Effects of Planck scale physics on neutrino mixing parameters in left-right symmetric models

Debasish Borah*

Department of Physics, Tezpur University, Tezpur 784 028, India

(Received 4 April 2013; published 15 May 2013)

Left-right symmetric models (LRSM) are extensions of the standard model by an enlarged gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L,}$ where automatic inclusion of right-handed fermions as $SU(2)_R$ doublets guarantees a natural seesaw origin of neutrino masses. Apart from the extended gauge symmetry, LRSM also has a built-in global discrete symmetry, called D-parity, which ensures equal gauge couplings for left and right sectors. Motivated by the fact that global symmetries are expected to be explicitly broken by theories of quantum gravity, here we study the effects of such gravity, or Planck scale physics, on neutrino masses and mixings by introducing explict D-parity breaking, Planck-scale-suppressed, higher-dimensional operators. Although such Planck-scale-suppressd operators have dimensions of at least six in generic LRSM, dimension five operators can also arise in the presence of additional scalar fields, which can be naturally accommodated within $SO(10)$ grand unified theory multiplets. We show that such corrections can give rise to significant changes in the predictions for neutrino mixing parameters compared to the ones predicted by tree-level seesaw formula if the left-right symmetry breaking scale is lower than 10^{14} GeV.

DOI: 10.1103/PhysRevD.87.095009 PACS numbers: 12.60.−i, 12.60.Cn, 14.60.Pq

I. INTRODUCTION

Left-right symmetric models (LRSM) [1] have been one of the most well-motivated extensions of the standard model (SM) and studied in great detail over the last few decades. Apart from explaining the origin of parity breaking in weak interactions spontaneously, LRSM can also explain the origin of tiny neutrino masses [2] naturally via a seesaw mechanism [3,4] without reference to very high scale physics such as grand unified theories (GUT). Supersymmetric versions of such models have several other motivations like protecting the scalar sector from quadratic divergences, providing a natural dark matter candidate among others. However, as studied previously in [5,6], generic supersymmetric left-right models are tightly constrained from consistent cosmology as well as successful gauge coupling unification point of view, and in quite a few cases these models do not give rise to successful unification and consistent cosmology simultaneously. Recently, nonsupersymmetric versions of LRSM were studied in the context of gauge coupling unification and consistent cosmology [7]. It was shown that minimal versions of LRSM cannot give rise to unification and consistent cosmology simultaneously, but suitable extensions of these models can give rise to both of these desired properties and at the same time allow the possibility of a low scale gauge symmetry.

Spontaneous breaking of exact discrete symmetries like parity (which we shall denote as D-parity hereafter) has cosmological implications, it leads to frustrated phase transitions, leaving behind a network of domain walls (DW). These domain walls, if not removed, will be in conflict with the observed Universe [8,9]. It was pointed out [10,11] that Planck-scale-suppressed nonrenormalizable operators can be a source of domain wall instability. The main theme of this was to assume exact parity symmetry at tree level and introduce explicit parity breaking terms of higher order. As pointed out in [10], any generic theories of quantum gravity should not respect global symmetries, whether discrete or continuous. Without worrying about the details of such symmetry breaking mechanisms, our purpose is to study the effects of such terms which arise only in the form of higher-dimensional operators. The role of such higher dimensional operators in destabilizing domain walls was studied in [5,7]. Here we intend to study the effects of such operators on the neutrino sector, namely the neutrino mixing parameters. We find that in generic LRSM, such operators which affect neutrino parameters can have dimension of at least six without significantly affecting the neutrino masses and mixings. However, in the presence of additional scalar fields, dimension five operators can arise and significantly affect the neutrino masses and mixings. In particular, we incorporate the presence of a gauge singlet scalar field, which can naturally fit inside several $SO(10)$ GUT representations. As discussed in [7], such singlet extension of minimal LRSM also leads to domain wall disappearance, which is not possible in the minimal versions. Here we study the effect of such higher-dimensional operators on neutrino mixing parameters and find that the corrections can be very significant if the left- right symmetry breaking scale is below 10^{14} GeV.

This paper is organized as follows. In Sec. II we discuss minimal LRSM with Higgs triplets and discuss how tiny neutrino mass arises in this model. In Sec. III we discuss the possible higher-dimensional and explicit

*dborah@tezu.ernet.in

parity breaking operators, which can affect neutrino masses. Then in Sec. IV, we present our numerical analysis of the effects of higher- dimensional operators on neutrino mixing parameters, and finally we conclude in Sec. V.

II. NEUTRINO MASS IN LRSM

The fermion content of minimal LRSM is

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim \left(3, 2, 1, \frac{1}{3}\right),$$

$$Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \sim \left(3^*, 1, 2, \frac{1}{3}\right),$$

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, 2, 1, -1),$$

$$\ell_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \sim (1, 1, 2, -1).$$

Similarly, the Higgs content of the minimal LRSM is

$$\Phi = \begin{pmatrix} \phi^0_{11} & \phi^+_{11} \\ \phi^-_{12} & \phi^0_{12} \end{pmatrix} \sim (1, 2, 2, 0)$$

$$\Delta_L = \begin{pmatrix} \delta^+_L/\sqrt{2} & \delta^{++}_L \\ \delta^0_L & -\delta^+_L/\sqrt{2} \end{pmatrix} \sim (1, 3, 1, 2),$$

$$\Delta_R = \begin{pmatrix} \delta^+_R/\sqrt{2} & \delta^{++}_R \\ \delta^0_R & -\delta^+_R/\sqrt{2} \end{pmatrix} \sim (1, 1, 3, 2),$$

where the numbers in brackets correspond to the quantum numbers with respect to the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In the symmetry breaking pattern, the scalar Δ_R acquires a vacuum expectation value (vev) to break the gauge symmetry of LRSM into that of the standard model and then to $U(1)$ of electromagnetism by the vev of Higgs bidoublet Φ,

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \overrightarrow{\langle \Delta_R \rangle} SU(2)_L \times U(1)_Y \overrightarrow{\langle \Phi \rangle} U(1)_{\mathrm{em}}.$$

The relevant Yukawa couplings which lead to small nonzero neutrino mass are given by

$$\mathcal{L}^{II}_\nu = y_{ij}\ell_{iL}\Phi\ell_{jR} + y'_{ij}\ell_{iL}\tilde{\Phi}\ell_{jR} + \mathrm{H.c.} + f_{ij}(\ell^T_{iR}Ci\sigma_2\Delta_R\ell_{jR} + (R \leftrightarrow L)) + \mathrm{H.c.}, \quad (1)$$

where $\tilde{\Phi} = \tau_2\Phi^*\tau_2$. In the above Yukawa Lagrangian, the indices $i, j = 1, 2, 3$ correspond to the three families of fermions. The Majorana Yukawa couplings f are the same for both left- and right-handed neutrinos because of left-right symmetry. These couplings f give rise to the Majorana mass terms of both left-handed and right-handed neutrinos after the triplet Higgs fields $\Delta_{L,R}$ acquire nonzero vev. These mass terms appear in the seesaw formula as discussed below. The resulting seesaw formula in this minimal model can be written as

$$m_{LL} = m^{II}_{LL} + m^I_{LL}, \quad (2)$$

where the usual type I seesaw formula is given by the expression

$$m^I_{LL} = -m_{LR}M^{-1}_{RR}m^T_{LR}. \quad (3)$$

Here m_{LR} is the Dirac neutrino mass matrix defined as $m_{LR} = y_{ij}\langle\Phi\rangle$. It should be noted that the Yukawa couplings y_{ij} in the definition of Dirac neutrino mass matrix are not the same as the ones introduced in the Yukawa Lagrangian (1), but the ones obtained at the electroweak scale after renormalization group evolution (RGE) effects are taken into account from the scale of left-right symmetry breaking down to the electroweak scale. Such RGE effects on neutrino parameters for type I and type II seesaw models have been studied in [12,13], respectively. However, in our present work we do not attempt to perform a systematic RGE study of neutrino parameters in LRSM. To simplify our analysis, we assume that the RGE effects on the neutrino Yukawa couplings from the left-right symmetry scale to the electroweak scale do not diminish the effects of higher-dimensional Planck-scale-suppressed operators on neutrino mass in LRSM (as we discuss in the next section).

In LRSM with Higgs triplets, M_{RR} can be expressed as $M_{RR} = v_Rf_R$ with v_R being the vev of the right-handed triplet Higgs field Δ_R, imparting Majorana masses to the right-handed neutrinos and f_R being the corresponding Yukawa coupling. The first term m^{II}_{LL} in Eq. (2) is due to the vev of $SU(2)_L$ Higgs triplet. Thus, $m^{II}_{LL} = f_Lv_L$ and $M_{RR} = f_Rv_R$, where $v_{L,R}$ denote the vev's and $f_{L,R}$ are symmetric 3×3 matrices. The left-right symmetry demands $f_R = f_L = f$. The induced vev for the left-handed triplet v_L can be shown for generic LRSM to be

$$v_L = \gamma\frac{M^2_W}{v_R}$$

with $M_W \sim 80.4$ GeV being the weak boson mass such that

$$|v_L| \ll M_W \ll |v_R|.$$

In general γ is a function of various couplings in the scalar potential of generic LRSM, and without any fine-tuning γ is expected to be of the order unity ($\gamma \sim 1$). The seesaw formula in Eq. (2) can now be expressed as

$$m_{LL} = \gamma(M_W/v_R)^2M_{RR} - m_{LR}M^{-1}_{RR}m^T_{LR}. \quad (4)$$

III. HIGHER-DIMENSIONAL OPERATORS IN LRSM

In the minimal LRSM discussed above, the next-to-leading-order terms contributing to neutrino masses can be written as

$$\mathcal{L}^{NR} = f_{gL}\ell^T_{iR}Ci\sigma_2\Delta_R\ell_{jR}\frac{\Delta^\dagger_R\Delta_R}{M^2_{\rm Pl}} + R \leftrightarrow L, \qquad (5)$$

where $M_{\rm Pl} \sim 10^{19}$ GeV is the Planck scale. Here $f_{gL} \neq f_{gR}$, and hence D-parity, is explicitly broken with the introduction of the higher-dimensional operators above. Now, using the tree-level Yukawa terms (1) as well as the higher-dimensional operators (5), the right-handed neutrino mass matrix can be written as

$$M_{RR} = f v_R + f_{gR}\frac{v_R^3}{M^2_{\rm Pl}}.$$

The first term on the right-hand side of Eq. (2) takes the form

$$\begin{aligned} m^{II}_{LL} &= f v_L + f_{gL}\frac{v_L^3}{M^2_{\rm Pl}} \Rightarrow m^{II}_{LL} \\ &= \gamma(M_W/v_R)^2\left(f v_R + f_{gL}\frac{v_L^2 v_R}{M^2_{\rm Pl}}\right) \Rightarrow m^{II}_{LL} \\ &= \gamma(M_W/v_R)^2\left(M_{RR} + f_{gL}\frac{v_L^2 v_R}{M^2_{\rm Pl}} - f_{gR}\frac{v_R^3}{M^2_{\rm Pl}}\right). \end{aligned}$$

Thus there arise two additional terms in the seesaw formula after the higher-dimensional operators are taken into account. These two terms are proportional to $v_L^2/(v_R M^2_{\rm Pl})$ and $v_R/M^2_{\rm Pl}$, respectively. We check that neither of these two correction terms can change the predictions of neutrino parameters from the ones predicted by the tree-level seesaw formula (4). This is obviously because of the $1/M^2_{\rm Pl}$ suppression in both the terms, which is almost negligible compared to the tree-level neutrino mass terms.

Now, let us consider the presence of an additional gauge singlet field σ in LRSM. Since a singlet like $\sigma(1, 1, 1, 0)$ can naturally fit inside several $SO(10)$ representations, we assume the vev of this singlet field to be of order $\langle\sigma\rangle \sim M_{\rm GUT} \sim 10^{16}$ GeV. In the presence of such a field, the nonleading terms contributing to neutrino masses can be of dimension five as follows:

$$\mathcal{L}^{NR} = f_{gL}\ell^T_{iR}Ci\sigma_2\Delta_R\ell_{jR}\frac{\sigma}{M_{\rm Pl}} + R \leftrightarrow L. \qquad (6)$$

Using the same analysis as in the case of minimal LRSM, here M_{RR} is found to be

$$M_{RR} = f v_R + f_{gR}\frac{\langle\sigma\rangle v_R}{M_{\rm Pl}}.$$

The type II seesaw term m^{II}_{LL} becomes

$$\begin{aligned} m^{II}_{LL} &= \gamma(M_W/v_R)^2\left(f v_R + f_{gL}\frac{\langle\sigma\rangle v_R}{M_{\rm Pl}}\right) \Rightarrow m^{II}_{LL} \\ &= \gamma(M_W/v_R)^2\left(M_{RR} + (f_{gL} - f_{gR})\frac{\langle\sigma\rangle v_R}{M_{\rm Pl}}\right). \end{aligned}$$

Without losing any generality, we assume $(f_{gL} - f_{gR})$ to be a Hermitian matrix of order one multiplied by a numerical factor f_g, which decides the overall strength of the corrected term. In the next section, we study the variation of neutrino mixing parameters as a function of this numerical factor f_g.

IV. NUMERICAL ANALYSIS

The latest global fit values for 3σ range of neutrino oscillation parameters [14] are as follows:

$$\begin{aligned} \Delta m^2_{21} &= (7.00\text{–}8.09) \times 10^{-5}\ \text{eV}^2 \\ \Delta m^2_{31}(\text{NH}) &= (2.27\text{–}2.69) \times 10^{-3}\ \text{eV}^2 \\ \Delta m^2_{23}(\text{IH}) &= (2.24\text{–}2.65) \times 10^{-3}\ \text{eV}^2 \\ \sin^2\theta_{12} &= 0.27\text{–}0.34 \qquad \sin^2\theta_{23} = 0.34\text{–}0.67 \\ \sin^2\theta_{13} &= 0.016\text{–}0.030, \end{aligned} \qquad (7)$$

where NH and IH refer to normal and inverted hierarchy, respectively. Unlike the tight constraints on the above parameters, the global fit 3σ range for the value of Dirac CP phase δ_{CP} extends over the entire 0–2π range. For illustrative purposes, here we take its value to be 300 degrees (same as the central value given in [14]).

For the purpose of our numerical analysis, we first fit the neutrino mass matrix m_{LL} using the best fit global parameters mentioned above. For both normal and inverted hierarchical neutrino mass patterns, we consider extremal Majorana phases such that the mass eigenvalues are either (m_1, m_2, m_3) or $(m_1, -m_2, m_3)$ denoted by $(+++)$ and $(+-+)$ respectively. We follow the same approach for numerical analysis as in [15], where the variation of neutrino mixing parameters with respect to the dimensionless parameter γ in the seesaw formula (4) was studied in detail.

After parametrizing the neutrino mass matrix for the tree-level seesaw formula (4) using the global fit neutrino data, we introduce the correction term (6) to the seesaw formula. As discussed above, this correction term is of the form

$$m^{\rm corr}_{LL} = \gamma(M_W/v_R)^2(f_{gL} - f_{gR})\frac{\langle\sigma\rangle v_R}{M_{\rm Pl}}.$$

Here we assume $(f_{gL} - f_{gR}) = f_g\mathcal{O}(1)$, where $\mathcal{O}(1)$ is a Hermitian matrix of order one.

We then vary the dimensionless parameter f_g from 10^{-5} to 1 and see the variations of neutrino mixing parameters. The results are shown in Figs. 1–9, for three different values of left-right symmetry breaking scales, $v_R = 10^{10}$, 10^{12}, 10^{14} GeV, and both normal and inverted hierarchies as well as both types of extremal Majorana phases. As seen from the figures, the changes in the neutrino mixing parameters from the best-fit values (corresponding to $f_g = 0$ in our case) become more and more significant as we go from $v_R = 10^{14}$ GeV to $v_R = 10^{10}$ GeV. In particular, for $v_R = 10^{14}$ GeV, almost all the neutrino parameters lie

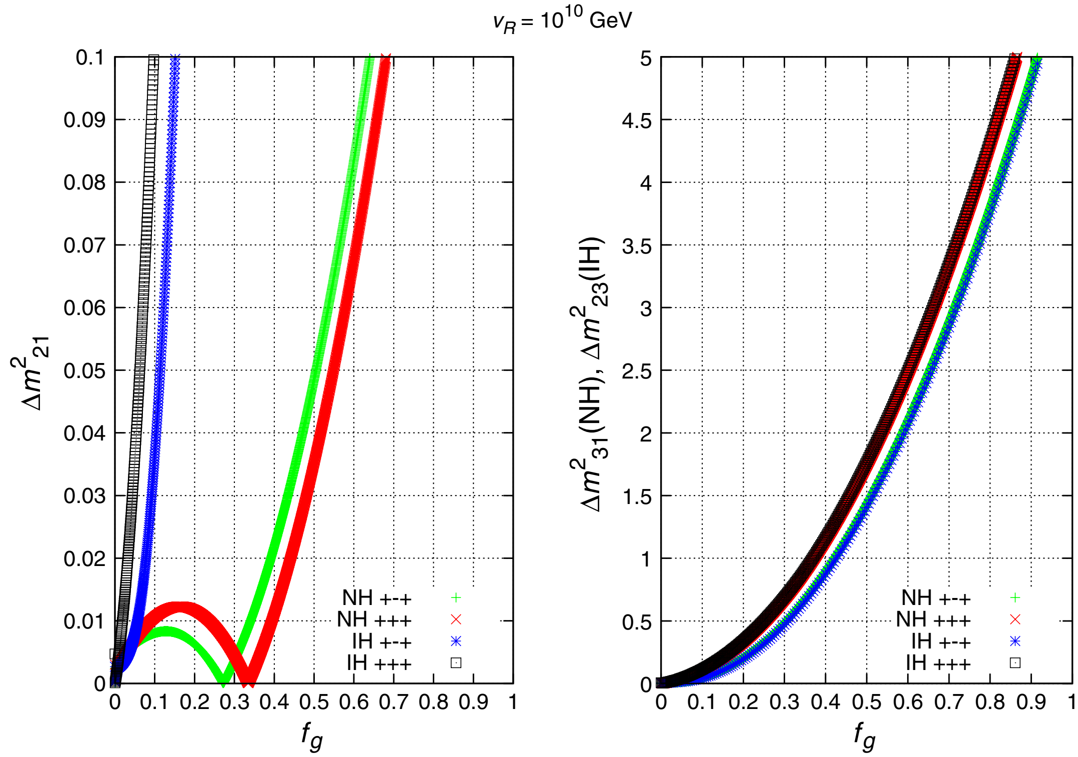

FIG. 1 (color online). Variations of Δm^2_{21} and Δm^2_{31}(NH), Δm^2_{23}(IH) as a function of f_g for $v_R = 10^{10}$ GeV.

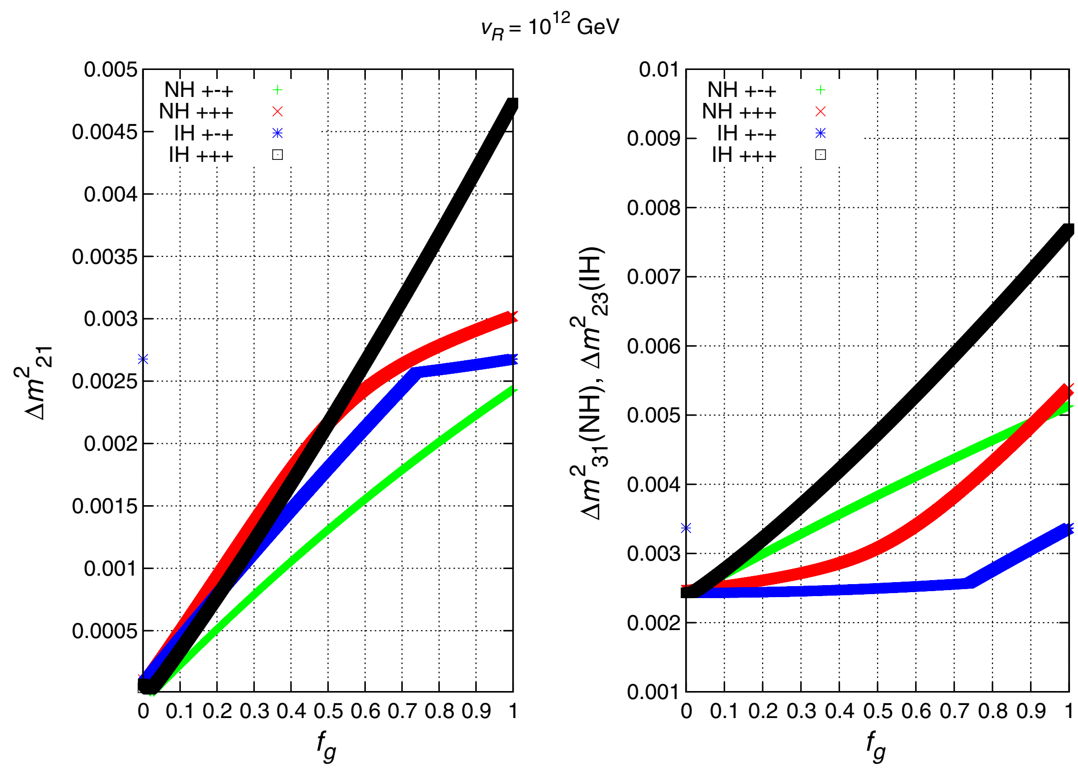

FIG. 2 (color online). Variations of Δm^2_{21} and Δm^2_{31}(NH), Δm^2_{23}(IH) as a function of f_g for $v_R = 10^{12}$ GeV.

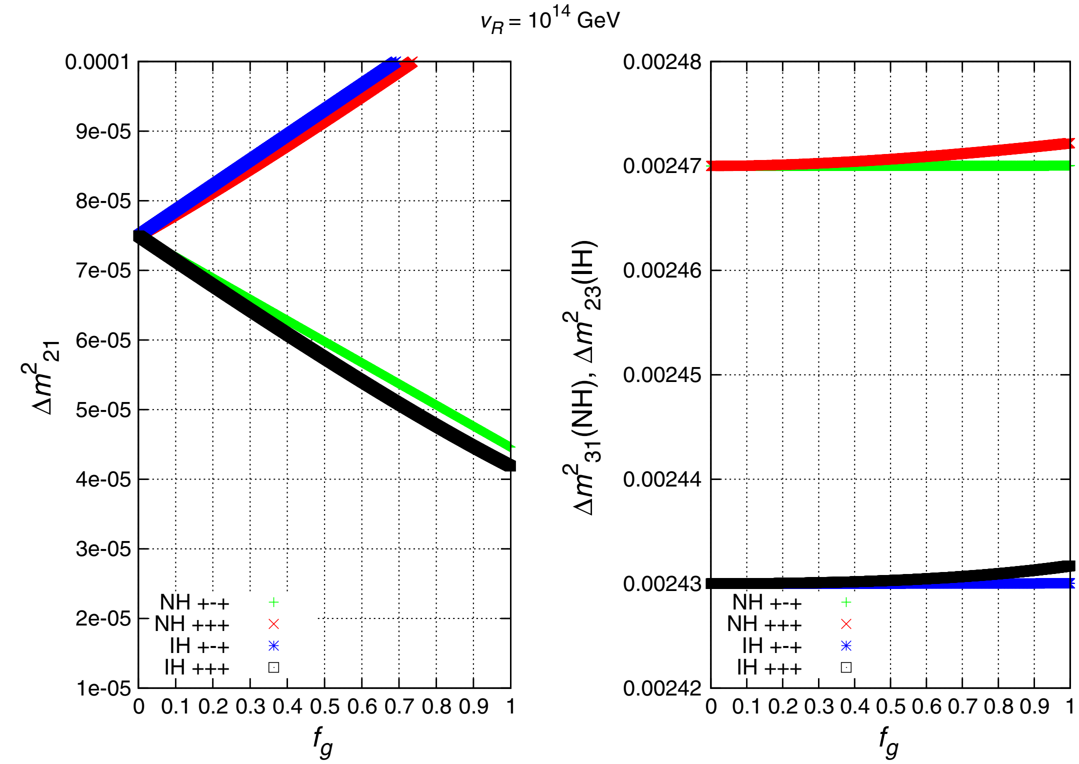

FIG. 3 (color online). Variations of Δm^2_{21} and Δm^2_{31}(NH), Δm^2_{23}(IH) as a function of f_g for $v_R = 10^{14}$ GeV.

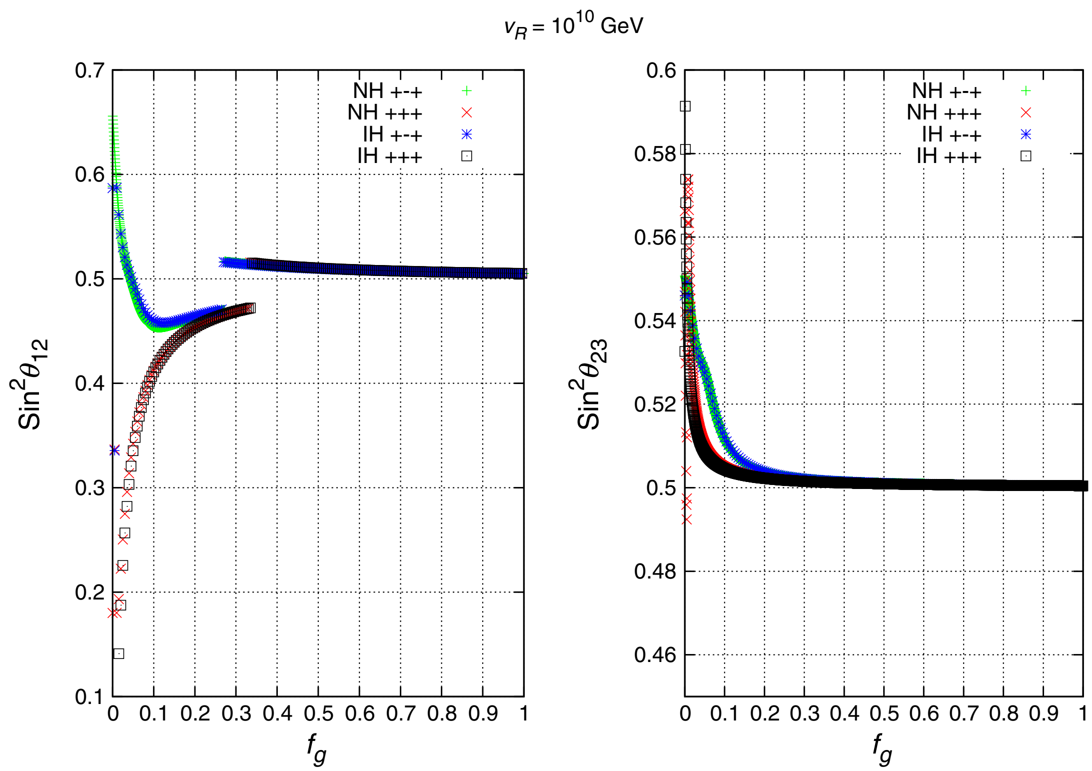

FIG. 4 (color online). Variations of $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ as a function of f_g for $v_R = 10^{10}$ GeV.

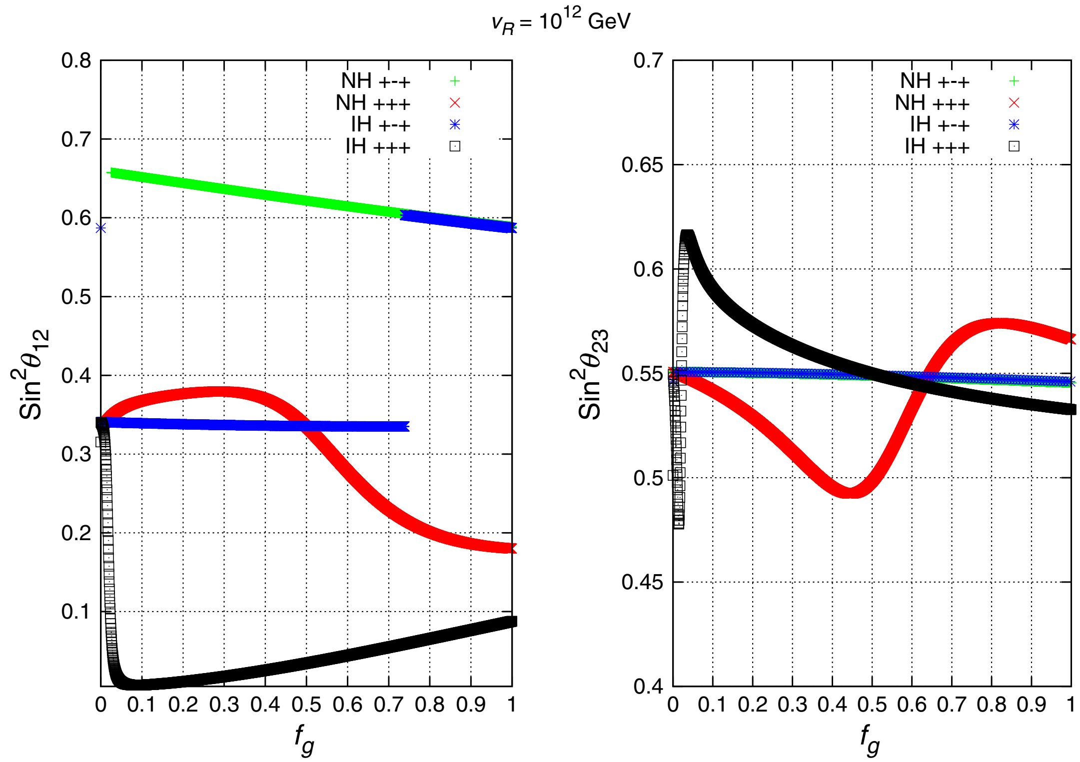

FIG. 5 (color online). Variations of $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ as a function of f_g for $v_R = 10^{12}$ GeV.

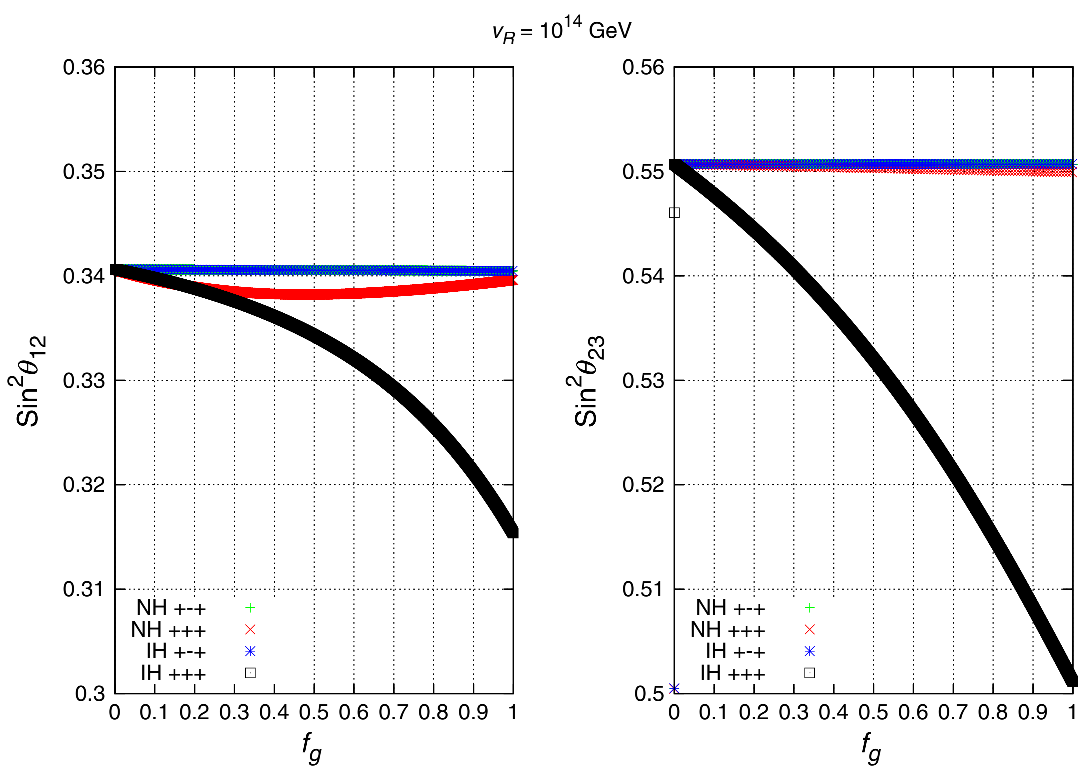

FIG. 6 (color online). Variations of $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ as a function of f_g for $v_R = 10^{14}$ GeV.

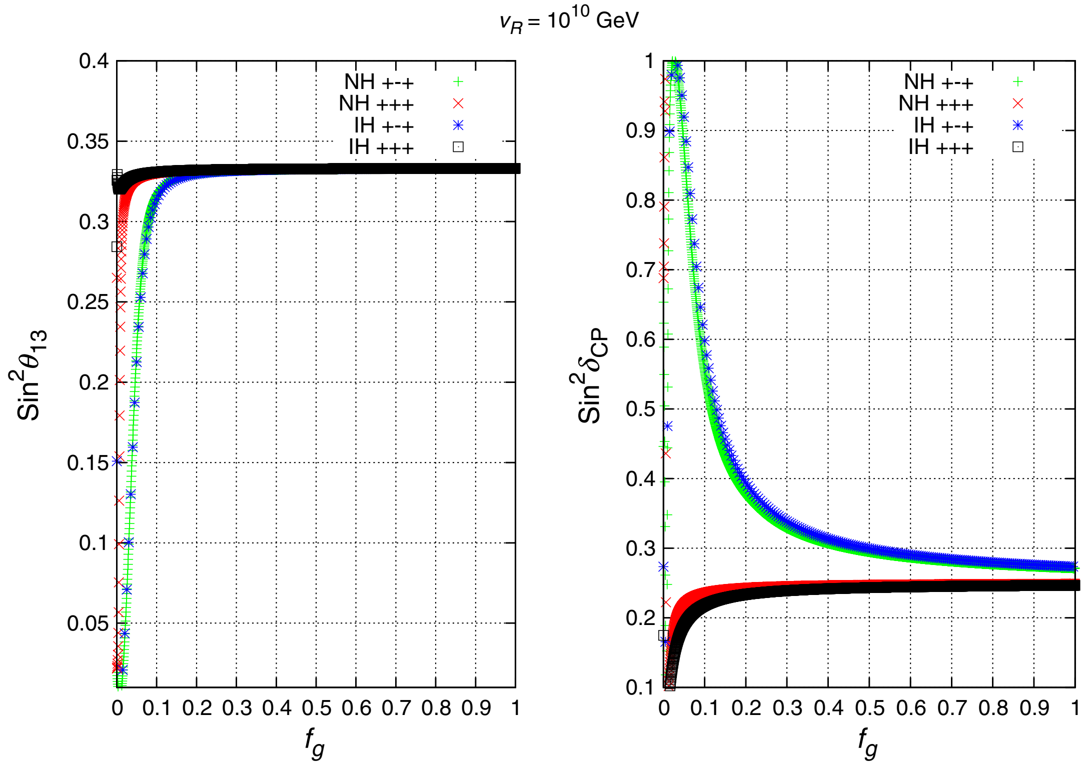

FIG. 7 (color online). Variations of $\sin^2\theta_{13}$ and $\sin^2\delta_{CP}$ as a function of f_g for $v_R = 10^{10}$ GeV.

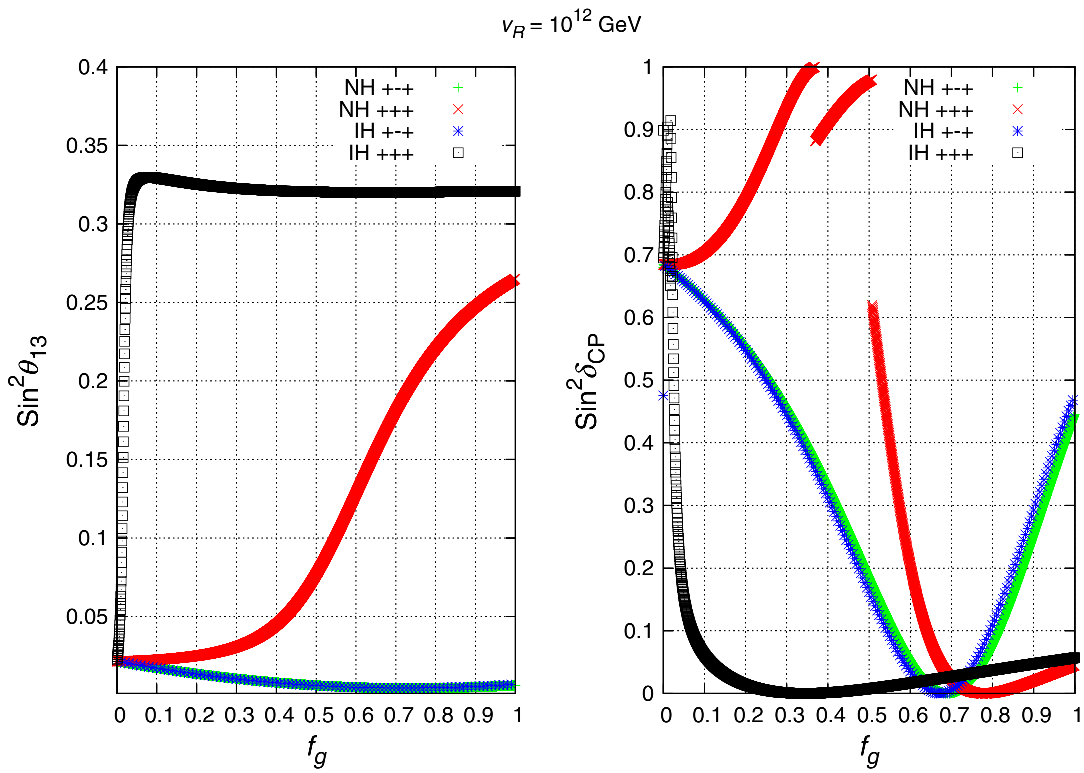

FIG. 8 (color online). Variations of $\sin^2\theta_{13}$ and $\sin^2\delta_{CP}$ as a function of f_g for $v_R = 10^{12}$ GeV.

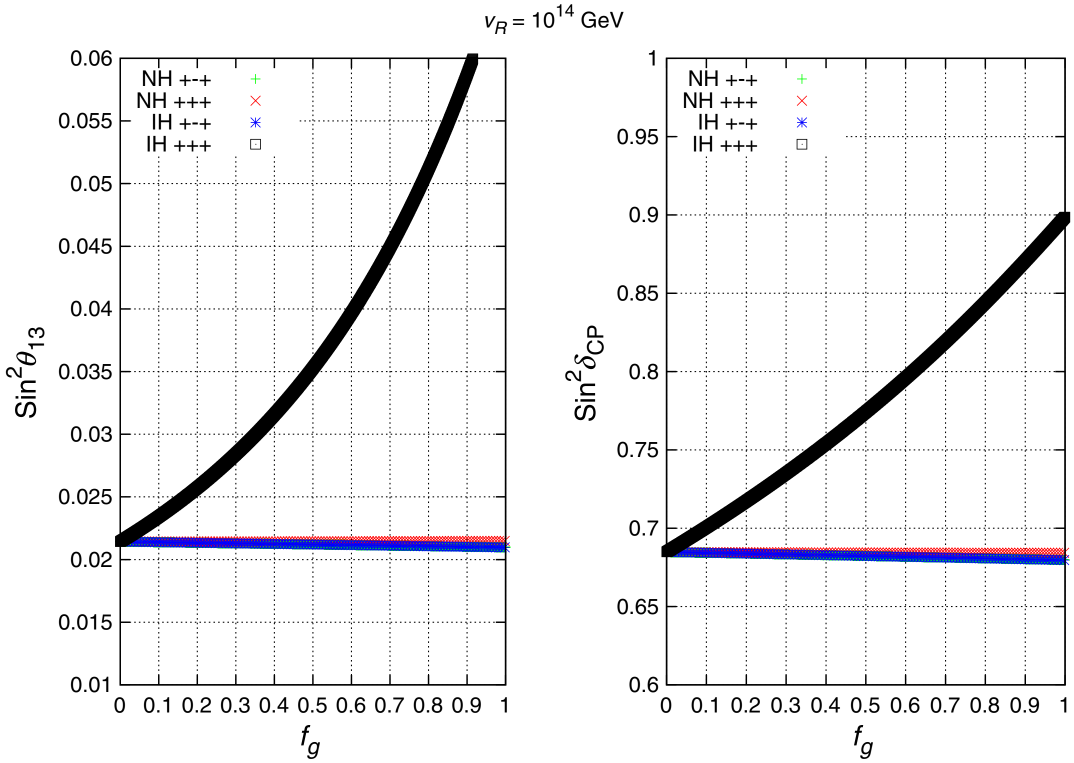

FIG. 9 (color online). Variations of $\sin^2\theta_{13}$ and $\sin^2\delta_{CP}$ as a function of f_g for $v_R = 10^{14}$ GeV.

within the 3σ allowed range for all possible values of f_g. Only Δm^2_{21} goes outside the 3σ range for $f_g > 0.1$ for all the models and θ_{13} deviates from the allowed range for $f_g > 0.35$ for IH$(+++)$ model. For $v_R = 10^{12}$ GeV, the mass squared differences lie within the allowed range only when $f_g < 0.01$, whereas for $v_R = 10^{10}$ GeV, they lie outside the 3σ range for the entire range of f_g parameters under study. Similarly, the mixing angles are also found to lie within the allowed range only for some small range of parameter space for lower values of v_R.

V. RESULTS AND CONCLUSION

We have studied the effects of higher-dimensional Planck-scale-suppressed operators on neutrino masses and mixings in left-right symmetric extension of standard models. These higher-dimensional correction terms arise due to the fact that no theory of quantum gravity respects global symmetries, whether continuous or discrete. Since left-right symmetric models have a built-in discrete global symmetry called D-parity, it is generic to introduce explicit D-parity breaking terms suppressed by the scale of gravity or the Planck scale. We have shown that in the minimal LRSM, the order of such higher-dimensional operators has dimension of at least six and hence is too small to affect neutrino masses and mixing. We then incorporate the presence of an additional gauge singlet scalar field, which allows dimension five Planck-suppressed operators to contribute to the neutrino mass matrix. Such a gauge singlet field can be naturally fit within several $SO(10)$ GUT multiplets. As discussed in our earlier work [7], these singlet scalar fields play a nontrivial role in destabilizing domain walls that arise in these models as a result of spontaneous D-parity breaking.

Sticking to the issue of neutrino mass alone in the present work, we then fit the tree-level neutrino mass matrix to the global best-fit neutrino data. After doing this, we introduce the higher-dimensional operators and see the variations in the neutrino mixing parameters with the changes in the overall coupling strength of these operators. We consider both normal and inverted hierarchies and two extremal Majorana phases in our work. Doing this exercise for three different left-right symmetry breaking scales, namely 10^{14}, 10^{12}, 10^{10} GeV, we show that the effects of these operators can be very significant for those models with left-right symmetry breaking scale below 10^{14} GeV. It should be noted that the purpose of our study is not to rule out or disfavor any particular model, but to emphasize the fact that fitting the tree-level seesaw formula with neutrino data is not enough in these models. The higher-dimensional opearators that violate D-parity explicitly can give rise to sizable contributions and hence must be taken into account in generic left-right symmetric models.

[1] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 2558 (1975); G. Senjanovic and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975); R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980); N. G. Deshpande, J. F. Gunion, B. Kayser, and F. I. Olness, Phys. Rev. D **44**, 837 (1991).
[2] S. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **86**, 5656 (2001); Q. R. Ahmad *et al.* (SNO), Phys. Rev. Lett. **89**, 011301 (2002); **89**, 011302 (2002); J. N. Bahcall and C. Pena-Garay, New J. Phys. **6**, 63 (2004); K. Nakamura *et al.*, J. Phys. G **37**, 075021 (2010).
[3] P. Minkowski, Phys. Lett. **67B**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, CERN, Print-80-0576; T. Yanagida, in *Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 1979*, edited by O. Sawada and A. Sugamoto (National Lab for High Energy Physics, Tsukuba, Japan, 1979), p. 109; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).
[4] R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981); G. Lazarides, Q. Shafi, and C Wetterich, Nucl. Phys. **B181**, 287 (1981); C. Watterich, Nucl. Phys. **B187**, 343 (1981); B. Brahmachari and R. N. Mohapatra, Phys. Rev. D **58**, 015001 (1998); R. N. Mohapatra, Nucl. Phys. B, Proc. Suppl. **138**, 257 (2005); S. Antusch and S. F. King, Phys. Lett. B **597**, 199 (2004).
[5] S. Mishra and U. A. Yajnik, Phys. Rev. D **81**, 045010 (2010); D. Borah and S. Mishra, Phys. Rev. D **84**, 055008 (2011).
[6] D. Borah, S. Patra, and U. Sarkar, Phys. Rev. D **83**, 035007 (2011); D. Borah and U. A. Yajnik, Phys. Rev. D **83**, 095004 (2011).
[7] D. Borah, Phys. Rev. D **86**, 096003 (2012).
[8] T. W. B. Kibble, Phys. Rep. **67**, 183 (1980).
[9] M. B. Hindmarsh and T. W. B. Kibble, Rep. Prog. Phys. **58**, 477 (1995).
[10] B. Rai and G. Senjanovic, Phys. Rev. D **49**, 2729 (1994).
[11] H. Lew and A. Riotto, Phys. Lett. B **309**, 258 (1993).
[12] S. Antusch, J. Kersten, M. Lindner, and M. Ratz, Phys. Lett. B **538**, 87 (2002).
[13] F. R. Joaquim, J. High Energy Phys. 06 (2010) 079.
[14] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, J. High Energy Phys. 12 (2012) 123.
[15] M. K. Das, D. Borah, and R. Mishra, Phys. Rev. D **86**, 095006 (2012); D. Borah and M. K. Das, Nucl. Phys. **B870**, 461 (2013).