PHYSICAL REVIEW D **87,** 103510 (2013)

Galaxy rotation curves driven by massive vector fields: Key to the theory of the dark sector

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(Received 21 March 2013; published 15 May 2013)

The nongauge vector field with as simple a Lagrangian as possible turned out to be an adequate tool for the macroscopic description of the main properties of dark matter. The dependence of the velocity of a star on the radius of the orbit $V(r)$—galaxy rotation curve—is derived analytically from the first principles completely within Einstein's general relativity. Milgrom's empirical modification of Newtonian dynamics in the nonrelativistic limit gets justified and specified in detail. In particular, the transition to a plateau is accompanied by damping oscillations. In the scale of a galaxy, and in the scale of the whole universe, the dark matter is described by a vector field with the same energy-momentum tensor. It is the evidence of the common physical nature. Now, when we have the general expression for the energy-momentum tensor of dark matter, it is possible to analyze its influence on the structure and evolution of superheavy stars and black holes.

DOI: 10.1103/PhysRevD.87.103510 PACS numbers: 95.35.+d, 04.20.Jb, 98.35.Gi, 98.62.Gq

I. INTRODUCTION

The "galaxy rotation curves" problem appeared after Oort discovered the Galactic halo, a group of stars orbiting the Milky Way outside the main disk [1]. In 1933, Zwicky [2] postulated "missing mass" to account for the orbital velocities of galaxies in clusters. Persistent investigations by Rubin and Ford [3] in the 1970s finally dispelled the skepticism about the existence of dark matter on the periphery of the galaxies.

Among numerous attempts to solve the problem of galaxy rotation curves, the most discussed one is an empirical explanation named modified Newtonian dynamics (MOND), proposed by Milgrom back in 1983 [4]. For a relativistic justification of MOND, Bekenstein [5], Sanders [6], Brownstein, and Moffat [7,8] introduce additional scalar, vector, or tensor fields. Although these (and many other) relativistic improvements of MOND are able to fit a large number of samples for about a hundred galaxies, the concern still remains. So far, we had neither a self-consistent description of the dark sector as a whole nor a direct derivation of MOND from the first principles within Einstein's general relativity. The survey [9] by Famaey and McGaugh reflects the current state of the research and contains the most comprehensive list of references.

From my point of view, the approach to the theory of the dark sector based on the use of vector fields in general relativity is very promising, and its abilities are not yet exhausted. Vector fields with the simplest Lagrangian,

$$L = a((\phi^K_{;K})^2 - m^2\phi^K\phi_K) - V_{0,} \tag{1}$$

allowed me to describe macroscopically the main features of the evolution of the Universe completely within the frames of Einstein's theory of general relativity [10]. The longitudinal nongauge massive vector field displays the repulsive elasticity. As a result, the big bang turns into a regular inflationlike state of maximum compression with the further accelerated expansion at late times. The parametric freedom of the theory allows me to forget the fine-tuning troubles. At the scales much larger than the distances between the galaxies, the Universe is homogeneous and isotropic. Its temporal evolution depends on time only. Currently, the characteristic rate of its expansion is determined by the Hubble parameter, which is of the order of the inverse time from the big bang. In the much smaller galactic scales, the situation is just the opposite. The space structure is essentially nonhomogeneous, while the influence of expansion is negligible.

In what follows, I present the macroscopic theory of dark matter, including the derivation of galaxy rotation curves, directly from the first principles within the minimal Einstein general relativity. In the galactic scale, the longitudinal nongauge vector field with the same Lagrangian (1) not only fits the observed rotation curves, but also opens a promising approach to understand the origin of the substance that we name dark matter. In the nonrelativistic limit, the expression (26), derived analytically, justifies and specifies the empirical modification of Newtonian dynamics by Milgrom [4].

II. VECTOR FIELD IN GENERAL RELATIVITY

In general relativity, the Lagrangian of a vector field ϕ_I consists of the scalar bilinear combinations of its covariant derivatives and a scalar potential $V(\phi^K\phi_K)$. A bilinear combination of the covariant derivatives is a four-index tensor $S_{IKLM} = \phi_{I;K}\phi_{L;M}$. The most general form of the scalar S, formed via contractions of S_{IKLM}, is $S = (ag^{IK}g^{LM} + bg^{IL}g^{KM} + cg^{IM}g^{KL})S_{IKLM}$, where a, b, and c are arbitrary constants. The general form of the Lagrangian of a vector field ϕ_I is

$$L = a(\phi^M_{;M})^2 + b\phi^L_{;M}\phi^{;M}_L + c\phi^L_{;M}\phi^M_{;L} - V(\phi_M\phi^M). \tag{2}$$

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The classification of vector fields ϕ_I is most convenient in terms of the symmetric $G_{IK} = \frac{1}{2}(\phi_{I;K} + \phi_{K;I})$ and antisymmetric $F_{IK} = \frac{1}{2}(\phi_{I;K} - \phi_{K;I})$ parts of the covariant derivatives. The Lagrangian (2) gets the form

$$L = a(G_M^M)^2 + (b+c)G_M^L G_L^M + (b-c)F_M^L F_L^M - V(\phi_M \phi^M).$$

The bilinear combination of antisymmetric derivatives $F_M^L F_L^M$ is the same as in electrodynamics. It becomes clear in the common notations $A_I = \phi_I/2$, $F_{IK} = A_{I;K} - A_{K;I}$.

The terms with symmetric covariant derivatives deserve special attention. In applications of the vector fields to elementary particles in flat space-time, the divergence $\frac{\partial \phi^K}{\partial x^K}$ is artificially set to zero [11]:

$$\frac{\partial \phi^K}{\partial x^K} = 0. \tag{3}$$

This restriction allows one to avoid the difficulty of a negative contribution to the energy. In the electromagnetic theory, it is referred to as the Lorentz gauge. The negative energy problem in the application to the galaxy rotation curves in view of a precaution against the instability of the vacuum was discussed by Bekenstein [5]. However, in general relativity (in curved space-time), the energy is not a scalar, and its sign is not invariant against the arbitrary coordinate transformations. From my point of view, considering vector fields in general relativity, it is worth getting rid of the restriction (3), using instead a more weak condition of regularity.

The covariant field equations

$$a\phi^K_{;K;I} + b\phi_{I;K}^{;K} + c\phi^K_{;I;K} = -V'\phi_I \tag{4}$$

and the energy-momentum tensor

$$\begin{aligned} T_{IK} = &-g_{IK}L + 2V'\phi_I\phi_K + 2ag_{IK}(\phi^M_{;M}\phi^L)_{;L} \\ &+ 2(b+c)[(G_{IK}\phi^L)_{;L} - G_K^L F_{IL} - G_I^L F_{KL}] \\ &+ 2(b-c)(2F^L{}_I F_{LK} - F^L{}_{K;L}\phi_I - F^L{}_{I;L}\phi_K) \end{aligned} \tag{5}$$

describe the behavior of the vector fields in the background of any arbitrary given metric g_{IK} [12]. Here, $V' \equiv \frac{dV(\phi_M\phi^M)}{d(\phi_M\phi^M)}$.

If the backreaction of the field on the curvature of space-time is essential, then the metric obeys the Einstein equations,

$$R_{IK} - \frac{1}{2}g_{IK}R + \Lambda g_{IK} = \varkappa T_{IK}, \tag{6}$$

with Eq. (5) added to T_{IK}. Here, Λ and $\varkappa$ are the cosmological and gravitational constants, respectively. With the account of the backreaction, the field equations (4) are not independent. They follow from the Einstein equations (6) with T_{IK} (5) due to the Bianchi identities. The field equations (4) are linear with respect to ϕ if the vector field is small, and the terms with the second and higher derivatives of the potential $V(\phi_M\phi^M)$ can be omitted.

III. DARK MATTER DESCRIBED BY A VECTOR FIELD

In curved space-time, there is no invariance against the order of covariant differentiation:

$$\phi^K_{;K;L} - \phi^K_{;L;K} = \phi^M R_{ML}.$$

In general relativity, there is no reason why the terms $\sim a$ in Eq. (2) and/or in Eq. (4) should be "less equal than others." In order to separate the dark matter from the ordinary one, it is reasonable to set $b = c = 0$. The case $a \neq 0$ is supposed to describe the dark matter only. The opposite case $a = 0$, and $b \neq 0$, $c \neq 0$, corresponds to either the electromagnetic field ($c = -b$) or to vector particles ($b \neq 0$, $c = 0$.) This way, the dark matter and the ordinary matter are separated from one another so that the ordinary matter is not taken into account twice. The dark matter is described by the Lagrangian

$$L_{\mathrm{dm}} = a(\phi^M_{;M})^2 - V(\phi_M\phi^M). \tag{7}$$

Thereafter, the field equation (4) and the energy-momentum tensor of the vector field (5) reduce to

$$a\frac{\partial \phi^M_{;M}}{\partial x^I} = -V'\phi_I, \tag{8}$$

$$T_{\mathrm{dm}\,IK} = g_{IK}[(\phi^M_{;M})^2/a + V] + 2V'(\phi_I\phi_K - g_{IK}\phi^M\phi_M). \tag{9}$$

Although the dark matter displays itself by curving the space-time, its physical nature remains unclear so far. We do not know the dependence $V(\phi_M\phi^M)$. If the vector ϕ_I remains small enough to neglect the second and higher derivatives of $V(\phi_M\phi^M)$, then the parameter

$$m^2 = \left| \frac{V'(0)}{a} \right|$$

characterizes the field. As usual, it is designated as the square of mass. In accordance with Eq. (8), the dimension of m is cm^{-1}. The covariant divergence $\phi^M_{;M}$ is a scalar, and in accordance with the Eq. (8), the massive ($m \neq 0$) field has a potential: it is a gradient of a scalar.

So far, there is no evidence of any direct interaction between dark and ordinary matter other than via gravitation. The gravitational interaction is described by Einstein equations (6) with

$$T_{IK} = T_{\mathrm{dm}\,IK} + T_{\mathrm{om}\,IK}, \tag{10}$$

where

$$T_{\mathrm{om}\,IK} = (\varepsilon + p)u_I u_K - pg_{IK} \tag{11}$$

is the well known energy-momentum tensor of macroscopic objects. The energy ε, pressure p, and temperature T of the ordinary matter obey the equation of state. If $T \ll \varepsilon$, the Einstein equations (6) with T_{IK} (10) together with the equation of state with $T = 0$ form a complete set. The field equation (8) is not independent. It is a consequence of the Einstein equations due to Bianci identities.

IV. GALAXY ROTATION CURVES

Applying general relativity to the galaxy rotation problem, it is reasonable to consider a static centrally symmetric metric

$$ds^2 = g_{IK}dx^I dx^K = e^{\nu(r)}(dx^0)^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2 \tag{12}$$

with two functions $\nu(r)$ and $\lambda(r)$ depending on only one coordinate—circular radius r. Real distribution of the stars and planets in a galaxy is neither static nor centrally symmetric. However, this simplification facilitates analyzing the problem and allows me to display the main results analytically. If a galaxy is concentrated around a supermassive black hole, the deviation from the central symmetry caused by the peripheral stars is small.

In the background of the centrally symmetric metric (12), the vector ϕ^I is longitudinal. In accordance with the field equation (8), its only nonzero component ϕ^r depends on r. In view of

$$g = \det g_{IK} = -e^{\lambda+\nu} r^4 \sin^2\theta,$$

$$\frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}}{\partial r} = \frac{2}{r} + \frac{\lambda' + \nu'}{2},$$

the covariant divergence

$$\phi^M_{;M} = \frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\phi^M)}{\partial x^M} = \frac{\partial\phi^r}{\partial r} + \left(\frac{2}{r} + \frac{\lambda' + \nu'}{2}\right)\phi^r. \tag{13}$$

In the "dust matter" approximation $p = 0$, and the only nonzero component of the energy-momentum tensor (11) is $T_{\mathrm{om}\,00} = \varepsilon g_{00}$. Whatever the distribution of the ordinary matter $\varepsilon(r)$ is, the covariant divergence $T^K_{\mathrm{om}\,I;K}$ is automatically zero. In the dust matter approximation, the curving of space-time by ordinary matter is taken into account, but the backreaction of the gravitational field on the distribution of matter is ignored. If $p = 0$, the energy $\varepsilon(r)$ is considered as a given function.

In the power series

$$V(\phi_M\phi^M) = V_0 + V'\phi_M\phi^M + O((\phi_M\phi^M)^2),$$

$V_0 = V(0)$, together with the cosmological constant Λ determine the expansion of the Universe. In the scale of galaxies, the role of expansion of the Universe as a whole is negligible, and one can set $\tilde{\Lambda} = \Lambda - \varkappa V_0 = 0$ in the Einstein equations. Omitting the second and higher derivatives of the potential $V(\phi_M\phi^M)$, we have the Einstein equations as follows (see Ref. [13], page 382 for the derivation of the left-hand sides):

$$-e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2} = \varkappa T^0_0 = \varkappa[(\phi^M_{;M})^2/a + V'e^{\lambda}(\phi^r)^2 + \varepsilon] \tag{14}$$

$$-e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2} = \varkappa T^r_r = \varkappa[(\phi^M_{;M})^2/a - V'e^{\lambda}(\phi^r)^2 - p] \tag{15}$$

$$-\frac{1}{2}e^{-\lambda}\left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu'\lambda'}{2}\right) = \varkappa[(\phi^M_{;M})^2/a + V'e^{\lambda}(\phi^r)^2 - p], \qquad I, K \neq 0, r. \tag{16}$$

Here, the prime stands for $\frac{d}{dr}$, except $V' = \frac{\partial V(\phi_M\phi^M)}{\partial(\phi_M\phi^M)}$. Among the four Eq. (8) and (14)–(16), for the unknowns ϕ^r, λ, and ν, any three are independent.

Extracting Eq. (15) from Eq. (14), we get a relation

$$\nu' + \lambda' = \varkappa r e^{\lambda}[2e^{\lambda}(\phi^r)^2 V' + \varepsilon + p]. \tag{17}$$

With account of Eqs. (13) and (17), the vector field equation (8) takes the form

$$\left[(\phi^r)' + \left(\frac{2}{r} + \varkappa r e^{2\lambda}(\phi^r)^2 V' + \frac{1}{2}re^{\lambda}(\varepsilon + p)\right)\phi^r\right]' = -m^2 e^{\lambda}\phi^r, \tag{18}$$

where $m^2 = -\frac{V'(0)}{a}$. The sign in the rhs of Eq. (18) corresponds to the case $V'(0) > 0$ $a < 0$. Negative a is taken in accordance with the requirements of regularity in application of the same Lagrangian (7) to the analysis of the role of dark matter in the evolution of the Universe [10]. It is convenient to set $a = -1$ in what follows. Hence, $V'(0) = m^2$. Equations (17) and (18) are derived with no assumptions concerning the strength of the gravitational field.

Excluding λ' from Eqs. (14) and (15), we get the following expression for ν':

$$\nu' = \varkappa r e^{\lambda}[m^2 e^{\lambda}(\phi^r)^2 + (\phi^M_{;M})^2 + p] + \frac{e^{\lambda} - 1}{r}.$$

In case of the dust matter approximation ($p = 0$),

$$\nu' = \varkappa r e^{\lambda}[m^2 e^{\lambda}(\phi^r)^2 + (\phi^M_{;M})^2] + \frac{e^{\lambda} - 1}{r}. \tag{19}$$

In a static centrally symmetric gravitational field, ν' determines the centripetal acceleration of a particle (see Ref. [13], page 323). Without dark matter, $\phi^r = 0$ (19) gives Newton's attractive potential far from the center:

$$\varphi_{\mathrm{N}}(r) = \frac{1}{2}c^2\nu(r) \sim -r^{-1}, \qquad r \to \infty.$$

The first term in the rhs of Eq. (19) appears due to the dark matter. Both terms have the same sign, and the presence of dark matter increases the attraction to the center.

The curvature of space-time caused by a galaxy is small. In the linear approximation, the influence of dark and ordinary matter can be separated from one another. For $\lambda \ll 1$, Eq. (19) reduces to

$$\nu' = \varkappa r[m^2(\phi^r)^2 + (\phi^M_{;M})^2] + \frac{\lambda}{r}, \tag{20}$$

where the first term does not contain ε. However, the contribution of dark matter comes from both additives. The vector field equation (18) and the Einstein equation (14) at $\lambda \ll 1$ are simplified:

$$(\phi^r)'' + \left(\left[\frac{2}{r} + m^2 r(\phi^r)^2 + \frac{1}{2}\varkappa r(\varepsilon + p)\right]\phi^r\right)' = -m^2\phi^r \tag{21}$$

$$\lambda' + \frac{\lambda}{r} = \varkappa r[-(\phi^M_{;M})^2 + m^2(\phi^r)^2 + \varepsilon]. \tag{22}$$

The boundary conditions for these equations,

$$\phi^r = \frac{1}{3}\phi_0' r, \qquad \lambda = \frac{1}{3}\varkappa(\varepsilon_0 - \phi_0'^2)r^2, \qquad r \to 0, \tag{23}$$

are determined by the requirement of regularity in the center. Here, $\varepsilon_0 = \varepsilon(0)$.

The term $\frac{1}{2}\varkappa r(\varepsilon + p)$ in Eq. (21) reflects the interaction of dark and ordinary matter via gravitation. If the curvature of space-time caused by the ordinary matter is small, this term is negligible compared to $2/r$. The nonlinear term $\varkappa m^2 r(\phi^r)^2$ is small compared to $2/r$ at $r \to 0$, but at $r \to \infty$, despite being small, it decreases only a little bit more quickly than $2/r$ This nonlinear term at $r \to \infty$ decreases as $(r \ln r)^{-1}$:

$$\varkappa m^2 r(\phi^r)^2 = \frac{2\sin^2 mr}{3r\ln\frac{r}{r^*}} \approx \frac{1}{3r\ln\frac{r}{r^*}}, \qquad r^* \sim \frac{1}{m}.$$

Neglecting both nonlinear terms in square brackets, the field equation (21) reduces to

$$\left((\phi^r)' + \frac{2}{r}\phi^r\right)' = -m^2\phi^r.$$

Its regular solution is

$$\phi^r = \frac{\phi_0'}{m^3 r^2}(\sin mr - mr\cos mr), \qquad \phi^M_{;M} = \phi_0'\frac{\sin mr}{mr}, \tag{24}$$

where $\phi_0' = \phi^M_{;M}(0)$. Substitution of Eq. (24) into Eq. (20) results in

$$\nu'(r) = \frac{\varkappa(\phi_0')^2}{m^2 r}f(mr) + \frac{\lambda}{r}, \qquad \lambda \ll 1.$$

Function $f(x)$,

$$f(x) = \left(1 - \frac{\sin 2x}{x} + \frac{\sin^2 x}{x^2}\right) = \begin{cases} x^2 - \frac{2}{9}x^4 + \dots, & x \to 0 \\ 1, & x \to \infty \end{cases}, \tag{25}$$

is presented in Fig. 1 (upper curve).

The balance of the centripetal $\frac{c^2\nu'}{2}$ and centrifugal $\frac{V^2}{r}$ accelerations determines the velocity V of a rotating object as a function of the radius r of its orbit:

$$V(r) = \sqrt{V_{\mathrm{pl}}^2 f(mr) + \frac{c^2}{2}\lambda(r)}, \tag{26}$$

$$V_{\mathrm{pl}} = \sqrt{\frac{\varkappa}{2}}\frac{c\phi_0'}{m}. \tag{27}$$

Far from the center, $\lambda(r)$ decreases as $1/r$, while $f(mr) \to 1$ The dependence $V(r)$ (26) turns at $r \gtrsim m^{-1}$ from linear to a plateau with damping oscillations. The plateau appears entirely due to the vector field. At the same time, the vector field contributes to $\lambda(r)$ as well. Regular at $r \to 0$, the solution of Eq. (22) is

$$\lambda(r) = 2\left(\frac{V_{\mathrm{pl}}}{c}\right)^2\Psi(mr) + \frac{\varkappa}{r}\int_0^r \varepsilon(r)r^2 dr, \qquad \lambda \ll 1. \tag{28}$$

The last term in Eq. (28) gives Newton's potential. The function

$$\Psi(x) = \frac{1}{x}\int_0^x \left(\frac{\sin^2 y}{y^2} - \frac{\sin 2y}{y} + \cos 2y\right)dy \tag{29}$$

is shown in Fig. 1 (lower curve). The radial dependence $V(r)/V_{\mathrm{pl}}$ at $\varepsilon \to 0$

$$V(r)/V_{\mathrm{pl}} = \sqrt{f(mr) + \Psi(mr)}, \tag{30}$$

is shown in Fig. 2. In the limit $\lambda \ll 1$, the galaxy rotation curve driven by the dark matter only is a universal

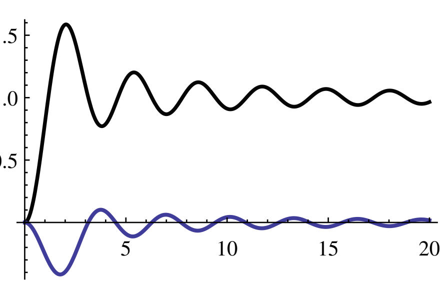

FIG. 1 (color online). Function $f(x)$ (25) on the upper curve and $\Psi(x)$ (29) on the lower curve.

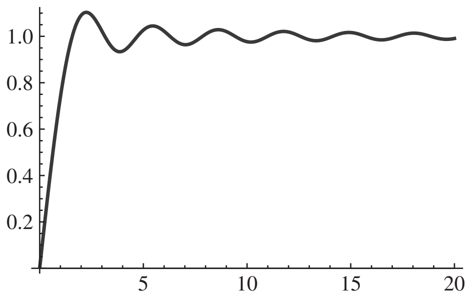

FIG. 2. Function $\sqrt{f(x) + \Psi(x)}$ in Eq. (30).

function (30). In dimensionless units, there are no parameters; see Fig. 2.

The limiting plateau value $V_{\rm pl}$ (27) is connected with the single parameter ϕ_0'/m. The period of oscillations is $\frac{2\pi}{m}$. The whole curve, including the damping oscillations, is determined by two physical parameters ϕ_0' and m.

The form of an observed curve allows me to restore the value of the parameter $\phi_0' = \phi^M_{;M}(0)$ at $r \to 0$ in the boundary conditions (23) As far as there is no evidence of any direct interaction of dark and ordinary matter, the origin of specific values ϕ_0' and m of a particular galaxy depends on what happens in the center. The values $V_{\rm pl}$ and m can differ from one galaxy to another. It looks like for each galaxy these values are driven by some heavy object (maybe a black hole, maybe a neutron star) located in the center (by the way, supporting the central symmetry of the gravitational field).

According to modern concepts, there is only some 5% of ordinary matter in the Universe, while the amount of dark matter is about 25%. If so, the main contribution comes from the dark matter, and the deviations from the universal curve (30) caused by the ordinary matter are small, especially if the major mass of a galaxy is provided by a black hole (or a neutron star) located in the center. In the case of spherical symmetry, the averaged distribution of stars $\varepsilon(r)$ outside the center could be restored from the deviations of the observed curves from the universal one.

Dark matter, described by a vector field with the Lagrangian (1), actually justifies the empirical Milgrom hypothesis of MOND—the modified Newton dynamics [4]. Newton's dynamics really gets modified by the vector field so that the rotation curve flattens out at the far periphery of a galaxy. Naturally, basing only on the intuitive arguments, it was scarcely possible to guess that the transition to a plateau is accompanied by damping oscillations.

However, the question of the origin of dark matter remains open. In other words, what makes $\phi_0' = \phi^K_{;K}(0)$ different from zero? Solutions of the linearized Einstein equations do not answer this question. In the dust matter approximation and weak gravitational field, ϕ_0' and m remain free parameters. The limiting plateau value (27) is determined by their single combination $\sqrt{\frac{\varkappa}{2}}\frac{c\phi_0'}{m}$. Is it the same as predicted by MOND, $V_{\rm pl} = (\varkappa M a_0)^{1/4}$, which is currently considered as actually observed? I admit that the answer to this question—the relation of $\frac{\phi_0'}{m}$ with a heavy mass M in the center—could be received by the self-consistent solution of the nonlinear Einstein equations. Interaction with dark matter via gravitation should affect the equilibrium structure of heavy stars and can shift the collapse boundary. If the gravitation is not weak, the parameters ϕ_0' and m in boundary conditions (23) should be determined self-consistently together with the structure of the heavy object in the center. If $p \neq 0$, the radial distribution of the ordinary matter and gravitational field are interdependent. It looks convenient to consider this problem in the approximation of cold degenerate relativistic gas and use the chemical potential μ_0 in the boundary conditions instead of ε_0. It is worth reconsidering the equilibrium [14] and collapse [15] of supermassive bodies taking the dark matter into account. However, it is a different story.

V. FITTING

The field itself is zero in the center, $\phi^r(0) = 0$, and the contributions of the dark matter and of the ordinary matter are introduced to the boundary conditions (23) by the values $(\phi_0')^2$ and ε_0, respectively.

When there is a plateau, the speed of rotation on the plateau $V_{\rm pl}$ (27), which is determined from the Doppler shift of spectral lines, provides us with information about the input parameter $\phi_0' = \phi^M_{;M}(0)$ in the boundary conditions. The parameter m is determined by scaling the radial coordinate so that the period of oscillations of $f(x)$ (25) fits the observations. While the distribution of dark matter is characterized unambiguously by the two parameters ϕ_0' and m, the situation with the density of the ordinary matter $\varepsilon(r)$ in galaxies is not that clear. Radiation coming from the galaxies does not carry information about cooled nonemitting stars and planets. Just the opposite, the strict fitting could provide us with the distribution of the ordinary matter in galaxies.

There are hundreds of graphs with rotation curves for different galaxies in the literature. It is impossible to present the fitting for all of them within one paper. It is reasonable to analyze the relative role of the major parameters of the theory and give some examples.

In the dust matter approximation and weak gravitational field, $\varepsilon(r)$ is an arbitrary given function. To demonstrate the relative role of dark and ordinary matter, I use the Gauss distribution for the density of dust matter,

$$\varepsilon(r) = \varepsilon_0 \exp(-r^2/r_0^2). \tag{31}$$

Qualitatively, a particular form of a monotonically decreasing function $\varepsilon(r)$ is not essential. (The existence of a hard core in the center is a special case.) ε_0 is the maximum density in the center, and r_0 is the mean radius of a galaxy.

The total mass of a galaxy $M \sim \varepsilon_0 r_0^3$. Although the dark and ordinary matter are input into the boundary conditions (23) via ϕ_0' and $\varepsilon_0 = \varepsilon(0)$, it looks more clear to demonstrate their relative role using $\varepsilon_0 r_0^3$ (proportional to the total rest energy of a galaxy) instead of ε_0.

In all Figs. 3 and 4, the dashed curve is the rotation curve (30) without ordinary matter. It is the same curve as in Fig. 2. In each case, the radial scales are specifically chosen to clarify the difference better. Solid lines in Fig. 3 are rotation curves with $m^2 r_0^2 = 1$, 10, and 0.1, respectively (the three cases in which the radius r_0 of a galaxy is equal, $\sqrt{10}$ times larger, and $\sqrt{10}$ times smaller than the period $\sim m^{-1}$ of oscillations, respectively). The ratio $\frac{\varepsilon_0}{(\phi_0')^2}(mr_0)^3 = 1$. The smaller $\frac{\varepsilon_0}{(\phi_0')^2}(mr_0)^3$ is, the less the difference between solid and dashed curves is.

Solid curves in Fig. 4 are rotation curves for a fixed $\frac{\varepsilon_0}{(\phi_0')^2}(mr_0)^3 = 10$, and $m^2 r_0^2 = 1$, 10, and 0.1, respectively. As $m^2 r_0^2$ grows, the oscillations are smoothed out, and when it decreases, the difference between the curves moves to the center.

One can find over a hundred graphs of galaxy rotation curves in the literature, including those displaying the transition to a plateau. One of them often referred to, marked UMa: NGC 3726, is shown in Figs. 5(a) and 5(b). Both graphs are taken from different places within the same list [7].

The points with error bars are the observations. The solid lines are the rotation curves determined from so-called metric-skew-tensor gravity (MSTG) [7,8,16]. The solid line in Fig. 5(a) practically coincides with MOND.

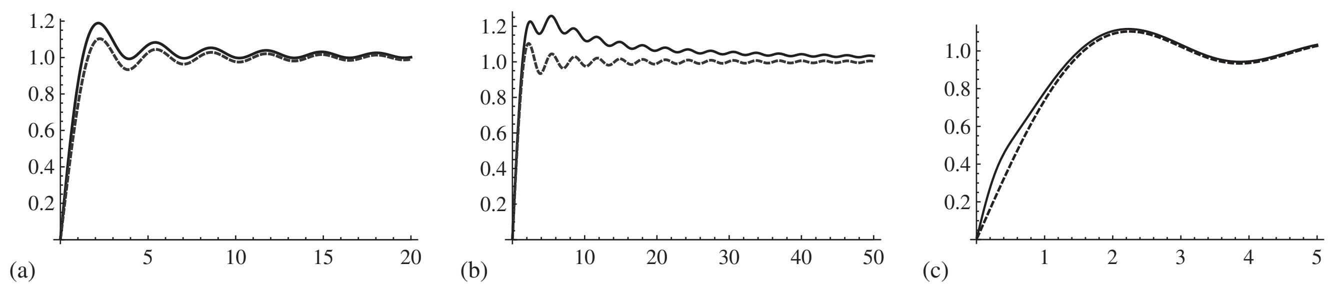

FIG. 3. Solid curve—(26) with (a) $\frac{\varepsilon_0}{(\phi_0')^2}(mr_0)^3 = 1$, $m^2 r_0^2 = 1$; (b) $\frac{\varepsilon_0}{(\phi_0')^2}(mr_0)^3 = 1$, $m^2 r_0^2 = 10$; (c) $\frac{\varepsilon_0}{(\phi_0')^2}(mr_0)^3 = 1$, $m^2 r_0^2 = 0.1$. Dashed curve—(30).

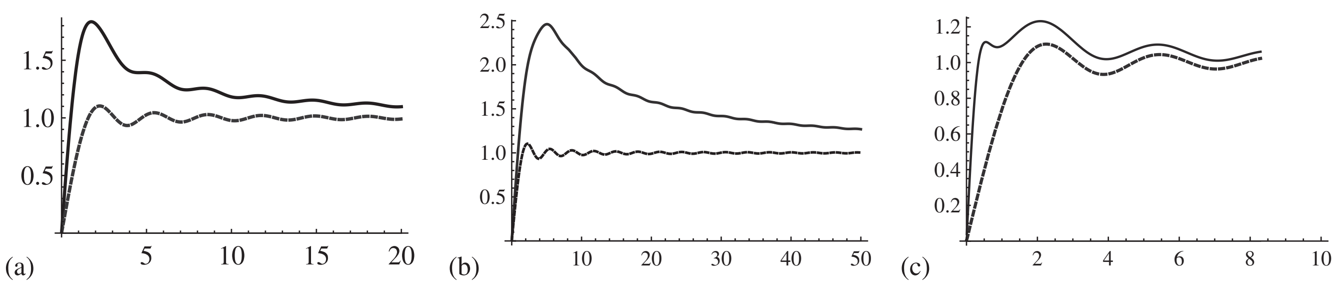

FIG. 4. Solid curve—(26) with (a) $\frac{\varepsilon_0}{(\phi_0')^2}(mr_0)^3 = 10$, $m^2 r_0^2 = 1$ (b) $\frac{\varepsilon_0}{(\phi_0')^2}(mr_0)^3 = 10$, $m^2 r_0^2 = 10$ (c) $\frac{\varepsilon_0}{(\phi_0')^2}(mr_0)^3 = 10$, $m^2 r_0^2 = 0.1$. Dashed curve—(30).

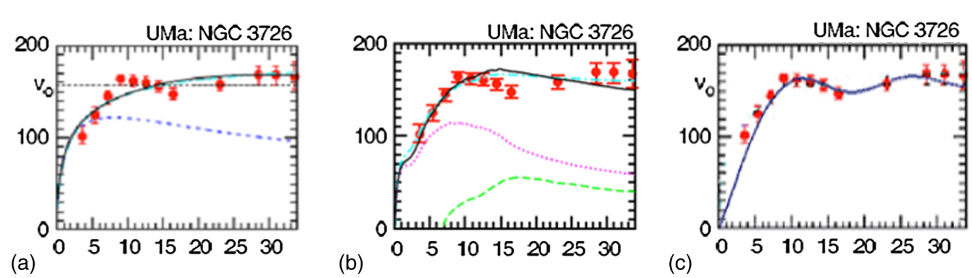

FIG. 5 (color online). (a) Fitting by MSTG, practically coinciding with MOND; (b) another fitting by MSTG, slightly different from MOND; and (c) points are observations, fitted by Eq. (26) together with Eqs. (28) and (31).

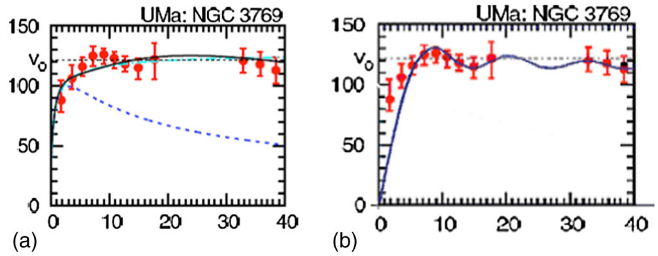

FIG. 6 (color online). (a) The solid line is fitting via MSTG and MOND. The dashed line is the Newton dynamics; (b) points are observations, and the solid curve is fitting by Eq. (26) together with Eqs. (28) and (31).

In Fig. 5(b), it is slightly different from MOND. Other dashed and dotted lines correspond to the ordinary Newton dynamics.

The curve in Fig. 5(c) shows how Eq. (26) together with Eqs. (28) and (31) fit the observations. The input parameters are $V_{\mathrm{pl}} = 158\,\frac{\mathrm{Km}}{\mathrm{sec}}$, $\frac{\varkappa\varepsilon_0}{m^2} = 0.00000005$, and $mr_0 = 3.78$.

Figure 6 is another example of the comparison of fitting by MSTG MOND [Fig. 6(a)] and in accordance with Eqs. (26), (28), and (31) [Fig. 6(b)]. The input parameters are $V_{\mathrm{pl}} = 120\,\frac{\mathrm{Km}}{\mathrm{sec}}$ $\frac{\varkappa\varepsilon_0}{m^2} = 0.00000005$, and $mr_0 = 2.24$. These examples clearly demonstrate the existence of oscillating features, which is the main observational signature of dark matter.

Frankly speaking, it is a surprise for me. I did not expect such a coincidence. Deviations at small radii can be related to the presence of an additional strongly gravitating compact object located at the center. As shown in Figs. 3(c) and 4(c) in case $r_0 \ll m^{-1}$, the initial growing part of the curve $V(r)$ shifts to the center. At the same time, if a heavy object in the center exists, it supports the central symmetry, and the gravitational field becomes only slightly distorted by other stars and planets of the galaxy.

VI. SUMMARY

The nongauge vector field with as simple a Lagrangian as possible (1) provides the macroscopic description of all major observed properties of the dark sector within Einstein's theory of general relativity.

In the galaxy scale, the field with the energy-momentum tensor (9) allows me to describe analytically the galaxy rotation curves in detail. The formulas (26)–(29) are derived completely within Einstein's theory. Thus, there is no need for any modifications of the general relativity to explain the observable plateau in rotation curves.

As I have shown previously [10], the vector fields with the same Lagrangian (1) are adequate tools for the macroscopic description of the main features of evolution of the Universe. In the scale of the whole Universe, the zero-mass field corresponds to the dark energy, and the massive one corresponds to dark matter. Price issue is the rejection of the prejudice (widely spread, unfortunately) that the energy should not be negative. Instead, I used a weaker condition of regularity. In general relativity, the energy is not a scalar, and its sign is not invariant against the arbitrary coordinate transformations. Described by the vector field with the same Lagrangian (1), the dark matter is of the same physical nature in both applications: to cosmology [10] and to galaxy rotation curves.

As a matter of fact, I agree with the Sanders's statement that "...the correct theory may well be one in which MOND reflects the influence of cosmology on local particle dynamics and arises only in a cosmological setting" [6]. However, it is evident that I do not share the Sanders's conclusion: "It goes without saying that this theory is not General Relativity, because in the context of General Relativity local particle dynamics is immune to the influence of cosmology" [6]. I have presented here the complete derivation from the Einstein equations (14)–(16) to the galaxy rotation curve (26).

There are attempts of applying the scalar, vector, and tensor fields in order "to explain the flat rotation curves of galaxies and cluster lensing without postulating exotic dark matter" [8]. In quantum physics, each elementary particle is a quantum of some field, and, vice versa, each field corresponds to its own quantum particle [17]. From my point of view, the various fields are just convenient mathematical instruments that we use for the description of physical phenomena, no matter how we name them.

According to the observations, the period of oscillations $\frac{2\pi}{m}$ [see Figs. 5(c) and 6(b)] is some 15 kpc. If in quantum mechanics it is the de Broglie wavelength $\lambda\!\!\!^{-} = \frac{\hbar}{mc}$, then the rest energy of a quantum particle, corresponding to the vector field, should be $mc^2 \sim 2.5 \times 10^{-27}$ eV.

The theory predicts the oscillating features with no baryonic counterparts in the rotation curves of the outer regions of galaxies. As this would be the main observational signature of the existence of dark matter, I persistently recommend this observational test.

A few words about fine-tuning. I have come across this situation three times. The first one was the widely used in the 1950s "Bennet pinch" [18]—a fine-tuned solution of equilibrium of a high current channel in which the magnetic attraction is balanced by the gas pressure of electric charges. In reality, it came out to be a boundary between the expansion and compression when the balance is broken [19]. For the second time, it was the conclusion of the existence of the limiting mass of an ultrarelativistic star by Chandrasekhar [20] and Landau [21]. The fine-tuned solution of equilibrium with an ultrarelativistic equation of state turned out to be the boundary of the gravitational collapse [15]. The third time it was the fine-tuned singular cosmological solution by Friedman [22], Robertson [23], and Walker [24]. In the 1930s, dark matter had not been taken seriously. With the account of dark matter, the Friedman-Robertson-Walker singular solution turned out to be a lower boundary of the regular oscillating cosmological solutions [10]. In all the cases, the requirement of regularity rules out the problem of fine-tuning.

From my point of view, it is time to reconsider the equilibrium [14] and collapse [15] of supermassive bodies, taking into account the dark matter.

ACKNOWLEDGMENTS

I am thankful to K. A. Bronnikov, M. Yu. Kagan, and V. I. Marchenko for discussions and to E. R. Podolyak for the help with analysis of the equations.

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