Stability analysis of chromo-natural inflation and possible evasion of Lyth's bound

Emanuela Dimastrogiovanni[1] and Marco Peloso[1]

[1]*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*

We perform the complete stability study of the model of chromo-natural inflation by Adshead and Wyman [Phys. Rev. Lett. **108**, 261302 (2012)], where, due to its coupling to a SU(2) vector, a pseudoscalar inflaton χ slowly rolls on a steep potential. As a typical example, one can consider an axion with a sub-Planckian decay constant f. The model was recently studied by Dimastrogiovanni, Fasiello, and Tolley [J. Cosmol. Astropart. Phys. **02**, 046 (2013)] in the $m_g \gg H$ limit, where m_g is the mass of the fluctuations of the vector field, and H the Hubble rate. We show that the inflationary solution is stable for $m_g > 2H$, while it otherwise experiences a strong instability due to scalar perturbations in the subhorizon regime. The tensor perturbations are instead enhanced at large m_g, while the vector ones remain perturbatively small. Depending on the parameters, this model can give a chiral gravity wave signal that can be detected in ongoing or forthcoming cosmic microwave background experiments. This detection can occur even if, during inflation, the inflaton spans an interval of size $\Delta\chi = O(f)$ which is some orders of magnitude below the Planck scale, evading a well known bound that holds for a free inflaton by Lyth [Phys. Rev. Lett. **78**, 1861 (1997)]. The spectral tilt of the scalar perturbations typically decreases with decreasing m_g. Therefore the simultaneous requirements of stability, sufficiently small tensor-to-scalar ratio, and sufficiently flat scalar spectrum can pose nontrivial bounds on the parameters of the model.

I. INTRODUCTION

Inflation is a successful paradigm for the physics of the early Universe [1]. Besides solving the classical problems of modern cosmology (e.g., the flatness, entropy, and horizon problems), it provides primordial perturbations in perfect agreement with the observations [2,3]. A challenge for inflationary models is to protect the required flatness of the inflaton potential against radiative corrections. This protection can be provided by an approximate shift symmetry as in models of natural inflation [4,5]. The symmetry can be broken by a controllably small amount; the best known example of this is the case of an axion field acquiring a potential from instantons. The application of this to inflation, however, requires a greater than Planckian axion decay constant f [6], which may not be stable against gravitational corrections [7] and which may be impossible to realize in string theory [8]. Proposed solutions to this problem include using two [9] or more [10] axions, which provide an effective large scale evolution even if the decay constants of the original axions are sub-Planckian, requiring nontrivial compactifications in string theory [11], suitably coupling the axion to a 4-form [12], modifying the axion kinetic term [13], and slowing down the axion evolution through particle production [14,15] as in warm inflation [16].

In particular, in the mechanism of [14] the dissipation occurs through the production of a U(1) field coupled to the inflaton χ through the interaction $\chi F\tilde{F}$ [where F is the U(1) field strength, and $\tilde{F}$ its dual]. Reference [17] showed that this coupling can also affect the background

evolution if the U(1) field is replaced by a SU(2) field with a nonvanishing vacuum expectation value (vev). Specifically, due to the interaction with the vev of the vector multiplet, the inflaton can be in slow roll even if its potential would otherwise (i.e., in absence of this interaction) be too steep to give inflation.[1] Such a model has been dubbed *chromo-natural inflation* in [17].[2]

The model is characterized by the action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P}{2} R - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2}(\partial\chi)^2 - V(\chi) + \frac{\lambda}{8f\sqrt{-g}} \chi \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} \right], \quad (1)$$

where χ is the axion inflaton, with the potential

$$V = \mu^4 \left[1 + \cos\frac{\chi}{f} \right] \quad (2)$$

---

[1]This mechanism has been extended to a Chern-Simons interaction in [18]. For recent reviews of vector fields in inflation, see [19].

[2]Chromo-natural inflation has trajectories in common with the so-called *gauge-flation* model [20,21]—a model characterized by a SU(2) field with a $(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$ term besides the usual $F_{\mu\nu}F^{\mu\nu}$ Yang-Mills term—in the limit in which the axion is close to the bottom of its potential [22,23]. Perturbations of gauge-flation were studied in the last reference among [19].

and $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\epsilon^{abc}A_\mu^b A_\nu^c$. We use the convention $\epsilon^{0123} = 1$ for the Levi-Civita tensor. The vector field has the vev[3]

$$A_0^a = 0, \qquad A_i^a = \delta_i^a a(t)Q(t), \qquad (3)$$

which is chosen to give isotropic expansion. We note that, for a generic theory with vector fields, an isotropic solution may be unstable against anisotropic perturbations. For instance, this can be the case when a dilaton-like coupling $f(\varphi)F^2$ of a scalar inflaton φ to a SU(2) field with the vev (3) or to an orthogonal U(1) triplet is arranged so as to produce a scale invariant spectrum for the vectors.[4] Isotropy is instead preserved if the vector fields are massive [27], as in the current model (if the mass arises from an explicit breaking, one should also check that the theory has no ghosts [28]).

Reference [17] performs a thorough analysis of the background evolution of the model. The study of the chromo-natural inflation theory at the perturbative level was recently performed in [29], for

$$m_g^2 \equiv 2g^2 Q^2 \gg H^2, \qquad (4)$$

where m_g is the mass of the vector field fluctuations in this limit [29] (as we also discuss below). When this condition is realized, the vector field can be integrated out while, at the same time, leaving its mark on the inflationary dynamics (this is an explicit realization of the so-called *gelaton* [30] mechanism). In this limit, chromo-natural inflation is equivalent to a single scalar field $P(X, \chi) = X + X^2/\Lambda^4 - V(\chi)$ theory [where $X \equiv -(\partial\chi)^2/2$ and $\Lambda^4 \equiv 8f^4 g^2/\lambda^4$]. The noncanonical kinetic term precisely encodes the effect of the gauge fields.

In [29] it is also shown that an effective field theory equivalence of chromo-natural inflation to a noncanonical $P(X, \chi)$ Lagrangian holds as long as $m_g^2 > 8H^2$. Beyond this limit, a general perturbative study of the dynamics of the gauge field and the axion becomes necessary in order to test the stability of the theory and formulate its predictions. This is precisely the scope of the present work. We perform a full linear order study of scalar, tensor and vector perturbations and we show that the inflationary background solution of the model is stable for $m_g > 2H$, and it is otherwise unstable.

A full phenomenological study of the model is beyond our purposes. Nonetheless, we explore the scalar and tensor modes' production for a given choice of $f \ll M_p$ and $\lambda \gg 1$ for which the coupling with the vector fields is crucial to ensure slow roll. Specifically, we choose $f = 10^{-2}M_p$, and $\lambda = 500$, while m_g/H is a free parameter controlled by the value of g. The stability condition $m_g/H > 2$ provides a lower bound for this ratio. Too small values are also excluded because they lead to a too red spectrum of the scalar modes (we obtained this numerically; this behavior is also seen in the analytic study of [29] in the regime of validity of their analysis). On the other hand, the amount of gravity wave signal increases with m_g/H, and a level which can be observed in the current or forthcoming experiments [31] can be obtained even if the inflaton in the model spans a range of some orders of magnitude below M_p, evading the so called Lyth bound [32] (in contrast to what was expected in [17]). Therefore, requiring a stable solution, with a sufficiently flat scalar power spectrum, and sufficiently small tensor modes provides constraints on the parameters of the model that go in opposite directions. For our choice $f = 10^{-2}M_p$ and $\lambda = 500$, we could not find any acceptable solution. We expect that the situation should improve at larger f, where the inflaton potential becomes flatter (and the model approaches conventional slow roll inflation).

The paper is organized as follows: in Sec. II we review the background evolution of the model for different (stable or unstable) ranges of the theory; in Sec. III we start the perturbation analysis by introducing the most general decomposition of the metric, the inflaton, and the gauge field fluctuations, identifying the physical degrees of freedom and verifying that scalar, vector and tensor perturbations decouple at linear order; we also quantize the system and define the cosmological correlators; in Secs. IV, V, and VI we study the fluctuations for, respectively, tensor, vector and scalar modes; finally in Sec. VII we draw our conclusions.

We supplement our perturbation analysis with three appendixes. In Appendix A we outline in detail the method that we use to compute the perturbations. We explicitly show that the presence of vector fields does not give rise to any additional conceptual difficulty with respect to the case of scalar field inflation. In Appendix B we write the quadratic action for the scalar perturbations of the model, disregarding the terms that involve metric perturbations. These terms are given in Appendix C, where we explicitly show that neglecting scalar metric fluctuations does not affect the stability analysis of the model (while it considerably simplifies the computations).

---

[3]We follow the standard convention of using greek letters for space-time indices, $i, j, k \dots$ for space indices, and $a, b, c, \dots$ for internal SU(2) indices. The index a should not be confused with the scale factor $a(t)$, that enters in the line element as $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j = a^2(\tau)[-d\tau^2 + \delta_{ij}dx^i dx^j]$. We denote by dot a derivative with respect to physical time t, and by prime a derivative with respect to conformal time τ.

[4]This effect originates from the sum of the IR modes that, in general, strongly breaks isotropy [24]. It is however possible that we live in a realization of inflation where this effect is small, as we believe that must be assumed in the computations of [25]. Analogous considerations may apply to the model of [26].

II. THE MODEL AND THE BACKGROUND SOLUTION

Chromo-natural inflation is described by the action (1) of a pseudoscalar field χ coupled to a SU(2) gauge

multiplet. The vector multiplet has the vev (3). The 00 component of Einstein equations reads

$$3M_P^2 H^2 = \frac{3}{2}(\dot{Q} + HQ)^2 + \frac{3}{2}g^2 Q^4 + \frac{\dot{\chi}^2}{2} + V(\chi). \quad (5)$$

Inflation can only occur if the potential $V(\chi)$ is the major contribution to the total energy density, i.e., $3H^2 M_P^2 \simeq V$; the parameters $\epsilon_1 \equiv Q^2/M_P^2$, $\epsilon_2 \equiv g^2 Q^4/H^2 M_P^2$, $\epsilon_Q \equiv \dot{Q}^2/H^2 M_P^2$ and $\epsilon_\chi \equiv \dot{\chi}^2/2H^2 M_P^2$ are all much smaller than unity.

The equations of motion for the inflaton and the gauge field are

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\mu^4}{f}\sin\left(\frac{\chi}{f}\right) = -\frac{3g\lambda}{f}Q^2(\dot{Q} + HQ), \quad (6)$$

$$\ddot{Q} + 3H\dot{Q} + (\dot{H} + 2H^2)Q + 2g^2 Q^3 = \frac{g\lambda}{f}Q^2\dot{\chi}. \quad (7)$$

If we neglect $\ddot{\chi}$, $\ddot{Q}$ and $\dot{H}$, one can solve Eqs. (6) and (7) for $\dot{\chi}$ and $\dot{Q}$ [17]

$$\dot{\chi} \simeq \frac{g\lambda f Q^2 H(\frac{2g^2 Q^3}{H} - HQ - \frac{fV_{,\chi}}{g\lambda Q^2})}{3f^2 H^2 + g^2\lambda^2 Q^4}, \quad (8)$$

$$\dot{Q} \simeq -\frac{HQ(2f^2 H^2 + 2g^2 f^2 Q^2 + g^2\lambda^2 Q^4) + \frac{g\lambda Q^2 f V_{,\chi}}{3}}{3f^2 H^2 + g^2\lambda^2 Q^4}. \quad (9)$$

In these equations we then assume

$$3f^2 H^2 \ll g^2\lambda^2 Q^4, \qquad \lambda^2 Q^2 \gg 2f^2, \quad (10)$$

and we obtain

$$\dot{\chi} \simeq \frac{fH}{g\lambda Q^2}\left(\frac{2g^2 Q^3}{H} - HQ - \frac{fV_{,\chi}}{g\lambda Q^2}\right), \quad (11)$$

$$\dot{Q} \simeq -HQ - \frac{fV_{,\chi}}{3g\lambda Q^2}. \quad (12)$$

Equation (12) can be rewritten as an equation of motion for the gauge field in terms of an effective potential

$$H\dot{Q} + \frac{\partial V_{\text{eff}}(Q)}{\partial Q} = 0, \qquad V_{\text{eff}} \equiv \frac{H^2 Q^2}{2} - \frac{fHV_\chi}{3g\lambda Q}, \quad (13)$$

where V_{eff} is minimized by

$$Q_{\text{min}} = \left(\frac{\mu^4 \sin(\chi/f)}{3g\lambda H}\right)^{1/3}. \quad (14)$$

In the minimum, the two terms of V_{eff} are parametrically equal to each other. Therefore, the same is true for the last two terms in (11). As a consequence, the first term on the right-hand side of (11) dominates over the other two in the regime $m_g^2 \gg H^2$ studied in [29]. This implies that [29]

$$m_g^2 = 2g^2 Q^2 \gg H^2 \Rightarrow m_g^2 \simeq g\lambda Q\frac{\dot{\chi}}{f}. \quad (15)$$

For general values of the parameters, inserting Q_{min}, Eq. (11) becomes

$$\dot{\chi} \simeq \frac{2}{3^{2/3}}\frac{f^{4/3}}{\lambda^{4/3}}\frac{3\frac{\lambda^{2/3}}{f^{2/3}}H^{8/3} + 3^{1/3}g^{4/3}(-V_{,\chi})^{2/3}}{g^{2/3}H^{1/3}(-V_{,\chi})^{1/3}}. \quad (16)$$

By looking at Eq. (6), one realizes that, when the gauge field settles in its minimum, it originates an actual damping term for the motion of the axion. Notice also that, when $Q = Q_{\text{min}}$, the kinetic energy of the fields can be disregarded [see e.g., Eq. (12)]; therefore the slow roll parameter (which would normally receive contributions also from ϵ_Q and ϵ_χ) will be, instead, mostly due to ϵ_1 and ϵ_2

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{Q^2}{M_P^2} + \frac{g^2 Q^4}{H^2 M_P^2}. \quad (17)$$

Finally we have

$$\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \simeq \frac{2g^2 Q^4}{H^2 M_P^2} + \frac{\dot{Q}}{HM_P^2\epsilon}\left(2Q + \frac{4g^2 Q^3}{H^2}\right). \quad (18)$$

From Eq. (16) we can compute the number of e-foldings

$$N \simeq \int dx \frac{\frac{3^{1/3}}{2}(\frac{\mu}{M_P})^{4/3}g^{2/3}\lambda^{4/3}(\sin x)^{1/3}(1 + \cos x)^{2/3}}{(\frac{\mu}{M_P})^{8/3}\lambda^{2/3}(1 + \cos x)^{4/3} + 3^{2/3}g^{4/3}(\sin x)^{2/3}}, \quad (19)$$

where we defined $x \equiv \chi/f$.

Let us introduce the parameter y

$$y \equiv \left(\frac{\lambda\mu^4}{3g^2 M_P^4}\right)^{2/3}, \quad (20)$$

in terms of which the expression (19) is rewritten as

$$N \simeq \frac{3\sqrt{y}\lambda}{2}\int\frac{dzz}{2y + z - yz^3}, \qquad z \equiv \left(1 - \cos\frac{\chi}{f}\right)^{1/3}. \quad (21)$$

An upper bound on N can be obtained by "pretending" that the slow roll approximation holds at all values of χ, corresponding to $0 \leq z \leq 2^{1/3}$. The resulting upper limit is maximized for $y \simeq 1$, where it evaluates to $N \lesssim 0.6\lambda$. In the $y \ll 1$ limit, the upper limit acquires the simple form $N < \frac{3\sqrt{y}\lambda}{2^{2/3}}$.

In terms of y, the slow roll solutions read

$$H \simeq \frac{\mu^2}{\sqrt{3}M_p}\sqrt{1+\cos x} \equiv \frac{\mu^2}{\sqrt{3}M_p}f_H(x),$$

$$Q \simeq M_p \frac{y^{1/4}}{\lambda^{1/2}}\left(\frac{\sin x}{\sqrt{1+\cos x}}\right)^{1/3} \equiv \frac{M_p}{\lambda^{1/2}}f_H(x)f_A(x), \qquad (22)$$

$$\dot{\chi} \simeq \frac{2f\mu^2}{\sqrt{3}M_p\lambda}\frac{y(1+\cos x)^{4/3}+(\sin x)^{2/3}}{\sqrt{y}(1+\cos x)^{1/6}(\sin x)^{1/3}} \simeq \frac{Hf}{\lambda}f_{\dot{\chi}}(x),$$

where we have defined

$$f_H \equiv \sqrt{1+\cos x}, \qquad f_A \equiv \frac{y^{1/4}(\sin x)^{1/3}}{(1+\cos x)^{2/3}},$$

$$f_{\dot{\chi}} \equiv 2\frac{y(1+\cos x)^{4/3}+(\sin x)^{2/3}}{\sqrt{y}(1+\cos x)^{2/3}(\sin x)^{1/3}}. \qquad (23)$$

Figure 1 allows us to appreciate the accuracy of these slow roll approximations. In the figure we show the evolution of $\dot{\chi}$ for four choices of y, comparing the exact evolution with the slow roll approximation (22).[5] In all cases, we take $f = 0.01M_p$, as the main purpose of this model is to allow slow roll for $f \ll M_p$. This is "compensated" by $\lambda \gg 1$, and in all cases we fixed for definiteness $\lambda = 500$. We note that fixing the value of y does specify a relation between μ and g, but does not fix these two values. As typical in inflationary models, the parameters can be specified only from fixing the normalization of the power spectrum of the scalar modes to the observed value $P_\zeta \simeq 2.5 \times 10^{-9}$ [2]. The evolutions shown in the figure cover 60 e-folds of inflation.

We note from the figure that $\dot{\chi}$ performs very small oscillations around the slow roll solution. We believe that they are due to the fact that $Q(t)$ is tracking a time-dependent minimum (14) (we do not show this here, but also this tracking is extremely accurate during the slow roll phase). The oscillations appear somewhat large in the logarithmic scale chosen, but we see that they do not lead to any net departure from the slow roll solution. Moreover, in Sec. VI we compare full numerical solutions of the scalar perturbations, obtained using the full numerical background solutions, with analytical solutions, for

---

[5]The initial conditions for the numerical evolution shown in the figure are chosen as follows: we fix an initial value for χ; we then employ (22) to have the initial value for $\dot{\chi}$ and Q. We set $\dot{Q} = 0$, and we then obtain the initial value for $\dot{a}$ by solving the 00 Einstein equations exactly at the initial time. We make sure that the initial value of χ leads to more than 60 e-folds of inflation (so that the slow roll solution can be achieved; we note that the system starts slightly displaced from the slow roll solution, since we set $\dot{Q} = 0$; the displacement is however very small, since $\dot{Q} \ll HQ$ in the slow roll solution, and the background evolution quickly approaches the slow roll solution). The evolutions shown in Fig. 1 are restricted to the final 60 e-folds of inflation.
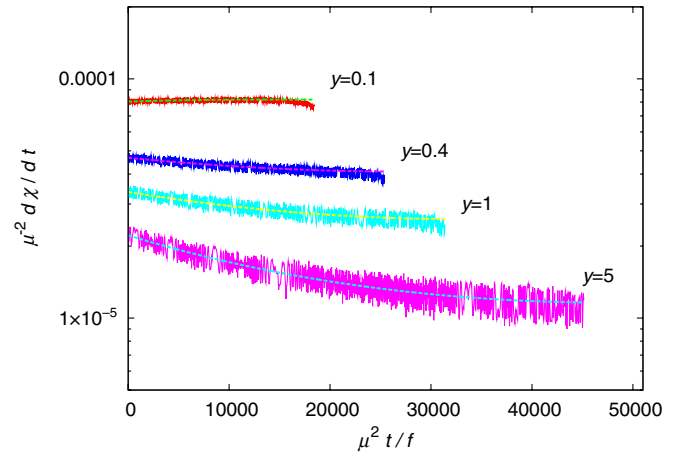

FIG. 1 (color online). Evolution of $\dot{\chi}$ during inflation for four different values of y. The other parameters are $\lambda = 500$ and $f = 10^{-2}M_p$. The evolutions shown correspond to 60 e-folds of inflation.

which the slow roll approximations are used, and we also find excellent agreement. As we discussed after Eq. (21), decreasing y in the $y < 1$ region, while keeping the other parameters (including the initial value of χ) fixed, decreases the amount of inflation. As a consequence, $y \ll 1$ can result in sufficient inflation only if the inflaton is initially close to the top of the potential. Another way to express this is to note that the inflaton rolls faster as y decreases (with the other parameters kept fixed), as it is clear both from the slow roll expression (22) and from the figure.

Perhaps the most surprising feature of the model is that, although all the solutions shown in Fig. 1 appear to be acceptable inflationary solutions (in all cases the slow roll solution appears to be an attractor; we note however that the background dynamics only probes homogeneous departures from the slow roll solution), and although they only differ from each other by the value of y, the background solution with $y = 5$ is unstable, while the other ones are stable. This emerges from the study of the scalar perturbations around these solutions that we perform below.

III. LINEARIZED EQUATIONS FOR THE PERTURBATIONS

In this section we discuss at the formal level how we compute, we quantize, and we solve the linearized theory for the perturbations of the model around the background solution discussed in the previous section. The discussion is divided in two subsections. In the first subsection we discuss how the perturbations can be divided into three groups, decoupled from one another at the linearized level. In the second subsection we give the form of the quadratic action and we discuss how we compute the corresponding linearized theory for the perturbations.

A. Decomposition

There are 23 perturbations in the system, one of the inflaton, 12 of the SU(2) vector field, and 10 of the metric, that we decompose as

$$
\chi = \chi + \delta\chi \qquad A_0^a = a(Y_a + \partial_a Y)
$$
$$
A_i^a = a[(Q + \delta Q)\delta_{ai} + \partial_i(M_a + \partial_a M)
$$
$$
+ \, \epsilon_{iac}(U_c + \partial_c U) + t_{ia}]
$$
$$
g_{00} = -a^2(1 - 2\phi) \qquad g_{0i} = a^2(B_i + \partial_i B)
$$
$$
g_{ij} = a^2[(1 + 2\psi)\delta_{ij} + 2\partial_i\partial_j E + \partial_i E_j + \partial_j E_i + h_{ij}].
$$
$$(24)$$

In this expression, $a = 1, 2, 3$ is the SU(2) index [we also denote by a the scale factor, as there is no ambiguity between it and a SU(2) index], while $i = 1, 2, 3$ ranges over the spatial coordinates. We denote as "tensor modes" the perturbations t_{ia} and h_{ij}, which we impose to be transverse ($\partial_i h_{ij} = \partial_i t_{ia} = \partial_a t_{ia} = 0$) and traceless ($t_{ii} = h_{ii} = 0$); due to these properties, the tensor sector contains four perturbations. We denote as "vector modes" the perturbations Y_a, M_a, U_c, B_i, E_i, which we impose to be transverse ($\partial_i Y_i = \cdots = \partial_i E_i = 0$); due to this, the vector sector contains ten perturbations. We denote as "scalar modes" the remaining nine perturbations.

We point out that the terms "tensor/vector/scalar" are appropriate for the perturbations of the metric and of the inflaton, as they indicate how these modes transform under a spatial rotation. We extend this terminology also to the perturbations of the vector field, following the notation of [21], even if, strictly speaking, these terms are inappropriate [given that the SU(2) index has been used in the decomposition]. We nonetheless adopt it since the fact that the vector vev is diagonal ($\langle A_i^a \rangle \propto \delta_i^a$) plus the transversality properties that we have imposed guarantee that the tensor/vector/scalars that we have defined above remain decoupled from one another at the linearized level [21].

We Fourier transform these modes

$$
\delta(t, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} \, e^{i\mathbf{k}\cdot\mathbf{x}} \delta(t, \mathbf{k}),
\qquad (25)
$$

where δ denotes any of the perturbations, and we study the theory in Fourier space. In our stability study, we solve for the perturbations at the linearized level, and therefore we study a mode with a given momentum $\mathbf{k}$ (modes of different momenta are coupled to one another at the nonlinear level). We can actually fix the orientation of $\mathbf{k}$ along the z axis without loss of generality. Starting from a general direction for $\mathbf{k}$, we rotate the coordinates so that $\mathbf{k} = k\hat{k}$, where $\hat{k}$ is the unit vector along the z axis, and $k > 0$. After a general rotation, $\langle A_i^a \rangle$ is no longer proportional to δ_i^a; however we can reobtain $\langle A_i^a \rangle = Q\delta_a^i$ through a global

SU(2) rotation. Therefore, we can set $k_x = k_y = 0$ without loss of generality. This choice simplifies our algebra.

We need to remove the redundancy associated to general coordinate and SU(2) transformations. Under an infinitesimal coordinate transformation with parameter $\xi^\mu = (\xi^0, \xi_i + \partial_i\xi)$,

$$
\psi \to \psi - \mathcal{H}\xi^0, \qquad E \to E - \xi, \qquad E_i \to E_i - \xi_i
$$
$$(26)$$

and we remove the freedom of infinitesimal coordinate transformations by setting $\psi = E = E_i = 0$, which corresponds to the so-called spatially flat gauge. Consider instead a SU(2) transformation with infinitesimal parameter $\alpha^a = \epsilon_a + \partial_a\epsilon$ (with ϵ_a transverse). Under this transformation,

$$
U \to U + gQ\epsilon, \qquad U_i \to U_i + gQ \qquad (27)
$$

and we can fix the SU(2) freedom by setting $U = U_i = 0$. Clearly, also other modes of the metric and of the gauge field change under these transformations, and different gauge choices can be made. Our choices are motivated by the fact that (i) they completely fix the freedom, and (ii) they preserve all the $\delta g_{0\mu}$ and δA_0^a modes. These perturbations are nondynamical, as they enter in the quadratic action of the perturbations without time derivative, and can be immediately integrated out. We describe this procedure in detail in Appendix A.

With our gauge choices, and with $k = k_z$, the decomposition (24) acquires the explicit form

$$
\chi = \chi + \delta\chi \qquad A_\mu^1 = a(Y_1, Q + \delta Q + t_+, +t_\times, \partial_z M_1)
$$
$$
A_\mu^2 = a(Y_2, t_\times, Q + \delta Q - t_+, \partial_z M_2)
$$
$$
A_\mu^3 = a(\partial_z Y, 0, 0, Q + \delta Q + \partial_z\partial_z M) \qquad (28)
$$

and

$$
g_{\mu\nu} = a^2 \begin{pmatrix} -1 + 2\phi & B_1 & B_2 & \partial_z B \\ & 1 + h_+ & h_\times & 0 \\ & & 1 - h_+ & 0 \\ & & & 1 \end{pmatrix}. \qquad (29)
$$

We verified explicitly that the scalar modes ($\delta\chi, Y, \delta Q, M, \phi, B$), the vector modes ($Y_{1,2}, M_{1,2}, B_{1,2}$) and the tensor modes ($t_+, t_\times, h_+, h_\times$) are decoupled from one another at the linearized level. Namely, the quadratic action for the perturbations splits into three decoupled parts:

$$
S_2 = S_{2,\text{scalar}} + S_{2,\text{vector}} + S_{2,\text{tensor}}. \qquad (30)
$$

B. Quantization of coupled systems and correlators

We have seen that the total action for the perturbations splits in a sum of three decoupled quadratic actions. Let us denote by $\mathcal{Y}$ the vector formed by the perturbations in one of these three systems. We can perform a transformation

$$\mathcal{Y}_i = \mathcal{M}_{ij}\Delta_j \qquad (31)$$

so that the action for the array Δ is of the type

$$S = \frac{1}{2}\int d\tau d^3k [\Delta'^{\dagger}\Delta' + \Delta'^{\dagger}K\Delta - \Delta^{\dagger}K\Delta' - \Delta^{\dagger}\Omega^2\Delta]. \qquad (32)$$

Hermitianity of the action implies that K is an anti-Hermitian matrix, and Ω^2 a Hermitian matrix; the matrices obtained in the current model are actually real. In the following we quantize the system (32), so as to obtain the initial conditions for the modes, and we give an expression for the correlators between the modes. This discussion summarizes the one of [33]. We first "rotate"

$$\psi \equiv R\Delta, \qquad (33)$$

where R is a unitary matrix [so that the kinetic term in (32) remains canonical], satisfying

$$R' = RK. \qquad (34)$$

As K is real, R can also be taken as real, and, therefore, orthogonal. We note that R is not uniquely determined by this condition, and we fix it by setting $R = 1$ at the initial time τ_{in}; the goal of the present discussion is also to understand when this initial time can be set. As we will see, the explicit solution for R is not needed. In terms of the vector ψ, the action becomes

$$S = \frac{1}{2}\int d\tau d^3k [\psi'^{\dagger}\psi' - \psi^{\dagger}\tilde{\Omega}^2\psi], \qquad (35)$$
$$\tilde{\Omega}^2 \equiv R(\Omega^2 + K^T K)R^T.$$

We then introduce the orthogonal matrix C satisfying

$$C^T\tilde{\Omega}^2 C = \text{diag}(\omega_1^2, \omega_2^2, \omega_3^2) \equiv \omega^2 \qquad (36)$$

and we decompose

$$\psi_i = C_{ij}[h_{jl}a_l + h_{jl}^*a_l^{\dagger}], \qquad (37)$$

where a_i ($a_i^{\dagger}$) destroys (creates) a quantum with the frequency ω_i. These operators satisfy the algebra

$$[a_i(\vec{k}), a_j^{\dagger}(\vec{p})] = \delta^{(3)}(\vec{k} - \vec{p})\delta_{ij}. \qquad (38)$$

For the systems that we study, in the deep subhorizon regime $(aH)_{\text{in}} \ll k$

$$\tilde{\Omega}_{\text{in}}^2 = (\Omega^2 + K^T K)_{\text{in}} \equiv k^2 1 + a^2 H^2 \mathcal{C} \qquad (39)$$

with $\mathcal{C}$ constant at leading order in slow roll. Therefore, provided that $\tilde{\Omega}_{\text{in}}^2 \simeq k^2 1$, we can set the initial conditions in the adiabatic vacuum

$$h \simeq \frac{e^{-i\int^{\tau} d\tau\omega}}{\sqrt{2\omega}}U = \frac{e^{-i\int_{\tau_{\text{in}}}^{\tau} d\tau\omega}}{\sqrt{2\omega}}, \qquad (40)$$

where U is a constant arbitrary orthogonal matrix. [At the practical level, Eq. (40) is an approximate solution of the equations of motion at early times; the closer $\tilde{\Omega}_{\text{in}}^2$ is to $k^2 1$, the better this approximation is. This determines how early τ_{in} needs to be taken.] The matrix U is unphysical, as it drops from the equations of motion for the modes, and from the physical correlators (which, as we will see, are given in terms of $hh^{\dagger}$). The freedom associated to U is the generalization to N fields of the freedom of changing by a constant phase the wave function in the single field case. In the final expression in (40) we have used the freedom associated to U to set the wave functions to be real at the initial time.

Combining (33) and (37), we have

$$\Delta_i = \mathcal{D}_{ij}a_j + \mathcal{D}_{ij}^*a_j^{\dagger}, \qquad \mathcal{D} = R^T Ch. \qquad (41)$$

Using the fact that $\tilde{\Omega}_{\text{in}}^2 \simeq k^2 1$ at τ_{in}, so that initially $C \simeq 1$, we arrive at

$$\mathcal{D}_{\text{in}} = \frac{1}{\sqrt{2k}}, \qquad \mathcal{D}_{\text{in}}' = -i\sqrt{\frac{k}{2}}. \qquad (42)$$

We start from these initial conditions and evolve the equations of motion for the modes following from (32)

$$\mathcal{D}'' + 2K\mathcal{D}' + (\Omega^2 + K')\mathcal{D} = 0. \qquad (43)$$

In this discussion, all the expressions are given in Fourier space. Let us denote by Y_i the original fields in real space,

$$Y_i = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}}\mathcal{Y}_i. \qquad (44)$$

We have the two-point correlation functions

$$\mathcal{C}_{ij}(\vec{x}, \vec{y}) = \frac{1}{2}\langle Y_i(\tau, \vec{x})Y_j(\tau, \vec{y}) + Y_j(\tau, \vec{y})Y_i(\tau, \vec{x})\rangle$$
$$\equiv \int \frac{dk}{k}\frac{\sin(kr)}{kr}\mathcal{P}_{ij}, \qquad r \equiv |\vec{x} - \vec{y}| \qquad (45)$$

where the power spectra are given by

$$\mathcal{P}_{ij}(k) = \frac{k^3}{2\pi^2}\text{Re}[\langle \mathcal{Y}\mathcal{Y}^{\dagger}\rangle_{ij}] = \frac{k^3}{2\pi^2}\text{Re}[(\mathcal{M}\mathcal{D}\mathcal{D}^{\dagger}\mathcal{M}^T)_{ij}]. \qquad (46)$$

These correlators are the theoretical prediction, to be confronted with the statistical average of the corresponding quantities. The power spectra (46) are the generalization to a system of N fields Y_i of the standard power spectrum of single field inflation.

To summarize, starting from the original fields $\mathcal{Y}_i$ in momentum space, we perform (31) to have a canonical kinetic term for Δ_i. We then decompose this field in terms of annihilation/creation operators of the physical quanta in the system (the particles of frequencies ω_i). We work in terms of the coefficients $\mathcal{D}_{ij}$ of this decomposition. We can

set the initial conditions (42) for these coefficients, provided that the initial time is chosen sufficiently early such that $\Omega^2 + K^T K \simeq k^2$. Starting from these initial conditions, the coefficients $\mathcal{D}_{ij}$ evolve according to (43). We will see that for some choice of parameters some of the perturbations become exponentially large on a time scale $\ll H^{-1}$, signaling an instability of the linearized theory. For values of parameters leading to stable solutions, the coefficients enter in observable quantities through the power spectra (46).

IV. TENSOR MODES

We introduce the two doublets

$$\Delta_L \equiv \frac{\Delta_+ + i\Delta_\times}{\sqrt{2}}, \qquad \Delta_R \equiv \frac{\Delta_+ - i\Delta_\times}{\sqrt{2}} \qquad (47)$$

for the left and right helicities, where

$$\Delta_+ \equiv \begin{pmatrix} \frac{aM_p}{\sqrt{2}} h_+ \\ \sqrt{2}a t_+ \end{pmatrix}, \qquad \Delta_\times \equiv \begin{pmatrix} \frac{aM_p}{\sqrt{2}} h_\times \\ \sqrt{2}a t_\times \end{pmatrix}. \qquad (48)$$

The action for the tensor modes splits into two separate actions, one for the left- and one for the right-helicity doublet, which are formally of the type (32). The action for the left-helicity doublet is characterized by

$$K_{12} = \frac{1}{M_p}(Q' + \mathcal{H}Q) \simeq aH\mathrm{O}\!\left(\frac{1}{\sqrt{\lambda}}\right) \qquad (49)$$

and

$$\Omega_{11}^2 = k^2 - 2\mathcal{H}^2 - \frac{1}{M_p^2}(Q' + \mathcal{H}Q)^2 + \frac{3g^2 a^2 Q^4}{M_p^2} + \frac{\chi'^2}{2M_p^2}$$

$$\simeq k^2 + a^2 H^2 \left[-2 + \mathrm{O}\!\left(\frac{1}{\lambda}\right) \right]$$

$$\Omega_{12}^2 = ak\frac{2gQ^2}{M_p} + \frac{\mathcal{H}}{M_p}(Q' + \mathcal{H}Q) - \frac{g\lambda Q^2 a\chi'}{fM_p}$$

$$\simeq akH\mathrm{O}\!\left(\frac{1}{\sqrt{\lambda}}\right) + a^2 H^2 \mathrm{O}\!\left(\frac{1}{\sqrt{\lambda}}\right)$$

$$\Omega_{22}^2 = k^2 - ak\left(2gQ + \frac{\lambda}{f}a\chi'\right) + g\frac{\lambda}{f}Qa\chi'$$

$$\simeq k^2 - akH\mathcal{A} + a^2 H^2 \mathcal{B}, \qquad (50)$$

where the first expression for each coefficient is exact, while the second one is obtained from the slow roll approximation (22) (we note that $\dot{Q} \ll HQ$). The quantities $\mathcal{A}$ and $\mathcal{B}$ are both of $\mathrm{O}(\lambda^0)$. Using the slow roll result (12) with Q given by (14), they evaluate to

$$\mathcal{A} \simeq \frac{4gQ}{H}\left(1 + \frac{H^2}{2g^2 Q^2}\right), \qquad \mathcal{B} \simeq \frac{2g^2 Q^2}{H^2}\left(1 + \frac{H^2}{g^2 Q^2}\right). \qquad (51)$$

The action for the right-helicity doublet is related to (50) by $k \to -k$, signaling the breaking of parity invariance induced by the evolution of the pseudoscalar inflaton.

The interactions between the gravity wave and vector field tensor perturbations are slow roll suppressed. However, the effective frequency squared Ω_{22}^2 of t_L turns negative for an intermediate interval of time next to horizon crossing.[6] This leads to a tachyonic growth of t_L in this interval of time, and, correspondingly, to a growth of h_L. The same growth does not occur in the right-helicity sector due to the opposite sign of the linear term in k. The situation is analogous to that first studied in [14], where the interaction $\chi F\tilde{F}$ between a vector field and a pseudoscalar rolling inflaton results in a tachyonic growth of the vector modes of a given helicity.

We note from (51) that the length of the tachyonic region increases with increasing

$$\frac{m_g}{H} = \frac{\sqrt{2}gQ}{H} \simeq \sqrt{\frac{2}{y}}\frac{\sin^{1/3}x}{(1 + \cos x)^{2/3}} \qquad (52)$$

(the last expression is the slow roll approximation). This corresponds to a larger tensor mode production with growing $\frac{m_g}{H}$. This corresponds to decreasing y in the numerical examples that we show below.

V. VECTOR MODES

The vector modes Y_1, Y_2, B_1, B_2 are nondynamical, and can be integrated out. Namely, they enter in the quadratic action of the perturbations without time derivatives, and therefore their equations of motion are algebraic equations in these variables (recall that we are in momentum space). When we solve these equations, we obtain an expression for the nondynamical modes in terms of the two dynamical modes M_1 and M_2. We then insert this expression back into the quadratic action for the vector modes, and obtain an action for $M_{1,2}$ only. In other words, the nondynamical modes do not introduce additional degrees of freedom, but are completely determined by the dynamical ones. After integrating out the nondynamical modes, we define

$$M_1 = F_1 V_1 + iF_2 V_2, \qquad M_2 = iF_1 V_1 + F_2 V_2, \qquad (53)$$

where

$$F_{1,2} \equiv \frac{\sqrt{M_p^2(\pm k + gaQ)^2 + g^2 a^2 Q^2(M_P^2 + 2Q^2)}}{\sqrt{2}gkM_p a^2 Q} \qquad (54)$$

(with the upper $+$ sign corresponding to F_1 and the lower $-$ sign corresponding to F_2). The modes $V_{1,2}$ are the canonical modes of the system, and are decoupled from each other at the linearized level:

---

[6] At late times, the k dependence becomes negligible, and the mass term for the two vector perturbations $t_{L/R}$ reproduces the value m_g given in [29].

$$S_{2,\text{vector}} = \frac{1}{2} \int d\tau d^3k [|V'_+|^2 - \Omega^2_{v+}|V_+|^2 + |V'_-|^2$$
$$- \Omega^2_{v-}|V_-|^2]. \tag{55}$$

An explicit computation gives

$$\frac{\Omega^2_{v\pm}}{a^2} = p^2 \pm \frac{\lambda}{f}\dot{\chi}p$$
$$+ \frac{M^4_p[c_4 p^4 \pm c_3 p^3 + c_2 p^2 \pm c_1 p + c_0]}{[M^2_p p^2 \pm 2g M^2_p Qp + 2g^2 Q^2(M^2_p + Q^2)]^2},$$
$$\tag{56}$$

where $p = \frac{k}{a}$ is the physical momentum, and the coefficients c_i are functions of background quantities. We note that

$$\Omega^2_{v+}(p) = \Omega^2_{v-}(-p). \tag{57}$$

We also note that the denominator in (56) is always positive, as it can be written as a sum of squares. The exact expressions for c_i are readily obtained, but they are not illuminating. We report here the leading expressions in the slow roll $\lambda \gg 1$ expansion:

$$c_4 = H^2\left[\frac{2g^2 Q^2}{H^2} + O\left(\frac{1}{\lambda}\right)\right]$$
$$c_3 = H^3\left[\frac{g^2 Q^2}{H^2}\left(6g\frac{Q}{H} + \frac{\lambda}{f}\frac{\dot{\chi}}{H}\right) + O\left(\frac{1}{\lambda}\right)\right]$$
$$c_2 = H^4\left[\frac{g^2 Q^2}{H^2}\left(3 + 4g\frac{Q}{H}\frac{\lambda}{f}\frac{\dot{\chi}}{H} + 8g^2\frac{Q^2}{H^2}\right) + O\left(\frac{1}{\lambda}\right)\right] \tag{58}$$
$$c_1 = H^5\left[\frac{g^4 Q^4}{H^4}\left(4g\frac{Q}{H} + 6\frac{\lambda}{f}\frac{\dot{\chi}}{H}\right) + O\left(\frac{1}{\lambda}\right)\right]$$
$$c_0 = H^6\left[4g^5\frac{Q^5}{H^5}\frac{\lambda}{f}\frac{\dot{\chi}}{H} + O\left(\frac{1}{\lambda}\right)\right].$$

Using (22), it can be immediately seen that, in each of the above expressions, the dominant term in the square parenthesis that we have written explicitly is a $O(1)$ coefficient. Moreover, the second term on the right-hand side of (56) evaluates to $\pm pHO(1)$. Therefore, at leading order in slow roll,

$$\Omega^2_{v\pm} \simeq k^2, \qquad p \gg H$$
$$\Omega^2_{v\pm} \simeq \frac{f_A f_{\dot{\chi}}}{y^{3/4}}a^2 H^2, \qquad p \ll H \tag{59}$$

in the deep subhorizon and superhorizon regimes, respectively. We conclude that the vector sector is stable.

We note that the superhorizon limit of (59) is precisely the slow roll expression of $a^2 m^2_g$ in the $m_g \gg H$ limit. The two dynamical vector modes originate from the perturbations of the vector field, and the expression for the mass in this limit coincides with that found in [29].

VI. SCALAR MODES

After gauge fixing, the system of scalar perturbations comprises one mode from the inflaton, $\delta\chi$; three from the SU(2) field, Y, δQ, M; and two from the metric, ϕ and B. Among these six modes, $\delta\chi$, δQ and M are dynamical, while the other three modes are nondynamical; namely they enter in the quadratic action without time derivatives, and they can be integrated out (see the discussion at the start of the previous section). The full quadratic action for the scalar modes is given in Eq. (C1).

Integrating out the metric perturbations makes the algebra extremely involved; for this reason, in the study presented in the main text we make the approximation of setting $\phi = B = 0$ from the start. We then integrate out Y, and we denote the resulting quadratic action by $S_{2,\text{scalar}}$. We also performed the full computation, which we present in Appendix C. We denote by $S_{2,\text{scalar-full}}$ the quadratic action obtained by including all the modes and by integrating out the three nondynamical ones. Both $S_{2,\text{scalar}}$ and $S_{2,\text{scalar-full}}$ are functionals of the three dynamical modes.

We expand all entries in these actions in slow roll. As we show in Appendix C, all entries of the matrices of the two actions agree at the leading order in this expansion at all scales (namely, for all values of H/p), with a single exception. The exception is the 11 coefficient of the frequency matrix Ω^2, for which the agreement is excellent only up to $H \lesssim 10p$. The discrepancy that takes place afterwards is surely completely irrelevant for the stability study that we perform here (as we will see, the instability, if present, manifests itself deeply inside the horizon). Not surprisingly, the metric perturbations do not affect the stability of the background solution. Very likely, this disagreement has also no significant consequence for the power spectra that we show below, since it manifests itself only after the power has frozen (see the appendix for a detailed study).

A closed equation for the scalar field perturbations (in the spatially flat gauge) in models of slow roll scalar field inflation can be found for example in Eq. (8.60) of [34]. In that equation it is manifest that the metric perturbations modify the evolution equation for the dynamical variables only in a slow roll suppressed way. The reason for this is that the field fluctuations are coupled to the perturbations of the metric only through the way that they locally affect the energy-momentum. However, local change in the potential energy is proportional to derivatives of the potential, which is suppressed for slow roll inflation, or to the kinetic energy of the field, which is also slow roll suppressed (this second suppression is even true at the background level).[7] The computations of Appendix C show that this suppression (with one exception) is present also in the current model. This can be more directly understood by inspection of the action (1). We are interested in $\delta g - \delta A$

_____

[7]We thank Toni Riotto and David Wands for discussions.

and $\delta g - \delta \chi$ couplings. Metric perturbations do not enter in the last term. Therefore, the $\delta g - \delta A$ coupling only arises from the $\sqrt{-g}F^2$ term. As this term is quadratic or higher in the vector field, such couplings are suppressed by at least one power of the vector field vev, which is a slow roll suppressed quantity [see Eq. (22)]. The $\delta g - \delta \chi$ coupling arises instead only from the $\sqrt{-g}[-\frac{1}{2}(\partial \chi)^2 - V(\chi)]$ term in (1), and it is therefore suppressed as shown in [34]. This can be verified explicitly from the interactions given in Eq. (C2). More accurately, performing this explicit check, one notices that one interaction term in (C2), namely $\frac{a^4}{2}V_{,\chi}(\phi^* \delta \chi + \text{H.c.})$, is actually not slow roll suppressed. In standard models of slow roll inflation this term is proportional to the slow roll parameter $\sqrt{\epsilon}$. In the current model instead the inflaton potential is no longer flat. This term does not present spatial derivatives, and it is therefore relevant only outside the horizon. This precisely explains the discrepancy in Ω_{11}^2 that we have mentioned in the previous paragraph.

We therefore set $\phi = B = 0$, and we obtain the action (B1) that we explicitly write in Appendix B. We integrate Y out of this action. The expression for Y in terms of the dynamical variables is

$$Y = \frac{1}{k^2 + 2g^2 a^2 Q^2}\left(-\delta Q' + k^2 M' + \frac{g\lambda a Q^2}{f}\delta \chi \right.$$
$$\left. - \mathcal{H}\delta Q + k^2 \mathcal{H}M \right) \tag{60}$$

which is of the type (A4). Inserting this solution back into the action (B1) we readily obtain the action for the dynamical modes of the system. This action is formally of the type (A6), and we can employ it to set the initial conditions and derive the equations of motion for the dynamical modes, as in the standard computations of scalar field inflation.

We define

$$\delta \chi \equiv \frac{\Delta_1}{a} \qquad \delta Q \equiv \frac{\Delta_2}{\sqrt{2}a}$$
$$\delta M \equiv \frac{gaQ\Delta_2 + \sqrt{k^2 + 2g^2 a^2 Q^2}\Delta_3}{\sqrt{2}gk^2 a^2 Q}. \tag{61}$$

In terms of the multiplet $\Delta = (\Delta_1, \Delta_2, \Delta_3)^T$, the quadratic action for the scalar modes acquires, up to a total derivative, the form (32). We denote as K_s and Ω_s^2 the 3×3 matrices entering in this action. These matrices have the following entries:

$$\frac{K_{s,12}}{a} = \frac{g\lambda Q^2}{\sqrt{2}f} \qquad \frac{K_{s,13}}{a} = -\frac{g^2 \lambda Q^3}{\sqrt{2}f\sqrt{p^2 + 2g^2 Q^2}} \qquad \frac{K_{s,23}}{a} = 0 \tag{62}$$

and

$$\frac{\Omega_{s,11}^2}{a^2} = p^2 + \frac{g^2 \lambda^2 p^2 Q^4}{f^2(p^2 + 2g^2 Q^2)} + \frac{g^2 Q^4}{M_p^2} + V_{,\chi\chi} - 2H^2$$
$$+ \frac{\dot{\chi}^2}{2M_p^2} + \frac{(\dot{Q} + HQ)^2}{M_p^2}$$

$$\frac{\Omega_{s,12}^2}{a^2} = \frac{3g\lambda HQ^2}{\sqrt{2}f} + \frac{\sqrt{2}g\lambda Q\dot{Q}}{f}$$

$$\frac{\Omega_{s,13}^2}{a^2} = -\frac{\sqrt{2}\lambda}{f}\left[\frac{g^2 HQ^3}{2\sqrt{p^2 + 2g^2 Q^2}} + \frac{2p^4 + 3g^2 p^2 Q^2 + 4g^4 Q^4}{2(p^2 + 2g^2 Q^2)^{3/2}}\right.$$
$$\left. \times (\dot{Q} + HQ)\right]$$

$$\frac{\Omega_{s,22}^2}{a^2} = p^2 + 4g^2 Q^2 - \frac{g\lambda Q\dot{\chi}}{f}$$

$$\frac{\Omega_{s,23}^2}{a^2} = -\sqrt{p^2 + 2g^2 Q^2}\left(2gQ - \frac{\lambda}{f}\dot{\chi}\right)$$

$$\frac{\Omega_{s,33}^2}{a^2} = p^2 + \frac{4g^2 Q^2(p^2 + g^2 Q^2)}{p^2 + 2g^2 Q^2} - \frac{g\lambda p^2 Q\dot{\chi}}{f(p^2 + 2g^2 Q^2)}$$
$$+ \frac{6g^2 p^2(\dot{Q} + HQ)^2}{(p^2 + 2g^2 Q^2)^2}. \tag{63}$$

We stress that these expressions are obtained by disregarding the scalar metric perturbations, but that they are otherwise exact. We can also verify that the eigenvalues of the $\{i, k\} = \{1, 2\}$ part of Ω_{ij}^2 are $\simeq a^2 m_g^2$ in the superhorizon regime and for $m_g \gg H$, in agreement with [29].

We solve the theory specified by this action following the steps outlined in Sec. III B. We assume that, after inflation, only the inflaton field provides a sizable contribution to reheating (we note that the energy in A_μ is much smaller than the inflaton energy during inflation). In this limit, we have the curvature perturbation $\zeta \simeq -\frac{H}{\dot{\chi}}\delta \chi$, with the power spectrum

$$P_\zeta = \frac{H^2 \mathcal{P}_{11}}{\dot{\chi}^2}, \tag{64}$$

where $\mathcal{P}$ is given in (46), with $\mathcal{Y}_i$ being the three fields on the left-hand side of (61).

In Fig. 2, we then present the time evolution of $\mathcal{P}_{11}$ for a single mode (a given k) and for the same choices of background parameters that we used in Fig. 1 for the background evolution. For definiteness, we considered in all cases the mode that leaves the horizon 60 e-folds before the end of inflation, and we denoted the corresponding comoving momentum by k_{60}. We choose the initial time of the evolution such that the mode is deeply inside the horizon at the start, and the last term in (39) is negligible. We observe that the choice $y = 5$ leads to an instability of the linearized theory, while the other three cases are stable, and are characterized by a power that freezes outside the horizon, as in the standard inflationary models.
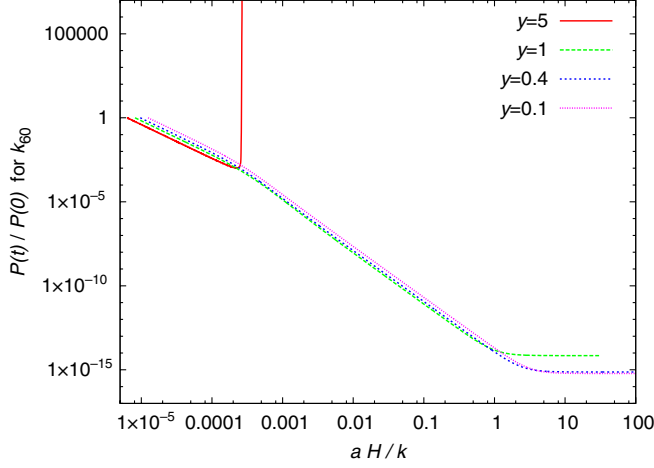
FIG. 2 (color online). Time evolution of the power (normalized to one at the initial time shown), for a mode that leaves the horizon 60 e-folds before the end of inflation, and for the same background evolutions shown in Fig. 1.

The instability of the $y = 5$ choice manifests itself at some scale inside the horizon, namely $p_{\text{unstable}} = k/a_{\text{unstable}} \gg H$. Even the stable solutions show a different evolution at this scale (note in the figure the small change in the slope of $P(t)$ for the stable cases at the same scale at which the $y = 5$ solution becomes unstable). We can see this analytically, by considering the approximated expressions of the scalar system inside the horizon. We will obtain the scale ΛH times a numerical factor, where

$$\Lambda \equiv \frac{\sqrt{\lambda}}{y^{1/4}} \frac{M_p}{f}, \qquad \Lambda \gg 1 \qquad (65)$$

(in the numerical examples that we have shown, $\Lambda = \sqrt{2} 10^3 y^{-1/4}$). Let us discuss the approximation in more detail. First of all, using the slow roll conditions (22), we find $g^2 Q^2 \simeq y^{-3/2} f_A^2 H^2 \ll \Lambda^2 H^2$ in all cases that we have studied. Therefore, we can disregard $g^2 Q^2$ in comparison to p^2 inside Eqs. (62) and (63). From this, and from the slow roll approximations (22), we obtain, in the subhorizon regime

$$\frac{K_{s,12}}{a} = \frac{g\lambda Q^2}{\sqrt{2}f} = O(\Lambda H) \qquad \frac{K_{s,13}}{a} O\left(\Lambda H \frac{H}{p}\right) \ll K_{12,s}$$
$$(66)$$

and

$$\frac{\Omega_{s,11}^2}{a^2} \simeq p^2 + \frac{g^2 \lambda^2 Q^4}{f^2} = O(\Lambda^2 H^2) \qquad \frac{\Omega_{s,12}^2}{a^2} = O(\Lambda H^2)$$

$$\frac{\Omega_{s,13}^2}{a^2} \simeq -\frac{\sqrt{2}\lambda p}{f} HQ = O(\Lambda H p)$$

$$\frac{\Omega_{s,22}^2}{a^2}, \frac{\Omega_{s,33}^2}{a^2} = p^2 + O(H^2) \qquad \frac{\Omega_{s,23}^2}{a^2} = O(Hp). \qquad (67)$$

We want to solve the evolution equations (43) until $p \sim \Lambda H$. We can do so by rewriting them as

$$\ddot{\mathcal{D}}_{ij} + \frac{K_{s,ik}}{a} \dot{\mathcal{D}}_{kj} + \frac{\Omega_{s,ik}^2}{a^2} \mathcal{D}_{kj} \simeq 0 \qquad (68)$$

by inserting only the terms written explicitly in (66) and (67), and by treating these terms as constant. All these approximations amount in considering only terms that contribute to the dynamics at $O(p^2, \Lambda Hp, \Lambda^2 H^2)$. We note for instance that $\dot{\mathcal{D}} = O(p\mathcal{D})$ in this regime, so that it is consistent to set $\Omega_{s,12}^2 = 0$ in the approximated equation, while retaining $K_{s,12}$ and the dominant term of $\Omega_{s,13}^2$. We also note that the fastest evolving coefficient in the matrices is p that evolves on a $O(H^{-1})$ time scale; therefore time derivatives of the terms in (66) and (67) introduce at most terms with an additional factor H, which are therefore suppressed in the $p > \Lambda H$ and $p \simeq \Lambda H$ regimes.

Performing these approximations, the system (68) reduces to a set of linear second order equations with constant coefficients. The equations split into three separate groups, one for the complex unknowns $\mathcal{D}_{i1}$ (with $i = 1, 2, 3$), one for the complex unknowns $\mathcal{D}_{i2}$, and one for the three complex unknowns $\mathcal{D}_{i3}$. In each group we need to solve three second order differential equations, and therefore we have six possible solutions; we note that the three groups have the identical set of equations, and they differ only in the initial conditions. Therefore the solutions are of the type

$$\mathcal{D} \simeq \sum_{a=1}^{6} C_a e^{\xi_a t}, \qquad (69)$$

where the matrices C_a are integration constants (so as to match the initial conditions) and ξ_a are constant numbers. The system is unstable if any of the ξ_a has a real and positive part. By solving the system, one can see that the only coefficient that can possibly be real is

$$\xi = \left[-p^2 - \frac{3g^2\lambda^2 Q^4}{2f^2} \right.$$
$$\left. + \frac{\lambda Q}{f}\sqrt{2p^2(H^2 + g^2 Q^2) + \frac{9}{4}\frac{\lambda^2 g^4 Q^6}{f^2}} \right]^{1/2} \qquad (70)$$

and therefore

$$\text{stability} \Leftrightarrow p^2 > \bar{p}^2 \equiv \frac{\lambda^2 Q^2}{f^2}(2H^2 - g^2 Q^2) \qquad (71)$$

where, using the slow roll condition,

$$\frac{\bar{p}^2}{H^2} = \Lambda^2 y^{1/2} f_A^2 f_H^2 \left(2 - \frac{f_A^2}{y^{3/2}}\right). \qquad (72)$$

If the expression in parenthesis is negative the background solution is stable. Otherwise, there is an instability at sufficiently large wavelengths. Therefore

$$\text{stability} \Leftrightarrow y < \frac{\sin^{2/3} x}{2(1 + \cos x)^{4/3}}. \qquad (73)$$

For any fixed y, this condition is violated at sufficiently small x. This provides an upper bound to the amount of inflation in the model (for any given choice of parameters). One needs to verify that this upper bound is comparable with the required amount of inflation. For the evolutions shown in Fig. 1, the inflaton field, at 60 e-folds before the end of inflation, evaluates to $\chi \simeq 2.45f$ for $y = 5$, to $\chi \simeq 2.20f$ for $y = 1$, to $\chi \simeq 2.00f$ for $y = 0.4$, and to $\chi \simeq 1.56f$ for $y = 0.1$. Correspondingly, the rhs of the condition (73) evaluates to $\simeq 2.62$ for $y = 5$, to $\simeq 1.43$ for $y = 1$, to ~ 0.95 for $y = 0.4$, and to $\simeq 0.49$ for $y = 0.1$. The criterion (73) therefore indicates that the choice $y = 5$ does not lead to a stable inflationary solution of 60 e-folds (recall that f and λ are fixed to $10^{-2} M_p$ and to 500, respectively, in this example), while the other choices do. This is in perfect agreement with Fig. 2.

Equation (71) confirms that the instability, if it exists, takes place at $p < \bar{p} = \Lambda H$ times an order 1 factor. For example, for the unstable $y = 5$ choice, the criterion (71) gives an instability starting at $\bar{p} \simeq 2.67 \Lambda H$, corresponding to $aH/\bar{p} \simeq 0.00025$, in excellent agreement with the evolution seen in the figure.

Moreover, let us verify that the instability is extremely fast. Let us assume that we have a background solution with $2H^2 > g^2 Q^2$. At any given moment, modes with $p \gtrsim \bar{p}$ have $\text{Re}\xi = 0$, and are therefore stable. Moreover, the instability is also negligible at very large scales, given that $\xi \to 0$ for $p \to 0$. However, for $p \lesssim \bar{p}$, a quick study of Eq. (70) shows that $\xi = O(\Lambda H)$ for p smaller than, but parametrically equal to $\bar{p}$. (For $2H^2 \gg g^2 Q^2$, the maximum of ξ is obtained for $p = \bar{p}/2$.) This corresponds to a very short instability time $\sim \frac{1}{H\Lambda} \ll H^{-1}$. Therefore, each mode experiences a strong instability while still inside the horizon.

The model (1) is a perfectly healthy model, as it simply describes an axion with a potential coupled to a SU(2) field. This model admits a stable Minkowski solution, with vanishing background fields. The instability that we have found is therefore an instability of the inflationary background solution. The starting assumption of the model is that the potential (2) is too steep to drive inflation, and that slow roll is obtained through the coupling with a SU(2) multiplet with a nonvanishing vev. Contrary to the mechanism of [14] [where the inflaton was coupled to a U(1) field with no vev], the additional friction provided by this coupling is not due to particle production, but it takes place at the background level. This requires that the gauge field is non-Abelian ($g \neq 0$), and it has a vev ($Q \neq 0$). Therefore, it is to be expected that, for any given value of H, one cannot achieve a stable inflationary background for arbitrary small gQ. As we discussed after Eq. (4), Ref. [29] showed that, in the $gQ > 2H$ regime, the model (1) can be
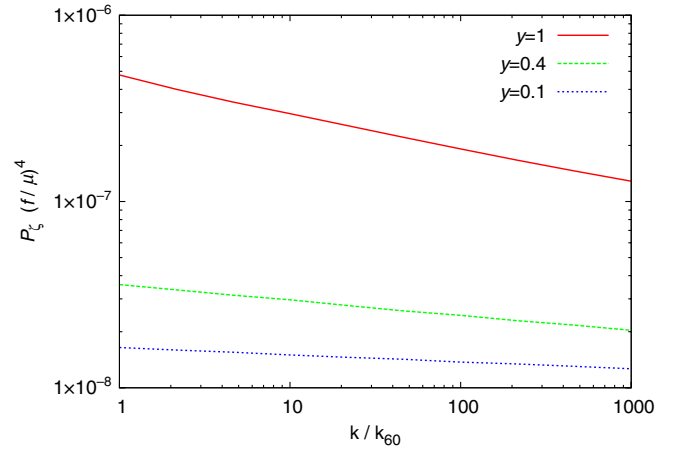

FIG. 3 (color online). Power spectra for $\lambda = 500$ and three different values of y. The spectral index (defined as $P_\zeta \propto k^{n_s - 1}$) is $n_s \simeq 0.81$ for $y = 1$, $n_s \simeq 0.92$ for $y = 0.4$ and $n_s \simeq 0.96$ for $y = 0.1$.

effectively described by a stable model with a single field. However, the single field description is no longer possible at smaller values of gQ [29], and therefore the instability can only be obtained and understood in a multifield description.

A background instability typically manifests itself through the presence of tachyonic modes, and our result shows that the model (1) is not an exception to this. The $\frac{\lambda}{f} \chi F \tilde{F}$ interaction needed for slow roll at the background level unavoidably leads to couplings between the perturbations, and possible vacuum amplification (i.e., the tachyonic instability). The vacuum amplification that we have obtained has the typical properties of the vacuum amplification of gauge modes due to the their coupling to axions that was found in [14] and to the amplification of gravity waves studied in Sec. IV.[8] As in these cases, it is due to a linear term in the momentum p, which is induced by the $\frac{\lambda}{f} \chi F \tilde{F}$ interaction. This is the $\Omega_{13,s}^2$ term in (67), which indeed vanishes for $\frac{\lambda}{f} \to 0$. When compared with the other terms in (68), this term is subdominant to the standard p^2 term at very large p, and to the p-independent mass term at very small p. If the mass term is sufficiently large, the interaction encoded by $\Omega_{13,s}^2$ never dominates the frequency of the modes, and the background solution is never tachyonic. This occurs for sufficiently large $m_g \propto gQ$, and indeed, the quantitative study of the eigenfrequency results precisely in the stability condition that we have found.

We conclude this section by showing in Fig. 3 the power spectrum P_ζ for the stable $y = 0.1, 0.4, 1$ configurations, which we discuss in the next section.

_____

[8]We thank Lorenzo Sorbo for discussions.

VII. CONCLUSIONS

We performed a complete study of the linear order quantum fluctuations for the chromo-natural inflation model. We separated the metric, gauge and axion fluctuations into scalar, vector and tensor modes, verifying explicitly that they decouple at the linearized level. We computed their equations of motion and worked out the quantization of the system. The tensor sector consists of the two gravity wave polarizations plus two modes from the gauge field. The gauge mode of one helicity becomes tachyonic for some finite interval of time next to horizon crossing, sourcing one gravity wave helicity. The vector sector consists of two dynamical modes that originate from the vector field, and that remain perturbatively small. The scalar sector contains three dynamical modes, one originating from the inflation and two from the gauge field.

We showed that, for some parameter choice, one of the eigenfrequencies of this system, that we denoted by ξ, can become imaginary inside the Hubble horizon, leading to a fast instability. Let us compare this with a standard result in inflation. Specifically, let us consider the gravity wave amplitude $\delta g_{ij} = a^2 h_{ij}^{TT}$. The canonical variable $h_c \propto a h^{TT}$ obeys $h_c'' + \Omega^2 h_c = 0$ with the dispersion relation $\Omega^2 = a^2 (p^2 - 2H^2)$. As a consequence $h_c \propto a$ outside the horizon, corresponding to a frozen amplitude of h^{TT}. Therefore, although the frequency of the canonical mode becomes tachyonic outside the horizon, its magnitude is not large enough to lead to a physical instability. The situation is analogous for a test scalar field or for the scalar perturbations in the standard case.

For the case at hand, the eigenfrequency ξ is given in Eq. (70). In this discussion, for illustrative purposes, let us approximate the full expression obtained in Eq. (70) with $\xi^2 \sim p^2 - c\Lambda^2 H^2$, where $\Lambda \gg 1$ is defined in (65), and c an order 1 factor. No instability appears if c is negative. Otherwise, an instability takes place for modes of physical momentum $p = O(\Lambda H)$. The time scale of the instability is $|\xi|^{-1} = O(\frac{1}{\Lambda H}) \ll H^{-1}$ (or, equivalently, the amplitude of the scalar modes grows proportionally to a large power of the scale factor) which indeed corresponds to a fast instability. This heuristic discussion reproduces the results obtained from the precise form of ξ (see Sec. VI for the precise computation). Moreover, we also performed a fully numerical and exact study of the scalar perturbations that confirms these analytical results.

The parameters of the model can be chosen so that ξ remains real inside the horizon (equivalently, $c < 0$ in the heuristic expression) so that the instability is avoided. This corresponds to choosing $m_g \equiv \sqrt{2} g Q > 2H$, where Q is the vev of the vector field, and g the SU(2) coupling. In the opposite case, the inflationary background solution is unstable. This instability reflects the fact that the inflationary mechanism of [17] requires both that the vector field is non-Abelian ($g \neq 0$) and that it has a nonvanishing vev ($Q \neq 0$), so that one cannot expect a stable inflationary

solution for arbitrarily small gQ. The strong tachyonic instability in the unstable regime is triggered by the $\chi F \tilde{F}$ coupling, precisely as the gauge field amplification that takes place in the mechanism of [14]. Contrary to the U(1) case of [14], the vector field is now massive, $m_g \propto gQ$, and a sufficiently high mass can shut off the tachyonic instability. This explains why the inflationary solution is stable if and only if the vector field is sufficiently heavy.

We have obtained the boundary of the stable region $m_g \gg H$ analyzed in [29] and our formalism can be readily employed to study the phenomenology of the boundary region. This study is beyond the purposes of the present work. For illustrative purposes, we have however computed the power spectrum of ζ for some sample choices of parameters. Specifically, Fig. 3 shows the power spectrum for an axion decay constant $f = 0.01 M_p$, for $\lambda = 500$ (we stress that the main motivation for the model is to provide inflation for a sub-Planckian axion decay constant, and that this can be obtained for sufficiently large λ), and for three choices of y. We note that, among those shown in Fig. 3, only the power spectrum obtained for $y = 0.1$ is sufficiently flat to meet the observational bounds [2,3]. We numerically found that the spectral tilt is a decreasing function of y in the $0.1 < y < 1$ interval (equivalently, it is an increasing function of m_g/H in this interval). This behavior can also be seen in the analytic relation given in [29] in the large m_g regime. Therefore, both the requirements of stability and of flatness of the power spectrum pose a lower bound on m_g/H.

On the other hand, the discussion around Eq. (52) leads to the conclusion that a large m_g/H leads to a detectable or ruled out gravity wave signal. The enhanced signal is parity violating, which results in nonvanishing temperature-B mode polarization (TB) and E mode–B mode polarization (EB) correlations. A measure of the net handedness of the tensor modes is

$$|\Delta \chi| \equiv \left| \frac{P_L - P_R}{P_L + P_R} \right|, \tag{74}$$

where $P_{L/R}$ is the power spectrum of the left-/right-helicity gravity wave modes. The corresponding observational bounds have been studied in [35,36]. In Fig. 4 we compare the bounds presented in [36] with the values of r and $\Delta \chi$ obtained in this model, for our choice of $f = 0.01 M_p$ and $\lambda = 500$, and for different values of y in the $0.35 < y < 0.7$ range (the tensor power spectra are obtained from the 11 element of (46) for the left- and right-helicity tensor sectors, following the procedure outlined in Sec. III B). Greater values of y do not lead to a visible gravity wave signal even in a cosmic variance limited experiment, while lower values are ruled out by the current limit $r < 0.13$ [3,37].

We point out that all values of y considered in this plot are actually ruled out because they lead to a too small value of n_s (the largest value $n_s \simeq 0.93$ is obtained for $y = 0.35$,
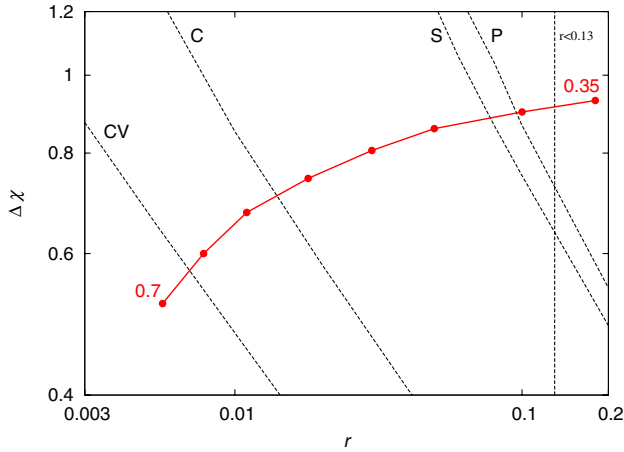
FIG. 4 (color online).   Red/solid curve: Values of $\Delta\chi$ and r obtained for $f = 0.01 M_p$ and $\lambda = 500$, and for different choices of y in the $0.35 < y < 0.7$ interval (successive points along the curve denote 0.05 increments in y); black/dotted vertical line: $r < 0.13$ bound from [3]; the other black/dotted curves are the 1σ detection lines for the Planck (P), SPIDER (S), CMB-Pol (C), and a cosmic-variance limited (CV) experiment. The signal needs to be above a line to be detectable at 1σ by that experiment. These experimental forecasts are an approximate copy of the lines shown in Fig. 2 of [36].

and n_s then further decreases at larger y). We stress that our choice of $f = 0.01 M_p$ and $\lambda = 500$ is only dictated by the requirement of $f \ll M_p, \lambda \gg 1$, so that the coupling to the gauge field plays a relevant role for the inflation dynamics (which, in turn, is the initial purpose of the model [17]). Obviously, these values can be changed, and our phenomenology discussion has the only purpose of understanding what kinds of limits can be imposed on the model. We expect that a viable region will exist at larger f (and smaller λ), as the model becomes closer to a free inflaton model slowly rolling on a flat potential.

At the theoretical level, it is interesting to note that the model can result in an observable gravity wave signal in the near future, even if the inflaton spans a range which is some orders of magnitude smaller than the Planck scale. This evades the Lyth bound [32], that states that $r \gtrsim 0.01$ is possible only for an excursion of the inflaton of $O(M_p)$ during the last ~ 60 e-folds of inflation. We note that the model we have studied evades the bound because of the $\chi F\tilde{F}$ interaction. Quite interestingly, the only other examples that we are aware of where $r > 0.01$ can be achieved even if the inflation evolution is orders of magnitude below the Planck scale[9] are those studied in Refs. [14,40], which are characterized by the same pseudoscalar interaction. As we already mentioned, Ref. [14] is also characterized by a $\sim f \ll M_p$ evolution due to the

---

[9]We note that inflationary potentials leading to $r \gtrsim 0.01$ have been constructed where the inflationary range is smaller than, but still of $O(M_p)$ [38,39].

damping from gauge field production. In this case, an interesting parity violating gravity wave signal can be generated [41] from the produced gauge quanta, but one needs to evade the simultaneous generation of too many non-Gaussian scalar density perturbations [42]. Reference [41] achieves this by considering a large number of gauge fields (this reduces non-Gaussianity by the central limit theorem) or by introducing a curvaton field. Reference [40] shows that $r \gtrsim 0.01$ can be obtained, and the non-Gaussianity limit can be respected, if the rolling scalar is not the inflaton.

An important difference is that, however, in the mechanism of [14,40] the tensor modes are produced at the nonlinear level by the vector fields produced by the rolling inflaton. For the model [17], the production occurs already at the linear level, due to its mixing with the vector mode induced by the vector vev and the non-Abelian structure ($g, Q \neq 0$). Quite likely, also in this model the vector modes t_L can source a significant amount of scalar density perturbations at the nonlinear level. This may reduce r from the level studied here, although a too large m_g will still likely be ruled out by the significant gravity wave production. It is possible that, for regimes resulting in acceptable r, the sourced scalar modes will lead to interesting levels of non-Gaussianity and primordial black holes as those obtained in [42–45]. It is also possible that, for some choice of parameters, the vector field production will "self-regulate" (with a consequent decrease of r) due to its backreaction on the inflationary dynamics, as visible in the background evolutions studied in [14,46] (a too large production may slow down the inflaton, and this may in turn decrease the vector production). All these interesting possibilities remain to be studied.

ACKNOWLEDGMENTS

Note added.—In the first version of this manuscript, we pointed out that (i) the inflationary solution of [17] is stable if and only if $m_g > 2H$, and that (ii) the tensor-to-scalar ratio r in this model is enhanced with respect to the case of a free inflaton, leading to violation of the Lyth's bound [32] for some choice of parameters. Both these claims are confirmed by the present analysis. The stability study is unchanged with respect to the first version. The study of the tensor modes, and the phenomenology considerations that follow from it, have instead been updated, to include the helicity violating terms in the tensor action that were erroneously missing in the first version. Such terms result

in a further increase of r and in a helicity violating gravity wave signal. The relevance of these terms was pointed out in [52], that appeared on the archive between the first and the current versions of this manuscript. Reference [52] agrees with our limit $m_g > 2H$ for the stability of the inflationary solution, and our revised analysis agrees with the effect of the helicity violating effects found in [52].

APPENDIX A: GENERAL FORMALISM FOR COSMOLOGICAL PERTURBATIONS AND ONE EXAMPLE

Our computation for the cosmological perturbations is algebraically more involved than those present in the literature for scalar field inflation. However, at the formal level, it does not differ from such computations. In this appendix we rigorously prove this claim by spelling out in detail the formal procedure that we follow, by showing how this procedure reproduces the standard computation of the perturbations of single scalar field inflation, and by showing that we follow the precise same steps for the computation of the perturbations for the model (1). The procedure is

(1) Perform a gauge choice that completely fixes the gauge freedom and preserves the manifestly non-dynamical modes in the system.

(2) Orient the coordinates such that the momentum of the mode studied in the linearized theory is along the z axis.

(3) Obtain the quadratic action for the perturbations with the above choices.

(4) Integrate out the nondynamical modes, so as to obtain the quadratic action for the dynamical modes.

(5) Obtain from this action the initial conditions in the subhorizon regime, and the linearized equations for the dynamical modes.

(6) Solve these equations, with these initial conditions.

Steps 1, 2, and 3 are described in detail in Sec. III A. The gauge we choose preserves the modes $\delta g_{0\mu}$ and δA_0^a, which are nondynamical.[10] Such variables enter without time derivatives in the quadratic action for the perturbations. These modes are nondynamical due to the structure of R and of F^2.[11]

In all generality, a Hermitian quadratic action for a set $\{X_i\}$ of dynamical modes, and a set $\{N_i\}$ of nondynamical modes, must be formally of the type (in Fourier space)

---

[10]We remark that choosing a different gauge that completely fixes the gauge freedom does not change the number of the dynamical or the nondynamical modes, but that, in this other gauge, the nondynamical modes may appear as linear combinations of the modes preserved. The gauge that we have chosen has simply the advantage of keeping manifest which modes are the nondynamical ones.

[11]The fact that the $\delta g_{0\mu}$ modes are nondynamical is the basis of the Arnowitt-Deser-Misner (ADM) formalism [47].

$$S = \int d\tau d^3k [a_{ij} X_i'^\dagger X_j' + (b_{ij} X_i'^\dagger X_j + \text{H.c.}) + c_{ij} X_i^\dagger X_j$$
$$+ (d_{ij} N_i^\dagger X_j' + \text{H.c.}) + (e_{ij} N_i^\dagger X_j + \text{H.c.}) + f_{ij} N_i^\dagger N_j], \tag{A1}$$

where the matrices formed by a_{ij}, c_{ij}, f_{ij} are Hermitian. Moreover, through an integration by parts, we can impose that the matrix formed by b_{ij} is anti-Hermitian. The coefficients $a_{ij}, \ldots, f_{ij}$ are functions of background quantities, and therefore are time dependent. The action (C1) is indeed of this type.

Once the explicit form of (A1) is obtained, it does no longer matter that some of the modes originated from a SU(2) multiplet. Therefore we do not expect any additional conceptual difficulty with respect to scalar field inflation. We now explicitly prove that this is indeed the case.

The linearized equations of motion following from (A1) are formally

$$\frac{\delta S}{\delta N_i^\dagger} = 0 \Rightarrow d_{ij} X_j' + e_{ij} X_j + f_{ij} N_j = 0 \tag{A2}$$

$$\frac{\delta S}{\delta X_i^\dagger} = 0 \Rightarrow (a_{ij} X_j' + b_{ij} X_j + d_{ji}^* N_j)'$$
$$- b_{ji}^* X_j' - c_{ij} X_j - e_{ji}^* N_j = 0 \tag{A3}$$

which are the linearized equations for the perturbations. We note that we can also obtain these equations by perturbing to linear order the equations of motion of the model, without computing the quadratic action of the perturbations. The quadratic action is only needed to set the initial conditions of the perturbations (typically, in the adiabatic vacuum).

Equations (A2) are the constraint equations of the system. In these equations, no second order time derivatives of the dynamical variables and no time derivatives of the nondynamical variables appear. We solve these equations by providing the nondynamical modes in terms of the dynamical modes and their first time derivative. In matrix form,

$$N = -f^{-1}(dX' + eX), \tag{A4}$$

where we note that the matrix f needs to be invertible [otherwise we would have a system of perturbations which cannot be solved; we verified that f is indeed invertible for the action (C1)]. We stress that both the dynamical fields and their first derivatives enter in this expression, but not the derivatives of the nondynamical modes. In other words, the constraint equations (A2) are algebraic in the nondynamical modes; therefore, the nondynamical modes are completely determined from the dynamical ones, without introducing additional physical degrees of freedom.

The action for the scalar field perturbations for the model (1) is given in Eq. (C1). In the main text we studied

the problem disregarding metric perturbations. There is therefore only one nondynamical mode in the system, denoted by Y. The constraint equation for this mode is $\frac{\delta S_{2,\text{scalar}}}{\delta Y^*} = 0$ (which is nothing but the scalar projection of the equation of motion for A_0^a, linearized to first order in the perturbations). This equation is solved by (60). We note that the solution (60) is precisely of the form (A4). It is straightforward to compute the equations for the three nondynamical modes Y, ϕ, B present in the full action (C1), and explicitly verify that these equations are also of the type (A4). Formally, this is not any different from the computation of standard scalar field inflation, where the constraint equations are solved by (A11). The fact that the solutions of the constraint equations of our model, and of standard scalar field inflation, are both of the form (A4), mathematically proves that no additional conceptual difficulty is present in the case at hand.

Inserting the solution (A4) into (A3), we obtain the linearized equations in terms of the dynamical modes only:

$$
(a - d^\dagger f^{-1} d) X'' + [(a - d^\dagger f^{-1} d)'
$$
$$
+ (b - d^\dagger f^{-1} e - \text{H.c.})] X' + [(b - d^\dagger f^{-1} e)'
$$
$$
- c + e^\dagger f^{-1} e] X = 0. \tag{A5}
$$

We can also insert the solution (A4) into the action (A1). This is the sense in which we integrate out the nondynamical modes from the action. We obtain

$$
S = \int d\tau d^3 k [X'^\dagger (a - d^\dagger f^{-1} d) X' + X^\dagger (c - e^\dagger f^{-1} e) X
$$
$$
+ (X'^\dagger (b - d^\dagger f^{-1} e) X + \text{H.c.})]. \tag{A6}
$$

This is the action of the dynamical modes of the system. Extremization of this action also leads precisely to Eq. (A5). By inverting the kinetic term, we can write canonical variables Δ, in terms of which the action (A6) can be cast in the form (32).

At the linearized level, the dynamical perturbations are determined by (A6), and solely by that. The initial conditions and the equations of motion (A5) that follow from this action are studied and solved in the main text.

Once the solution for the dynamical modes has been obtained, it can be inserted into Eq. (A4) to provide the explicit solution also for the nondynamical variables.

1. One example

We now show how the steps outlined above lead to the standard result for single scalar field inflation. We decompose the metric as given in Eq. (29). In this illustrative example we only focus on the scalar modes of the system. So, we have the line element

$$
ds^2 = a^2(\tau)[-(1 - 2\phi) d\tau^2 + 2\partial_z B d\tau dz + \delta_{ij} dx^i dx^j]. \tag{A7}
$$

We consider single scalar field inflation with the Lagrangian

$$
\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi), \qquad \varphi = \varphi^{(0)} + \delta\varphi \tag{A8}
$$

and arbitrary potential V. Therefore, we have the single dynamical mode $X = \{\delta\varphi\}$, and the two nondynamical modes $N = \{\phi, B\}$. In a theory with n scalar fields we would end up with the n dynamical modes $X = \{\delta\varphi_1, \ldots, \delta\varphi_n\}$. We insert the above line element and Lagrangian into the action

$$
S = \int d^4 x \sqrt{-g} \left[\frac{M_p^2}{2} R + \mathcal{L} \right], \tag{A9}
$$

we expand this action at quadratic order in the perturbations, and we Fourier transform. We obtain

$$
S = \int \frac{d\tau d^3 k a^2}{2} [|\delta\varphi'|^2 - (k^2 + a^2 V_{,\varphi\varphi})|\delta\varphi|^2
$$
$$
+ \varphi^{(0)'}(\phi^* \delta\varphi' + \text{H.c.}) + a^2 V_{,\varphi}(\phi^* \delta\varphi + \text{H.c.})
$$
$$
- k^2 \varphi^{(0)'}(B^* \delta\varphi + \text{H.c.}) + (\varphi^{(0)'2} - 6 M_p^2 \mathcal{H}^2)|\phi|^2
$$
$$
- 2 k^2 M_p^2 \mathcal{H}(\phi^* B + \text{H.c.})], \tag{A10}
$$

where we recall that $\mathcal{H} = \frac{a'}{a}$. As it must be, this action is indeed of the form (A1). In particular, the nondynamical fields enter in the action without time derivatives.

The constraint equations are obtained by extremization of (A10). These equations are solved by

$$
\phi = -\frac{\varphi^{(0)'}}{2 M_p^2 \mathcal{H}} \delta\varphi
$$
$$
B = \frac{1}{2 k^2 M_p^2} \left[\frac{\varphi^{(0)'}}{\mathcal{H}} \delta\varphi' + \left(3\varphi^{(0)'} + \frac{a^2 V_{,\varphi}}{\mathcal{H}} - \frac{\varphi^{(0)'3}}{2 M_p^2 \mathcal{H}^2} \right) \delta\varphi \right]
$$
$$
\tag{A11}
$$

which are indeed of the type (A4). Namely, the nondynamical quantities are given by linear combinations of dynamical variables and their first time derivatives. The constraint equations solved by (A11) are nothing but the linearized 00 and the 0i components of the Einstein equations of the system [48]. We could have equivalently obtained them without computation of the quadratic action.

Inserting the solutions (A11) back into the quadratic action leads to

$$
S = \int d\tau d^3 k \frac{a^2}{2} \left[|\delta\varphi'|^2 - \left(k^2 + a^2 V_{,\varphi\varphi} + 2 \frac{a^2 V_{,\varphi} \varphi^{(0)'}}{M_p^2 \mathcal{H}} \right. \right.
$$
$$
\left. \left. + \frac{3\varphi^{(0)'2}}{M_p^2} - \frac{\varphi^{(0)'4}}{2 M_p^4 \mathcal{H}^2} \right) |\delta\varphi|^2 \right] \tag{A12}
$$

which is indeed of the form (A6). Finally, the Mukhanov-Sasaki variable [49,50] in the spatially flat gauge reads $v = a\delta\varphi$. In terms of v, this action reduces to the standard well known result [48]

$$S = \frac{1}{2} \int d\tau d^3k \left[|v'|^2 - \left(k^2 - \frac{z''}{z} \right) |v|^2 \right], \qquad z \equiv \frac{a\varphi^{(0)\prime}}{\mathcal{H}}.$$

$$(A13)$$

The computation of the cosmological perturbations performed in this work exactly follows the steps outlined with this example. Most of the procedure that we have employed is standard in cosmological perturbations theory (see for instance [51], which also uses the spatially flat gauge, and integrates out the $\delta g_{0\mu}$ modes as we do). The only differences of our computation with respect to single scalar field inflation are the presence of (i vector fields in the original action, and of (ii) more than one dynamical field. Concerning (i), Eq. (28) shows how the vector multiplet is decomposed. This decomposition leads to a quadratic action for the perturbations which is precisely of the

type (A1), after which the next steps in the method can be followed. Concerning (ii), the presence of more than one dynamical field is also something commonly encountered in scalar field inflation. Indeed, it is straightforward to generalize the discussion leading from Eq. (A8) to Eq. (A13) to the presence of more scalar fields [34].

APPENDIX B: EXPLICIT FORM OF THE QUADRATIC ACTION FOR THE SCALAR MODES WITHOUT METRIC PERTURBATIONS

In this appendix we write the explicit expression for the quadratic action of the scalar perturbations without metric perturbations. We start from the scalar perturbations $\delta\chi$, Q, M, Y entering in the decomposition (28), and we insert these fields into the action (1). We expand the action to second order in the perturbations, and obtain

$$
\begin{aligned}
S_{\text{no}\,\delta g} = \int d\tau d^3k \frac{a^2}{2} \Bigg\{ & k^2(k^2 + 2g^2a^2Q^2)|Y|^2 + \left[k^2 Y^* \left(\delta Q' - k^2 M' - \frac{g\lambda}{f} aQ^2 \delta\chi + \mathcal{H}\delta Q - k^2\mathcal{H}M \right) + \text{H.c.} \right] + |\delta\chi'|^2 \\
& + 3|\delta Q'|^2 + k^4|M'|^2 - k^2(\delta Q^{*\prime}M' + \text{H.c.}) + \frac{3g\lambda aQ^2}{2f} \left[\delta Q^*\delta\chi' - \delta\chi\delta Q^{*\prime} + \frac{k^2}{3}(\delta\chi^*M' - M\delta\chi^{*\prime}) + \text{H.c.} \right] \\
& - (k^2 + a^2 V_{,\chi\chi})|\delta\chi|^2 - \left[2k^2 + 3g^2a^2Q^2 \left(6 - \frac{Q^2}{M_p^2} \right) + 6\mathcal{H}^2 - \frac{3}{M_p^2}(Q' + \mathcal{H}Q)^2 - \frac{6g\lambda aQ\chi'}{f} - \frac{3\chi'^2}{2M_p^2} \right]|\delta Q|^2 \\
& - k^4 \left[g^2a^2Q^2 \left(2 - \frac{Q^2}{M_p^2} \right) + 2\mathcal{H}^2 - \frac{(Q' + \mathcal{H}Q)^2}{M_p^2} - \frac{\chi'^2}{2M_p^2} \right]|M|^2 - \frac{3g\lambda aQ}{2f}(2Q' + 3\mathcal{H}Q)\left(\delta\chi^*\delta Q - \frac{k^2}{3}\delta\chi^*M + \text{H.c.} \right) \\
& - k^2 \left[-g^2a^2Q^2 \left(6 - \frac{Q^2}{M_p^2} \right) - 2\mathcal{H}^2 + \frac{(Q' + \mathcal{H}Q)^2}{M_p^2} + \frac{2g\lambda aQ\chi'}{f} + \frac{\chi'^2}{2M_p^2} \right](\delta Q^*M + \text{H.c.}) \Bigg\}.
\end{aligned}
$$

$$(B1)$$

We note that this action is of the type (A1), and therefore we can proceed with the computation of the perturbations following the steps outlined in Appendix A. The results of the computation are presented in Sec. VI.

APPENDIX C: INCLUDING SCALAR METRIC PERTURBATIONS

In Sec. VI we studied the system of linear perturbations for the model, disregarding the perturbations of the metric. Here we summarize the results for the full system, and we confirm the accuracy of the approximation made in the main text.

We start from the full set of six scalar perturbations (after gauge fixing), including the two modes from the metric perturbations (ϕ and B) that we have (artificially) set to zero in the main text. The full action is

$$S_{2,\text{scalar-full}} = S_{\text{no}\,\delta g} + S_{\delta g} \tag{C1}$$

where the first term is explicitly given in (B1), while the explicit expression for the second term is

$$
\begin{aligned}
S_{\delta g} = \int d\tau d^3k \frac{a^2}{2} \Bigg\{ & (-6M_p^2\mathcal{H}^2 + 3(Q' + \mathcal{H}Q)^2 + \chi'^2)|\phi|^2 + 2k^2g^2a^2Q^4|B|^2 - 2k^2M_p^2\mathcal{H}(\phi^*B + \text{H.c.}) \\
& + [\phi^*(\chi'\delta\chi' + (Q' + \mathcal{H}Q)(3\delta Q' - k^2M' + k^2Y) + a^2V_{,\chi}\delta\chi + (\mathcal{H}(Q' + \mathcal{H}Q) + 2g^2a^2Q^3)(3\delta Q - k^2M)) + \text{H.c.}] \\
& - \left[2k^2B^*\left(g^2a^2Q^3Y + (Q' + \mathcal{H}Q)\delta Q + \frac{\chi'\delta\chi}{2} \right) + \text{H.c.} \right] \Bigg\}.
\end{aligned}
$$

$$(C2)$$

The new term (C2) collects all the dependence of the full action on the metric perturbations ϕ and B. Both these modes are nondynamical, and we integrate them out, together with the other nondynamical perturbation Y. We are left with three dynamical modes, and we rotate them as in Eq. (61) of the main text. In this way, we obtain the full quadratic action for the perturbations. It is formally of the type

$$S_{2,\text{scalar-full}} = \frac{1}{2} \int d\tau d^3k [\Delta'^\dagger C_{\text{s,f}} \Delta' + \Delta'^\dagger K_{\text{s,f}} \Delta$$
$$- \Delta^\dagger K_{\text{s,f}} \Delta' - \Delta^\dagger \Omega_{\text{s,f}}^2 \Delta]. \tag{C3}$$

Namely, the modes (61) are not the exact canonical scalar variables. However, as we now show, they provide a very good approximation to the canonical modes in the slow roll regime. More in general, the matrices of the full action (C3) are extremely involved. We studied them in slow roll approximation. Specifically, using the slow roll approximation (22), we can cast all the elements of these matrices in the form

$$\frac{\sum_i c_i k^{p_{\alpha_i}} H^{\beta_i}}{\sum_j d_j k^{p_{\alpha_j}} H^{\beta_j}}, \tag{C4}$$

where the coefficients c_i and d_j only depend on the parameters of the model, and on slowly evolving background quantities. We computed the leading order expression for these coefficients in the slow roll approximation. For example, we obtain

$$(C_{\text{s,f}})_{11} = 1 + \frac{g^2 Q^4 \dot{\chi}^2 / M_p^4}{2H^2 p^2 + 4g^2 H^2 Q^2 [1 + O(\frac{1}{\lambda})]}$$
$$= 1 + \frac{O(\frac{f^2}{M_p^2 y^{3/2} \lambda^3}) H^4}{2H^2 p^2 + O(\frac{1}{y^{3/2}})[1 + O(\frac{1}{\lambda})] H^4}. \tag{C5}$$

This expression is extremely close to 1, since, parametrically, the second term is $\lambda^{-3} \ll 1$ outside the horizon, and even more suppressed inside the horizon. In fact, the kinetic matrix differs from the identity only up to slow roll suppressed quantities:

$$(C_{\text{s,f}})_{12} = \frac{O(\lambda^{1/2}) H^2}{O(y^{3/2} \lambda^3) p^2 + O(\lambda^3) H^2}$$

$$(C_{\text{s,f}})_{13} = \frac{O(\lambda^{3/2}) H \sqrt{H^2 + O(y^{3/2}) p^2}}{O(y^{3/2} \lambda^3) p^2 + O(\lambda^3) H^2}$$

$$(C_{\text{s,f}})_{22} = 1 + \frac{O(\lambda) H^2}{O(y^{3/2} \lambda^3) p^2 + O(\lambda^3) H^2} \tag{C6}$$

$$(C_{\text{s,f}})_{23} = \frac{O(\lambda^2) H \sqrt{H^2 + O(y^{3/2}) p^2}}{O(y^{3/2} \lambda^3) p^2 + O(\lambda^3) H^2}$$

$$(C_{\text{s,f}})_{33} = 1 + \frac{O(\lambda^2) H^2}{O(y^{3/2} \lambda^3) p^2 + O(\lambda^3) H^2}.$$

Therefore, up to very small slow roll corrections, the modes (61) are also the canonical variable of the full scalar system.

Performing the same procedure on $K_{\text{s,f}}$, we obtain

$$\left(\frac{K_{\text{s,f}}}{a}\right)_{12} = \frac{g\lambda Q^2}{\sqrt{2} f} \frac{p^2 + m_g^2 [1 + O(\frac{1}{\lambda})]}{p^2 + m_g^2 [1 + O(\frac{1}{\lambda})]}$$
$$= \left(\frac{K_{\text{s}}}{a}\right)_{12} \left[1 + O\left(\frac{\lambda^{-1}}{1 + y^{3/2} p^2 / H^2}\right)\right]$$

$$\left(\frac{K_{\text{s,f}}}{a}\right)_{13} = -\frac{g^2 \lambda Q^3}{\sqrt{2} f \sqrt{p^2 + m_g^2}} \frac{(p^2 + m_g^2)[1 + O(\frac{1}{\lambda})]}{p^2 + m_g^2 [1 + O(\frac{1}{\lambda})]}$$
$$= \left(\frac{K_{\text{s}}}{a}\right)_{13} [1 + O(\lambda^{-1})]$$

$$\left(\frac{K_{\text{s,f}}}{a}\right)_{23} = \frac{H^4}{(p^2 + m_g^2)^{3/2}} \left[O\left(\frac{1}{y^{9/4} \lambda}\right) \frac{p^2}{H^2} + O\left(\frac{1}{y^{15/4} \lambda}\right)\right]$$
$$\sim O\left(\frac{H}{\sqrt{p^2 + H^2}} \frac{f}{M_p} \frac{1}{\lambda^{3/2}}\right) \times \left(\frac{K_{\text{s,f}}}{a}\right)_{12}, \tag{C7}$$

where we recall that $m_g^2 \equiv 2g^2 Q^2$. The 12 and 13 entries are in excellent agreement with those given in the main text. We recall that the 13 entry is much smaller than the 12 entry inside the horizon, and it is negligible in the stability study. We note that, for the full system, the 23 element is nonvanishing, while $(K_{\text{s}})_{23} = 0$. However, this element is strongly slow roll suppressed with respect to the other two, and completely negligible.

Proceeding in the same way (for brevity, we omit here the powers of y), we obtain

$$\frac{(\Omega_{\text{s,f}}^2)_{12}}{(\Omega_{\text{s}}^2)_{12}} = 1 + \frac{\sum_{i=0}^2 O(H^{2i} p^{4-2i})}{(p^2 + m_g^2)^2 \lambda}$$

$$\frac{(\Omega_{\text{s,f}}^2)_{13}}{(\Omega_{\text{s}}^2)_{13}} = 1 + \frac{1}{\lambda} \frac{H^2}{p^2 + m_g^2} \frac{\sum_{i=0}^2 O(H^{2i} p^{4-2i})}{\sum_{i=0}^2 O(H^{2i} p^{4-2i})}$$

$$\frac{(\Omega_{\text{s,f}}^2)_{22} - k^2}{(\Omega_{\text{s}}^2)_{22} - k^2} = 1 + O\left(\frac{1}{\lambda}\right) \tag{C8}$$

$$\frac{(\Omega_{\text{s,f}}^2)_{23}}{(\Omega_{\text{s}}^2)_{23}} = 1 + \frac{\sum_{i=0}^3 O(H^{2i} p^{6-2i})}{(p^2 + m_g^2)^3 \lambda}$$

$$\frac{(\Omega_{\text{s,f}}^2)_{33} - k^2}{(\Omega_{\text{s}}^2)_{33} - k^2} = 1 + \frac{\sum_{i=0}^3 O(H^{2i} p^{6-2i})}{(p^2 + m_g^2) \lambda \sum_{i=0}^2 O(H^{2i} p^{4-2i})}$$

and we see that, for all these entries, the expressions of $\Omega_{\text{s,f}}^2$ and of Ω_{s}^2 agree at all scales (namely, for any value of H/p) up to subdominant $O(\lambda^{-1})$ corrections. For the 11 entry, we obtain

$$\frac{(\Omega_{\text{s,f}}^2)_{11} - k^2}{(\Omega_{\text{s}}^2)_{11} - k^2} = 1 + \frac{H^2}{p^2 \lambda} \frac{O(p^2) + O(H^2)}{p^2 + m_g^2}. \tag{C9}$$

Also on this entry, the two matrices are in perfect agreement during the full subhorizon regime. However, while for the other entries the agreement continues also in the superhorizon regime, the 11 entries differ from each other for $H^2 > O(\lambda p^2)$. By evaluating the coefficients in (C9),

we found that the disagreement starts only at $H \gtrsim 10p$ for all the evolutions studied in the main text.

To conclude, all the matrix elements, up to one exception, of the system of scalar perturbations studied in the main text (in which we made the approximation of disregarding the scalar metric perturbations) agree at all scales (namely, for any value of H/p) with the corresponding entries of the full system up to subdominant terms in a slow roll expansion. The single exception is the 11 entry of

Ω^2, for which the agreement persists during the entire subhorizon regime, at horizon crossing, and also up to $H \lesssim 10p$, but not further. This proves that the stability study performed in the main text is valid also when the metric perturbations are included, given that the instability, when present, manifests itself deeply inside the horizon. Most likely, this guarantees that also the power spectra shown in the main text are accurate, since the disagreement manifests itself only after the powers have frozen (see Fig. 2).

---

[1] A. D. Linde, Contemp. Concepts Phys. **5**, 1 (1990).

[2] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **192**, 18 (2011).

[3] G. Hinshaw, D. Larson, E. Komatsu, D. N. Spergel, C. L. Bennett, J. Dunkley, M. R. Nolta, M. Halpern *et al.*, arXiv:1212.5226.

[4] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).

[5] K. Freese and W. H. Kinney, Phys. Rev. D **70**, 083512 (2004).

[6] C. Savage, K. Freese, and W. H. Kinney, Phys. Rev. D **74**, 123511 (2006).

[7] R. Kallosh, A. D. Linde, D. A. Linde, and L. Susskind, Phys. Rev. D **52**, 912 (1995).

[8] T. Banks, M. Dine, P. J. Fox, and E. Gorbatov, J. Cosmol. Astropart. Phys. 06 (2003) 001.

[9] J. E. Kim, H. P. Nilles, and M. Peloso, J. Cosmol. Astropart. Phys. 01 (2005) 005; M. Berg, E. Pajer, and S. Sjors, Phys. Rev. D **81**, 103535 (2010).

[10] S. Dimopoulos, S. Kachru, J. McGreevy, and J. G. Wacker, J. Cosmol. Astropart. Phys. 08 (2008) 003.

[11] R. Kallosh, Lect. Notes Phys. **738**, 119 (2008); R. Kallosh, N. Sivanandam, and M. Soroush, Phys. Rev. D **77**, 043501 (2008); E. Silverstein and A. Westphal, Phys. Rev. D **78**, 106003 (2008); L. McAllister, E. Silverstein, and A. Westphal, Phys. Rev. D **82**, 046003 (2010).

[12] N. Kaloper and L. Sorbo, Phys. Rev. Lett. **102**, 121301 (2009).

[13] C. Germani and A. Kehagias, Phys. Rev. Lett. **106**, 161302 (2011); J. Ohashi and S. Tsujikawa, J. Cosmol. Astropart. Phys. 10 (2012) 035; D. Maity, Phys. Lett. B **720**, 389 (2013).

[14] M. M. Anber and L. Sorbo, Phys. Rev. D **81**, 043534 (2010).

[15] S. Mohanty and A. Nautiyal, Phys. Rev. D **78**, 123515 (2008); L. Visinelli, J. Cosmol. Astropart. Phys. 09 (2011) 013.

[16] A. Berera, Phys. Rev. Lett. **75**, 3218 (1995).

[17] P. Adshead and M. Wyman, Phys. Rev. Lett. **108**, 261302 (2012).

[18] E. Martinec, P. Adshead, and M. Wyman, J. High Energy Phys. 02 (2013) 027.

[19] E. Dimastrogiovanni, N. Bartolo, S. Matarrese, and A. Riotto, Adv. Astron. **2010**, 752670 (2010); K. Dimopoulos, Int. J. Mod. Phys. D **21**, 1250023

(2012); **21**, 1292003(E) (2012); J. Soda, Classical Quantum Gravity **29**, 083001 (2012); A. Maleknejad, M. M. Sheikh-Jabbari, and J. Soda, arXiv:1212.2921.

[20] A. Maleknejad and M. M. Sheikh-Jabbari, Phys. Lett. B **717**, 6 (2012).

[21] A. Maleknejad and M. M. Sheikh-Jabbari, Phys. Rev. D **84**, 043515 (2011).

[22] P. Adshead and M. Wyman, Phys. Rev. D **86**, 043530 (2012).

[23] M. M. Sheikh-Jabbari, Phys. Lett. B **717**, 6 (2012).

[24] N. Bartolo, S. Matarrese, M. Peloso, and A. Ricciardone, Phys. Rev. D **87**, 023504 (2013).

[25] M.-a. Watanabe, S. Kanno, and J. Soda, Phys. Rev. Lett. **102**, 191302 (2009); S. Kanno, J. Soda, and M.-a. Watanabe, J. Cosmol. Astropart. Phys. 12 (2010) 024; K. Murata and J. Soda, J. Cosmol. Astropart. Phys. 06 (2011) 037; K. Yamamoto, Phys. Rev. D **85**, 123504 (2012); K.-i. Maeda and K. Yamamoto, Phys. Rev. D **87**, 023528 (2013); H. Funakoshi and K. Yamamoto, arXiv:1212.2615; K. Dimopoulos, M. Karciauskas, and J. M. Wagstaff, Phys. Rev. D **81**, 023522 (2010).

[26] K. Dimopoulos, M. Karciauskas, and J. M. Wagstaff, Phys. Lett. B **683**, 298 (2010); J. M. Wagstaff and K. Dimopoulos, Phys. Rev. D **83**, 023523 (2011); R. Namba, Phys. Rev. D **86**, 083518 (2012).

[27] J. A. R. Cembranos, C. Hallabrin, A. L. Maroto, and S. J. N. Jareno, Phys. Rev. D **86**, 021301 (2012); J. A. R. Cembranos, A. L. Maroto, and S. J. N. Jareno, Phys. Rev. D **87**, 043523 (2013).

[28] B. Himmetoglu, C. R. Contaldi, and M. Peloso, Phys. Rev. Lett. **102**, 111301 (2009); Phys. Rev. D **79**, 063517 (2009); **80**, 123530 (2009).

[29] E. Dimastrogiovanni, M. Fasiello, and A. J. Tolley, J. Cosmol. Astropart. Phys. 02 (2013) 046.

[30] A. J. Tolley and M. Wyman, Phys. Rev. D **81**, 043502 (2010).

[31] D. Baumann *et al.* (CMBPol Study Team Collaboration), AIP Conf. Proc. **1141**, 10 (2009).

[32] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997).

[33] A. E. Gumrukcuoglu, B. Himmetoglu, and M. Peloso, Phys. Rev. D **81**, 063528 (2010).

[34] K. A. Malik and D. Wands, Phys. Rep. **475**, 1 (2009).

[35] S. Saito, K. Ichiki, and A. Taruya, J. Cosmol. Astropart. Phys. 09 (2007) 002.

[36] V. Gluscevic and M. Kamionkowski, Phys. Rev. D **81**, 123529 (2010).

[37] R. Keisler, C. L. Reichardt, K. A. Aird, B. A. Benson, L. E. Bleem, J. E. Carlstrom, C. L. Chang, H. M. Cho *et al.*, Astrophys. J. **743**, 28 (2011).

[38] I. Ben-Dayan and R. Brustein, J. Cosmol. Astropart. Phys. 09 (2010) 007.

[39] S. Hotchkiss, A. Mazumdar, and S. Nadathur, J. Cosmol. Astropart. Phys. 02 (2012) 008.

[40] N. Barnaby, J. Moxon, R. Namba, M. Peloso, G. Shiu, and P. Zhou, Phys. Rev. D **86**, 103508 (2012).

[41] L. Sorbo, J. Cosmol. Astropart. Phys. 06 (2011) 003.

[42] N. Barnaby and M. Peloso, Phys. Rev. Lett. **106**, 181301 (2011).

[43] N. Barnaby, R. Namba, and M. Peloso, J. Cosmol. Astropart. Phys. 04 (2011) 009.

[44] P. D. Meerburg and E. Pajer, J. Cosmol. Astropart. Phys. 02 (2013) 017.

[45] A. Linde, S. Mooij, and E. Pajer, arXiv:1212.1693.

[46] N. Barnaby, E. Pajer, and M. Peloso, Phys. Rev. D **85**, 023525 (2012).

[47] R. L. Arnowitt, S. Deser, and C. W. Misner, Gen. Relativ. Gravit. **40**, 1997 (2008).

[48] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).

[49] V. F. Mukhanov, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 402 (1985) [JETP Lett. **41**, 493 (1985)].

[50] M. Sasaki, Prog. Theor. Phys. **76**, 1036 (1986).

[51] J. M. Maldacena, J. High Energy Phys. 05 (2003) 013.

[52] P. Adshead, E. Martinec, and M. Wyman, arXiv:1301.2598.