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# Exploring the electromagnetic form factors of the scalar $K_0^*(1430)$, $f_0(1500)$ and $a_0(1450)$ mesons in light-cone sum rules

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We calculate the electromagnetic form factors of the scalar $K_0^*(1430)$, $f_0(1500)$ and $a_0(1450)$ mesons by considering them as regular quark-antiquark states and using their distribution amplitudes within the light-cone QCD sum rules approach.

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## I. INTRODUCTION

Investigation of the electromagnetic form factors of the scalar mesons is one of the most useful tools for uncovering their mysterious structures. Generally, having knowledge of the electromagnetic form factors of the mesons plays an important role in our understanding of their structure, the way they respond to the electromagnetic fields, as well as their geometric shapes and charge distributions. Moreover, these form factors can be used in the calculation of their multipole moments, such as the electric and magnetic dipole, quadrupole and octupole moments.

In the literature there have been several studies on the electromagnetic properties of the vector, axial vector and pseudoscalar mesons and also baryons via different models [1–11]. However, there are relatively few works devoted to the electromagnetic properties of the scalar mesons. In the present work, making use of their distribution amplitudes, we calculate the electromagnetic form factors of the scalar $K_0^*(1430)$, $f_0(1500)$ and $a_0(1450)$ mesons in the framework of the light-cone QCD sum rules, a fruitful hybrid of the Shifman-Vainshtein-Zakharov technique [12] and the theory of a hard exclusive process. In this model, the basic idea is to expand the products of the currents near the light cone. This approach has been very useful for many years in calculating various hadronic transition form factors.

The layout of the article is as follows. In Sec. II, we obtain the light-cone QCD sum rules for the electromagnetic form factors of the scalar mesons under consideration. Section III is devoted to the numerical analysis of the form factors and a discussion of the results obtained.

## II. THEORETICAL FRAMEWORK

In order to obtain the sum rules for the electromagnetic form factors of the scalar meson ($M$) with momentum $p$, we start by considering the correlation function

$$\Pi_\mu(p,q) = i\int d^4x e^{iqx}\langle M(p)|j_\mu^{\rm el}(x)j^M(0)|0\rangle, \qquad (2.1)$$

where $q$ is the momentum of the electromagnetic current and $j^M$ stands for the interpolating currents of the scalar mesons. Considering these mesons as the regular quark-antiquark states, the interpolating currents for the members under consideration are given by [13–16]

$$\begin{aligned} j^{K_0^*}(x) &= \bar{s}(x)d(x),\\ j^{f_0}(x) &= \frac{1}{\sqrt{2}}\left(\frac{\bar{u}(x)u(x)+\bar{d}(x)d(x)}{\sqrt{2}} - \bar{s}(x)s(x)\right),\\ j^{a_0}(x) &= \frac{1}{\sqrt{2}}\left(\frac{-\bar{u}(x)u(x)+\bar{d}(x)d(x)}{\sqrt{2}} + \bar{s}(x)s(x)\right). \end{aligned} \qquad (2.2)$$

In the following, some remarks about the interpolating currents of the scalar mesons under consideration and their structures are in order. The above current for the $K_0^*$ is exact; however, the interpolating currents in Eq. (2.2) for the $f_0$ and $a_0$ are special cases of the following general interpolating currents [17]:

$$\begin{aligned} j^{f_0}(x) &= \cos\theta\frac{|\bar{d}(x)d(x)\rangle + |\bar{u}(x)u(x)\rangle}{\sqrt{2}} - \sin\theta|\bar{s}(x)s(x)\rangle,\\ j^{a_0}(x) &= \sin\theta\frac{|\bar{d}(x)d(x)\rangle - |\bar{u}(x)u(x)\rangle}{\sqrt{2}} + \cos\theta|\bar{s}(x)s(x)\rangle. \end{aligned} \qquad (2.3)$$

The light scalar mesons can also be considered as diquark-antidiquark (tetraquarks) states bound by color forces [18]. In this picture, the diquarks are considered in color $\bar{\mathbf{3}}$, spin $S=0$ and flavor $\bar{\mathbf{3}}$; and antidiquarks are considered in the conjugate representations. The diquark-antidiquark bound states naturally reproduce the SU(3) nonet structure. In this scenario, the $f_0$ and $a_0$ mesons are indicated by the explicit quark composition:

$$\begin{aligned} f_0 &= \frac{[su][\bar{s}\,\bar{u}] + [sd][\bar{s}\,\bar{d}]}{\sqrt{2}},\\ a_0 &= [su][\bar{s}\,\bar{d}]; \frac{[su][\bar{s}\,\bar{u}] - [sd][\bar{s}\,\bar{d}]}{\sqrt{2}}; [sd][\bar{s}\,\bar{u}]. \end{aligned} \qquad (2.4)$$

In some other scenarios, the scalar mesons are considered as good candidates for the scalar glueball and hybrids [19],

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as well as hadron molecules [20]. We will consider the currents in Eqs. (2.2) and (2.3) representing the scalar mesons as quark-antiquark mesons but not the tetraquark, glueball, hybrid and hadron molecule representations in the present work. Note that, in the following, we will only give the steps of our calculations for the currents of Eqs. (2.2); however, we will consider the currents of Eqs. (2.3) with different angles when we discuss the dependence of the electromagnetic form factors on $Q^2$.

We also take the electromagnetic current as

$$j^{\rm el}_\mu(x) = \sum_{q=u,d,s} e_q \bar{q}(x)\gamma_\mu q(x). \tag{2.5}$$

From the general philosophy of the QCD sum rules, we calculate the aforesaid correlation function in two different languages. First, on the physical side, we calculate it in terms of the hadronic parameters as well as the electromagnetic form factors. Next, on the QCD side, we evaluate it in terms of the QCD degrees of freedom and distribution amplitudes using the operator product expansion in the deep Euclidean region. We then match these two different representations to each other and apply the Borel transformation as well as continuum subtraction. This procedure leads to the QCD sum rules for the form factors.

On the physical side, by inserting a complete set of intermediate mesonic states between the currents in Eq. (2.1), we get

$$\Pi_\mu = \frac{\langle M(p)|j^{\rm el}_\mu|M(p+q)\rangle\langle M(p+q)|j^M|0\rangle}{(p+q)^2 - m_M^2}. \tag{2.6}$$

The matrix elements entering Eq. (2.6) are defined as

$$\begin{aligned}\langle M(p)|j^{\rm el}_\mu|M(q+p)\rangle &= F^{\rm el}_M(2p+q)_\mu,\\ \langle M(p+q)|j^M|0\rangle &= m_M\bar{f}_M,\end{aligned} \tag{2.7}$$

where $F^{\rm el}_M$ is the electromagnetic form factor and $\bar{f}_M$ is the scalar meson's decay constant. By substituting Eq. (2.7) into Eq. (2.6) we end up with the following relation for the phenomenological part of the correlation function:

$$\Pi_\mu(p,q) = m_M\bar{f}_M F^{\rm el}_M \frac{2p_\mu + q_\mu}{(p+q)^2 - m_M^2}. \tag{2.8}$$

This relation shows that $\Pi_\mu(p,q)$ can be decomposed into two different structures,

$$\Pi_\mu(p,q) = \Pi_1(q^2)p_\mu + \Pi_2(q^2)q_\mu, \tag{2.9}$$

where

$$\begin{aligned}\Pi_1(q^2) &= 2m_M\bar{f}_M\frac{F^{\rm el}_M}{(p+q)^2 - m_M^2},\\ \Pi_2(q^2) &= m_M f_M\frac{F^{\rm el}_M}{(p+q)^2 - m_M^2}.\end{aligned} \tag{2.10}$$

The results are independent of which structure, $q_\mu$ or $p_\mu$, is selected, so for convenience, we will focus only on the $p_\mu$ structure.

On the QCD side, we write each invariant amplitude $\Pi_i(q^2)$ as a dispersion relation of the form

$$\Pi_i(q^2) = \int ds\frac{\rho_i(s,q^2)}{s-(p+q)^2} + \text{subtraction terms}, \tag{2.11}$$

where $\rho_i(s,q^2)$ are spectral densities. Our main task in the following is to calculate the spectral density corresponding to the structure $p_\mu$ for each scalar meson. For this purpose, by putting the interpolating and electromagnetic currents in Eq. (2.1) and contracting the quark pairs using the Wick theorem (for instance, for the $K_0^*$ meson), we get

$$\begin{aligned}\Pi_\mu(p,q) = i\int d^4x e^{iqx}\{&\langle M(p)|e_s\bar{s}(x)\gamma_\mu S_s(x)d(0)|0\rangle\\ &+ e_d\bar{s}(0)\gamma_\mu S_d(-x)d(x)|0\rangle\},\end{aligned} \tag{2.12}$$

where $S_q(x)$ is the propagator of the light quark and it is given by

$$\begin{aligned}S_q(x) = &\frac{i\not{x}}{2\pi^2x^4} - \frac{m_q}{4\pi^2x^2} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q}{4}\not{x}\right)\\ &- \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\left(1 - i\frac{m_q}{6}\not{x}\right)\\ &- ig_s\int_0^1 du\Big[\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}(ux)\sigma_{\mu\nu}\\ &- ux^\mu G_{\mu\nu}(ux)\gamma^\nu\frac{i}{4\pi^2x^2}\\ &- i\frac{m_q}{32\pi^2}G_{\mu\nu}\sigma^{\mu\nu}\left(\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right)\Big].\end{aligned} \tag{2.13}$$

Before substituting the expression of the propagator into Eq. (2.12), we use the expansion

$$\gamma_\mu S_q = \frac{1}{4}\sum_i \Gamma_i {\rm Tr}[\gamma_\mu S_q\Gamma_i], \tag{2.14}$$

where $\Gamma_i = (I, \gamma_5, \gamma_{\alpha'}, \gamma_{\alpha'}\gamma_5, \sigma_{\alpha'\beta'})$ is a complete set of Dirac matrices.

To proceed, we substitute the expression of the light-quark propagator and define the matrix elements of the nonlocal operators between the vacuum and the mesonic state in terms of the "wave functions" or "light-cone distribution amplitudes" of the scalar mesons as [21–23]

$$\begin{aligned}\langle M(p)|\bar{q}_2(x)\gamma_\mu q_1(y)|0\rangle &= p_\mu\int_0^1 du e^{i(up\cdot x+\bar{u}p\cdot y)}\phi_M(u,\mu),\\ \langle M(p)|\bar{q}_2(x)q_1(y)|0\rangle &= m_M\int_0^1 du e^{i(up\cdot x+\bar{u}p\cdot y)}\phi_M^s(u,\mu),\\ \langle M(p)|\bar{q}_2(x)\sigma_{\mu\nu}q_1(y)|0\rangle &= -m_M(p_\mu z_\nu - p_\nu z_\mu)\\ &\quad\times\int_0^1 du e^{i(up\cdot x+\bar{u}p\cdot y)}\phi_M^\sigma(u,\mu),\end{aligned} \tag{2.15}$$

where $u$ is the momentum fraction carried by the quark $q$, $\bar{u} = 1 - u$ and $z = x - y$. Here $\phi_M(u, \mu)$ is the leading twist-2 wave function, while $\phi_M^s(u, \mu)$ and $\phi_M^\sigma(u, \mu)$ are the twist-3 wave functions. The function $\phi_M(u, \mu)$ is antisymmetric, while $\phi_M^s(u, \mu)$ and $\phi_M^\sigma(u, \mu)$ are symmetric under the replacement of $u \to 1 - u$. The renormalizations of $\phi_M(u, \mu)$, $\phi_M^\sigma(u, \mu)$ and $\phi_M^\sigma(u, \mu)$ are given by the following equations:

$$\int_0^1 du\phi_M(u, \mu) = f_M, \quad \int_0^1 du\phi_M^s(u, \mu) = \int_0^1 du\phi_M^\sigma(u, \mu) = \bar{f}_M, \tag{2.16}$$

wherein

$$\phi_M(u, \mu) = \bar{f}_M(\mu) 6u(1-u)\left[B_0(\mu) + \sum_{m=1}^{\infty} B_m(\mu) C_m^{3/2}(2u-1)\right], \tag{2.17}$$

$$\phi_M^s(u, \mu) = \bar{f}_M(\mu)\left[1 + \sum_{m=1}^{\infty} a_m(\mu) C_m^{1/2}(2u-1)\right], \tag{2.18}$$

$$\phi_M^\sigma(u, \mu) = \bar{f}_M(\mu) 6u(1-u)\left[1 + \sum_{m=1}^{\infty} b_m(\mu) C_m^{3/2}(2u-1)\right]. \tag{2.19}$$

The vector-current and the scalar-current decay constants $(f_M, \bar{f}_M)$ of the scalar mesons can be joined by

$$\bar{f}_M = \mu_M f_M, \qquad \mu_M = \frac{m_M}{m_2(\mu) - m_1(\mu)}, \tag{2.20}$$

with $m_1$ and $m_2$ being the masses of the quark content of the meson. The Gegenbauer moments for the twist-2 and twist-3 distribution amplitudes of the scalar $K_0^*$, $f_0$ and $a_0$ at the scale $\mu = 1$ GeV are given in Tables I, II, and III [21–23]. The zeroth Gegenbauer moment $B_0(\mu)$ for the twist-2 distribution amplitude $\phi_M(u, \mu)$ is also given by $B_0 = \mu_M^{-1}$.

The next step is to apply the Borel transformation as well as continuum subtraction in order to suppress the contributions of the higher states and the continuum and also eliminate the subtraction terms. For this we need to define

$$\frac{1}{(q+up)^2} = \frac{1}{u}\left[\frac{1}{s(u) - u(p+q)^2}\right], \tag{2.21}$$

where

$$s(u) = \left(\frac{u-1}{u}\right)q^2 + (1-u)p^2 = \frac{\bar{u}}{u}(m_M^2 u + Q^2), \tag{2.22}$$

TABLE I. The Gegenbauer moments for the twist-2 distribution amplitude $\phi_M$ at the scale $\mu = 1$ GeV.

| Meson | $B_1$ | $B_3$ |
|---|---|---|
| $K_0^*$ | $-0.57 \pm 0.13$ | $-0.42 \pm 0.22$ |
| $f_0(1500)$ | $-0.48 \pm 0.11$ | $-0.37 \pm 0.20$ |
| $a_0(1450)$ | $-0.58 \pm 0.12$ | $-0.49 \pm 0.15$ |

TABLE II. The Gegenbauer moments for the twist-3 distribution amplitude $\phi_M^s$ at the scale $\mu = 1$ GeV.

| Meson | $a_1(\times 10^{-2})$ | $a_2$ | $a_4$ |
|---|---|---|---|
| $K_0^*$ | $1.8 \sim 4.2$ | $-0.33 \sim -0.025$ | $\cdots$ |
| $f_0(1500)$ | 0 | $-0.33 \sim -0.18$ | $0.28 \sim 0.79$ |
| $a_0(1450)$ | 0 | $-0.33 \sim -0.18$ | $-0.11 \sim 0.39$ |

TABLE III. The Gegenbauer moments for the twist-3 distribution amplitude $\phi_M^\sigma$ at the scale $\mu = 1$ GeV.

| Meson | $b_1(\times 10^{-2})$ | $b_2$ | $b_4$ |
|---|---|---|---|
| $K_0^*$ | $3.7 \sim 5.5$ | $0 \sim 0.15$ | $\cdots$ |
| $f_0(1500)$ | 0 | $-0.15 \sim -0.088$ | $0.044 \sim 0.16$ |
| $a_0(1450)$ | 0 | $0 \sim 0.058$ | $0.070 \sim 0.20$ |

with $Q^2 = -q^2$, and use the Borel transformations with respect to $(p+q)^2$ via the relations

$$\mathcal{B}_{M^2}((p+q)^2)^k = 0, \quad k > 0, \quad \mathcal{B}_{M^2}\left(\frac{1}{[m_M^2 - (p+q)^2]^k}\right) = \frac{1}{\Gamma(k)}\frac{e^{-m_M^2/M^2}}{M^{2(k-1)}}, \tag{2.23}$$

where $M^2$ is the Borel parameter. The contributions of the higher states and continuum are subtracted using the quark hadron duality assumption (for details see [12,24]), which converts the range of the integrals to $u_0 \leq u \leq 1$, where

$$u_0 = -\frac{1}{2m_M^2}\left[(s_0 + Q^2 - m_M^2) - \sqrt{(s_0 + Q^2 - m_M^2)^2 + 4m_M^2 Q^2}\right], \tag{2.24}$$

and $s_0$ is the threshold of the lowest continuum state.

Finally, as it was mentioned before, by matching the results of the physical and the QCD sides of the correlation function, the electromagnetic form factors of the scalar mesons under consideration are found as

$$
\begin{aligned}
Q^2 F_{K_0^*}(Q^2) = {} & e^{m_{K_0^*}^2/M^2} \frac{Q^2}{m_{K_0^*}} \Big\{ \frac{1}{4} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \frac{1}{u} \Big[ e_s m_s \phi_{K_0^*}(u) + e_d m_d \phi_{K_0^*}(\bar{u}) \Big] \\
& - \frac{1}{2} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \frac{1}{u^2 M^2} \pi^2 \Big[ e_s \langle s\bar{s} \rangle \phi_{K_0^*}(u) + e_d \langle d\bar{d} \rangle \phi_{K_0^*}(\bar{u}) \Big] \\
& + \frac{5}{16} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \frac{m_0^2}{u^3 M^4} \pi^2 \Big[ e_s \langle s\bar{s} \rangle \phi_{K_0^*}(u) + e_d \langle d\bar{d} \rangle \phi_{K_0^*}(\bar{u}) \Big] \Big\} \\
& + e^{m_{K_0^*}^2/M^2} Q^2 \Big\{ - \frac{1}{2} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \Big[ e_s \phi^s_{K_0^*}(u) - e_d \phi^s_{K_0^*}(\bar{u}) \Big] \\
& + \frac{1}{4} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \pi^2 \frac{1}{u^2 M^4} \Big[ e_s m_s \langle s\bar{s} \rangle \phi^s_{K_0^*}(u) - e_d m_d \langle d\bar{d} \rangle \phi^s_{K_0^*}(\bar{u}) \Big] \\
& - \frac{5}{12} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \pi^2 \frac{m_0^2}{u^3 M^6} \Big[ e_s m_s \langle s\bar{s} \rangle \phi^s_{K_0^*}(u) - e_d m_d \langle d\bar{d} \rangle \phi^s_{K_0^*}(\bar{u}) \Big] \Big\} \\
& - 2 e^{m_{K_0^*}^2/M^2} Q^2 \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \Big[ e_s \phi^\sigma_{K_0^*}(u) - e_d \phi^\sigma_{K_0^*}(\bar{u}) \Big]. \\
& + \frac{5}{2} e^{m_{K_0^*}^2/M^2} Q^2 \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \frac{\pi^2}{u^3 M^4} \Big[ e_s m_s \langle s\bar{s} \rangle \phi^\sigma_{K_0^*}(u) - e_d m_d \langle d\bar{d} \rangle \phi^\sigma_{K_0^*}(\bar{u}) \Big],
\end{aligned} \tag{2.25}
$$

$$
\begin{aligned}
Q^2 F_{f_0}(Q^2) = {} & \frac{1}{\sqrt{2}} e^{m_{f_0}^2/M^2} \frac{Q^2}{m_{f_0}} \Big\{ \frac{1}{4} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \frac{1}{u} \Big[ -(e_s m_s \phi_{f_0}(u) + e_s m_s \phi_{f_0}(\bar{u})) + \frac{1}{\sqrt{2}} (e_u m_u \phi_{f_0}(u) + e_u m_u \phi_{f_0}(\bar{u}) \\
& + e_d m_d \phi_{f_0}(u) + e_d m_d \phi_{f_0}(\bar{u})) \Big] - \frac{1}{2} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \frac{1}{u^2 M^2} \pi^2 \Big[ -(e_s \langle s\bar{s} \rangle \phi_{f_0}(u) + e_s \langle s\bar{s} \rangle \phi_{f_0}(\bar{u})) \\
& + \frac{1}{\sqrt{2}} (e_u \langle u\bar{u} \rangle \phi_{f_0}(u) + e_u \langle u\bar{u} \rangle \phi_{f_0}(\bar{u}) + e_d \langle d\bar{d} \rangle \phi_{f_0}(u) + e_d \langle d\bar{d} \rangle \phi_{f_0}(\bar{u})) \Big] \\
& + \frac{5}{8} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \frac{m_0^2}{u^3 M^4} \pi^2 \Big[ -(e_s \langle s\bar{s} \rangle \phi_{f_0}(u) + e_s \langle s\bar{s} \rangle \phi_{f_0}(\bar{u})) + \frac{1}{\sqrt{2}} (e_u \langle u\bar{u} \rangle \phi_{f_0}(u) + e_u \langle u\bar{u} \rangle \phi_{f_0}(\bar{u}) \\
& + e_d \langle d\bar{d} \rangle \phi_{f_0}(u) + e_d \langle d\bar{d} \rangle \phi_{f_0}(\bar{u})) \Big] \Big\} + \frac{1}{\sqrt{2}} e^{m_{f_0}^2/M^2} Q^2 \Big\{ - \frac{1}{2} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \Big[ -(e_s \phi^s_{f_0}(u) - e_s \phi^s_{f_0}(\bar{u})) \\
& + \frac{1}{\sqrt{2}} (e_u \phi^s_{f_0}(u) - e_u \phi^s_{f_0}(\bar{u}) + e_d \phi^s_{f_0}(u) - e_d \phi^s_{f_0}(\bar{u})) \Big] \\
& + \frac{1}{4} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \pi^2 \frac{1}{u^2 M^4} \Big[ -(e_s m_s \langle s\bar{s} \rangle \phi^s_{f_0}(u) - e_s m_s \langle s\bar{s} \rangle \phi^s_{f_0}(\bar{u})) + \frac{1}{\sqrt{2}} (e_u m_u \langle u\bar{u} \rangle \phi^s_{f_0}(u) \\
& - e_u m_u \langle u\bar{u} \rangle \phi^s_{f_0}(\bar{u}) + e_d m_d \langle d\bar{d} \rangle \phi^s_{f_0}(u) - e_d m_d \langle d\bar{d} \rangle \phi^s_{f_0}(\bar{u})) \Big] \\
& - \frac{5}{12} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \pi^2 m_0^2 \frac{1}{u^3 M^6} \Big[ -(e_s \langle s\bar{s} \rangle \phi^s_{f_0}(u) - e_s \langle s\bar{s} \rangle \phi^s_{f_0}(\bar{u})) + \frac{1}{\sqrt{2}} (e_u \langle u\bar{u} \rangle \phi^s_{f_0}(u) \\
& - e_u \langle u\bar{u} \rangle \phi^s_{f_0}(\bar{u}) + e_d \langle d\bar{d} \rangle \phi^s_{f_0}(u) - e_d \langle d\bar{d} \rangle \phi^s_{f_0}(\bar{u})) \Big] \Big\} \\
& + \frac{1}{\sqrt{2}} e^{m_{f_0}^2/M^2} Q^2 \Big\{ -2 \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \Big[ -(e_s \phi^\sigma_{f_0}(u) - e_s \phi^\sigma_{f_0}(\bar{u})) + \frac{1}{\sqrt{2}} (e_u \phi^\sigma_{f_0}(u) - e_u \phi^\sigma_{f_0}(\bar{u}) \\
& + e_d \phi^\sigma_{f_0}(u) - e_d \phi^\sigma_{f_0}(\bar{u})) \Big] + \frac{5}{2} \int_{u_0}^1 du e^{-\frac{s(u,Q^2)}{M^2}} \frac{\pi^2}{u^3 M^4} \Big[ -(e_s m_s \langle s\bar{s} \rangle \phi^\sigma_{f_0}(u) - e_s m_s \langle s\bar{s} \rangle \phi^\sigma_{f_0}(\bar{u})) \\
& + \frac{1}{\sqrt{2}} (e_u m_u \langle u\bar{u} \rangle \phi^\sigma_{f_0}(u) - e_u m_u \langle u\bar{u} \rangle \phi^\sigma_{f_0}(\bar{u}) + e_d m_d \langle d\bar{d} \rangle \phi^\sigma_{f_0}(u) - e_d m_d \langle d\bar{d} \rangle \phi^\sigma_{f_0}(\bar{u})) \Big] \Big\},
\end{aligned} \tag{2.26}
$$

and

$$
\begin{aligned}
Q^2F_{a_0}(Q^2) &= \frac{1}{\sqrt{2}}e^{m_{a_0}^2/M^2}\frac{Q^2}{m_{a_0}}\bigg\{\frac{1}{4}\int_{u_0}^1 due^{-\frac{s(u,Q^2)}{M^2}}\frac{1}{u}\bigg[(e_s m_s\phi_{a_0}(u)+e_s m_s\phi_{a_0}(\bar{u}))+\frac{1}{\sqrt{2}}(e_u m_u\phi_{a_0}(u)+e_u m_u\phi_{a_0}(\bar{u})\\
&\quad -e_d m_d\phi_{a_0}(u)-e_d m_d\phi_{a_0}(\bar{u}))\bigg]-\frac{1}{2}\int_{u_0}^1 due^{-\frac{s(u,Q^2)}{M^2}}\frac{1}{u^2M^2}\pi^2\bigg[(e_s\langle s\bar{s}\rangle\phi_{a_0}(u)+e_s\langle s\bar{s}\rangle\phi_{a_0}(\bar{u}))\\
&\quad +\frac{1}{\sqrt{2}}(e_u\langle u\bar{u}\rangle\phi_{a_0}(u)+e_u\langle u\bar{u}\rangle\phi_{a_0}(\bar{u})-e_d\langle d\bar{d}\rangle\phi_{a_0}(u)-e_d\langle d\bar{d}\rangle\phi_{a_0}(\bar{u}))\bigg]\\
&\quad +\frac{5}{8}\int_{u_0}^1 due^{-\frac{s(u,Q^2)}{M^2}}\frac{m_0^2}{u^3M^4}\pi^2\bigg[-(e_s\langle s\bar{s}\rangle\phi_{a_0}(u)+e_s\langle s\bar{s}\rangle\phi_{a_0}(\bar{u}))+\frac{1}{\sqrt{2}}(e_u\langle u\bar{u}\rangle\phi_{a_0}(u)+e_u\langle u\bar{u}\rangle\phi_{a_0}(\bar{u})\\
&\quad -e_d\langle d\bar{d}\rangle\phi_{a_0}(u)-e_d\langle d\bar{d}\rangle\phi_{a_0}(\bar{u}))\bigg]\bigg\}+\frac{1}{\sqrt{2}}e^{m_{a_0}^2/M^2}Q^2\bigg\{-\frac{1}{2}\int_{u_0}^1 due^{-\frac{s(u,Q^2)}{M^2}}\bigg[(e_s\phi_{a_0}^s(u)-e_s\phi_{a_0}^s(\bar{u}))\\
&\quad +\frac{1}{\sqrt{2}}(e_u\phi_{a_0}^s(u)-e_u\phi_{a_0}^s(\bar{u})-e_d\phi_{a_0}^s(u)+e_d\phi_{a_0}^s(\bar{u}))\bigg]\\
&\quad +\frac{1}{4}\int_{u_0}^1 due^{-\frac{s(u,Q^2)}{M^2}}\pi^2\frac{1}{u^2M^4}\bigg[(e_s m_s\langle s\bar{s}\rangle\phi_{a_0}^s(u)-e_s m_s\langle s\bar{s}\rangle\phi_{a_0}^s(\bar{u}))+\frac{1}{\sqrt{2}}(e_u m_u\langle u\bar{u}\rangle\phi_{a_0}^s(u)\\
&\quad -e_u m_u\langle u\bar{u}\rangle\phi_{a_0}^s(\bar{u})-e_d m_d\langle d\bar{d}\rangle\phi_{a_0}^s(u)+e_d m_d\langle d\bar{d}\rangle\phi_{a_0}^s(\bar{u}))\bigg]\\
&\quad -\frac{5}{12}\int_{u_0}^1 due^{-\frac{s(u,Q^2)}{M^2}}\pi^2 m_0^2\frac{1}{u^3M^6}\bigg[(e_s\langle s\bar{s}\rangle\phi_{a_0}^s(u)-e_s\langle s\bar{s}\rangle\phi_{a_0}^s(\bar{u}))+\frac{1}{\sqrt{2}}(e_u\langle u\bar{u}\rangle\phi_{a_0}^s(u)\\
&\quad -e_u\langle u\bar{u}\rangle\phi_{a_0}^s(\bar{u})-e_d\langle d\bar{d}\rangle\phi_{a_0}^s(u)+e_d\langle d\bar{d}\rangle\phi_{a_0}^s(\bar{u}))\bigg]\bigg\}\\
&\quad +\frac{1}{\sqrt{2}}e^{m_{a_0}^2/M^2}Q^2\bigg\{-2\int_{u_0}^1 due^{-\frac{s(u,Q^2)}{M^2}}\bigg[(e_s\phi_{a_0}^\sigma(u)-e_s\phi_{a_0}^\sigma(\bar{u}))+\frac{1}{\sqrt{2}}(e_u\phi_{a_0}^\sigma(u)-e_u\phi_{a_0}^\sigma(\bar{u})\\
&\quad -e_d\phi_{a_0}^\sigma(u)+e_d\phi_{a_0}^\sigma(\bar{u}))\bigg]+\frac{5}{2}\int_{u_0}^1 due^{-\frac{s(u,Q^2)}{M^2}}\frac{\pi^2}{u^3M^4}\bigg[-(e_s m_s\langle s\bar{s}\rangle\phi_{a_0}^\sigma(u)-e_s m_s\langle s\bar{s}\rangle\phi_{a_0}^\sigma(\bar{u}))\\
&\quad +\frac{1}{\sqrt{2}}(e_u m_u\langle u\bar{u}\rangle\phi_{a_0}^\sigma(u)-e_u m_u\langle u\bar{u}\rangle\phi_{a_0}^\sigma(\bar{u})+e_d m_d\langle d\bar{d}\rangle\phi_{a_0}^\sigma(u)-e_d m_d\langle d\bar{d}\rangle\phi_{a_0}^\sigma(\bar{u}))\bigg]\bigg\}. \qquad (2.27)
\end{aligned}
$$

## III. NUMERICAL ANALYSIS

Having explicit expressions for the electromagnetic form factors of the scalar $K_0^*$, $f_0$ and $a_0$ mesons, we numerically analyze these form factors in this section. The main input parameters that entered the sum rules are distribution amplitudes of the scalar mesons which are parametrized in terms of the Gegenbauer moments presented in the previous section as well as the Gegenbauer polynomials $C_n^k(\xi)$. The first four polynomials are given as [21,22,25]

$$
\begin{aligned}
&C_0^{1/2}(\xi)=1, \qquad C_1^{1/2}(\xi)=\xi, \qquad C_2^{1/2}(\xi)=\frac{1}{2}(3\xi^2-1), \qquad C_3^{1/2}(\xi)=\frac{1}{2}\xi(5\xi^2-3),\\
&C_0^{3/2}(\xi)=1, \qquad C_1^{3/2}(\xi)=3\xi, \qquad C_2^{3/2}(\xi)=\frac{3}{2}(5\xi^2-1), \qquad C_3^{3/2}(\xi)=\frac{5}{2}\xi(7\xi^2-3).
\end{aligned}
$$

For the other required input parameters we choose

$$
\begin{aligned}
&m_{K_0^*(1430)}=1425\ \text{MeV}, \qquad m_{f_0(1500)}=1505\ \text{MeV}, \qquad \langle\bar{u}u\rangle=-(0.243)^3\ \text{GeV}^3, \qquad \langle\bar{s}s\rangle=0.8\langle\bar{u}u\rangle,\\
&m_0^2=(0.8\pm0.2)\ \text{GeV}^2, \qquad m_s=142\ \text{MeV}, \qquad \bar{f}_{K_0^*}=(445\pm50)\ \text{MeV}, \qquad \bar{f}_{f_0}=(490\pm50)\ \text{MeV}.\\
&\bar{f}_{a_0}=(460\pm50)\ \text{MeV}, \qquad m_{a_0(1450)}=1474\ \text{MeV}. \qquad (3.1)
\end{aligned}
$$

Besides the above input parameters, the sum rules for the electromagnetic form factors also include two auxiliary parameters: the Borel mass parameter $M^2$ and the continuum threshold $s_0$. The continuum threshold is not totally arbitrary, but it is correlated with the energy of the first excited state. Our numerical results depict that in the intervals $s_0=(3.5\text{–}4.5)\ \text{GeV}^2$, $s_0=(3.2\text{–}4.2)\ \text{GeV}^2$ and $s_0=(3\text{–}4)\ \text{GeV}^2$, respectively, for the scalar $f_0(1500)$, $a_0(1450)$ and $K_0^*(1430)$ mesons, our results weakly depend on the continuum threshold.

The working region for $M^2$ is found by demanding not only that the contributions of the higher states and

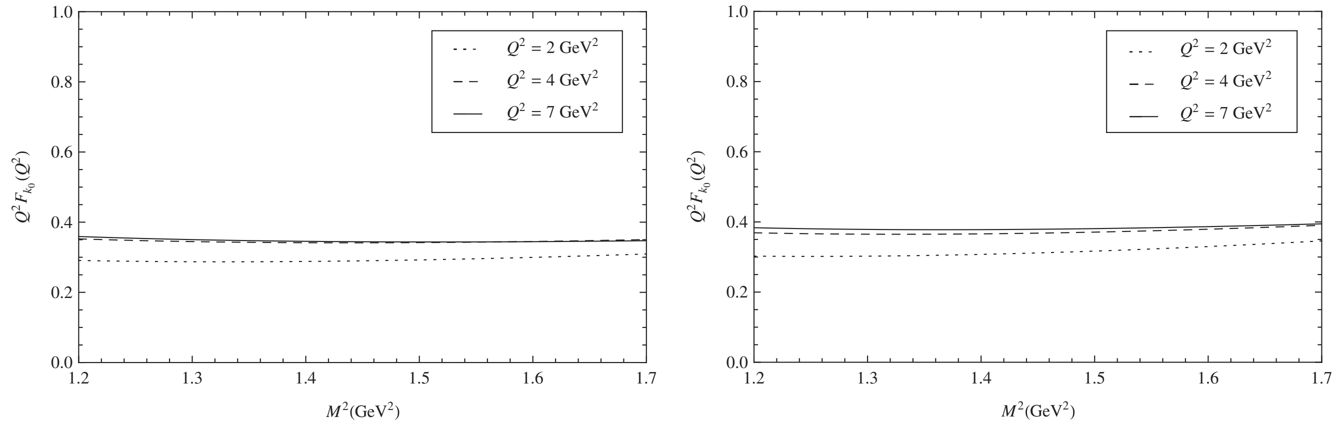


FIG. 1. Dependence of the $K_0^*(1430)$ electromagnetic form factor on the Borel mass parameter at $s_0 = 3$ GeV$^2$ (left panel) and at $s_0 = 4$ GeV$^2$ (right panel).

continuum are suppressed but also that the highest twist constitutes a small percentage of the total results; i.e., the series of light-cone sum rules are convergent. Our numerical calculations lead to the working region 1.2 GeV$^2 \leq M^2 \leq 1.7$ GeV$^2$ for all the scalar mesons under consideration.

We present the dependence of the electromagnetic form factors of the mesons under consideration on the Borel mass parameters in Figs. 1–3 at different fixed values of the $Q^2$ and $s_0$. From these figures we see that the results weakly depend on the auxiliary parameters $M^2$ and $s_0$ in their working regions. We also conclude that the absolute values of the electromagnetic form factors of the scalar $K_0^*$ meson differ from those of the $f_0$ and $a_0$ by an order of magnitude at any fixed values of the $Q^2$ and $s_0$.

We finally depict the dependence of the electromagnetic form factors of the $K_0^*$, $f_0$ and $a_0$ scalar mesons on $Q^2$ in Fig. 4 at different values of the Borel mass square and continuum threshold. As it is also clear from these figures, the light-cone QCD sum rules do not give reliable results near $Q^2 = 0$. Starting from $Q^2 = 1$ GeV$^2$, the absolute value of the $Q^2$ times electromagnetic form factor of the $K_0^*$ grows increasing the values of $Q^2$ up to $Q^2 = 4$ GeV$^2$ then becomes approximately unchanged after this point. In the case of $f_0$ and $a_0$ scalar mesons, which show similar behavior, the absolute values of the $Q^2$ times electromagnetic form factors first decrease starting from $Q^2 = 1$ GeV$^2$; then after reaching a minimum, they grow. From these figures we also see that, in the case of the $K_0^*$ meson, the dependence of the results on $s_0$ is sensible at higher values of $Q^2$, while for the $f_0$ and $a_0$ scalar mesons we see the inverse situation. This can be related to the internal structure of these mesons. In order to compare the above results with the predictions of the currents in Eq. (2.3), we plot the dependence of the electromagnetic form factors on $Q^2$ at different angles in Fig. 5 at the same fixed values of auxiliary parameters as in Fig. 4. From this figure we see that the results considerably depend on the choices of the interpolating currents.

In summary, we have studied the electromagnetic form factors of the $K_0^*$, $f_0$ and $a_0$ scalar mesons by considering them as the regular quark-antiquark states using their

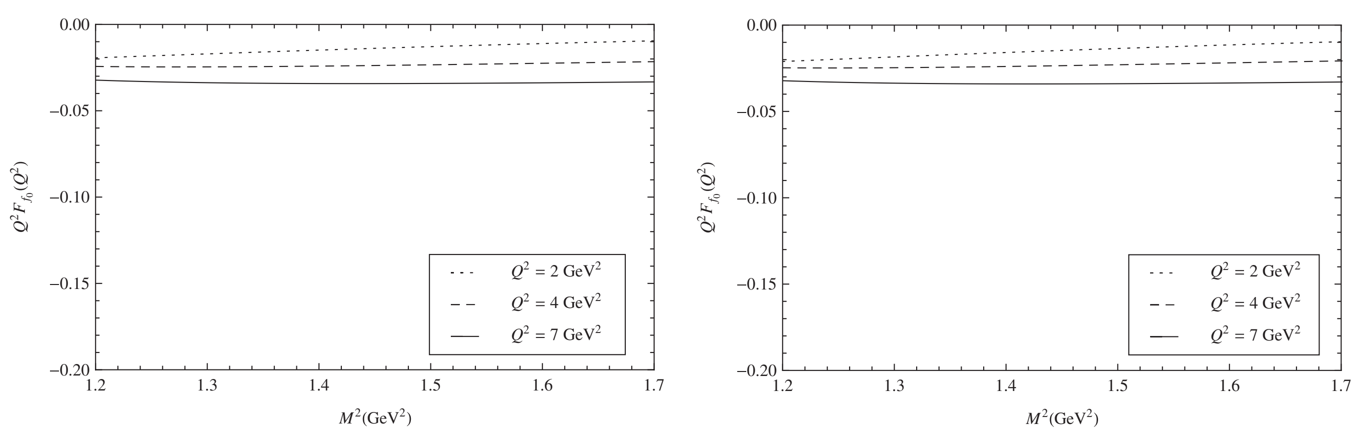


FIG. 2. Dependence of the $f_0(1500)$ electromagnetic form factor on the Borel mass parameter at $s_0 = 3.5$ GeV$^2$ (left panel) and at $s_0 = 4$ GeV$^2$ (right panel).

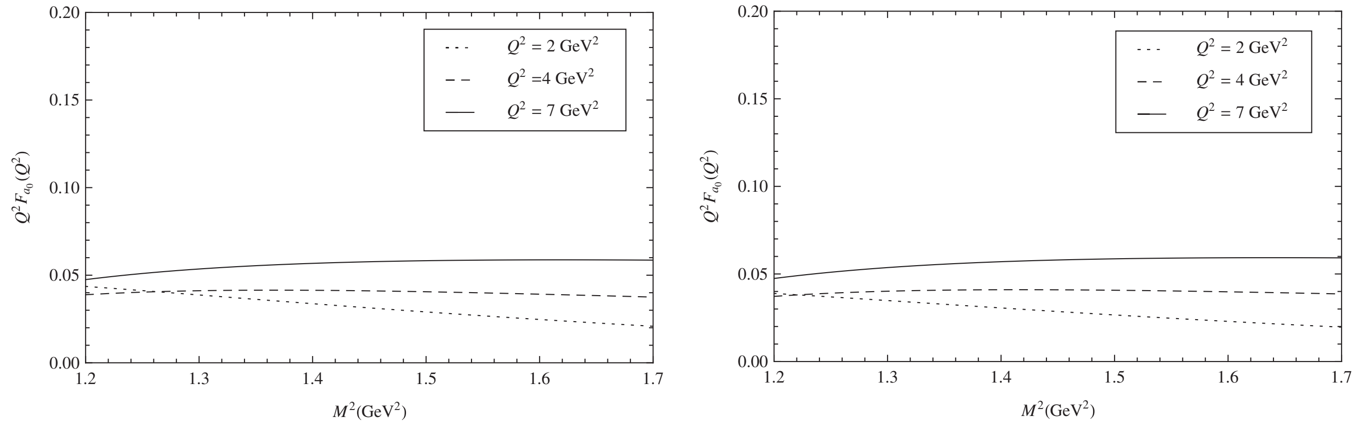


FIG. 3. Dependence of the $a_0(1450)$ electromagnetic form factor on the Borel mass parameter at $s_0 = 3.5$ GeV$^2$ (left panel) and at $s_0 = 4$ GeV$^2$ (right panel).

distribution amplitudes via light-cone QCD sum rules. We observed that different interpolating currents lead to different results that considerably differ from each other. For a long time, the understanding of the scalar mesons has been problematic and still a subject of debate from both theoretical and experimental sides. To complete the analysis from the phenomenological and theoretical points of view, one may calculate the electromagnetic properties of these mesons by also considering them as tetraquarks, glueballs, hybrids or hadron molecules. From the experimental side, the studies of the identification and spectroscopy of the scalar mesons are continued [26]. Considering the developments at the LHC, we hope we will be able to complete the experimental studies on the spectroscopy of these mesons. Although the experimental measurement of the electromagnetic form factors of the

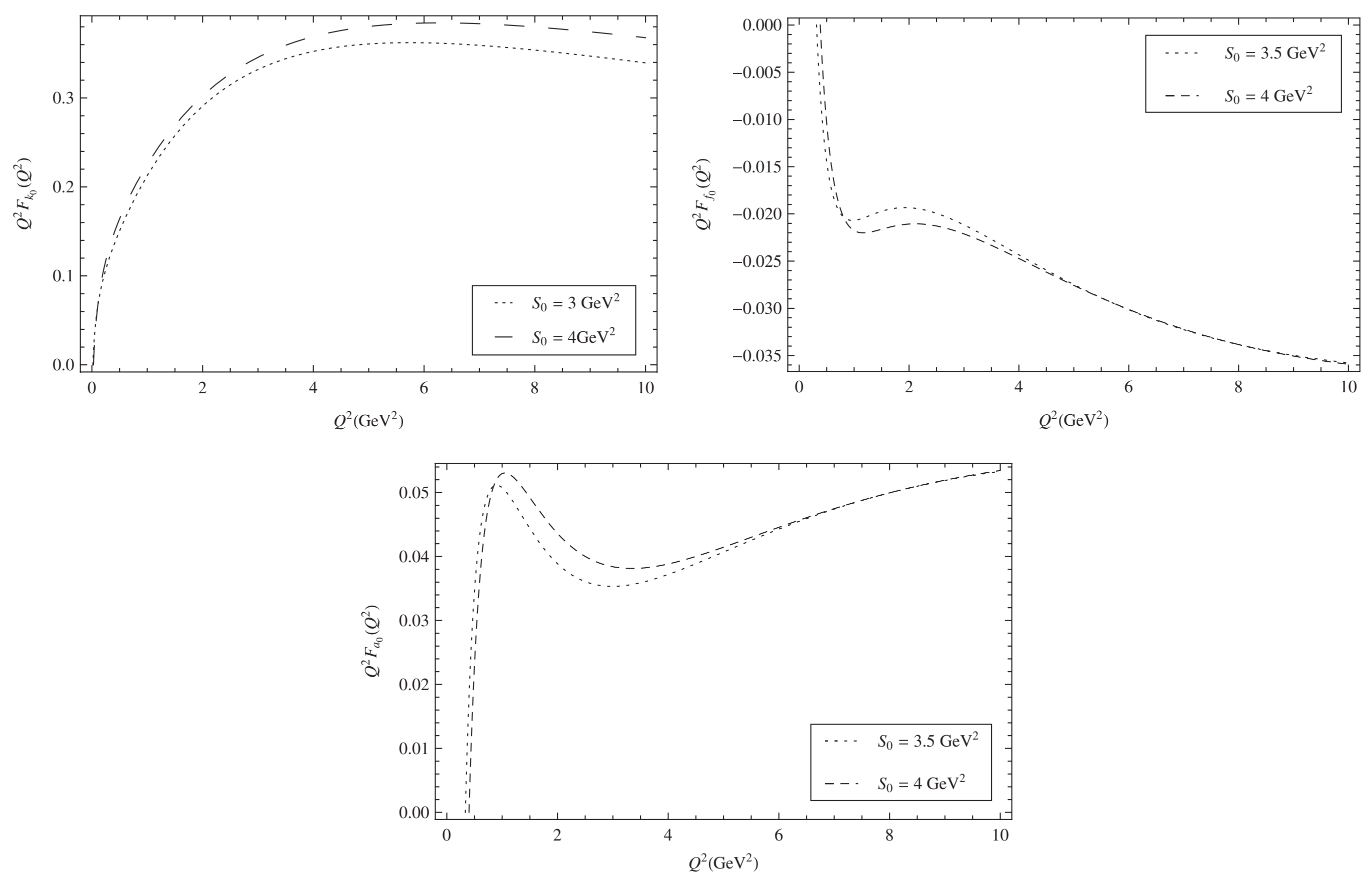


FIG. 4. Dependence of the electromagnetic form factors of the scalar mesons on $Q^2$ at $M^2 = 1.2$ GeV$^2$.

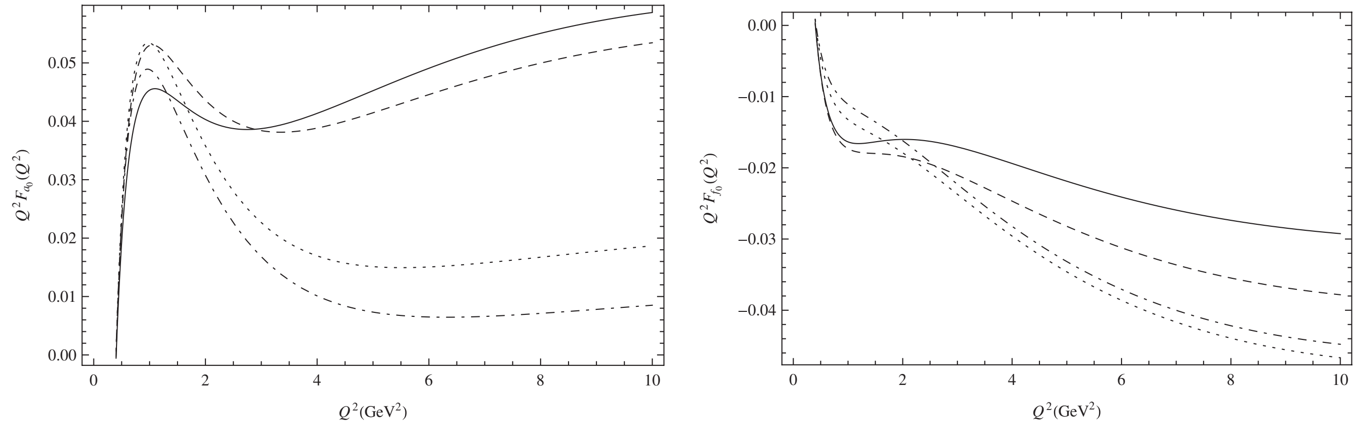


FIG. 5. Dependence of the electromagnetic form factors of the $f_0$ and $a_0$ scalar mesons on $Q^2$ at the same fixed values of auxiliary parameters as in Fig. 4. In these figures the solid, dotted and dotted-dashed lines correspond to the currents in Eq. (2.3) for $\theta = 30$, $\theta = 90$ and $\theta = 100$, respectively, while the dashed lines stand for the currents in Eq. (2.2).

scalar mesons seems to be difficult in the near future, we hope to accomplish this goal; we have made good progress in measuring the electromagnetic properties of hadrons at MAMI at Mainz, ELSA at Bonn, LEGS at Brookhaven, GRAAL at Grenoble and the GlueX experiment at the JLab accelerator (see, for instance, [27]). Any measurement on the electromagnetic form factors, together with the experimental results on the spectroscopy of the scalar mesons, and comparison with the theoretical predictions can give valuable information about the nature of the scalar mesons. This will help us to uncover the mysterious internal structure of these states whether they are regular quark-antiquark states, tetraquarks, glueballs, hybrids or hadron molecules.

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