Semileptonic decays $B/B_s \to (\eta, \eta', G)(l^+l^-, l\bar{\nu}, \nu\bar{\nu})$ in the perturbative QCD approach beyond the leading order

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In this paper we make a systematic study of the semileptonic decays $B/B_s \to (\eta, \eta', G)(l^+l^-, l\bar{\nu}, \nu\bar{\nu})$ by employing the perturbative QCD (pQCD) factorization approach. The next-to-leading-order (NLO) contributions to the relevant form factors are included, and the ordinary η-η' mixing scheme and the η-η'-G mixing scheme are considered separately, where G denotes a pseudoscalar glueball. The numerical results and the phenomenological analysis indicate that (a) the NLO contributions to the relevant form factors provide 25% enhancement to the leading-order pQCD predictions for the branching ratios $\mathrm{Br}(B^- \to \eta^{(\prime)} l^- \bar{\nu}_l)$, leading to a good agreement between the predictions and the data; (b) for all considered decays, the pQCD results are basically consistent with those from other different theoretical models; (c) the pQCD predictions in the two considered mixing schemes agree well with each other within theoretical errors. The outcomes presented here can be tested by LHCb and forthcoming Super-B experiments.

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The pseudoscalar mesons η and η' are rather different from other light pseudoscalar mesons π or K, not only for the large mass of the η' meson but also for their mixing and possible gluonic components. These features caught more attention recently because of the so-called $B \to K\eta^{(\prime)}$ puzzle. The two-body charmless hadronic B meson decays involving the $\eta^{(\prime)}$ final states and several different η-η' mixing schemes have been investigated intensively in the standard model (SM) (see, for instance, Refs. [1–6]) and in the new physics models beyond the SM (see, for instance, Refs. [7,8]). The semileptonic B/B_s meson decays, studied in Refs. [9–17], also play an important role in improving our understanding about the nature of the η and η' mesons.

Many precision measurements of the branching ratios and CP asymmetries for relevant B meson decays, such as $B \to K\eta^{(\prime)}$ and $B_s \to J/\Psi\eta^{(\prime)}$, have become available [18] and have been interpreted successfully within the SM in Refs. [6,19] by employing the perturbative QCD (pQCD) factorization approach [20]. As for the $B_{d,s} \to (\eta, \eta', G) \times (l^+l^-, l\bar{\nu}, \nu\bar{\nu})$ semileptonic decays, which will be studied in this paper, only two of them, $B^+ \to \eta^{(\prime)} l^+ \nu_l$, were observed so far [18,21]. For the others, experimental data are not yet available, and we have to wait for measurements at LHCb or at the forthcoming Super-B factory.

On the theory side, the previous predictions given in Refs. [9,10], where the form factors from QCD sum rules or lattice QCD were adopted as inputs, basically agree with the measured values. The B meson semileptonic decays, on the other hand, are frequently used to extract the corresponding $B \to (P, V)$ transition form factors, with P and V denoting the light pseudoscalar and vector mesons, respectively, such as $\pi, K, \rho, \ldots$ when relevant data are available.

Analogous to Ref. [22], we will make a systematic pQCD study of the semileptonic decays $B/B_s \to (\eta, \eta', G) \times (l^+l^-, l\bar{\nu}, \nu\bar{\nu})$ here, where G stands for a physical pseudoscalar glueball, and compare our pQCD predictions with existing calculations and numerical results. Based on the assumption of the $SU(3)$ flavor symmetry, we will extend the next-to-leading-order (NLO) pQCD calculation for the $B \to \pi$ form factors in Ref. [23] to the $B_{(s)} \to \eta_{q,s}$ cases, $\eta_{q,s}$ being the flavor eigenstates of the light and strange quarks, respectively. The $B_{(s)} \to \eta_g$ transitions form factors will also be calculated in the pQCD factorization approach, where η_g represents the unmixed pseudoscalar glueball. The relevant Feynman diagrams for the $B_{(s)} \to \eta_{q,s}$ and $B_{(s)} \to G$ transitions are displayed in Fig. 1.

We will consider two different meson mixing schemes in our calculation and compare the corresponding numerical results. The first mixing scheme is the conventional Feldmann-Kroll-Stech (FKS) η-η' mixing scheme [24], in which the physical states η and η' are written as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \tag{1}$$

with the flavor states $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$, and the mixing angle ϕ [24]. The three input parameters f_q, f_s, and ϕ in the FKS mixing scheme have been extracted from data of relevant exclusive processes [24]: $f_q = (1.07 \pm 0.02)f_\pi$, $f_s = (1.34 \pm 0.06)f_\pi$, and $\phi = 39.3° \pm 1.0°$ with $f_\pi = 0.13$ GeV.

In the second mixing scheme, i.e., the η-η'-G mixing scheme defined in Ref. [25], the physical states η, η', and G are related to η_q, η_s, and η_g through the rotation

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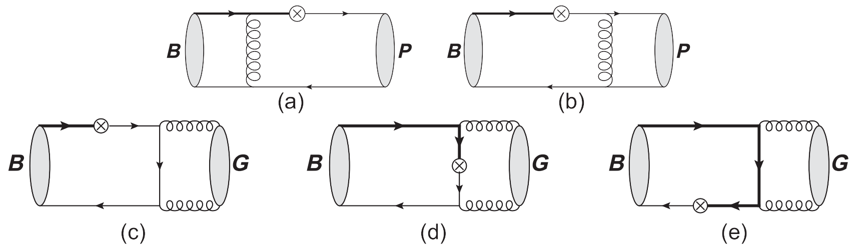

FIG. 1. Feynman diagrams: (a), (b) for the $B_{(s)} \to \eta_{q,s}$ transitions, and (c)–(e) for the $B_{(s)} \to G$ transitions. The symbol ⊗ refers to the weak vertex where the final-state lepton pairs are emitted, and P and G stand for the $\eta_{q,s}$ mesons and the glueball, respectively.

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \end{pmatrix} = U(\theta, \phi, \phi_G) \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |\eta_g\rangle \end{pmatrix}, \tag{2}$$

where the mixing matrix $U(\theta, \phi, \phi_G)$ has been given in Eq. (4) of Ref. [25] with $\phi = \theta + 54.7°$ and $\phi_G \sim 30°$.

In the numerical analysis, we assume the same functional form for the B and B_s meson distribution amplitudes we did in Ref. [22]. For the η_q and η_s mesons, we adopt the distribution amplitudes given in Refs. [26,27]. The relevant Gegenbauer moments take the values $a_2^{\eta_{q(s)}} = 0.115$, $a_4^{\eta_{q(s)}} = -0.015$, $\eta_3^{\eta_{q(s)}} = 0.013$, and $\omega_3^{\eta_{q(s)}} = -3$ [26]. The leading-twist distribution amplitude of the unmixed glueball η_g is defined by [28,29]

$$\langle \eta_g(p_2)|A^a_{[\mu}(z)A^b_{\nu]}(0)|0\rangle = \frac{f_{\eta_g} C_F \delta^{ab}}{96} \epsilon_{\mu\nu\rho\sigma} \frac{n^\rho p_2^\sigma}{n \cdot p_2} \times \int_0^1 dx e^{ixp_2\cdot z} \frac{\phi^G(x)}{x(1-x)}, \tag{3}$$

with the decay constant $f_{\eta_g} = f_s$ ($\sqrt{2} f_q$) for the $B_s \to \eta_g$ ($B \to \eta_g$) transition. The function $\phi^G(x)$ is expressed as [30]

$$\phi^G(x) = B_2^{q(s)} 5x^2(1-x)^2(2x-1), \tag{4}$$

where the coefficients B_2^q and B_2^s for the $B \to \eta_q$ and $B_s \to \eta_s$ transitions, respectively, take the value $B_2^q = B_2^s \equiv B_2 = 4.6 \pm 2.5$ [28,29].

In the B meson (for simplicity, B denotes both the B and B_s mesons here) rest frame, the momentum p_1 of the B meson and p_2 of the final-state light pseudoscalar meson are written as $p_1 = \frac{m_B}{\sqrt{2}}(1, 1, 0_T)$ and $p_2 = \frac{m_B}{\sqrt{2}}\rho(0, 1, 0_T)$ with the energy fraction $\rho = 1 - q^2/m_B^2$, where the lepton pair momentum is defined by $q = p_1 - p_2$. The light spectator momenta k_1 in the B meson and k_2 in the final-state meson are parametrized as $k_1 = (x_1, 0, k_{1T})\frac{m_B}{\sqrt{2}}$ and $k_2 = (0, x_2\rho, k_{2T})\frac{m_B}{\sqrt{2}}$, respectively.

For the $B \to P$ transition, the relevant form factors $F_{0,+}(q^2)$ and $F_T(q^2)$ have been defined, for example, in Ref. [31] with the relation $F_0(0) = F_+(0)$. For convenience, one usually considers the auxiliary form factors $f_1(q^2)$ and $f_2(q^2)$ defined via

$$\langle P(p_2)|\bar{b}(0)\gamma_\mu q(0)|B(p_1)\rangle = f_1(q^2)p_{1\mu} + f_2(q^2)p_{2\mu}, \tag{5}$$

in terms of which $F_+(q^2)$ and $F_0(q^2)$ are written as

$$F_+(q^2) = \frac{1}{2}[f_1(q^2) + f_2(q^2)],$$
$$F_0(q^2) = \frac{1}{2} f_1(q^2)\left[1 + \frac{q^2}{m_B^2 - m_P^2}\right] + \frac{1}{2} f_2(q^2)\left[1 - \frac{q^2}{m_B^2 - m_P^2}\right]. \tag{6}$$

The authors in Ref. [23] derived the k_T-dependent NLO hard kernel H for the $B \to \pi$ transition form factors. Here we quote their results directly and extend the expressions to the $B_{(s)} \to \eta_{q(s)}$ transitions under the assumption of $SU(3)$ flavor symmetry. The hard kernel H is given, at NLO, by [23]

$$H = H^{(0)}(\alpha_s) + H^{(1)}(\alpha_s^2) = [1 + F(x_1, x_2, \mu, \mu_f, \eta, \zeta_1)]H^{(0)}(\alpha_s), \tag{7}$$

where the expression of the NLO factor $F(x_1, x_2, \mu, \mu_f, \eta, \zeta_1)$ can be found in Eq. (3) of Ref. [22].

Employing the pQCD factorization approach with the inclusion of the Sudakov factors and the threshold resummation effects [22], we obtain the form factors $f_{1,2}(q^2)$ and $F_T(q^2)$ for the considered decays. For the $B_s \to \eta_s$ and $B \to \eta_q$ transitions, for instance, the form factor $f_1^{B_s\to\eta_s}(q^2)$ is of the form

$$f_1^{B_s\to\eta_s}(q^2) = \sqrt{2} f_1^{B\to\eta_q}(q^2) = 16\pi C_F m_B^2 \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1)\{[r_0(\phi^p(x_2) - \phi^t(x_2)) \cdot h_1(x_1, x_2, b_1, b_2) - r_0 x_1 \rho m_B^2 \phi^\sigma(x_2) h_2(x_1, x_2, b_1, b_2)] \cdot \alpha_s(t_1) \exp[-S_{B\eta}(t_1)] + [x_1(\rho\phi^a(x_2) - 2r_0\phi^p(x_2)) + 4r_0 x_1 \phi^p(x_2)] h_1(x_2, x_1, b_2, b_1) \cdot \alpha_s(t_2) \exp[-S_{B\eta}(t_2)]\}, \tag{8}$$

with $C_F = 4/3$, $r_0 = m_0^{\eta_s}/m_{B_s}$, $\rho = 1 - q^2/m_{B_s}^2$ ($r_0 = m_0^{\eta_q}/m_B$, $\rho = 1 - q^2/m_B^2$) for the $B_s \to \eta_s$ ($B \to \eta_q$) transition. We choose $m_{\eta_q} = 0.18 \pm 0.08$ GeV and $m_{\eta_s} = 0.69$ GeV as in Ref. [11].

The hard functions $h_{1,2}$, the scales $t_{1,2}$, and the Sudakov factors $\exp[-S_{B\eta}(t)]$ are the same as in Refs. [22,32].

One should note that the form factors in Eq. (8) represent the leading-order (LO) pQCD predictions. To include the NLO corrections, the coupling constant α_s in Eq. (8) is changed into $\alpha_s \cdot F(x_1, x_2, \rho, \mu_f, \mu, \zeta_1)$, with the NLO factor $F(x_1, x_2, \rho, \mu_f, \mu, \zeta_1)$ being given in Ref. [22]. For the $B_{(s)} \to \eta_g$ transitions, only the LO expressions are available now. The formulas for the differential $b \to u l^- \bar{\nu}_l$, $b \to s l^+ l^-$, and $b \to s \nu \bar{\nu}$ decay widths are provided in Ref. [22].

The following input parameters (the masses and decay constants are all in units of GeV) [18,19,24] are adopted in the numerical analysis: $\Lambda^{(f=4)}_{\bar{M}S} = 0.287$, $f_B = 0.21$, $f_{B_s} = 0.23$, $m_0^q = 1.50$, $m_0^s = 1.90$, $m_B = 5.279$, $m_{B_s^0} = 5.3663$, $\tau_{B^\pm} = 1.638\,\text{ps}$, $\tau_{B^0} = 1.525\,\text{ps}$, $\tau_{B_s^0} = 1.472\,\text{ps}$, $m_\tau = 1.777$, $m_b = 4.8$, $m_W = 80.4$, $m_t = 172$, $m_\eta = 0.548$, and $m_{\eta'} = 0.958$. For the relevant Cabibbo-Kobayashi-Maskawa matrix elements, we take $|V_{tb}| = 0.999$, $|V_{ts}| = 0.0403$, and $|V_{td}/V_{ts}| = 0.211$ [18].

As explained in Ref. [22], the pQCD predictions for the form factors $F_{0,+,T}(q^2)$ are reliable only at small q^2, such as the region $0 \le q^2 \le m_\tau^2$. To get the form factors at larger q^2, one has to make an extrapolation from the lower q^2 region. Therefore, we perform the pQCD calculations for the $B/B_s \to (\eta_q, \eta_s, \eta_g)$ transition form factors in the range $0 \le q^2 \le m_\tau^2$, and then apply the extrapolation using the pole model parametrization

$$F_i(q^2) = \frac{F_i(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}, \tag{9}$$

where F_i denotes a function among $F_{0,+,T}$, and a, b are the constants to be determined by the fitting procedure.

In Table I we list the LO and NLO pQCD predictions for the form factors $F_{0,+,T}(0)$ involved in the $B \to \eta_q$ and $B_s \to \eta_s$ transitions. The three errors of $F_{0,+,T}(0)$ come from the uncertainties of $\omega_B = 0.40 \pm 0.04\,\text{GeV}$ or $\omega_{B_s} = 0.50 \pm 0.05\,\text{GeV}$, $f_B = 0.21 \pm 0.02\,\text{GeV}$ or $f_{B_s} = 0.23 \pm 0.03\,\text{GeV}$, and $a_2^{\eta_q,\eta_s} = 0.115 \pm 0.115$, respectively. The errors from the variations of V_{ts} or $|V_{td}/V_{ts}|$ and $f_q = (1.07 \pm 0.02) f_\pi$ or $f_s = (1.34 \pm 0.06) f_\pi$ are very small and have been neglected. One can see that the NLO contribution to the form factors $F_{0,+,T}(0)$ in the $B/B_s \to (\eta_q, \eta_s)$ transitions can provide about 12% enhancement to the LO ones. The NLO pQCD predictions for $F_{0,+,T}^{B_{(s)} \to \eta_{q,s}}(0)$ agree well with the values obtained from other methods.

The pQCD predictions (in units of 10^{-2}) for the form factors $F_{0,+,T}(0)$ in the $B/B_s \to \eta_g$ transitions are also shown in Table I. The sources of the first two errors are the same as those for the $B \to \eta_q$ and $B_s \to \eta_s$ transitions, while the third one comes from the uncertainty of $B_2 = 4.6 \pm 2.5$. It is easy to see that the form factors $F_{0,+,T}(0)$ in the $B/B_s \to \eta_g$ transitions are of order 10^{-3}, so the corresponding contributions to the decay rates are negligible. Using the relevant formulas and the input parameters given above, it is straightforward to calculate the branching ratios of the considered decays.

In the FKS mixing scheme, the LO and NLO pQCD predictions for the branching ratios of the considered decays with $l = (e, \mu)$ are listed in Table II. We show only the central values of the LO pQCD predictions in column two, and the central values and the major theoretical errors simultaneously in column three. The first error arises from the uncertainty of $\omega_B = 0.40 \pm 0.04$ or $\omega_{B_s} = 0.50 \pm 0.05$, the second one is from the uncertainty of $f_B = 0.21 \pm 0.02$ or $f_{B_s} = 0.23 \pm 0.03$, and the third one is induced by the variations of $a_2^{\eta_q,\eta_s} = 0.115 \pm 0.115$. One can see from Table II the following:

(i) For all the considered decays, the inclusion of the NLO contribution to the $B_{(s)} \to \eta_{q,s}$ transition form factors provides about 25% enhancement to the branching ratios. The pQCD predictions for the $B^- \to \eta^{(\prime)} l^- \bar{\nu}_l$ decay rates then become well consistent with the measured values and other known theoretical predictions [9,11].

TABLE I. pQCD predictions for the form factors $F_{0,+,T}(0)$ in the $B \to \eta_q$ and $B_s \to \eta_s$, and $B/B_s \to \eta_g$ transitions at LO and NLO.

		$F_i(0)$		$F_i(0)(10^{-2})$
$F_0^{B\to\eta_q}$	LO	$0.17 \pm 0.02 \pm 0.02 \pm 0.01$	$F_0^{B\to\eta_g}$	$0.14^{+0.02}_{-0.01} \pm 0.01 \pm 0.08$
	NLO	$0.19^{+0.03}_{-0.02} \pm 0.02 \pm 0.01$		
$F_+^{B\to\eta_q}$	LO	$0.17 \pm 0.02 \pm 0.02 \pm 0.01$	$F_+^{B\to\eta_g}$	$0.14^{+0.02}_{-0.01} \pm 0.01 \pm 0.08$
	NLO	$0.19^{+0.03}_{-0.02} \pm 0.02 \pm 0.01$		
$F_T^{B\to\eta_q}$	LO	$0.15 \pm 0.02 \pm 0.01 \pm 0.01$	$F_T^{B\to\eta_g}$	$0.10 \pm 0.01 \pm 0.01 \pm 0.06$
	NLO	$0.17 \pm 0.02 \pm 0.02 \pm 0.01$		
$F_0^{B_s\to\eta_s}$	LO	$0.27^{+0.04}_{-0.03} \pm 0.04 \pm 0.01$	$F_0^{B_s\to\eta_g}$	$0.11 \pm 0.01 \pm 0.01 \pm 0.06$
	NLO	$0.31^{+0.05}_{-0.04} \pm 0.04 \pm 0.01$		
$F_+^{B_s\to\eta_s}$	LO	$0.27^{+0.04}_{-0.03} \pm 0.04 \pm 0.01$	$F_+^{B_s\to\eta_g}$	$0.11 \pm 0.01 \pm 0.01 \pm 0.06$
	NLO	$0.31^{+0.05}_{-0.04} \pm 0.04 \pm 0.01$		
$F_T^{B_s\to\eta_s}$	LO	$0.27 \pm 0.04 \pm 0.04 \pm 0.01$	$F_T^{B_s\to\eta_g}$	$0.08 \pm 0.01 \pm 0.01 \pm 0.04$
	NLO	$0.31^{+0.05}_{-0.04} \pm 0.04 \pm 0.01$		

TABLE II. LO and NLO pQCD predictions for the branching ratios of the considered decays with errors. The relevant data [18] and other theoretical predictions [9,12–16] are listed in the last two columns.

Decay modes	LO	pQCD$_{\rm NLO}$	Set A	Set B	Others	Data [18]
$Br(B^- \to \eta l^- \bar\nu_l)(10^{-4})$	0.33	$0.41^{+0.12}_{-0.09} \pm 0.08^{+0.04}_{-0.03}$	$0.33^{+0.12}_{-0.10}$	$0.37^{+0.14}_{-0.11}$	0.43 ± 0.08 [9]	0.39 ± 0.08
$Br(B^- \to \eta \tau^- \bar\nu_\tau)(10^{-4})$	0.19	$0.24^{+0.07+0.05}_{-0.05-0.04} \pm 0.02$	$0.20^{+0.07}_{-0.05}$	$0.23^{+0.08}_{-0.06}$	$0.29^{+0.07}_{-0.06}$ [14]	
$Br(B^- \to \eta' l^- \bar\nu_l)(10^{-4})$	0.16	$0.20^{+0.06}_{-0.04} \pm 0.04 \pm 0.02$	$0.16^{+0.06}_{-0.05}$	$0.17^{+0.06}_{-0.05}$	0.21 ± 0.04 [9]	0.23 ± 0.08
$Br(B^- \to \eta' \tau^- \bar\nu_\tau)(10^{-4})$	0.08	$0.10^{+0.03}_{-0.02} \pm 0.02 \pm 0.01$	$0.08^{+0.03}_{-0.02}$	$0.09^{+0.03}_{-0.02}$	$0.13^{+0.03}_{-0.02}$ [14]	
$Br(\bar B^0 \to \eta l^+ l^-)(10^{-8})$	0.39	$0.48^{+0.14+0.10+0.05}_{-0.10-0.09-0.04}$	$0.39^{+0.14}_{-0.11}$	$0.45^{+0.17}_{-0.13}$	0.6 [12]	
$Br(\bar B^0 \to \eta \tau^+ \tau^-)(10^{-9})$	0.83	$0.98^{+0.28+0.19+0.08}_{-0.20-0.18-0.06}$	$0.80^{+0.28}_{-0.22}$	$0.92^{+0.33}_{-0.26}$	1.1 ± 0.1 [15]	
$Br(\bar B^0 \to \eta \nu \bar\nu)(10^{-9})$	0.31	$0.38^{+0.11+0.08+0.03}_{-0.08-0.07-0.03}$	$0.31^{+0.11}_{-0.09}$	$0.36^{+0.13}_{-0.11}$		
$Br(\bar B^0 \to \eta' l^+ l^-)(10^{-8})$	0.18	$0.24^{+0.07+0.05}_{-0.05-0.04} \pm 0.02$	$0.19^{+0.07}_{-0.05}$	$0.20^{+0.08}_{-0.06}$	0.3 [12]	
$Br(\bar B^0 \to \eta' \tau^+ \tau^-)(10^{-9})$	0.21	$0.25^{+0.07+0.05+0.02}_{-0.05-0.04-0.01}$	$0.20^{+0.07}_{-0.05}$	$0.21^{+0.08}_{-0.06}$		
$Br(\bar B^0 \to \eta' \nu \bar\nu)(10^{-9})$	0.14	$0.18^{+0.05+0.04+0.02}_{-0.04-0.03-0.02}$	$0.14^{+0.05}_{-0.04}$	$0.16^{+0.06}_{-0.05}$		
$Br(\bar B^0_s \to \eta l^+ l^-)(10^{-7})$	1.68	$2.07^{+0.65+0.57+0.10}_{-0.51-0.50-0.09}$	$2.59^{+1.09}_{-0.90}$	$2.20^{+0.93}_{-0.77}$	3.7 [12]; 2.4 [13,16]	
$Br(\bar B^0_s \to \eta \tau^+ \tau^-)(10^{-7})$	0.39	$0.45^{+0.15+0.13}_{-0.11-0.11} \pm 0.02$	$0.56^{+0.25}_{-0.21}$	$0.48^{+0.21}_{-0.18}$	0.58 [13]; 0.34 [15]	
$Br(\bar B^0_s \to \eta \nu \bar\nu)(10^{-6})$	1.33	$1.62^{+0.54+0.45+0.11}_{-0.38-0.39-0.10}$	$2.03^{+0.89}_{-0.69}$	$1.72^{+0.76}_{-0.59}$	1.7 [13]; 1.4 [16]	
$Br(\bar B^0_s \to \eta' l^+ l^-)(10^{-7})$	1.77	$2.18^{+0.73+0.61}_{-0.52-0.53} \pm 0.15$	$1.45^{+0.64}_{-0.50}$	$1.92^{+0.85}_{-0.67}$	3.35 [12]; 1.8 [13]	
$Br(\bar B^0_s \to \eta' \tau^+ \tau^-)(10^{-7})$	0.23	$0.27^{+0.09}_{-0.07} \pm 0.07 \pm 0.01$	$0.18^{+0.07}_{-0.06}$	$0.24^{+0.10}_{-0.09}$	0.26 [13]; 0.28 [16]	
$Br(\bar B^0_s \to \eta' \nu \bar\nu)(10^{-6})$	1.39	$1.71^{+0.57+0.47}_{-0.41-0.42} \pm 0.12$	$1.14^{+0.47}_{-0.40}$	$1.50^{+0.62}_{-0.53}$	1.3 [13,16]	

(ii) For all neutral current processes, the NLO pQCD predictions basically agree with other known theoretical predictions [12–15].

(iii) Because of $Br(\bar B^0_s \to \eta^{(\prime)} \nu \bar\nu) \approx 1.7 \times 10^{-6}$, these decays may be observed at the LHCb. Other neutral decay modes with the decay rates at the 10^{-7}–10^{-9} level may be very hard, if not impossible, to measure.

In the η-η'-G mixing scheme the physical states η, η', and G are related to the flavor states η_q, η_s, and η_g through the mixing matrix $U(\theta, \phi, \phi_G)$ [25]. The $B/B_s \to \eta_g$ transition form factors are 2 orders of magnitude smaller than the $B/B_s \to \eta_q$, η_s ones as indicated in Table I, so the former contributions to the $B/B_s \to \eta, \eta', G$ decays can be neglected safely. In Table II we also list the NLO pQCD predictions for the branching ratios in the η-η'-G mixing scheme with two sets of mixing angles (ϕ, ϕ_G): Set A with $(\phi, \phi_G) = (43.7°, 33°)$ [19], and Set B with $(\phi, \phi_G) = (40°, 22°)$ [25]. The theoretical errors from the uncertainties of ω_B, ω_{B_s}, f_B, f_{B_s}, and the Gegenbauer moments are added in quadrature. The pQCD predictions for the $B_{(s)} \to G(ll, l\nu, \nu\bar\nu)$ branching ratios in the η-η'-G mixing scheme are listed in Table III. One can see the following from Tables II and III:

(i) The pQCD predictions in the two-state mixing schemes, and in the three-state mixing scheme with the two sets of mixing angles, are all similar within theoretical errors.

(ii) The pQCD predictions for $Br(B^- \to G l^- \bar\nu_l)$ and $Br(\bar B^0_{(s)} \to G l^+ l^-)$ are about 2 orders of magnitude smaller than those of the corresponding $B/B_s \to \eta^{(\prime)}$ transitions as expected.

TABLE III. pQCD predictions for the $B^- \to G l^- \bar\nu_l (\tau^- \bar\nu_\tau)$ and $\bar B^0_{(s)} \to G l^+ l^- (\tau^+ \tau^-, \nu\bar\nu)$ branching ratios in the η-η'-G mixing scheme for $\phi_G = 33°$ or $22°$.

Decay modes	$\phi_G = 33°$	$\phi_G = 22°$
$Br(B^- \to G l^- \bar\nu_l)(10^{-5})$	$0.64^{+0.23}_{-0.22}$	$0.30^{+0.11}_{-0.10}$
$Br(B^- \to G \tau^- \bar\nu_\tau)(10^{-5})$	$0.25^{+0.09}_{-0.08}$	0.12 ± 0.04
$Br(\bar B^0 \to G l^+ l^-)(10^{-9})$	$0.76^{+0.28l}_{-0.23}$	$0.36^{+0.13}_{-0.11}$
$Br(\bar B^0 \to G \tau^+ \tau^-)(10^{-10})$	$0.18^{+0.07}_{-0.05}$	$0.08^{+0.03}_{-0.02}$
$Br(\bar B^0 \to G \nu \bar\nu)(10^{-9})$	$5.89^{+2.16}_{-1.74}$	$2.79^{+1.02}_{-0.82}$
$Br(\bar B^0_s \to G l^+ l^-)(10^{-7})$	$0.24^{+0.11}_{-0.09}$	$0.11^{+0.05}_{-0.04}$
$Br(\bar B^0_s \to G \tau^+ \tau^-)(10^{-9})$	$0.88^{+0.38}_{-0.30}$	$0.42^{+0.18}_{-0.14}$
$Br(\bar B^0_s \to G \nu \bar\nu)(10^{-7})$	$1.85^{+0.82}_{-0.65}$	$0.88^{+0.39}_{-0.31}$

In summary, we have studied the semileptonic decays $B/B_s \to (\eta, \eta', G)(l^+ l^-, l\bar\nu, \nu\bar\nu)$ in this work by employing the pQCD factorization approach beyond LO. Based on the numerical calculations and the phenomenological analysis, the following points have been observed:

(i) The NLO contributions can provide 25% enhancement to the LO pQCD predictions for $Br(B^- \to \eta^{(\prime)} l^- \bar\nu_l)$, leading to a good agreement between the pQCD predictions and the data.

(ii) For all the considered decays, the pQCD results are basically consistent with those from other different theoretical models.

(iii) The pQCD predictions in the two considered meson mixing schemes agree well with each other within theoretical errors.

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