Two Higgs doublets model in gauge-Higgs unification framework

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(Received 23 June 2012; published 10 May 2013)

We discuss the realization of a two Higgs doublets model in the framework of a six-dimensional gauge-Higgs unification model with a simple Lie group G_M. Two Higgs $SU(2)_L$ doublets can emerge at the low energy effective theory, and the quartic coupling terms in the scalar potential, which are essential for the electroweak symmetry breaking, are now G_M gauge invariant and permissive. A realistic two Higgs doublets model can possibly be obtained only when two of the root vectors associated with the would-be Higgs doublets and the root vector for $SU(2)_L$ form an isosceles triangle with a vertex angle either of $\pi/3$, $\pi/2$, or $2\pi/3$. Moreover, depending on G_M, the scalar potential of the resulting two Higgs doublets model can admit only a few limited forms. The mass spectrum of the physical Higgs and the weak mixing angle are briefly discussed.

DOI: 10.1103/PhysRevD.87.095005 PACS numbers: 12.60.Fr, 11.10.Kk, 11.15.−q, 12.10.Dm

I. INTRODUCTION

In the gauge-Higgs unification (GHU) models, the vector fields of gauge group $G_M \supset SU(2)_L \times U(1)_Y$ propagate in (4 + d)-dimensional spacetime. The gauge components in the d compactified extra spatial dimensions behave like scalar fields below the compactification scale [1]. With properly chosen gauge symmetry and orbifolding boundary conditions, an effective scalar $SU(2)_L$ doublet can emerge at the low energy and play the role of the Standard Model (SM) Higgs. Hence, we have no need of introducing a fundamental scalar. Due to the higher dimensional gauge symmetry, the d extra scalar fields are massless. The spontaneous electroweak symmetry breaking (EWSB) in SM can be triggered by the quantum corrections with the Wilson loop in the nonsimple connected space [2]. The notorious gauge hierarchy problem associated with a fundamental Higgs boson can be thus alleviated. For instance, a Higgs doublet could arise from a five-dimensional $SU(3)$ electroweak gauge theory on the S_1/Z_2 orbifold [3]. However, for $d = 1$, the quartic coupling term in the scalar potential must be generated by some symmetry-breaking quantum corrections for it vanishes at tree level as well. When $d \geq 2$, the quartic coupling terms in the scalar potential, which arise from the square of field strength, are gauge invariant by construction. Moreover, it is possible to generate multi-scalars at the low energy [4].

In this paper, we focus on the realization of two Higgs doublets model (2HDM) in six-dimensional GHU models, bearing in mind that (1) 2HDM predicts $\rho \equiv M_W^2/(M_Z^2 \cos^2\theta_W) = 1$ after EWSB at tree level, and (2) $d = 2$ is the minimal requirement to yield two Higgs [not limited to $SU(2)_L$ doublets]. We shall exhaust all possible simple Lie groups for G_M and examine the resulting quartic coupling terms of the Higgs potential, denoted as V_4, which is now completely determined by group theory at tree evel. It is a delightful surprise to us that V_4 can admit only a few forms for all possible Lie groups. Our key finding is that, to successfully generate a 2HDM at low energy, only the root vectors of G_M associated with the would-be Higgs doublets and the root vector for $SU(2)_L$ form an isosceles triangle with vertex angle either of $\pi/3$, $\pi/2$, or $2\pi/3$. Moreover, V_4 solely depends on the vertex angle. Our result is summarized in Table II. On the other hand, the quadratic terms of the Higgs potential, denoted as V_2, are assumed to be generated by some symmetry-breaking mechanism and they cannot be fixed by the gauge symmetry. However, by using the physical Higgs mass spectrum, one can parametrize V_2 phenomenologically and bypass the question of their origin. Finally, it is a well-known difficulty to construct a GHU model with the weak mixing $\sin^2\theta_W$ close to 1/4 at tree level [5,6]. In the phenomenology section, we discuss two possible remedies,

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by including either the brane kinetic term (BKT) [7,8] or an extra $U(1)$ factor, to address this problem and the consequent modification to the Higgs mass spectrum.

II. GROUP THEORY ANALYSIS

Following [6], we adopt the standard convention for the Lie group that $[H_i, H_j] = 0, [\vec{H}, E_\alpha] = \vec{\alpha} E_\alpha, [E_\alpha, E_{-\alpha}] = \vec{\alpha}\cdot\vec{H}$, and $[E_\alpha, E_\beta] = N_{\alpha,\beta}E_{\alpha+\beta}$. Here, H and E_α are the Cartan and root generators of G_M, respectively. The structure constant, $N_{\alpha,\beta}$, is given by $N^2_{\alpha,\beta} = n(m+1)(\vec{\alpha}\cdot\vec{\alpha})/2$ [9]. Moreover, we take the following normalization for H and E: $\mathrm{tr}H_iH_j = \delta_{ij}$, $\mathrm{tr}E_\alpha E_\beta = \delta_{\alpha+\beta,0}$, and $\mathrm{tr}E_\alpha H_i = 0$. The two extra compactified spatial coordinates, x^5 and x^6, can be conveniently described by a complex coordinate $z = (x^5 + ix^6)/\sqrt{2}$ and its conjugate $\bar{z}$. Accordingly, we work with $A_z \equiv (A_5 - iA_6)/\sqrt{2}$, the associated gauge field components in z, where $A_z = A^a_z T^a$ and T^a is the group generator.

By imposing the proper orbifolding boundary conditions, the remaining zero modes of A_z, the would-be scalars, are the gauge components associated with the unbroken group generators $E_{\beta,\gamma}$ while their four-dimensional (4D) gauge components possess no zero modes. Unless further stated, the notation A_z is recycled to collectively signify these zero modes which carry one mass dimension:

$$A_z = \frac{1}{2}h_u E_\beta + \frac{1}{2}h_d E_\gamma + \frac{1}{2}h'_u E_{-\beta} + \frac{1}{2}h'_d E_{-\gamma}. \quad (1)$$

And two would-be Higgs doublets can be built up in the following way:

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} h'_u \\ h'_d \end{pmatrix}, \qquad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} -h_d \\ h_u \end{pmatrix}. \quad (2)$$

The SM $SU(2)_L$ and $U(1)_Y$ groups must be embedded into G_M. If the root vector of the would-be SM $SU(2)_L$ is denoted as $\vec{\alpha}$, the corresponding generators read $J_0 = \frac{1}{|\vec{\alpha}|^2}\vec{\alpha}\cdot\vec{H}$, $J_+ = \frac{\sqrt{2}}{|\vec{\alpha}|}E_\alpha$, and $J_- = (J_+)^\dagger$. The would-be hypercharge generator Y has to be a linear combination of Cartan generators and denoted as $\vec{y}\cdot\vec{H}$. Since the SM group is $SU(2)_L \times U(1)_Y$, hence $\vec{y}\cdot\vec{\alpha} = 0$. The SM gauge bosons correspond to the zero modes of the gauge fields associated with these generators given as

$$A_\mu = W^+_\mu E_\alpha + W^-_\mu E_{-\alpha} + W^0_\mu \vec{\alpha}\cdot\vec{H} + B_\mu \vec{y}\cdot\vec{H}. \quad (3)$$

In [5], the phenomenologically viable embedding of the SM electroweak groups into G_M has been studied, where the normalization of $\vec{y}$ is fixed such that the SM Higgs doublet carries a hypercharge $1/2$ ($Q = T_3 + Y$). The root vectors $\vec{\alpha}$ and $\vec{y}$ we adopt from [5] are listed in the first two columns in Table II. From the commutators of $E_{\beta,\gamma}$, one has

$$\vec{\gamma} + \vec{\alpha} = \vec{\beta} \quad \text{or} \quad (\vec{\beta} - \vec{\alpha} = \vec{\gamma}). \quad (4)$$

TABLE I. Vertex angles, isosceles triangles, and candidate simple Lie groups for the possible realizations of 2HDM.

$\theta_{\vec{\beta},\vec{\gamma}}$	$\lvert\vec{\beta}\rvert$	Candidate groups	Type of triangle
60°	$\lvert\vec{\alpha}\rvert$	A_n, D_n, G_2, F_4, $E_{6,7,8}$	equilateral
90°	$\frac{\lvert\vec{\alpha}\rvert}{\sqrt{2}}$	B_n, C_n, F_4	right isosceles
120°	$\frac{\lvert\vec{\alpha}\rvert}{\sqrt{3}}$	G_2	obtuse isosceles

Since h_u and h_d transform into each other within one $SU(2)_L$ doublet, the magnitudes of the two root vectors should be the same, $|\vec{\beta}| = |\vec{\gamma}|$. The same requirement applies to the pair of h'_u and h'_d. From Eq. (4), the three root vectors, $\vec{\alpha}$, $\vec{\beta}$, and $\vec{\gamma}$ form an isosceles triangle lying on a plane in the root space. A trivial geometrical relation follows that

$$|\vec{\beta}|\sin\frac{\theta}{2} = \frac{|\vec{\alpha}|}{2}, \quad (5)$$

where θ is an angle between $\vec{\beta}$ and $\vec{\gamma}$. For a simple Lie group, θ can take only three possible values: either $\pi/3$, $\pi/2$, or $2\pi/3$. Hence, the original group theory problem of embedding the 2HDM in the GHU model with gauge symmetry G_M is now equivalent to looking for the existence of any equilateral, right isosceles, or obtuse isosceles triangles in the root diagram of G_M.

Once the root $\vec{\alpha}$ is given, one only needs to look up the corresponding Dynkin diagram and find out which simple root is adjacent to $\vec{\alpha}$. Next, one looks for the special isosceles triangle in the two-dimensional space spanned by the two simple roots. In Table I, we list all possible realizations of 2HDM by employing Eqs. (4) and (5) in various Lie groups. Note that all groups, except G_2 and F_4, have only one possible angle between the two root vectors for the would-be Higgs doublets.

We illustrate the finding by four rank-2 groups, A_2, B_2, C_2, and G_2, which can be diagrammatically summarized in the self-explanatory Fig. 1. Here, the triangles formed by the corresponding root vectors, $\vec{\alpha}$, $\vec{\beta}$, and $\vec{\gamma}$, are highlighted by color shades. Note that there are two distinct isosceles triangles that can be drawn with $\vec{\alpha}$ in the root diagram for G_2. Therefore, there are four possible forms for V_4 (two reds, two yellows, red with yellow on one side, and red with yellow on the opposite sides) in the G_2-based GHU model. Since the root $\vec{\alpha}$ for SM $SU(2)_L$ is always at the end of the Dynkin diagram, the analysis for G_M with a rank higher than two is no more complicated than those displayed in Fig. 1.

III. HIGGS POTENTIAL AND WEAK MIXING

The scalar quartic coupling terms in 2HDM arise from the six-dimensional gauge field strength square, $g^2_{4D}\mathrm{tr}[A_Z, A^\dagger_Z]^2$,

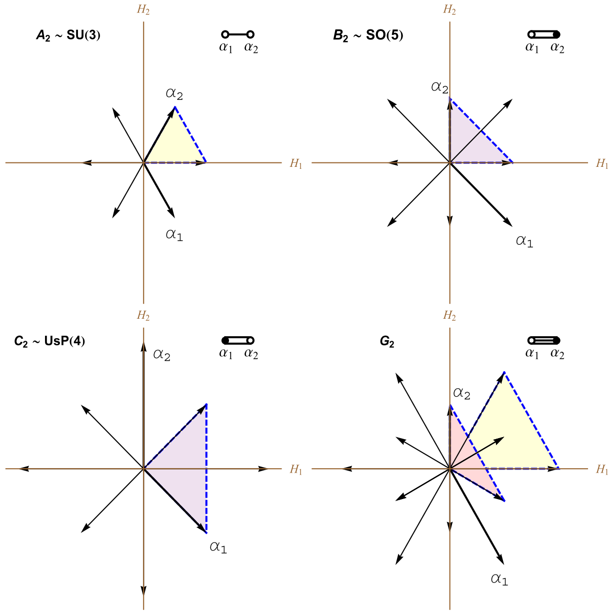

FIG. 1 (color online). Root diagrams, Dynkin diagrams, and corresponding triangles of A_2 (left upper), B_2 (right upper), C_2 (left bottom), and G_2 (right bottom) where yellow, purple, and red triangles represent the equilateral, right isosceles, and obtuse isosceles triangles, respectively.

$$V_4 = \frac{g_{4D}^2\vec{\beta}^2}{4}\{(|h_d|^2 - |h_u'|^2)^2 + (|h_u|^2 + |h_d'|^2)^2 + 2(\tilde{N}^2_{-\gamma,\beta} + 1)(|h_u|^2|h_d|^2 + |h_u'|^2|h_d'|^2) + 2(\tilde{N}^2_{\gamma,\beta} - \cos\theta)(|h_d|^2|h_d'|^2 + |h_u|^2|h_u'|^2) + 2(\tilde{N}^2_{-\gamma,\beta} + \tilde{N}^2_{\gamma,\beta})(h_d h_u^* h_u' h_d'^* + h_d^* h_u h_u'^* h_d')\}, \quad (6)$$

where $\tilde{N}^2_{\gamma,\beta} \equiv N^2_{\gamma,\beta}/\vec{\beta}^2$. Also, Eqs. (4) and (5) and the relations $N_{\gamma,\beta} = -N_{\beta,\gamma} = -N_{-\alpha,-\gamma}$ have been used to arrive in the above expression. Unsurprisingly, the resulting Higgs potential is completely determined by the gauge group G_M for a given root vector $\vec{\alpha}$ for SM $SU(2)_L$.

Equation (6) can be written in a much more compact form as

$$V_4(H_1, H_2) = \frac{1}{2}\lambda(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}N\lambda|H_1^\dagger H_2|^2, \quad (7)$$

where $\lambda = |\vec{\beta}|^2 g_{4D}^2$ and N is an integer depending on the group G_M, as shown in Table II. In Eq. (7), H_1 and H_2 have the desired hypercharge, either $Y = 1/2$ or $Y = -1/2$. Geometrically speaking, two identical or the mirror pair root triangles are adopted for the two would-be Higgs doublets. The cross coupling between H_1 and H_2, $\frac{1}{2}N\lambda$, can be calculated from the ladder chain of root vectors. For G_2 and F_4, there are two possible root triangles that can be adopted for 2HDM. For each realization, $\vec{y}$ has to be normalized accordingly, thus two possible N's for G_2 and F_4. Although the origins are totally different, Eq. (7) is accidentally identical to the D-term Higgs potential in the minimal supersymmetric SM (MSSM) if one substitutes $\frac{1}{2}\lambda \Rightarrow \frac{g^2+g'^2}{8}$ and $\frac{1}{2}N\lambda \Rightarrow \frac{g^2}{4}$. It is thus expected that the 2HDM in GHU and the MSSM share a similar physical Higgs mass spectrum.

Among all the Lie groups, the G_2-based GHU models have the richest 2HDM phenomenology. In addition to the quartic coupling terms given in Eq. (7), the 2HDM based on G_2 can have two extra possible forms for V_4:

$$V_4(H_1, H_2) = \frac{1}{2}\lambda\left[\frac{1}{3}(H_1^\dagger H_1)^2 + (H_2^\dagger H_2)^2 + N_1(H_1^\dagger H_1)(H_2^\dagger H_2) + N_2(H_1^\dagger \sigma^a H_2)(H_2^\dagger \sigma^a H_1)\right], \quad (8)$$

where (N_1, N_2) is either $(-2, +1)$ (as discussed in [10]) or $(+4, -2)$, corresponding to the yellow and red triangles in

TABLE II. The candidate simple Lie groups, based on which the six-dimensional GHU model is phenomenologically viable [5], their root vectors $\vec{\alpha}$ and $\vec{y}$ for $SU(2)_L$ and $U(1)_Y$, respectively, and all other relevant numbers; see text. Here, α^k is the kth simple root labeled by the Dynkin diagram, and $\tilde{\mu}_k$ is the rescaled kth fundamental weights such that $\vec{\alpha}^i \cdot \tilde{\mu}_j = \delta_{ij}$. Note that the SM quark representation cannot be accommodated in the GHU models based on the C_n or D_n groups [5]. However, C_n and D_n are listed here for the sake of comparison and completeness.

Group	α	y	$\tan\theta_W$	$\tilde{N}^2_{-\gamma,\beta}$	$\tilde{N}^2_{\gamma,\beta}$	$\theta_{\beta,\gamma}$	N
A: $SU(3l)$	α^1	$\tilde{\mu}_2/2$	$\sqrt{3l/(3l-2)}$	1/2	0	60°	1
B: $SO(2n+1)$	α^1	$\tilde{\mu}_2/6$	$\sqrt{3}$	1	1	90°	4
C: $USp(2n)$	α^n	$\tilde{\mu}_{n-1}/2$	$\sqrt{1/(n-1)}$	1	1	90°	4
D: $SO(2n)$	α^1, $\alpha^{n,\,n-1}$	$\tilde{\mu}_2/2$, $\tilde{\mu}_{n-2}/2$	$\sqrt{2/(n-1)}$	1/2	0	60°	1
G_2	α^1	$\tilde{\mu}_2/2$, $\tilde{\mu}_2/6$	$\sqrt{1/3}$, $\sqrt{3}$	1/2, 3/2	0, 2	60°, 120°	1, 7
F_4	α^1	$\tilde{\mu}_2/2$, $\tilde{\mu}_2/6$	$\sqrt{1/3}$, $\sqrt{3}$	1/2, 1	0, 1	60°, 90°	1, 4
E_6	$\alpha^{1,5}$	$\tilde{\mu}_{2,3}/2$	$\sqrt{3/5}$	1/2	0	60°	1
E_7	$\alpha^{1,7}$	$\tilde{\mu}_{2,3}/6$	$\sqrt{3}$,$\sqrt{3/2}$	1/2	0	60°	1
E_8	$\alpha^{1,8}$	$\tilde{\mu}_{2,3}/6$	$\sqrt{9/7}$,$\sqrt{3/5}$	1/2	0	60°	1

Fig. 1 on the opposite sides or on the same side, respectively. For G_2, $\vec{y}$ aligns with the medians of two possible root triangles. Hence, only one of the two Higgs doublets can acquire the desired hypercharge. In Eq. (8), $H_1(H_2)$ carries hypercharge $1/2(3/2)$. Therefore, only H_1 is responsible for the EWSB, and H_2 is not the canonical Higgs doublet with $|Y| = 1/2$. We note in passing that since the two possible root triangles of F_4 lie on different planes, a not so interesting V_4 with $(N_1, N_2) = (0, 0)$ can emerge.

As mentioned in the Introduction, the Higgs quadratic couplings in the GHU model must be generated radiatively by some symmetry-breaking mechanism [11]. The resulting quadratic couplings are highly model dependent. Here, we take a bottom-up approach and treat all quadratic couplings as phenomenological parameters. Then the full $SU(2)$ invariant scalar potential of 2HDM reads

$$V = m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_{12}^2(H_1^\dagger H_2 + H_2^\dagger H_1) + V_4. \tag{9}$$

Based on this, a textbook EWSB analysis can be performed straightforwardly. We denote the vacuum expected values as $\langle H_1^0\rangle = v_1$ and $\langle H_1^0\rangle = v_2$ with $\sqrt{v_1^2 + v_2^2} \sim 246$ GeV, and $\tan\beta = v_2/v_1$. The masses of CP odd pseudoscalar A_0 and charged Higgs H^+ are given by $M_{A_0}^2 = m_{12}^2/\sin\beta\cos\beta$ and $M_{H^+}^2 = M_{A_0}^2 - \frac{1}{4}N\lambda v^2$, respectively. Similar to MSSM, the tree-level mass of the lightest neutral Higgs has an upper bound, $M_h \leq \sqrt{\lambda v^2}(\sqrt{7\lambda v^2}/2)$ for $N = 1$ and 4 ($N = 7$), which may cause a phenomenological problem. However, just like in MSSM, there are many heavy degrees of freedom beyond SM in the GHU model. Similar to the positive δM_h due to the stop loops in the MSSM [12], it is well known that the inclusion of all the radiative corrections from either the bulk fields and their Kaluza-Klein excitations, the brane-localized fields, or the brane kinematic terms could largely enhance M_h from the above tree-level prediction [11]. However, a general discussion on this issue is still lacking. Such a model-independent discussion on the radiative corrections to M_h is beyond the scope of the present paper; here we just assume that a careful consideration which includes the radiative corrections to V_4 can rescue the lightest neutral Higgs mass problem as in MSSM. The investigation along this line will be presented elsewhere [13].

Finally, we address the weak mixing angle problem in GHU which is independent of d. Adding the BKT to the GHU model is one of the remedies to obtain a realistic $\sin^2\theta_W$ close to the experimental value. We use the following BKT which involves gauge zero modes on the brane only:

$$\mathcal{L}_{\mathrm{B.K}} = -\frac{1}{4}\int d^4x dx^5 dx^6 \delta(x^5)\delta(x^6) \times [c_1(F_{\mu\nu}^{(0)a})^2 + c_2(F_{\mu\nu}^{(0)b})^2], \tag{10}$$

where the superscript (0) represents the zero mode, c_1, c_2 are free parameters with mass dimension -2, and a, b are the group indices for SM $SU(2)_L$ and $U(1)_Y$, respectively. The 4D effective gauge couplings are modified:

$$g_{4D}^i \to \frac{g_{4D}^i}{\sqrt{Z_i}}, \quad \text{with} \quad Z_i = 1 + \frac{c_i}{Z_0^2}, \qquad (i = 1 \quad \text{or} \quad 2), \tag{11}$$

where Z_0^2 is the volume of the two extra dimensions. As a result, the weak mixing angle becomes

$$\tan\theta_W \to \sqrt{\frac{Z_1}{Z_2}}\tan\theta_W. \tag{12}$$

From the experimental value $\tan\theta_W = 0.5356$ [14], the ratio of Z_1/Z_2 can be fixed for a given G_M. Moreover, the upper bound of the lightest neutral Higgs mass gets a factor of $\sqrt{Z_1}$ enhancement and this problem can be alleviated as well.

Adding an additional $U(1)'$ gauge group is an alternative to obtain a realistic weak mixing angle in the GHU model. The mixing between SM $U(1)_Y$ and $U(1)'$ leads to an effective $Z_2 \neq 1$ (with $Z_1 = 1$) as in the previous case. The extra $U(1)$ factor introduces a new electrically neutral gauge boson whose mass shall be arranged to be the order of the compactification scale. The original $\tan\theta_W$ for each group without $U(1)'$ can be found in Table II. Note that to obtain the realistic θ_W, $SU(3)$, $SO(2n+1)$, G_2, F_4, and E_7 require the largest Z_2 among all the groups.

Before concluding, we remark on the new scalar boson recently observed near 126 GeV with the diphoton excess at the Large Hadron Collider [15,16]. Although most of its physical properties seem to be consistent with the elementary Higgs boson in the SM, the diphoton excess may indicate the existence of new physics beyond the SM. As discussed, $M_h \sim 126$ GeV could be easily accommodated in a realistic 2HDM in the GHU models with some construction dependent extensions. We note in passing that the diphoton excess, if it persists, could also be explained in this GHU framework for there are many charged heavy degrees of freedom.

In summary, we perform a general group theory analysis on the realization of 2HDM in the GHU model with a six-dimensional gauge G_M symmetry, where G_M is a simple Lie group. We showed that a 2HDM at low energy can possibly be made if the three root vectors associated with the would-be Higgs doublets and the SM $SU(2)_L$ form isosceles triangles with a vertex angle either of $\pi/3$, $\pi/2$, or $2\pi/3$. The quartic coupling terms in the 4D effective 2HDM potential are completely determined by the group G_M at tree level. Only a few potential forms can be admitted for all possible Lie groups, as shown in Table II. Moreover, which form to be admitted depends solely on the vertex angle. We observed that, among all possible Lie groups, the 2HDM based on the GHU model with the G_2 gauge symmetry has the richest structure in the Higgs potential. We discuss the mass spectrum of physical Higgs bosons and two possible remedies to obtain a realistic weak mixing angle in GHU models as well. Finally, we briefly comment on how to accommodate the recently observed ~126 GeV scalar boson at the Large Hadron Collider in this GHU framework.

ACKNOWLEDGMENTS

W. F. C. was supported by the Taiwan NSC, Grant No. 99-2112-M-007-006-MY3. S. K. K was supported by NRF and MEST of Korea, Grant No. 2011-0029758. J. P. was supported by the Taiwan NSC, Grants No. 100-2811-M-007-030 and No. 099-2811-M-007-077. J. P. thanks D. W. Jeong for valuable comments and discussion.

[1] N. S. Manton, Nucl. Phys. **B158**, 141 (1979); D. B. Fairlie, Phys. Lett. **82B**, 97 (1979); P. Forgacs and N. S. Manton, Commun. Math. Phys. **72**, 15 (1980).
[2] Y. Hosotani, Ann. Phys. (N.Y.) **190**, 233 (1989); Phys. Lett. **126B**, 309 (1983); **129B**, 193 (1983).
[3] I. Antoniadis and K. Benakli, Phys. Lett. B **326**, 69 (1994).
[4] C. A. Scrucca, M. Sernoe, A. Wulzer, and L. Silvestrini, J. High Energy Phys. 02 (2004) 049.
[5] A. Aranda and J. Wudka, Phys. Rev. D **82**, 096005 (2010).
[6] B. Grzadkowski and J. Wudka, Phys. Rev. Lett. **97**, 211602 (2006).
[7] G. Burdman and Y. Nomura, Nucl. Phys. **B656**, 3 (2003); K. Agashe, R. Contino, and A. Pomarol, Nucl. Phys. **B719**, 165 (2005); A. Aranda and J. L. Diaz-Cruz, Phys. Lett. B **633**, 591 (2006); G. Panico, M. Safari, and M. Serone, J. High Energy Phys. 02 (2011) 103.
[8] J. Park and S. K. Kang, J. High Energy Phys. 04 (2012) 101.
[9] R. Gilmore, *Lie Groups, Lie Algebras, and Some of their Applications* (Dover, New York, 2005).
[10] C. Csaki, C. Grojean, and H. Murayama, Phys. Rev. D **67**, 085012 (2003).
[11] I. Antoniadis, K. Benakli, and M. Quiros, New J. Phys. **3**, 20 (2001); C. A. Scrucca, M. Serone, and L. Silvestrini, Nucl. Phys. **B669**, 128 (2003); G. Cacciapaglia, C. Csaki, and S. C. Park, J. High Energy Phys. 03 (2006) 099.
[12] H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991); Y. Okada, M. Yamaguchi, and T. Yanagida, Prog. Theor. Phys. **85**, 1 (1991); Phys. Lett. B **262**, 54 (1991); J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. B **257**, 83 (1991); **262**, 477 (1991).
[13] W.-F. Chang, S K. Kang, and J. Park (work in progress).
[14] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
[15] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012).
[16] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**, 30 (2012).