RAPID COMMUNICATIONS

PHYSICAL REVIEW D **87,** 101902(R) (2013)

Supergravity dual of *c*-extremization

Parinya Karndumri[1,2] and Eoin Ó Colgáin[3]

[1]*Department of Physics, Faculty of Science, Chulalongkorn University, Bangkok 10330, Thailand*
[2]*Thailand Center of Excellence in Physics, CHE, Ministry of Education, Bangkok 10400, Thailand*
[3]*Departamento de Física, Universidad de Oviedo, Oviedo 33007, Spain*

(Received 2 March 2013; published 10 May 2013)

Recently, a general principle, called c-extremization, which determines the exact R symmetry of two-dimensional conformal field theories with $\mathcal{N} = (0, 2)$ supersymmetry, was identified. In this work we show that the supergravity dual corresponds to the extremization of the T tensor of $\mathcal{N} = 2$ gauged supergravity in three dimensions. To support this claim, we demonstrate that the expected central charge of conformal field theories arising from twisted compactifications of four-dimensional $\mathcal{N} = 4$ Super-Yang-Mills on Riemann surfaces, whose gravity dual is a reduction of five-dimensional $U(1)^3$ gauged supergravity, is recovered in the three-dimensional framework.

DOI: 10.1103/PhysRevD.87.101902 PACS numbers: 11.25.Tq, 11.30.Pb

I. INTRODUCTION

As arguably the most concrete example of the holographic principle [1], the AdS/CFT correspondence [2] states that any solution of string theory with an anti-de Sitter (AdS) factor should be equivalent to a conformal field theory (CFT) in one space-time dimension lower. This correspondence and its generalizations have proved instrumental in offering unrivaled insights into the nonperturbative regime of quantum field theories and the quantum nature of black holes.

Within this context, backgrounds with AdS_3 factors are particularly appealing since, in contrast to higher dimensions, the conformal group in two dimensions is infinite dimensional, and as a result the CFTs are much more tractable. Indeed, it is a well-known fact [3] that the entropy of a class of five-dimensional black holes can be derived from the central charge of $\mathcal{N} = (4, 4)$ CFTs dual to $\mathrm{AdS}_3 \times S^3 \times CY_2$ backgrounds of type IIB supergravity.

Moreover, what makes three dimensions historically well-suited to holography is the pioneering pre-AdS/CFT observation [4], which states that any consistent theory of quantum gravity in three dimensions with AdS asymptotics defines a two-dimensional CFT. It is in this spirit that we have witnessed a resurgence in variants of general relativity, notably topologically massive gravity [5] and new massive gravity [6]. This interest extends to various (warped) AdS_3 black holes [7,8], solutions which also crop up in three-dimensional supergravity [9,10], and the microscopic degrees of freedom of the dual field theory. Remarkably, it has recently been suggested that topologically massive gravity may act as a conduit to holography in asymptotically flat space-times [11].

In this paper, working directly with three-dimensional gauged supergravity, without recourse to higher-dimensional string theory constructions, we show how the exact R symmetry and central charge of AdS_3 vacua dual to $\mathcal{N} = (0, 2)$ CFTs may be identified. As such, our prescription provides a supergravity dual for c-extremization [12,13], a recently identified lower-dimensional counterpart of a maximization [14]. Since the R symmetry can mix with flavor symmetries for supersymmetric theories flowing to IR fixed points, these respective principles extremize polynomials constructed from 't Hooft anomalies, which are recognized invariances of renormalization group flows, to determine the exact R symmetry.

Via AdS/CFT, a maximization [14] can be recast in terms of volume minimization of Sasaki-Einstein manifolds [15,16] so that the Reeb vector dual to the R symmetry is picked out from a linear combination of candidate $U(1)$ isometries. Subsequent studies [17,18] have shown a maximization and volume minimization to be formally equivalent. More generally, a maximization has an interpretation in terms of the minimization of the Killing prepotential of $\mathcal{N} = 2$ gauged supergravity in five dimensions [19], a fact put to use in Ref. [20] to identify the R symmetry for a family of supergravity solutions [21,22] based on wrapped M5-branes.

Recent developments beg the question of what is the holographic dual description for c-extremization. To address this problem, we retrace the arguments of Ref. [19] in the natural language of three-dimensional gauged supergravity, and, in the so-called T tensor of $\mathcal{N} = 2$ gauged supergravity, we identify a function that, when extremized, determines the R symmetry and central charge. As we shall see, when the $SO(2)_R \sim U(1)_R$ R symmetry is gauged, the scalar potential is only a function of T, meaning that the extremization of T naturally leads to AdS_3 vacua.

II. REVIEW OF *c*-EXTREMIZATION

In a nonconformal $\mathcal{N} = (0, 2)$ supersymmetric theory with $U(1)_R$ R symmetry, the R symmetry is not uniquely defined, and mixing of $U(1)_R$ with the other Abelian flavor symmetries is permitted. At a conformal fixed point, this changes, and an exact superconformal R symmetry is picked out. To identify this exact R symmetry at the superconformal fixed point, Refs. [12,13] introduced a "trial R current,"

$$\Omega^{\rm tr}_\mu(t) = J^r_\mu + \sum_{M(\neq r)} t_M J^M_\mu, \qquad (1)$$

where J^r_μ is a choice of the R symmetry current and $J^M_\mu (M \neq r)$ are Abelian flavor symmetry currents. From $\Omega^{\rm tr}_\mu(t)$ one constructs a quadratic function $c^{\rm tr}_R(t)$, which is proportional to the 't Hooft anomaly of $\Omega^{\rm tr}_\mu(t)$:

$$c^{\rm tr}_R(t) = 3\left(k^{rr} + 2\sum_{M(\neq r)} t_M k^{rM} + \sum_{M,N(\neq r)} t_M t_N k^{MN}\right), \qquad (2)$$

where k^{MN} are the 't Hooft anomaly coefficients. Recall that these anomalies arise in the context of theories with $U(1)^P$ global symmetry when the theory is coupled to nondynamical vector fields A^M_μ, $M = 1, \ldots, P$, in a curved background with metric $g_{\mu\nu}$. The anomalous violations of current conservation are then given by

$$\nabla^\mu J^M_\mu = \sum_N \frac{k^{MN}}{8\pi} F^N_{\mu\nu}\epsilon^{\mu\nu},$$
$$\nabla_\mu T^{\mu\nu} = \frac{k}{96\pi} g^{\nu\alpha}\epsilon^{\mu\rho}\partial_\mu\partial_\beta\Gamma^\beta_{\alpha\rho},$$

where $F^M = dA^M$, $T_{\mu\nu}$ is the stress tensor and $\Gamma^\beta_{\alpha\rho}$ is the Levi-Civita connection for $g_{\mu\nu}$.

The trial c function (2) can be motivated from a study of the $\mathcal{N} = 2$ superconformal algebra [12,13]. In particular, for supercharges $\mathcal{Q}$ with R charge one, the algebra fixes a relation between the central charge c_R and the R symmetry anomaly $c_R = 3k^{rr}$. In addition, it can be shown in a renormalization scheme where all currents are primary fields that there are no mixed anomalies between the superconformal R current and flavor currents. This imposes the constraint $k^{rM} = 0$, $\forall\ M \neq r$ and leads to the extremality condition

$$\frac{\partial c^{\rm tr}_R}{\partial t^M}(t_0) = 0, \quad \forall\ M \neq r. \qquad (3)$$

Since $c^{\rm tr}_R(t)$ is quadratic, there is a unique solution t_0.

III. $\mathcal{N} = 2$ SUPERGRAVITY

Here, following the notation of Ref. [23], we present a succinct review of $\mathcal{N} = 2$ gauged supergravity in three dimensions. The field content comprises scalar fields ϕ^i and spinor fields χ^i, both with $i = 1, \ldots, d$; a dreibein $e_\mu{}^a$; the spin connecton ω^{ab}_μ; and two gravitini ψ^I_μ, $I = 1, 2$, which transform under the R-symmetry group $SO(2)_R$.

The target space for scalars is a Kähler manifold. As such, it is convenient to decompose the d real fields into $d/2$ complex ones and their corresponding complex conjugates, $\phi^i \to (\phi^i, \bar\phi^{\bar i})$. The Kähler manifold can then be locally written in terms of a metric $g_{i\bar i} = \partial_i\partial_{\bar i}\mathcal{K}$, where $\mathcal{K}(\phi, \bar\phi)$ is the Kähler potential.

As explained in Ref. [23], a subgroup of isometries may be gauged through the introduction of an embedding tensor Θ_{MN} that defines the Killing vectors that generate the gauge group $X^i = g\Theta_{MN}\Lambda^N(x)X^{Ni}$, where g is the gauge coupling constant and $\Lambda^N(x)$ denotes the gauge group parameters. As is customary, the embedding tensor appears along with gauge fields A^M_μ in the definition of covariant derivative

$$\mathcal{D}_\mu\phi^i = \partial_\mu\phi^i + g\Theta_{MN}A^M_\mu X^{Ni} \qquad (4)$$

and also appears in the (Abelian) Chern-Simons (CS) term in the Lagrangian

$$\mathcal{L}_{\rm CS} = \frac{1}{2} g\epsilon^{\mu\nu\rho}A^M_\mu\Theta_{MN}\partial_\nu A^N_\rho. \qquad (5)$$

The embedding tensor also crops up in the T tensor $T = 2\mathcal{V}^M\Theta_{MN}\mathcal{V}^N$, where $\mathcal{V}$ is the moment map of the gauged isometries. We observe here that the T tensor is quadratic in the moment maps, so structurally it bears some resemblance to the trial c function (2).

Lastly, the scalar potential of the gauged theory may be expressed in terms of a real superpotential F:

$$V = -g^2(8F^2 - 8g^{i\bar i}\partial_i F\partial_{\bar i}F), \qquad (6)$$

where one can choose F to be one of the eigenvalues of the gravitino mass matrix $F = -T \pm e^{\mathcal{K}/2}|W|$, where W is the holomorphic superpotential satisfying $\partial_i\bar W = \partial_{\bar i}W = 0$. The potential tells us that, even in the absence of gauging, one can generate a cosmological constant with constant W. An alternative way to do this involves gauging the R-symmetry group, in which case T is a nonzero constant with $W = 0$. When the R symmetry is gauged, W must vanish since it transforms nontrivially under $SO(2)_R$.

A. Dual of c-extremization

Now that we have discussed the rudiments of $\mathcal{N} = 2$ gauged supergravity, we can recast the argument of Ref. [19] in terms of three-dimensional language. We start by noting that the embedding tensor Θ_{MN} encodes the CS terms, and, as observed in Ref. [13], these correspond to the 't Hooft anomalies k^{MN}. We will introduce the exact relationship for wrapped D3-brane geometries later.

Next, we remark that for two-dimensional superconformal theories, the corresponding AdS_3 dual geometry will preserve four supersymmetries. In particular, one can verify that the Killing spinor equations [23] are satisfied when $\partial_i T = 0$. Going further, from an analysis of the anticommutator of the supercharges acting on the scalars, one can infer that the superconformal R symmetry is

$$R = \tilde s^M Q_M = t\mathcal{V}^M Q_M, \qquad (7)$$

where Q_M, $M = 1, \ldots, P$, are charges corresponding to the currents J^M_μ and t is a constant of proportionality.

As in Ref. [19], the gauge transformation for the gravitino [23]

$$\mathcal{D}_\mu \psi^I_\nu = \partial_\mu \psi^I_\nu + g\Theta_{MN} A^M_\mu \mathcal{V}^{NIJ} \psi^J_\nu \ldots \tag{8}$$

allows us to use the fact that the gravitino has R charge one to fix the constant of proportionality, i.e., $\tilde{s}^M \Theta_{MN} \mathcal{V}^N = 1$, leading to

$$\tilde{s}^M = 2T^{-1}\mathcal{V}^M, \tag{9}$$

where T is the T tensor we introduced earlier. We are now in a position to propose the supergravity trial c function,

$$c_R \propto \tilde{s}^N \Theta_{MN} \tilde{s}^M = 2T^{-1}. \tag{10}$$

Observe that this trial function is extremized when $\partial_i T = 0$, which is precisely the condition for a supersymmetric AdS_3 vacuum. Furthermore, for D3-branes wrapped on a Riemann surface Σ, we can infer the constant of proportionality from Eq. (3.15) of Ref. [13],

$$k^{MN} = \frac{\eta_\Sigma d_G}{2}\Theta_{MN}, \tag{11}$$

where d_G is the dimension of the gauge group G and η_Σ is related to the volume of the Riemann surface $\frac{1}{2\pi}\,\mathrm{vol}_\Sigma = \eta_\Sigma$. We now recall that the trial c function (2) is of the form $c_R \sim 3k^{MN} \sim \frac{3}{2}\eta_\Sigma d_G \Theta_{MN}$, where we have used Eq. (11). This suggests that the trial c function from the supergravity perspective should be

$$c_R = \frac{3\eta_\Sigma d_G}{T}. \tag{12}$$

In the next section, we show that this formula recovers the expected central charge for the wrapped D3-brane geometries discussed in Refs. [12,13].

IV. AdS_3 VACUA FROM D3-BRANES

In this section, to back up our claim, we revisit the initial example of c-extremization presented in Ref. [12] (later in Ref. [13]) but recast it here in the language of three-dimensional gauged supergravity. Our point of departure will be five-dimensional $U(1)^3$ gauged supergravity, which, in turn, may be embedded into type IIB supergravity in ten dimensions [24]. The action reads

$$e^{-1}\mathcal{L}_5 = R - \frac{1}{2}\sum_i^2 (\partial\varphi_i)^2 - \frac{1}{4}\sum_i^3 X_i^{-2} F^i_{\rho\sigma}F^{i\rho\sigma} + V_5 + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda}F^1_{\mu\nu}F^2_{\rho\sigma}A^3_\lambda, \tag{13}$$

where e is the determinant of the Vierbein, A^i denotes the gauge fields, V_5 labels the potential

$$V_5 = 4\sum_i^3 X_i^{-1}, \tag{14}$$

and, for completeness, we define the constrained scalars

$$X_1 = e^{-\frac{1}{2}(\frac{2}{\sqrt{6}}\varphi_1 + \sqrt{2}\varphi_2)}, \qquad X_2 = e^{-\frac{1}{2}(\frac{2}{\sqrt{6}}\varphi_1 - \sqrt{2}\varphi_2)}, \tag{15}$$

with X_3 following from the constraint $X_1X_2X_3 = 1$. Observe also that, for simplicity, we have set the gauge coupling of the $U(1)^3$ theory to unity, $g = 1$. This theory permits the following chain of further consistent truncations: $\{\varphi_2 = 0, F^1 = F^2\} \to U(1)^2$ gauging and $\{\varphi_1 = \varphi_2 = 0, F^1 = F^2 = F^3\} \to$ minimal gauged supergravity, where, in the latter case, the retained gauge field is the graviphoton.

To establish a connection to three dimensions, we adopt the following ansatz for five-dimensional space-time:

$$ds_5^2 = e^{-4A}ds_3^2 + e^{2A}ds^2(\Sigma), \tag{16}$$

where A is a scalar warp factor and Σ is a Riemann surface with constant curvature $\kappa = -1, 0, 1$. In tandem, we take an appropriate ansatz for the field strengths:

$$F^i = -a_i \mathrm{vol}_\Sigma + G^i, \tag{17}$$

where closure of F^i implies that a_i are constants and that associated to each G^i we have gauge potential B^i, $G^i = dB^i$. In addition, we make the natural assumption that the scalars φ_i do not depend on the coordinates of the Riemann surface.

Plugging the ansatz into the five-dimensional equations of motion and reconstructing the Lagrangian, or alternatively performing the reduction at the level of the action, one finds a three-dimensional theory of the form

$$e_3^{-1}\mathcal{L}_3 = R - 6(\partial A)^2 - \frac{1}{2}\sum_i^2 (\partial\varphi_i)^2 - \frac{e^{4A}}{4}\sum_i^3 X_i^{-2} G^i_{\rho\sigma}G^{i\rho\sigma} + V_3 - \frac{1}{4}\epsilon^{\mu\nu\rho}|\epsilon_{ijk}|a_i B^j_\mu \wedge G^k_{\nu\rho}, \tag{18}$$

where the final line corresponds to the topological CS term, and the new potential is

$$V_3 = \sum_i^3 \left[4\frac{e^{-4A}}{X_i} - \frac{1}{2}\frac{e^{-8A}}{X_i^2}a_i^2 \right] + 2\kappa e^{-6A}. \tag{19}$$

We can now dualize the gauge fields to bring the action to the canonical form of a nonlinear sigma model coupled to gravity [23]. To do this, we redefine the field strengths

$$G^i = X_i^2 e^{-4A} * DY_i, \qquad DY_i = dY_i - \frac{1}{2}|\epsilon_{ijk}|a_j B^k \tag{20}$$

and rewrite the fields $e^{W_i} = e^{2A}X_i^{-1}$. This rewriting has the added bonus that the scalars are then canonically normalized. In performing this action, the CS terms remain, and one can check that varying the gauge fields leads to the duality relations (20).

The structure of $\mathcal{N} = 2$ supergravity is now manifest. In particular, one can see that the scalar manifold corresponds to the coset $[SU(1,1)/U(1)]^3$, where each factor is parametrized by a complex coordinate

$$z_i = e^{W_i} + iY_i. \tag{21}$$

This is in line with expectations, since in Ref. [9], the same coset appears when ungauged five-dimensional supergravity is reduced on an S^2. However, one important distinction here is that the R symmetry is gauged so $W = 0$. To make the Kähler structure of the scalar target space more explicit, we can introduce a Kähler potential

$$\mathcal{K} = -\sum_i^3 \log(\mathrm{Re} z_i). \tag{22}$$

Now that we understand the scalar manifold, it is relatively easy to extract the T tensor,

$$T = \sum_i^3 \left[\frac{1}{2}e^{-W_i} - \frac{1}{4}e^{\mathcal{K}}\sum_i^3 a_i e^{W_i}\right], \tag{23}$$

and check that it reproduces the expected terms in the potential (19). The required gauging of the R symmetry can also be verified from reducing the Killing spinor equations from five dimensions.

We can now minimize the potential with the supersymmetry condition $a_1 + a_2 + a_3 = -\kappa$ [25] leading to the general supersymmetric AdS_3 vacuum presented in Refs. [12,13]. This is also a critical point of T as expected for supersymmetric critical points.

In terms of T, the AdS_3 radius is now $\ell = 1/(2T)$. One can then determine the central charge by using the standard holographic prescription [4,26]:

$$c_R = \frac{3\ell}{2G^{(3)}}, \tag{24}$$

resulting in the expression

$$c_R = -12\eta_\Sigma N^2 \frac{a_1 a_2 a_3}{\Theta}, \qquad \Theta = a_1^2 + a_2^2 + a_3^2 - 2(a_1a_2 + a_1a_3 + a_2a_3), \tag{25}$$

which is, as expected, in perfect agreement with Refs. [12,13]. As an added bonus, one can also confirm that the exact superconformal R symmetry (7) and (9) agrees with Ref. [13]:

$$T_R = \sum_{i=1}^3 \frac{2a_i(2a_i + \kappa)}{\Theta} T_i, \tag{26}$$

where T_i are the generators of the $SO(2)^3$ global symmetry [27]. While the canonical R symmetry can be identified from higher dimensions [28], we believe this is the first statement purely in three-dimensional supergravity.

V. SUMMARY

In this work, we have proposed a natural three-dimensional supergravity description of c-extremization for CFTs with $\mathcal{N} = (0,2)$ supersymmetry. In light of the work of Ref. [19], it is not too surprising that the T tensor is the function being extremized. From the gravity perspective, it is already understood [29] that the holographic c function should be inversely proportional to the real superpotential, and for certain three-dimensional flows, this is the case [30]. However, the fact that we also recover the R symmetry is certainly novel, and it means that one can identify the R symmetry directly in three-dimensional supergravity without recourse to higher dimensions. The task remains to identify the gauged supergravities corresponding to the wrapped M5-brane examples presented in Ref. [13]. We also hope to identify three-dimensional gauged supergravities that arise from dimensional reductions of generic wrapped-brane geometries, such as those discussed in Refs. [31–33].

ACKNOWLEDGMENTS

We have enjoyed discussion with N. Bobev, M. P. García del Moral, D. Rodríguez-Gómez, and H. Samtleben. The work of P. K. is partially supported by Thailand Center of Excellence in Physics through the ThEP/CU/2-RE3/11 project and Chulalongkorn University through Ratchadapisek Sompote Endowment Fund. E. Ó C. acknowledges support from the research Grant No. MICINN-09- FPA2012-35043-C02-02.

[1] G 't Hooft, *Salamfestschrift* (World Scientific, Singapore, 1993); L. Susskind, J. Math. Phys. (N.Y.) **36**, 6377 (1995).
[2] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[3] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).
[4] J. D. Brown and M. Henneaux, Commun. Math. Phys. **104**, 207 (1986).
[5] S. Deser, R. Jackiw, and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982).
[6] E. A. Bergshoeff, O. Hohm, and P. K. Townsend, Phys. Rev. Lett. **102**, 201301 (2009).
[7] W. Li, W. Song, and A. Strominger, J. High Energy Phys. 04 (2008) 082.
[8] D. Anninos, W. Li, M. Padi, W. Song, and A. Strominger, J. High Energy Phys. 03 (2009) 130.
[9] E. Ó Colgáin and H. Samtleben, J. High Energy Phys. 02 (2011) 031.
[10] S. Detournay and M. Guica, arXiv:1212.6792.
[11] A. Bagchi, S. Detournay, and D. Grumiller, Phys. Rev. Lett. **109**, 151301 (2012).
[12] F. Benini and N. Bobev, Phys. Rev. Lett. **110**, 061601 (2013).
[13] F. Benini and N. Bobev, arXiv:1302.4451.
[14] K. A. Intriligator and B. Wecht, Nucl. Phys. **B667**, 183 (2003).

[15] D. Martelli, J. Sparks, and S.-T. Yau, Commun. Math. Phys. **268**, 39 (2006).
[16] D. Martelli, J. Sparks, and S.-T. Yau, Commun. Math. Phys. **280**, 611 (2008).
[17] A. Butti and A. Zaffaroni, J. High Energy Phys. 11 (2005) 019.
[18] R. Eager, arXiv:1011.1809.
[19] Y. Tachikawa, Nucl. Phys. **B733**, 188 (2006).
[20] P. Szepietowski, J. High Energy Phys. 12 (2012) 018.
[21] I. Bah, C. Beem, N. Bobev, and B. Wecht, Phys. Rev. D **85**, 121901 (2012).
[22] I. Bah, C. Beem, N. Bobev, and B. Wecht, J. High Energy Phys. 06 (2012) 005.
[23] B. de Wit, I. Herger, and H. Samtleben, Nucl. Phys. **B671**, 175 (2003).
[24] M. Cvetic, M. J. Duff, P. Hoxha, J. T. Liu, H. Lü, J. X. Lu, R. Martinez-Acosta, C. N. Pope, H. Sati, and T. A. Tran, Nucl. Phys. **B558**, 96 (1999).
[25] J. M. Maldacena and C. Nunez, Int. J. Mod. Phys. A **16**, 822 (2001).
[26] M. Henningson and K. Skenderis, J. High Energy Phys. 07 (1998) 023.
[27] Here we have used the explicit form of the moment maps $\mathcal{V}_i = \frac{1}{4}e^{-W_i}$, $i = 1, 2, 3$.
[28] N. Kim, J. High Energy Phys. 01 (2006) 094.
[29] D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner, Adv. Theor. Math. Phys. **3**, 363 (1999).
[30] M. Berg and H. Samtleben, J. High Energy Phys. 05 (2002) 006.
[31] J. P. Gauntlett, O. A. P. Mac Conamhna, T. Mateos, and D. Waldram, J. High Energy Phys. 11 (2006) 053.
[32] P. Figueras, O. A. P. Mac Conamhna, and E. Ó Colgáin, Phys. Rev. D **76**, 046007 (2007).
[33] J. P. Gauntlett and O. A. P. Mac Conamhna, Classical Quantum Gravity **24**, 6267 (2007).