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Limits on lepton flavor violation from $\mu^- - e^-$ conversion

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We revisit the status of lepton flavor violating (LFV) μeqq contact interactions from the view point of $\mu^- - e^-$ conversion in nuclei. We consider their contribution to this LFV process via the two mechanisms on the hadronic level: direct nucleon and meson exchange ones. In the former case the quarks are embedded directly into the nucleons while in the latter in mesons which then interact with nucleons in a nucleus. We revise and in some cases reevaluate the hadronic parameters relevant for both mechanisms and calculate the contribution of the above mentioned contact interactions in coherent $\mu^- - e^-$ conversion in various nuclei. Then we update our previous upper bounds and derive new ones for the scales of the μeqq contact interactions from the experimental limits on the capture rates of $\mu^- - e^-$ conversion. We compare these limits with the ones derived in the literature from other LFV processes and comment on the prospects of LHC searches related to the contact μeqq interactions.

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I. INTRODUCTION

Neutrino oscillations give the first and so far unique evidence for lepton flavor violation (LFV), forbidden in the Standard Model (SM). Knowing that *lepton flavor* is a nonconserving quantity, it is natural to expect LFV effects also in the sector of charged leptons, although so far these effects have not been experimentally observed. Theoretically LFV in the neutrino sector, originating from the nondiagonal neutrino mass matrix, is transmitted to the charged lepton sector at the loop level, in the form of penguin and box diagrams with virtual neutrinos. However, these effects are Glashow-Iliopoulos-Maini-like suppressed down to the level of 10^{-50}, being far beyond the experimental reach. On the other hand, the charged lepton sector may receive other LFV contributions from physics beyond the SM, attributed to a certain high-energy scale $\Lambda_{\rm LFV}$, which is not *a priori* necessarily very high and which may provide observable LFV phenomena.

Thus, searching for lepton flavor violation in reactions with charged leptons offers a good opportunity for getting information on possible physics beyond the SM. Muon-to-electron conversion in nuclei

$$\mu^- + (A,Z) \to e^- + (A,Z)^* \qquad (1)$$

is well known to be one of the most sensitive probes of LFV and of underlying physics beyond the SM (for reviews, see [1–4]). Up to now there have been undertaken significant efforts aimed at searching for LFV via this processes in various nuclei with negative results [1], thus setting upper limits on the $\mu^- - e^-$ conversion rate

$$R^A_{\mu e} = \frac{\Gamma(\mu^- + (A,Z) \to e^- + (A,Z))}{\Gamma(\mu^- + (A,Z) \to \nu_\mu + (A,Z-1))}. \qquad (2)$$

The SINDRUM II experiment at PSI has set stringent upper bounds on $\mu^- - e^-$ conversion rate $R_{\mu e} \le 4.3 \times 10^{-12}$, 7.0×10^{-13}, 4.6×10^{-11} in ^{48}Ti [5], ^{197}Au [6] and ^{208}Pb [7] as stopping targets, respectively. Several new proposals for $\mu^- - e^-$ experiments are aimed at a significant improvement of the SINDRUM II sensitivity. Among them we mention the planned nearest future DeeMe experiment at J-PARC [8], the next generation muon-to-electron conversion experiment by Mu2e Collaboration at Fermilab [9,10] and COMET at J-PARC [11] with planned sensitivities around 10^{-14}, 7×10^{-17} and 10^{-16}, respectively, as well as the more distant future proposal PRISM/PRIME [12] at J-PARC, with estimated sensitivity 10^{-18}.

As is known from previous studies (see, for instance, Refs. [1,3,13] and references therein) and as will be also discussed later in the present paper, these experimental bounds allow setting stringent bounds on the mechanisms of $\mu^- - e^-$ conversion [13], on LFV decays of vector mesons [14,15] and, in general, on the underlying theories of LFV [3].

The theoretical studies of $\mu^- - e^-$ conversion, presented in the literature, cover various aspects of this LFV process: the adequate treatment of structure effects [4,16,17] of the nucleus participating in the reaction and

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the underlying mechanisms of LFV at the quark level within different scenarios of physics beyond the SM (see [3] and references therein).

As is known there are two categories of $\mu^- - e^-$ conversion mechanisms: photonic and nonphotonic. In the photonic case photon connects the LFV leptonic and the electromagnetic nuclear vertices. The nonphotonic mechanisms are induced by the four-fermion lepton-quark LFV contact interactions. These mechanisms significantly differ from each other, receiving different contributions from new physics and requiring different description of the nucleon and the nuclear structure.

In the present paper we analyze nonphotonic mechanism. We revisit some of the results of Refs. [13,18,19] using improved values of hadronic parameters. Then we significantly extend our previous analysis made in these papers including all the possible contact terms contributing to $\mu^- - e^-$ conversion. To this end we evaluate nucleon form factors for the heavy quark currents and take into account contributions of heavy vector mesons. Finally we update our previous bounds on the μeqq four-fermion contact interactions and derive new ones from the experimental data on $\mu^- - e^-$ conversion rates in various nuclei.

The paper is organized as follows. In Sec. II we specify all the above mentioned LFV contact interactions contributing to coherent $\mu^- - e^-$ conversion and briefly describe their hadronization within direct nucleon and meson exchange mechanisms. In Sec. III we consider the existing and future $\mu^- - e^-$ conversion data and extract limits on the generic LFV parameters and on the equivalent mass scales $\Lambda^{(q)}_{\mu e}$ of the μeqq contact interactions. Then we compare our limits with the existing ones in the literature and comment on a possible experimental reach of the LHC experiments in terms of the mass scales of these interactions.

II. MODEL-INDEPENDENT FRAMEWORK

The effective Lagrangian $\mathcal{L}^{lq}_{\text{eff}}$ describing the coherent $\mu^- - e^-$ conversion at the quark level can be written in the form [16,18]

$$\mathcal{L}^{lq}_{\text{eff}} = \frac{1}{\Lambda^2_{\text{LFV}}}\Big[(\eta^{(q)}_{VV} j^V_\mu + \eta^{(q)}_{AV} j^A_\mu) J^{V\mu}_q + (\eta^{(q)}_{SS} j^S + \eta^{(q)}_{PS} j^P) J^S_q\Big], \tag{3}$$

where the lepton and the quark currents are defined as

$$\begin{aligned} j^V_\mu &= \bar{e}\gamma_\mu \mu, \qquad j^A_\mu = \bar{e}\gamma_\mu\gamma_5 \mu, \qquad j^S = \bar{e}\mu, \\ j^P &= \bar{e}\gamma_5 \mu, \qquad J^{V\mu}_q = \bar{q}\gamma^\mu q, \qquad J^S_q = \bar{q}q. \end{aligned} \tag{4}$$

In Eq. (3) the summation is understood over all the quark flavors $q = \{u, d, s, b, c, t\}$. The dimension-1 mass parameter Λ_{LFV} is a high-energy scale of LFV connected to new physics. The dimensionless LFV parameters η^q in Eq. (3) refer to a specific LFV model. We start with studying $\mu^- - e^-$ conversion in a model-independent way based on the Lagrangian (3) and will extract upper limits on the parameters η^q from the experimental bounds on this LFV process. We consider the dominant coherent mode of $\mu^- - e^-$ conversion, therefore, in Eq. (3) we disregarded the terms with the axial-vector and pseudoscalar quark currents which are irrelevant in this case [4,20].

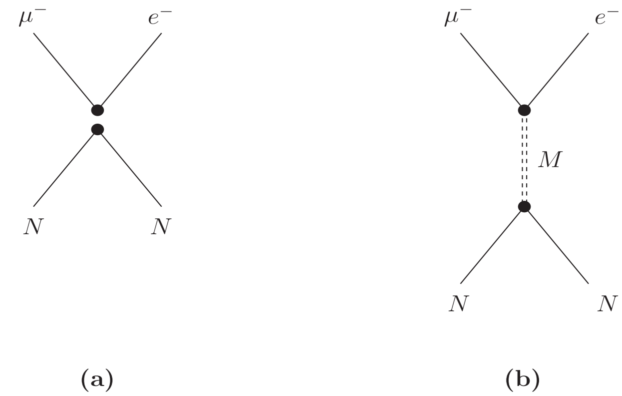

FIG. 1. Diagrams contributing to the nuclear $\mu^- - e^-$ conversion: direct nucleon (a) and meson exchange mechanism (b).

To this end one needs to translate the LFV lepton-quark Lagrangian (3) to the corresponding LFV lepton-nucleon Lagrangian. This implies a certain hadronization prescription. Because of the absence of a well-defined theory of hadronization we rely on some reasonable assumptions and models. Following Refs. [18,19] we consider two mechanisms of nuclear $\mu^- - e^-$ conversion: direct nucleon mechanism (DNM) and vector-meson exchange mechanism (MEM), which are shown in Fig. 1.

In the case of the DNM the quark currents are directly embedded into the nucleon currents [Fig. 1(a)]. The MEM consists of two stages [Fig. 1(b)]. First, the quark currents are embedded into the interpolating meson fields which then interact with the nucleon currents. It is well known that only vector [13,18] and scalar [19] mesons contribute to coherent $\mu^- - e^-$ conversion while axial and pseudoscalar mesons contribute to a subdominant incoherent channel of this process. At present the relative strengths for the DNM and the MEM cannot be reliably determined. Therefore, in our analysis we assume that only one of the mechanism is operative at a time and estimate its contribution to $\mu^- - e^-$ conversion.

A. Direct nucleon mechanism

In the case of the DNM, schematically represented by the diagram in Fig. 1(a), the quark fields from Eq. (3) are embedded in the effective nucleon fields N. Then the effective Lagrangian of $\mu^- - e^-$ conversion can be written in a general Lorentz covariant form with the isospin structure of the $\mu - e$-transition operator [16]:

$$\mathcal{L}_{\rm eff}^{lN} = \frac{1}{\Lambda_{\rm LFV}^2}[j_\mu^h(\alpha_{hV}^{(0)} J^{V\mu(0)} + \alpha_{hV}^{(3)} J^{V\mu(3)}) + j^r(\alpha_{rS}^{(0)} J^{S(0)} + \alpha_{rS}^{(3)} J^{S(3)})], \quad (5)$$

where the summation runs over the double indices $h = V$, A and $r = S$, P. The isosinglet $J^{(0)}$ and isotriplet $J^{(3)}$ nucleon currents are defined as

$$J^{V\mu(k)} = \bar{N}\gamma^\mu\tau^k N, \qquad J^{S(k)} = \bar{N}\tau^k N, \quad (6)$$

where N is the nucleon isospin doublet, $k = 0$, 3 and $\tau_0 \equiv \hat{I}$ is the 2×2 unit matrix.

This Lagrangian is supposed to originate from the quark level Lagrangian in Eq. (3), and therefore must correspond to the same order $1/\Lambda_{\rm LFV}^2$ in inverse powers of the LFV scale.

Next we relate the coefficients α in Eq. (5) to the "fundamental" LFV parameters η of the quark level Lagrangian (3). Towards this end we apply the on-mass-shell matching condition [21]

$$\langle e^- N|\mathcal{L}_{\rm eff}^q|\mu^- N\rangle \approx \langle e^- N|\mathcal{L}_{\rm eff}^N|\mu^- N\rangle, \quad (7)$$

in terms of the matrix elements of the Lagrangians (3) and (5) between the initial and final states of $\mu^- - e^-$ conversion at the nucleon level.

In order to solve this equation in Ref. [16], various relations for the matrix elements of quark operators between nucleon states were used

$$\langle N|\bar{q}\Gamma_K q|N\rangle = G_K^{(q,N)}\bar{N}\Gamma_K N, \quad (8)$$

with $q = \{u, d, s\}$, $N = \{p, n\}$. In the following we neglect the q^2 dependence of the nucleon form factors $G_K^{(q,N)}$, because the maximal momentum transfer in $\mu - e$ conversion is significantly smaller than the typical scale of nucleon structure $\Lambda \sim 1$ GeV. Following Ref. [16] we find relations between the LFV parameters of the Lagrangians (3) and (5):

$$\text{DNM:}\ \alpha_{hV}^{(3)} = \frac{(G_V^u - G_V^d)}{2}\eta_{hV}^{(3)}, \qquad \alpha_{hV}^{(0)} = \frac{(G_V^u + G_V^d)}{2}\eta_{hV}^{(0)}, \quad (9)$$

$$\alpha_{rS}^{(3)} = \frac{(G_S^u - G_S^d)}{2}\eta_{rS}^{(3)},$$
$$\alpha_{rS}^{(0)} = \frac{(G_S^u + G_S^d)}{2}\eta_{rS}^{(0)} + \eta_{rS}^{(s)}G_S^s + \eta_{rS}^{(c)}G_S^c + \eta_{rS}^{(b)}G_S^b + \eta_{rS}^{(t)}G_S^t, \quad (10)$$

where $h = V, A$, $r = S, P$ and $\eta^{(0,3)} = \eta^{(u)} \pm \eta^{(d)}$.

Here the nucleon form factors have the following values [16,19,22]:

$$G_V^u = 2; \quad G_V^d = 1; \quad G_S^u = 3.74[5.1]; \quad G_S^d = 2.69[4.3]; \quad G_S^s = 0.64[2.5]; \quad (11)$$

$$G_S^c = 0.06; \quad G_S^b = 0.02; \quad G_S^t = 5\times 10^{-4}. \quad (12)$$

The values of the vector form factors G_V^q are exact and are equal to the total number of the corresponding specie q of quark in the proton. For this reason $G_V^{s,c,b,t} = 0$. For the scalar form factors $G_S^{u,d,s}$ we use the conservative values derived in Ref. [19]. They are deduced from the values of the meson-nucleon sigma terms of Refs. [23–25] which are extracted from the data on the basis of dispersion analysis of πN scattering data taking into account chiral symmetry constraints. In the square brackets we also show the significantly larger values of the scalar form factors derived in Ref. [16] within the QCD picture of baryon masses as based on [26,27]. The latter approach also allows for an estimate of the heavy quark scalar form factors $G_S^{c,b,t}$ of the nucleon. The value of G_S^b was calculated in Ref. [22]. Here we calculated in the same approach values of the remaining scalar form factors G_S^c and G_S^t. The resulting values of $G_S^{c,b,t}$ are shown in Eq. (12). For a discussion of theoretical uncertainties and the possible error bars see, for instance, Ref. [28].

B. Meson exchange mechanism

This mechanism is described by the diagram in Fig. 1(b). As we already mentioned, the mesons that can contribute to this mechanism are the unflavored vector and scalar ones. The lightest vector mesons are the isotriplet $\rho(770)$ and the two isosinglet $\omega(782)$, $\phi(1020)$ mesons. In our analysis we adopt the ideal singlet-octet mixing, corresponding to the following quark content of the ω and ϕ mesons [29]: $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$, $\phi = -s\bar{s}$.

Contributions of the heavy vector mesons J/Ψ and Υ are significantly suppressed in comparison with the above specified light vector mesons but these mesons probe the heavy quark vector currents of the nucleon inaccessible in the DNM and therefore are worth to be taken into account.

The properties of scalar mesons are not yet well experimentally established [29]. However their phenomenological role in the $\mu^- - e^-$ conversion could be important, because they contribute as well as the vector mesons to the experimentally most interesting coherent mode of this rare process. The isosinglet $f_0(500)$ and the isotriplet $a_0(980)$ states are the lightest unflavored scalar mesons. The $f_0(500)$ meson has been considered in the context of the nonlinear realization of chiral symmetry as a wide resonance in the $\pi\pi$ system (see, e.g., in Refs. [30–32]). There should also be mentioned a model-independent study of scalar mesons using uniformizing-variable method based on analyticity and unitarity of the S matrix [33]. In our analysis we neglect a possible small strangeness content of the isosinglet meson and take it in the form: $f_0(500) = (\bar{u}u + \bar{d}d)/\sqrt{2}$.

The upper vertex of diagram Fig. 1(b) is described by the LFV effective lepton-meson Lagrangian [18]:

$$\mathcal{L}^{lM} = \frac{\Lambda_H^2}{\Lambda_{\rm LFV}^2}\Big[(\xi_V^{M_V} j_\mu^V + \xi_A^{M_V} j_\mu^A)M_V^\mu + (\xi_S^{M_S} j^S + \xi_P^{M_S} j^P)M_S + \frac{1}{\Lambda_H^2}\{\text{Derivative terms}\}\Big], \quad (13)$$

with $M_V = \rho,\ \omega,\ \phi,\ J/\Psi,\ \Upsilon$ and $M_S = f_0,\ a_0$ mesons. The unknown dimensionless coefficients ξ are to be determined from the hadronization prescription. Since we suppose that this Lagrangian originates from the quark-lepton Lagrangian (3), all its terms are suppressed by a factor $\Lambda_{\rm LFV}^{-2}$ with respect to the large LFV mass scale $\Lambda_{\rm LFV}$. Another mass scale in the problem is the hadronic scale $\Lambda_H \sim 1$ GeV. It is introduced in the Lagrangian of Eq. (13) in order to adjust physical dimensions of its terms. Typical momenta involved in $\mu^- - e^-$ conversion are $q \sim m_\mu$ where m_μ is the muon mass. Thus, from naive dimensional counting one expects that the contribution of the derivative terms to $\mu^- - e^-$ conversion is suppressed by a factor $(m_\mu/\Lambda_H)^2 \sim 10^{-2}$ in comparison with the nonderivative terms. Therefore, we retain in Eq. (13) only the dominant nonderivative terms. For a more detailed discussion of the role of the derivative terms see Ref. [18].

We relate the parameters of Lagrangians (13) and (3) with the help of the on-mass-shell matching condition proposed in Refs. [13,18,19]:

$$\langle \mu^+ e^- | \mathcal{L}_{\rm eff}^{lq} | M\rangle \approx \langle \mu^+ e^- | \mathcal{L}_{\rm eff}^{lM} | M\rangle, \quad (14)$$

with $|M = \rho,\ \omega,\ \phi,\ J/\Psi,\ \Upsilon,\ a_0,\ f_0\rangle$ corresponding to meson states on their mass shells. This equation can be solved using the well-known quark current matrix elements for vector and scalar mesons

$$\langle 0|\bar{u}\gamma_\mu u|\rho^0(p,\epsilon)\rangle = -\langle 0|\bar{d}\gamma_\mu d|\rho^0(p,\epsilon)\rangle = m_\rho^2 f_\rho \epsilon_\mu(p), \quad (15)$$

$$\langle 0|\bar{u}\gamma_\mu u|\omega(p,\epsilon)\rangle = \langle 0|\bar{d}\gamma_\mu d|\omega(p,\epsilon)\rangle = 3m_\omega^2 f_\omega \epsilon_\mu(p), \quad (16)$$

$$\langle 0|\bar{s}\gamma_\mu s|\phi(p,\epsilon)\rangle = -3m_\phi^2 f_\phi \epsilon_\mu(p), \quad (17)$$

$$\langle 0|\bar{c}\gamma_\mu c|J/\Psi(p,\epsilon)\rangle = m_{J/\Psi}^2 f_{J/\Psi} \epsilon_\mu(p), \quad (18)$$

$$\langle 0|\bar{b}\gamma_\mu b|\Upsilon(p,\epsilon)\rangle = m_\Upsilon^2 f_\Upsilon \epsilon_\mu(p), \quad (19)$$

$$\langle 0|\bar{u}u|f_0(p)\rangle = \langle 0|\bar{d}d|f_0(p)\rangle = m_{f_0}^2 f_{f_0}, \quad (20)$$

$$\langle 0|\bar{u}u|a_0^0(p)\rangle = -\langle 0|\bar{d}d|a_0(p)\rangle = m_{a_0}^2 f_{a_0}. \quad (21)$$

Here p, m_M and f_M are the 4-momentum, mass and dimensionless decay constant of the meson M, respectively; ϵ_μ is the vector-meson polarization state vector.

The current central values of the meson decay constants f_V and masses m_V are [29]

$$f_\rho = 0.2, \quad f_\omega = 0.059, \quad f_\phi = 0.074, \quad f_{J/\Psi} = 0.134, \quad f_\Upsilon = 0.08, \quad (22)$$

$$m_\rho = 771.1 \text{ MeV}, \quad m_\omega = 782.6 \text{ MeV}, \quad m_\phi = 1019.5 \text{ MeV}, \quad (23)$$

$$m_{J/\Psi} = 3097 \text{ MeV}, \quad m_\Upsilon = 9460 \text{ MeV}, \quad m_{f_0} = 500 \text{ MeV}, \quad m_{a_0} = 984.7 \text{ MeV}. \quad (24)$$

The decay constants f_{f_0} and f_{a_0} in Eqs. (20) and (21), are not yet known experimentally. In Ref. [19] we evaluated them on the basis of the linear σ model in the case of f_0 meson [34,35] and with the help of the QCD sum rules for a_0 meson [36]. The result is [19]

$$f_{f_0} = 0.28, \quad f_{a_0} = 0.19. \quad (25)$$

Following Refs. [18,19] we find the solution of Eq. (14) in the form

$$\xi_h^\rho = \left(\frac{m_\rho}{\Lambda_H}\right)^2 f_\rho \eta_{hV}^{(3)}, \quad \xi_h^\omega = 3\left(\frac{m_\omega}{\Lambda_H}\right)^2 f_\omega \eta_{hV}^{(0)}, \quad \xi_h^\phi = -3\left(\frac{m_\phi}{\Lambda_H}\right)^2 f_\phi \eta_{hV}^{(s)}, \quad (26)$$

$$\xi_h^{J/\Psi} = \left(\frac{m_{J/\Psi}}{\Lambda_H}\right)^2 f_{J/\Psi} \eta_{hV}^{(c)}, \quad \xi_h^\Upsilon = \left(\frac{m_\Upsilon}{\Lambda_H}\right)^2 f_\Upsilon \eta_{hV}^{(b)}, \quad (27)$$

$$\xi_r^{a_0} = \left(\frac{m_{a_0}}{\Lambda_H}\right)^2 f_{a_0} \eta_{rS}^{(3)}, \quad \xi_r^{f_0} = \left(\frac{m_{f_0}}{\Lambda_H}\right)^2 f_{f_0} \eta_{rS}^{(0)}, \quad (28)$$

where as before $h = V, A$, $r = S, P$ and $\eta^{(0,3)} = \eta^{(u)} \pm \eta^{(d)}$.

The lower vertex of the diagram in Fig. 1(b) corresponds to the conventional strong isospin invariant effective Lagrangian [37–39]:

$$\mathcal{L}^{MN} = \bar{N}\Big(\frac{1}{2} g_{\rho NN}\gamma^\mu \rho_\mu^k \tau^k + \frac{1}{2} g_{M_V^{(0)}NN}\gamma^\mu M_{V\mu}^{(0)} + g_{a_0NN} a_0^k \tau^k + g_{f_0NN} f_0\Big)N. \quad (29)$$

Here $M_V^{(0)} = \omega,\ \phi,\ J/\Psi,\ \Upsilon$ are the isosinglet vector mesons. In this Lagrangian we again neglected the derivative terms, irrelevant for coherent $\mu^- - e^-$ conversion. For the light vector-meson-nucleon couplings we use numerical values taken from an updated dispersive analysis [18,38,40]

$$g_{\rho NN} = 4.0, \qquad g_{\omega NN} = 41.8,$$
$$-18.3 \leq g_{\phi NN} \leq -0.24. \tag{30}$$

In Ref. [15] the couplings $g_{J/\Psi NN}$ and $g_{\Upsilon NN}$ were extracted from the QCD analysis [41,42] of the existing data [29] on the decay rates $\Gamma(J/\Psi \to \bar{p}p)$ and $\Gamma(\Upsilon \to \bar{p}p)$. Their values are

$$g_{J/\Psi NN} = 1.6 \times 10^{-3}, \qquad g_{\Upsilon NN} = 5.6 \times 10^{-6}. \tag{31}$$

In the literature there are also estimations of the scalar meson-nucleon couplings. In our analysis we use

$$g_{f_0 NN} \simeq 8.5[5.0], \qquad g_{a_0 NN} \simeq g_{f_0 NN}. \tag{32}$$

The first number is an empirical value of the scalar meson coupling $g_{f_0 NN}$ to provide the needed intermediate range nucleon-nucleon attraction according to Ref. [43]. The last approximate relation was obtained in the chiral unitary approach in Ref. [32]. The value of the scalar f_0 meson coupling calculated in this approach is given in square brackets. For our analysis we consider the empirical values of Ref. [43] as more reliable, but we will also show our results for the smaller values of Ref. [32].

The meson exchange contribution to $\mu^- - e^-$ conversion corresponding to the diagram in Fig. 1(b) is of second order in the Lagrangian $\mathcal{L}^{lM} + \mathcal{L}^{MN}$. Considering coherent $\mu^- - e^-$ conversion we ignore, as justified before, all derivative terms involving nucleon and lepton fields. Furthermore we neglect the kinetic energy of the final nucleus, the muon binding energy and the electron mass. In this approximation the squared momentum, transferred to the nucleus, has a constant value $q^2 \approx -m_\mu^2$ and the meson propagators contract to δ functions. Thus the meson exchange contributions in Fig. 1(b) result in effective lepton-nucleon four-fermion operators of the same type as in Eq. (5). For the corresponding α parameters we find

$$\text{MEM:}\ \alpha_{hV}^{(3)} = -\beta_\rho \eta_{hV}^{(3)},$$
$$\alpha_{hV}^{(0)} = -\beta_\omega \eta_{hV}^{(0)} - \beta_\phi \eta_{hV}^{(s)} - \beta_{J/\Psi} \eta_{hV}^{(c)} - \beta_\Upsilon \eta_{hV}^{(b)}, \tag{33}$$

$$\alpha_{rS}^{(3)} = \beta_{a_0} \eta_{rS}^{(3)}, \qquad \alpha_{rS}^{(0)} = \beta_{f_0} \eta_{rS}^{(0)}, \tag{34}$$

with $h = V, A$, $r = S, P$ and the coefficients

$$\beta_\rho = \frac{1}{2}\frac{g_{\rho NN} f_\rho m_\rho^2}{m_\rho^2 + m_\mu^2}, \qquad \beta_\omega = \frac{3}{2}\frac{g_{\omega NN} f_\omega m_\omega^2}{m_\omega^2 + m_\mu^2},$$
$$\beta_\phi = -\frac{3}{2}\frac{g_{\phi NN} f_\phi m_\phi^2}{m_\phi^2 + m_\mu^2}, \tag{35}$$

$$\beta_{J/\Psi} = \frac{g_{J/\Psi NN} f_{J/\Psi} m_{J/\Psi}^2}{m_{J/\Psi}^2 + m_\mu^2}, \qquad \beta_\Upsilon = \frac{g_{\Upsilon NN} f_\Upsilon m_\Upsilon^2}{m_\Upsilon^2 + m_\mu^2}, \tag{36}$$

$$\beta_{a_0} = \frac{g_{a_0 NN} f_{a_0} m_{a_0}^2}{m_{a_0}^2 + m_\mu^2}, \qquad \beta_{f_0} = \frac{g_{f_0 NN} f_{f_0} m_{f_0}^2}{m_{f_0}^2 + m_\mu^2}. \tag{37}$$

Substituting the values of the meson parameters from Eqs. (22)–(25) and (30)–(32) we obtain

$$\beta_\rho = 0.39, \quad \beta_\omega = 3.63, \quad 0.03 \leq \beta_\phi \leq 2.0, \tag{38}$$

$$\beta_{J/\Psi} = 2 \times 10^{-4}, \qquad \beta_\Upsilon = 5 \times 10^{-7}, \tag{39}$$

$$\beta_{a_0} = 1.58[0.93], \qquad \beta_{f_0} = 2.24[1.32]. \tag{40}$$

The coefficients β_{a_0}, β_{f_0} are estimated for the two values of $g_{f_0 NN}$, $g_{a_0 NN}$ shown in Eq. (32). In order to give a more transparent comparison of the two studied mechanisms DNM and MEM in Table I we display the coefficients of proportionality between the α and η parameters for both cases.

We note that the contributions of $\eta_{hV}^{(0)}$ and $\eta_{rS}^{(3)}$ are significantly enhanced in the MEM in comparison with the DNM. Also, the heavy J/Ψ and Υ mesons involve charmed and bottom quarks to contribute to $\mu^- - e^-$ conversion via vector currents. This effect is absent in the DNM. Thus taking into account the MEM allows setting new limits from $\mu^- - e^-$ conversion on the parameters of the underlying LFV models beyond the SM.

III. LIMITS ON LFV PARAMETERS FROM $\mu^- - e^-$ CONVERSION

The branching ratio of $\mu^- - e^-$ conversion can be written in the form [4,20]:

$$R_{\mu e^-}^{\text{coh}} = \frac{\mathcal{Q}}{2\pi \Lambda_{\text{LFV}}^4} \frac{p_e E_e (\mathcal{M}_p + \mathcal{M}_n)^2}{\Gamma_{\mu c}}, \tag{41}$$

with p_e, E_e being 3-momentum and energy of the final electron, respectively. Here $\mathcal{M}_{p,n}$ are the nuclear $\mu^- - e^-$

TABLE I. The absolute values of the coefficients in the relation $\alpha_I^{(k)} = a_I^{(k,A)} \eta_I^{(A)}$ schematically encoding Eqs. (9) and (10) for DNM and (33) and (34), for MEM. Here $k = 3, 0$; $A = 3, 0, s, c, b, t$ and $I = hV, rS$. For the definition of the values in square brackets see Eqs. (11), (12), and (38)–(40).

Mechanism	$a_{hV}^{(3,3)}$	$a_{rS}^{(3,3)}$	$a_{hV}^{(0,0)}$	$a_{hV}^{(0,s)}$	$a_{hV}^{(0,c)}$	$a_{hV}^{(0,b)}$	$a_{rS}^{(0,0)}$	$a_{rS}^{(0,0)}$	$a_{rS}^{(0,s)}$	$a_{rS}^{(0,c)}$	$a_{rS}^{(0,b)}$	$a_{rS}^{(0,t)}$
DNM	0.5	0.52 [0.4]	1.5	⋯	⋯	⋯	⋯	3.2 [4.7]	0.64 [2.5]	0.06	0.02	5×10^{-4}
MEM	0.39	1.58 [0.93]	3.63	0.03–2.0	2×10^{-4}	5×10^{-7}	2.24 [1.32]	⋯	⋯	⋯	⋯	⋯

transition matrix elements and $\Gamma_{\mu c}$ is the total rate of the ordinary muon capture. The factor $\mathcal{Q}$ can be expressed in terms of the parameters of Lagrangian (5) as

$$\mathcal{Q} = |\alpha_{VV}^{(0)} + \alpha_{VV}^{(3)}\phi|^2 + |\alpha_{AV}^{(0)} + \alpha_{AV}^{(3)}\phi|^2 + |\alpha_{SS}^{(0)} + \alpha_{SS}^{(3)}\phi|^2 + |\alpha_{PS}^{(0)} + \alpha_{PS}^{(3)}\phi|^2 + 2\,\mathrm{Re}\{(\alpha_{VV}^{(0)} + \alpha_{VV}^{(3)}\phi)(\alpha_{SS}^{(0)} + \alpha_{SS}^{(3)}\phi)^* + (\alpha_{AV}^{(0)} + \alpha_{AV}^{(3)}\phi)(\alpha_{PS}^{(0)} + \alpha_{PS}^{(3)}\phi)^*\}. \quad (42)$$

This expression involves the nuclear structure factor

$$\phi = (\mathcal{M}_p - \mathcal{M}_n)/(\mathcal{M}_p + \mathcal{M}_n). \quad (43)$$

The nuclear matrix elements $\mathcal{M}_{p,n}$ have been calculated in Refs. [16,17] for ^{27}Al, ^{48}Ti, ^{197}Au and ^{208}Pb. Their values are presented in Table II together with data for the total rates $\Gamma_{\mu c}$ of ordinary muon capture [44] and the 3-momentum p_e of the final electron.

With the parameters from Table II we find that the presently most stringent limits on the dimensionless lepton-nucleon LFV couplings α of the Lagrangian (5) result from the SINDRUM II searches for $\mu^- - e^-$ conversion on ^{198}Au [6]. Here we show these limits and the limits corresponding to the future experiment PRISM/PRIME [12] with titanium ^{48}Ti target aiming at the sensitivity of 10^{-18}. We have for these two cases:

TABLE II. $\mu^- - e^-$ nuclear matrix elements, $\mathcal{M}_{p,n}$, and other quantities from Eq. (41).

Nucleus	p_e (fm^{-1})	$\Gamma_{\mu c}$ ($\times 10^6$s^{-1})	$\mathcal{M}_p$ (fm$^{-3/2}$)	$\mathcal{M}_n$ (fm$^{-3/2}$)
^{27}Al	0.531	0.71	0.047	0.045
^{48}Ti	0.529	2.60	0.104	0.127
^{197}Au	0.485	13.07	0.395	0.516
^{208}Pb	0.482	13.45	0.414	0.566

$$R_{\mu e}^{\mathrm{Au}} \leq 4.3 \times 10^{-12}\,[6]: \quad \alpha_{hV,rS}^{(k)}\left(\frac{1\ \mathrm{GeV}}{\Lambda_{\mathrm{LFV}}}\right)^2 \leq 8.5 \times 10^{-13} B^{(k)}(\mathrm{Au}), \quad (44)$$

$$R_{\mu e}^{\mathrm{Ti}} \lesssim 10^{-18}\,[12]: \quad \alpha_{hV,rS}^{(k)}\left(\frac{1\ \mathrm{GeV}}{\Lambda_{\mathrm{LFV}}}\right)^2 \leq 1.6 \times 10^{-15} B^{(k)}(\mathrm{Ti}), \quad (45)$$

with $k = 0, 3$, $h = A, V$, $r = S, P$ and $B^{(0,3)}(Ti) = (1, 10)$, $B^{(0,3)}(Au) = (1, 7.5)$.

The limits in Eqs. (44) and (45) can be used for derivation of individual bounds on the terms contributing to the coefficients α. Following the common practice we assume the absence of substantial cancellations between different terms. Thus, from Eqs. (9), (10), (33), and (34), we deduce upper limits on the dimensionless couplings of the four-fermion quark-lepton LFV contact terms of the Lagrangian (3) for the two studied mechanisms of hadronization: for the DNM and for the MEM. These limits are listed in Tables III and IV.

In Tables III and IV, we also show lower limits on the individual mass scales, $\Lambda_{\mu e}^{ij}$, of the quark-lepton contact operators in Eq. (3). In the convenn ttional definition these scales are related to our notations as

TABLE III. Upper limits on the LFV parameters η of the quark-lepton contact operators in Eq. (3), and lower limits on their individual mass scales, $\Lambda_{\mu e}$, defined in Eq. (46), inferred from the SINDRUM II data for ^{198}Au [6]. We show the limits both for the DNM and the MEM. For $\eta_{hV}^{(s)}$ in the 3rd column and for $\Lambda_{\mu e}^{(s)hV}$ in the last column we show the two limits corresponding to the upper and lower bounds in Eq. (30). The limits in the square brackets correspond to the options shown in Eqs. (11) and (32). All the limits are extracted assuming that only one η-term in Eqs. (9), (10), (33), and (34) is dominant.

η	DNM $\times\left(\frac{\Lambda_{\mathrm{LFV}}}{1\ \mathrm{GeV}}\right)^2$	MEM $\times\left(\frac{\Lambda_{\mathrm{LFV}}}{1\ \mathrm{GeV}}\right)^2$	$\Lambda_{\mu e}$ in TeV	DNM	MEM
$\eta_{hV}^{(3)}$	1.3×10^{-11}	1.6×10^{-11}	$\Lambda_{\mu e}^{(3)hV}$	10^3	900
$\eta_{hV}^{(0)}$	5.7×10^{-13}	2.4×10^{-13}	$\Lambda_{\mu e}^{(0)hV}$	4.7×10^3	7.2×10^3
$\eta_{hV}^{(s)}$	No limits	2.8×10^{-11}; 4.3×10^{-13}	$\Lambda_{\mu e}^{(s)hV}$	No limits	770; 5.4×10^3
$\eta_{hV}^{(c)}$	No limits	4.3×10^{-9}	$\Lambda_{\mu e}^{(c)hV}$	No limits	54
$\eta_{hV}^{(b)}$	No limits	1.6×10^{-6}	$\Lambda_{\mu e}^{(b)hV}$	No limits	3
$\eta_{rS}^{(3)}$	$1.2 \times 10^{-11}[1.6 \times 10^{-11}]$	$4.0 \times 10^{-12}[6.9 \times 10^{-12}]$	$\Lambda_{\mu e}^{(3)rS}$	$10^3[900]$	$1.8 \times 10^3[1.4 \times 10^3]$
$\eta_{rS}^{(0)}$	$2.7 \times 10^{-13}[1.8 \times 10^{-13}]$	$3.7 \times 10^{-13}[6.4 \times 10^{-13}]$	$\Lambda_{\mu e}^{(0)rS}$	$6.8 \times 10^3[8.4 \times 10^3]$	$5.8 \times 10^3[4.4 \times 10^3]$
$\eta_{rS}^{(s)}$	$1.3 \times 10^{-12}[3.4 \times 10^{-13}]$	No limits	$\Lambda_{\mu e}^{(s)rS}$	$3 \times 10^3[6 \times 10^3]$	No limits
$\eta_{rS}^{(c)}$	1.4×10^{-11}	No limits	$\Lambda_{\mu e}^{(c)rS}$	950	No limits
$\eta_{rS}^{(b)}$	4.3×10^{-11}	No limits	$\Lambda_{\mu e}^{(b)rS}$	540	No limits
$\eta_{rS}^{(t)}$	1.7×10^{-9}	No limits	$\Lambda_{\mu e}^{(t)rS}$	90	No limits

TABLE IV. The same as in Table III but for the expected sensitivities of the future experiment PRISM/PRIME [12] with titanium ^{48}Ti.

η	DNM $\times \left(\frac{\Lambda_{\rm LFV}}{1\ \rm GeV}\right)^2$	MEM $\times \left(\frac{\Lambda_{\rm LFV}}{1\ \rm GeV}\right)^2$	$\Lambda_{\mu e}$ in TeV	DNM	MEM
$\eta_{hV}^{(3)}$	2.1×10^{-14}	2.6×10^{-14}	$\Lambda_{\mu e}^{(3)hV}$	2.5×10^4	2.3×10^4
$\eta_{hV}^{(0)}$	1.1×10^{-15}	4.8×10^{-16}	$\Lambda_{\mu e}^{(0)hV}$	9.4×10^4	1.4×10^5
$\eta_{hV}^{(s)}$	No limits	5.6×10^{-14}; 8.6×10^{-16}	$\Lambda_{\mu e}^{(s)hV}$	No limits	1.5×10^4; 1.1×10^5
$\eta_{hV}^{(c)}$	No limits	8.6×10^{-12}	$\Lambda_{\mu e}^{(c)hV}$	No limits	1.1×10^3
$\eta_{hV}^{(b)}$	No limits	3.2×10^{-9}	$\Lambda_{\mu e}^{(b)hV}$	No limits	60
$\eta_{rS}^{(3)}$	$2.0\times10^{-14}[2.6\times10^{-14}]$	$6.4\times10^{-15}[1.1\times10^{-14}]$	$\Lambda_{\mu e}^{(3)rS}$	$2.5\times10^4[2.3\times10^4]$	$4.5\times10^4[3.5\times10^4]$
$\eta_{rS}^{(0)}$	$5.4\times10^{-16}[3.6\times10^{-16}]$	$7.4\times10^{-16}[1.3\times10^{-15}]$	$\Lambda_{\mu e}^{(0)rS}$	$1.4\times10^5[1.7\times10^5]$	$1.2\times10^5[8.8\times10^4]$
$\eta_{rS}^{(s)}$	$2.6\times10^{-15}[6.8\times10^{-15}]$	No limits	$\Lambda_{\mu e}^{(s)rS}$	$6\times10^4[1.2\times10^5]$	No limits
$\eta_{rS}^{(c)}$	2.8×10^{-14}	No limits	$\Lambda_{\mu e}^{(c)rS}$	1.9×10^4	No limits
$\eta_{rS}^{(b)}$	8.6×10^{-14}	No limits	$\Lambda_{\mu e}^{(b)rS}$	1.1×10^4	No limits
$\eta_{rS}^{(t)}$	3.4×10^{-12}	No limits	$\Lambda_{\mu e}^{(t)rS}$	1.8×10^3	No limits

$$|\eta_z^{(a)}|\left(\frac{1\ \rm GeV}{\Lambda_{\rm LFV}}\right)^2 = 4\pi\left(\frac{1\ \rm GeV}{\Lambda_{\mu e}^{(a)z}}\right)^2, \quad (46)$$

with $a = 0, 3, s, c, b, t$ and $z = hV, rS$, where $h = A, V$ and $r = P, S$ as defined before.

Let us compare our limits for the mass scales $\Lambda_{\mu e}$ with similar limits existing in the literature. It is a custom to refer to $\pi^- \to e^-\nu_e$ as the process which provides the most stringent limits on the lepton flavor conserving contact terms involving pseudoscalar and scalar quark currents [45]. Note, the latter does not contribute directly to this process, but due to the gauge invariance with respect to the SM group one can relate the couplings of the scalar and pseudoscalar lepton-quark contact operators. The updated upper limit from $\pi^- \to e^-\nu_e$ [29] on the corresponding mass scale is $\Lambda_{ee} \geq 500$ TeV. This limit is not related to our limits for the LFV mass scales $\Lambda_{\mu e}$. However, it can be taken as a reference value, illustrating the present situation with the (pseudo-)scalar contact terms. Limits on $\Lambda_{\mu e}$ of the LFV (pseudo-)scalar contact terms were derived in the literature from the experimental bounds on $\pi^+ \to \mu^+\nu_e$, $\pi^0 \to \mu^\pm e^\mp$ [46]. Typical limits from these processes are $\Lambda_{\mu e} \geq$ few TeV.

As to the vector lepton-quark contact interactions, the corresponding scales can be extracted from the experimental limits [29] on $M_V \to \mu^\pm e^\mp$ for $M_V = \rho, \omega, \phi, J/\Psi, \Upsilon$. But, in Refs. [14,15] it was shown that $\mu^- - e^-$ conversion is much more sensitive probe of the LFV physics than vector meson decays. Therefore limits on $\Lambda_{\mu e}$ from $\mu^- - e^-$ conversion must be much better than limits from these decays.

Recently the ATLAS Collaboration reported results of an analysis of Drell-Yan e^-e^+ and $\mu^-\mu^+$ dileptons from the data collected in 2011 at the LHC with $\sqrt{s} = 7$ TeV [47]. They set lower limits on the scale of the lepton flavor conserving lepton-quark vector contact interactions with the typical values $\Lambda_{ee} \gtrsim 10$ TeV and $\Lambda_{\mu\mu} \gtrsim 5$ TeV. The LHC experiments are also able to constrain the LFV lepton-quark contact interaction scales $\Lambda_{ll'}$ from the measurement of Drell-Yan cross sections in the high dilepton mass region [48]. In this case typical expected limits are $\Lambda_{ll'} \geq 35$ TeV. A comparison of the above mentioned limits existing in the literature with the ones in Table III, extracted from $\mu^- - e^-$ conversion, shows that our limits are more stringent with the only possible exception of the scale of the $bb\mu e$ vector contact interaction. However, as seen from Table IV, the future PRISM/PRIME experiment [12] would be able to set such limits on the scales of all the contact LFV interactions of the type $qq\mu e$ that look hardly accessible for other experiments including those at the LHC.

IV. SUMMARY

In this paper we analyzed the nuclear $\mu^- - e^-$ conversion using general framework of effective Lagrangians without referring to any particular LFV model beyond the SM. We examined two hadronization mechanisms of the underlying effective quark-lepton LFV Lagrangian (3): the DNM and the MEM mechanisms.

Using experimental upper bounds on the $\mu^- - e^-$ conversion rate we extracted lower limits on the mass scales $\Lambda_{\mu e}$ of the LFV lepton-quark contact vector and scalar terms $qq\mu e$ involved in this process for all quark flavors $q = u, d, s, c, b, t$. We showed that these limits are more stringent than the similar ones existing in the literature, including the limits from the present experimental data on meson decays and the limits expected from the future experiments at the LHC.

We demonstrated that neither of the two hadronization mechanisms, DNM and MEM, should be overlooked in analysis of $\mu^- - e^-$ conversion due to their complementarity. As seen from Tables II and III in some cases it is the DNM which is only able to set limits on the corresponding LFV parameters while in some other cases it is the MEM. Also MEM improves the limits on the LFV parameters $\Lambda_{\mu e}^{(0)hV}$ and $\Lambda_{\mu e}^{(3)rS}$ in comparison with the conventional DNM mechanism. This fact may have an appreciable impact on the phenomenology of the LFV physics beyond the Standard Model.

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[1] W. J. Marciano, T. Mori, and J. M. Roney, Annu. Rev. Nucl. Part. Sci. **58**, 315 (2008).
[2] Y. Kuno and Y. Okada, Rev. Mod. Phys. **73**, 151 (2001).
[3] A. Czarnecki, W. Marciano, and K. Melnikov, AIP Conf. Proc. **549**, 938 (2000).
[4] T. Kosmas, G. Leontaris, and J. Vergados, Prog. Part. Nucl. Phys. **33**, 397 (1994).
[5] C. Dohmen *et al.* (SINDRUM II Collaboration), Phys. Lett. B **317**, 631 (1993).
[6] W. H. Bertl *et al.* (SINDRUM II Collaboration), Eur. Phys. J. C **47**, 337 (2006).
[7] W. Honecker *et al.* (SINDRUM II Collaboration), Phys. Rev. Lett. **76**, 200 (1996).
[8] M. Aoki, Proc. Sci., ICHEP2010 (2010) 279.
[9] R. Carey *et al.* (Mu2e Collaboration) (unpublished).
[10] R. K. Kutschke, arXiv:1112.0242.
[11] Y. Cui *et al.* (COMET Collaboration) (unpublished).
[12] H. Witte *et al.*, Conf. Proc. **C1205201**, 79 (2012).
[13] A. Faessler, Th. Gutsche, S. Kovalenko, V. E. Lyubovitskij, I. Schmidt, and F. Šimkovic, Phys. Rev. D **70**, 055008 (2004).
[14] T. Gutsche, J. C. Helo, S. Kovalenko, and V. E. Lyubovitskij, Phys. Rev. D **81**, 037702 (2010).
[15] T. Gutsche, J. C. Helo, S. Kovalenko, and V. E. Lyubovitskij, Phys. Rev. D **83**, 115015 (2011).
[16] T. Kosmas, S. Kovalenko, and I. Schmidt, Phys. Lett. B **511**, 203 (2001).
[17] A. Faessler, T. Kosmas, S. Kovalenko, and J. Vergados, Nucl. Phys. **B587**, 25 (2000).
[18] A. Faessler, Th. Gutsche, S. Kovalenko, V. E. Lyubovitskij, I. Schmidt, and F. Šimkovic, Phys. Lett. B **590**, 57 (2004).
[19] A. Faessler, T. Gutsche, S. Kovalenko, V. E. Lyubovitskij, and I. Schmidt, Phys. Rev. D **72**, 075006 (2005).
[20] T. Kosmas, Z. Ren, and A. Faessler, Nucl. Phys. **A665**, 183 (2000).
[21] A. Faessler, S. Kovalenko, F. Simkovic, and J. Schwieger, Phys. Rev. Lett. **78**, 183 (1997).
[22] T. Kosmas, S. Kovalenko, and I. Schmidt, Phys. Lett. B **519**, 78 (2001).
[23] J. Gasser, H. Leutwyler, and M. Sainio, Phys. Lett. B **253**, 252 (1991).
[24] J. Gasser and M. E. Sainio, arXiv:hep-ph/0002283.
[25] M. Sainio, PiN Newslett. **16**, 138 (2002).
[26] M. A. Shifman, A. Vainshtein, and V. I. Zakharov, Phys. Lett. **78B**, 443 (1978).
[27] T. P. Cheng, Phys. Rev. D **38**, 2869 (1988).
[28] A. Corsetti and P. Nath, Phys. Rev. D **64**, 125010 (2001).
[29] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[30] G. Colangelo, J. Gasser, and H. Leutwyler, Nucl. Phys. **B603**, 125 (2001).
[31] J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. Lett. **80**, 3452 (1998).
[32] E. Oset, H. Toki, M. Mizobe, and T. T. Takahashi, Prog. Theor. Phys. **103**, 351 (2000).
[33] Y. S. Surovtsev, P. Bydzovsky, and V. E. Lyubovitskij, Phys. Rev. D **85**, 036002 (2012); Y. S. Surovtsev, P. Bydzovsky, R. Kaminski, V. E. Lyubovitskij, and M. Nagy, Phys. Rev. D **86**, 116002 (2012).
[34] R. Delbourgo and M. Scadron, Mod. Phys. Lett. A **10**, 251 (1995).
[35] R. Delbourgo, M. Scadron, and A. Rawlinson, Mod. Phys. Lett. A **13**, 1893 (1998).
[36] K. Maltman, Phys. Lett. B **462**, 14 (1999).
[37] S. Weinberg, Phys. Rev. **166**, 1568 (1968).
[38] P. Mergell, U. G. Meissner, and D. Drechsel, Nucl. Phys. **A596**, 367 (1996).
[39] B. Kubis and U.-G. Meissner, Nucl. Phys. **A679**, 698 (2001).
[40] U.-G. Meissner, V. Mull, J. Speth, and J. van Orden, Phys. Lett. B **408**, 381 (1997).
[41] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981).
[42] V. Chernyak and A. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
[43] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).
[44] T. Suzuki, D. F. Measday, and J. P. Roalsvig, Phys. Rev. C **35**, 2212 (1987).
[45] O. U. Shanker, Nucl. Phys. **B204**, 375 (1982).
[46] J. E. Kim, P. Ko, and D.-G. Lee, Phys. Rev. D **56**, 100 (1997).
[47] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **712**, 40 (2012).
[48] N. Krasnikov and V. Matveev, Phys. Usp. **47**, 643 (2004).