamma \to T_{\gamma 0}$) ratio of neutrino to photon temperatures changes,

$$\frac{\Sigma m'_\nu}{\Sigma m_\nu} = \frac{4}{11}\left(\frac{T_\gamma}{T_\nu}\right)_0^3 = \left(\frac{3}{N_{\rm eff}^0}\right)^{3/4}. \tag{38}$$

Here, for simplicity, it is assumed that the ΔN_ν extra neutrinos decouple along with the SM neutrinos (e.g., they are sterile neutrinos) so that $\Sigma m'_\nu$ is the sum of the SM and sterile neutrino masses. Since for light WIMPs $N_{\rm eff}^0 \leq 3$, this allows $\Sigma m'_\nu \geq \Sigma m_\nu$, permitting more massive SM neutrinos to be compatible with current CMB/LSS constraints. Compared to the examples discussed earlier (Secs. II B and III C), for the case considered here of light WIMPs, with or without sterile neutrinos, the deviation of $N_{\rm eff}^0$ from 3 is less dramatic, resulting in relatively smaller differences between the sum of the neutrino masses with and without the light WIMP ($1 \leq \Sigma m'_\nu/\Sigma m_\nu \lesssim 1.6$). The neutrino mass ratios, $\Sigma m'_\nu/\Sigma m_\nu$, are shown as functions of the light WIMP mass in Fig. 11 for Majorana and Dirac fermion WIMPs as well as for a scalar WIMP.

V. DARK RADIATION WITHOUT DARK RADIATION: "TRULY WEAK" LIGHT WIMPS

As a novel alternative to the case discussed above in Sec. IV, consider the consequences of a "truly weak" light WIMP that couples only to the standard model neutrinos, but not to the photons or the $e^\pm$ pairs [23,24,37]. Assume there are no equivalent neutrinos ($\Delta N_\nu = 0$). In this case the WIMP annihilation heats the neutrinos (but not the photons), while the annihilation of the $e^\pm$ pairs heats the photons (but not the decoupled neutrinos). After the both the $e^\pm$ pairs and the light WIMPs have annihilated ($T_\gamma \to T_{\gamma 0}$) the ratio of neutrino to photon temperatures can be found by considerations of entropy conservation. In this case, the entropies (in a comoving volume) of the photons and $e^\pm$ pairs ($S_{\gamma e}$) and of the neutrinos and the WIMPs ($S_{\nu\chi}$), are conserved individually. As a result,

$$\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \frac{1 + 4\tilde{g}_\chi\phi_{\chi d}/21}{1 + 7\phi_{ed}/4} = \frac{1 + 4\tilde{g}_\chi\phi_{\chi d}/21}{2.738},$$
$$\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = 1.0045\left(1 + \frac{4\tilde{g}_\chi\phi_{\chi d}}{21}\right). \tag{39}$$

For sufficiently massive WIMPs, for which $\phi_{\chi d} \to 0$, the usual result, $N_{\rm eff} = 3.018$, is recovered. But for very light WIMPs, for which $\phi_{\chi d} \to 1$,

$$\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = 1.0045\left(1 + \frac{4\tilde{g}_\chi}{21}\right),$$
$$N_{\rm eff} = N_{\rm eff}^0 = 3.018\left(1 + \frac{4\tilde{g}_\chi}{21}\right)^{4/3}. \tag{40}$$

The effective number of neutrinos is shown as a function of the WIMP mass for a truly weak WIMP in Fig. 12 for

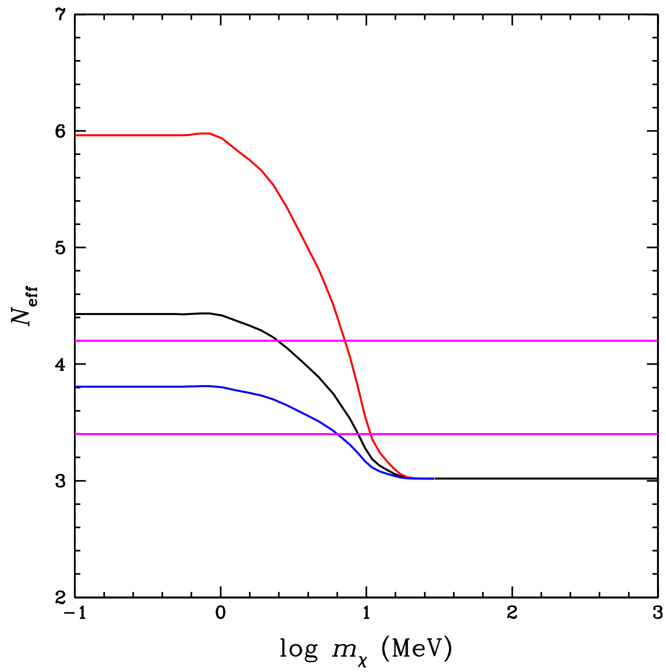

FIG. 12 (color online). The effective number of neutrinos, $N_{\rm eff}$, as a function of the WIMP mass for "truly weak" WIMPs that only couple to the standard model neutrinos, but not to the photons or the $e^{\pm}$ pairs. The intermediate, black curve is for a Majorana WIMP, the upper, red curve is for a Dirac WIMP, and the lower, blue curve is for a scalar WIMP. The horizontal band (purple) corresponds to the $\pm 1\sigma$ band consistent with the WMAP9 data. The lower, red curve is for a Dirac WIMP, the intermediate, black curve is for a Majorana WIMP, and the upper, blue curve is for a scalar WIMP.

Majorana and Dirac WIMPs as well as for a scalar WIMP. Also shown in Fig. 12 is the $\pm 1\sigma$ band consistent with the WMAP9 value of $N_{\rm eff}$ [19].

The current, pre-Planck CMB estimates suggesting that $N_{\rm eff} > 3$ [19–22] are not inconsistent with the absence of equivalent neutrinos ($\Delta N_\nu = 0$). In contrast to the "standard" WIMP case, for light WIMPs that couple only to neutrinos, an observational determination of $N_{\rm eff} > 3$ could lead to the mistaken conclusion that $\Delta N_\nu > 0$, even in the absence of dark radiation or equivalent neutrinos—"Dark radiation without dark radiation."

A. Neutrino masses in the presence of a truly weak light WIMP

Here, too (see Secs. II B, III C, and IV C), the constraint on the sum of the neutrino masses is modified by the presence of a light WIMP that only couples to neutrinos and not to photons. In this case,

$$\frac{\Sigma m'_\nu}{\Sigma m_\nu} = \left(\frac{3}{N_{\rm eff}}\right)^{3/4} \lesssim 1, \tag{41}$$

tightening the constraint on the sum of the neutrino masses.

Since $N_{\rm eff} > 3$ for truly weak light WIMPs, their presence isn't favorable for the existence of sterile neutrinos.

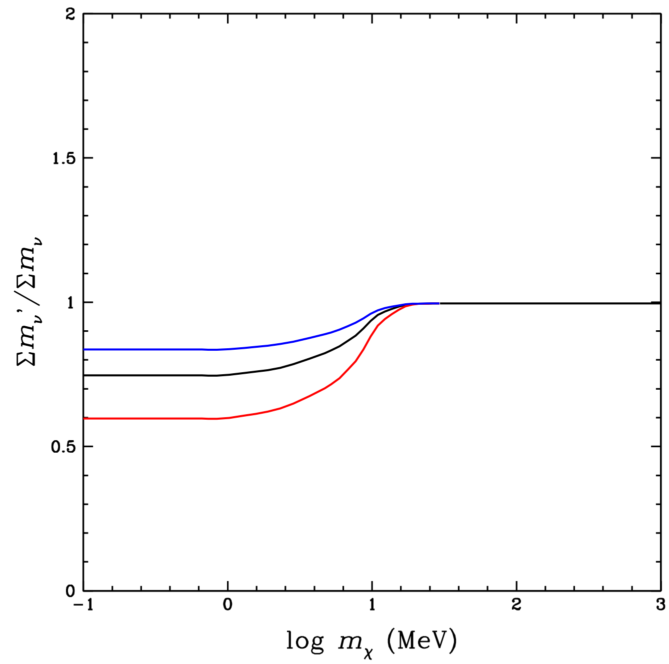

FIG. 13 (color online). As in Fig. 11, but for WIMPs that couple only to the SM neutrinos, not to photons.

However, if sterile neutrinos are present, $\Sigma m'_\nu$ is the sum of the SM and sterile neutrino masses. As for the case of the "normally" coupled light WIMPs (Sec. IV C), the deviation of $N_{\rm eff}$ from 3 is not very large, resulting in relatively smaller differences between the sum of the neutrino masses with and without the light WIMP ($0.6 \lesssim \Sigma m'_\nu/\Sigma m_\nu \leq 1$). $\Sigma m'_\nu/\Sigma m_\nu$ is shown as a function of the truly weak, light WIMP mass in Fig. 13.

VI. SUMMARY AND CONCLUSIONS

At late times in the early, radiation-dominated Universe, after all the SM particles and light WIMPs (χ), if present, have annihilated, the energy density consists of the contributions from the photons (γ) and the three SM neutrinos (ν), possibly supplemented by the contribution from ΔN_ν equivalent neutrinos (ξ). At these late times ($T_{\gamma 0} \ll \min\{m_e, m_\chi\}$) the ratio, by number, of one species of SM neutrino to the photons is

$$\left(\frac{n_\nu}{n_\gamma}\right)_0 = \frac{3}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3 = \frac{3}{11}\left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^3\right]. \tag{42}$$

Since at least some of the SM neutrinos are sufficiently massive to be nonrelativistic at present, the neutrino contribution to the present Universe mass density is $\rho_{\nu 0} = \Sigma m_\nu n_{\nu 0}$. In the absence of light WIMPs and equivalent neutrinos, $\Sigma m_\nu = 94.12\Omega_\nu h^2$, where CMB and LSS data bound the neutrino mass density, $\Omega_\nu \leq \Omega_{\rm HDM}$. An observational constraint on $\Omega_{\rm HDM}h^2$ leads to an upper bound to Σm_ν. Since the cosmological constraint on the sum of the neutrino masses depends on the frozen out ratio of the number densities of neutrinos to photons, in the more general cases allowing for light WIMPs and equivalent neutrinos the cosmological constraint on the sum of the neutrino masses is modified. That is, the simple numerical factor connecting Σm_ν and $\Omega_\nu h^2$ is modified, with the conversion factor now depending on the properties of the WIMP and the equivalent neutrinos. In the presence of a WIMP, the late time ratio of SM neutrino and photon temperatures $(T_\nu/T_\gamma)_0$ depends on the WIMP mass (m_χ) as well as on the SM and equivalent neutrino decoupling temperatures ($T_{\nu d}$ and $T_{\xi d}$, respectively), while the late time ratio of the equivalent neutrino and SM neutrino temperatures $(T_\xi/T_\nu)_0$ depends on the equivalent neutrino decoupling temperature. As a result, $(n_\nu/n_\gamma)_0$ is a function of $T_{\xi d}$ and m_χ (and of $T_{\nu d}$).

The neutrino (SM and equivalent neutrinos) contributions to the late time radiation energy density are measured by the effective number of neutrinos, $N_{\rm eff}$,

$$N_{\rm eff} \equiv \left[\frac{11}{4}\left(\frac{T_\nu}{T_\gamma}\right)^3_0\right]^{4/3}\left[3 + \Delta N_\nu\left(\frac{T_\xi}{T_\nu}\right)^4_0\right] \equiv N^0_{\rm eff}\left(1 + \frac{\Delta N^*_\nu}{3}\right). \tag{43}$$

In general, $N_{\rm eff}$ is a function of ΔN_ν, $T_{\nu d}$, $T_{\xi d}$, and m_χ, leading to degeneracies among them for any observationally determined value of $N_{\rm eff}$.

In Sec. II the standard, textbook discussion of neutrino decoupling (freeze out) in the early Universe (no equivalent neutrinos ($\Delta N_\nu = 0$), no light WIMPs) was reviewed, noting how $(n_\nu/n_\gamma)_0$ and $N_{\rm eff}$ depend on the choice of the SM neutrino decoupling temperature. As may be seen in Figs. 1–3, the earlier the neutrinos decouple (the weaker the weak interactions), the cooler they are relative to the photons and the smaller are $(n_\nu/n_\gamma)_0$ (allowing for larger neutrino masses) and $N_{\rm eff}$. Conversely, the stronger the weak interactions, the later the neutrinos decouple and the larger are the frozen out values of $(n_\nu/n_\gamma)_0$ and $N_{\rm eff}$. As the discussion in Sec. II and Fig. 1 in particular show, if the neutrino decoupling temperature were a free parameter, allowed to vary from $T_{\nu d} \gg m_t$ to $T_{\nu d} \ll m_e$, the frozen out ratio of neutrinos (one species) to photons would vary by a factor of $\sim$50, from $(n_\nu/n_\gamma)_0 \sim 0.015$ to $(n_\nu/n_\gamma)_0 \sim 0.75$. The effect on the neutrino mass constraint of this variation in the abundance of neutrinos relative to photons is shown in Fig. 3. Allowing $T_{\nu d}$ to be a free parameter, the effective number of neutrinos could assume any value from $N_{\rm eff} \sim 0.06$ to $N_{\rm eff} \sim 11.56$ (see Fig. 1). In reality, the neutrino decoupling temperature is determined empirically to be $T_{\nu d} \approx 2$ MeV [14–16]. In the standard, textbook analyses some simplifying assumptions are made (instantaneous decoupling; massless electrons), leading to $(T_\nu/T_\gamma)^3_0 = 4/11$, so that $(n_\nu/n_\gamma)_0 = 3/11$ and $N_{\rm eff} = 3$. However, in Sec. II it was noted that for the best estimate of $T_{\nu d}$ and assuming the neutrinos decouple instantaneously, there is a small difference from the canonical results (see Fig. 2); $(T_\nu/T_\gamma)^3_0 \rightarrow 1.006(4/11)$, so that $(n_\nu/n_\gamma)_0 \rightarrow 1.006(3/11)$ and $N_{\rm eff} \rightarrow 3.018$.

With these results as prologue, in Sec. III the standard model of particle physics was extended to allow for the presence of ΔN_ν equivalent neutrinos (ξ). Fixing the SM neutrino decoupling temperature at $T_{\nu d} = 2$ MeV, the connection between the equivalent neutrino decoupling temperature ($T_{\xi d}$) and the late time ratio of the SM neutrino to photon temperatures was explored along with the changes to the corresponding values of $(n_\nu/n_\gamma)_0$ (and its implication for the constraint on the sum of the neutrino masses) and $N_{\rm eff}$ (see Figs. 4–6). As may be seen in Fig. 4, depending on when an equivalent neutrino decouples (how weakly it interacts with the SM particles), one equivalent neutrino (i.e., a very light, Majorana fermion) need not contribute $\Delta N_\nu = 1$ to $N_{\rm eff}$. In the equivalent neutrino contribution to $N_{\rm eff}$ there is a degeneracy between ΔN_ν and $T_{\xi d}$. As $T_{\xi d}$ decreases from $\gg m_t$ to $\ll m_e$, the contribution to $N_{\rm eff}$ from one equivalent neutrino increases from 0.05 to 3.85, while the contribution from the SM neutrinos increases from 3.02 to 4.82, and $N_{\rm eff}$ increases from 3.07 to 8.67 (7.02 for a scalar equivalent neutrino); see Figs. 4 and 5. As noted in Sec. III, sterile neutrinos are a special case of the more general equivalent neutrinos. Sterile neutrinos, very light Majorana fermions that decouple along with the SM neutrinos ($T_{\xi d} = T_{\nu d} = 2$ MeV), simplify the connection between $N_{\rm eff}$ and ΔN_ν by eliminating the degeneracy between ΔN_ν and $T_{\xi d}$. In this case, $N_{\rm eff} = 3.018(1 + \Delta N_\nu/3)$, corresponding to $N_{\rm eff} = 4.02(5.03)$ for one (two) sterile neutrinos.

In Secs. IV and V, the connections between a light WIMP and equivalent neutrinos were explored. A relatively light WIMP, whether or not it qualifies as a dark matter candidate, will annihilate late during the early evolution of the Universe, heating the SM particles, including possibly the neutrinos (SM and equivalent). For a sufficiently light WIMP ($m_\chi \lesssim 20$ MeV) without enhanced coupling to the SM neutrinos, late time annihilation may heat the photons relative to the decoupled neutrinos, reducing $(T_\nu/T_\gamma)_0$ below what it would be in the absence of the WIMP, resulting in $N_{\rm eff} < 3$, even in the presence of the three SM neutrinos. This allows for additional equivalent neutrinos, $\Delta N_\nu > 0$, even if observations should determine that $N_{\rm eff} \approx 3$ [35,36]. Indeed, as found in Sec. IV and as shown in Fig. 7, for a very light ($m_\chi \lesssim m_e$) Majorana fermion WIMP, $N_{\rm eff} \rightarrow 2.09$, allowing for the consistency of one or even two sterile neutrinos with current CMB constraints [19,20,22] (see Fig. 8). As noted in Sec. IV B and illustrated in Figs. 9 and 10, in the presence of a light WIMP there is a degeneracy between the WIMP mass and the combination $\Delta N^*_\nu \equiv \Delta N_\nu (T_\xi/T_\nu)^4_0$. Depending on the observationally determined value of $N_{\rm eff}$, there may be several combinations of m_χ, ΔN_ν, and $T_{\xi d}$ that are consistent with the same value of $N_{\rm eff}$. To illustrate this point the case of three right-handed neutrinos [18], where $\Delta N_\nu = 3$ and $\Delta N^*_\nu = 3(T_\xi/T_\nu)^4_0$, was revisited. It was noted that for $m_\chi \lesssim 1$ MeV and $N_{\rm eff} = 3$, $\Delta N_\nu^* \approx 1.3$, requiring $(T_\xi/T_\nu)_0^4 \approx 0.4$, which can be achieved provided that $T_{\xi d} \approx 120$ MeV $\approx 60 T_{\nu d}$ [18]. Such a high decoupling temperature for the three right-handed neutrinos could result from their being coupled to a heavier Z boson, $M_{Z'}/M_Z \approx 8$ or $M_{Z'} \approx 0.7$ TeV. In general, in the presence of a sufficiently light WIMP, $N_{\rm eff} = 3$ is no guarantee of the absence of equivalent neutrinos.

The discussion in Sec. V considered the effects of a "truly weak" light WIMP, a particle that couples only to the SM neutrinos but not to the other SM particles (in particular, it does not couple to the photons and the $e^\pm$ pairs) [23,24,37]. Before the SM neutrinos decouple ($T_\gamma \geq T_{\nu d}$), $T_\nu = T_\gamma$. However, when $T_\gamma < T_{\nu d}$ $e^\pm$ annihilation heats the photons but not the decoupled SM neutrinos. In contrast, when the truly weak WIMP annihilates it heats the SM neutrinos but not the photons, bringing the late time neutrino and photon temperatures closer together. As was the case in § IV, the simple connection between $N_{\rm eff}$ and ΔN_ν is broken. In the presence of a truly weak, light WIMP it is possible to have $N_{\rm eff} > 3$ even if $\Delta N_\nu = 0$: Dark radiation without dark radiation.

As the key points presented here have shown, there's more to a measurement of the effective number of neutrinos than meets the eye, at least at first sight. In the presence of a sufficiently light WIMP, $N_{\rm eff}$ depends on the WIMP mass, m_χ, as well as its nature (fermion or boson) and its coupling, or not, to the SM neutrinos, on the number of equivalent neutrinos, ΔN_ν (and on their nature as well), and on the equivalent neutrino decoupling temperature, $T_{\xi d}$. A measurement of $N_{\rm eff} = 3$, within the observational uncertainties, is not evidence for the absence of equivalent neutrinos, sterile or otherwise. Conversely, a measurement of $N_{\rm eff} > 3$, accounting for the observational uncertainties, does not, by itself, establish the presence of equivalent neutrinos or dark radiation.

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