PHYSICAL REVIEW D **87,** 094035 (2013)

Are there infinitely many decompositions of the nucleon spin?

M. Wakamatsu*

Department of Physics, Faculty of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan
(Received 20 February 2013; published 31 May 2013)

We discuss the uniqueness or nonuniqueness problem of the decomposition of the gluon field into the physical and pure-gauge components, which is the basis of the two recently proposed physically inequivalent gauge-invariant decompositions of the nucleon spin. It is crucially important to recognize the fact that the standard gauge-fixing procedure is essentially a process of projecting out the physical components of the massless gauge field. A complexity of the non-Abelian gauge theory as compared with the Abelian case is that a closed expression for the physical component can be given only with use of the nonlocal Wilson line, which is generally path dependent. It is known that—by choosing an infinitely long straight-line path in space and time, the direction of which is characterized by a constant four-vector n^μ—one can cover a class of gauge called the general axial gauge, containing three popular gauges, i.e., the temporal, the light-cone, and the spatial axial gauge. Within this general axial gauge, we calculate the one-loop evolution matrix for the quark and gluon longitudinal spins in the nucleon. We find that the final answer is exactly the same independent of the choices of n^μ, which amounts to proving the gauge independence and path independence simultaneously, although within a restricted class of gauges and paths. By drawing on all of these findings together with well-established knowledge from gauge theories, we argue against the rapidly spreading view in the community that there are infinitely many decompositions of the nucleon spin.

DOI: 10.1103/PhysRevD.87.094035 PACS numbers: 12.38.−t, 12.20.−m, 14.20.Dh, 03.50.De

I. INTRODUCTION

Is a gauge-invariant complete decomposition of the nucleon spin possible? This is a fundamentally important question of QCD as a color gauge theory. The reason is that the gauge invariance is generally believed to be a *necessary condition* of *observability*. Unfortunately, this is quite a delicate problem, which is still under intense debate [1–36]. In a series of papers [16–19], we have established the fact that there are two physically inequivalent gauge-equivalent decompositions of the nucleon spin, which we call decompositions (I) and (II). The decompositions (I) and (II) are respectively characterized by two different orbital angular momenta (OAMs) for both quarks and gluons, i.e., the "dynamical" OAMs and the generalized "canonical" OAMs. We also clarified the fact that the difference between the above two kinds of orbital angular momenta is characterized by a quantity which we call the "potential angular momentum," the QED correspondent of which is nothing but the angular momentum carried by the electromagnetic field or potential, which plays a key role in the famous Feynman paradox of classical electrodynamics [16,37]. The basic assumption for obtaining these two gauge-invariant decompositions of the nucleon spin is that the total gluon field can be decomposed into two parts, i.e., the physical component and the pure-gauge component, as $A^\mu(x) = A^\mu_{\rm phys}(x) + A^\mu_{\rm pure}(x)$. In the course of deriving the above two gauge-invariant decompositions of the nucleon spin, these two components are supposed to obey the following general conditions, i.e., the pure-gauge condition for the pure-gauge component, $F^{\mu\nu}_{\rm pure} \equiv \partial^\mu A^\nu_{\rm pure} - \partial^\nu A^\mu_{\rm pure} - ig[A^\mu_{\rm pure}, A^\nu_{\rm pure}] = 0$, supplemented with the homogeneous (or covariant) and inhomogeneous gauge transformation properties, respectively, for the physical and pure-gauge components of the gluon field under general gauge transformations of QCD.

A natural question is whether these general conditions are enough to uniquely fix the above decomposition. The answer is evidently no! Note, however, that the above decomposition is proposed as a covariant generalization of Chen *et al.*'s decomposition given in a noncovariant form as $\boldsymbol{A}(x) = \boldsymbol{A}_{\rm phys}(x) + \boldsymbol{A}_{\rm pure}(x)$ [8,9]. One must acknowledge the fact that, at least in the QED case, this decomposition is nothing new. It just corresponds to the standardly known transverse-longitudinal decomposition of the three-vector potential of the photon field, i.e., $\boldsymbol{A}(x) = \boldsymbol{A}_\perp(x) + \boldsymbol{A}_\parallel(x)$ satisfying the properties $\nabla \cdot \boldsymbol{A}_\perp = 0$ and $\nabla \times \boldsymbol{A}_\parallel = 0$ [38,39]. It is a well-established fact that this decomposition is *unique* once the Lorentz frame of reference is specified [39]. As we shall see later, a physically essential element here is the transversality condition $\nabla \cdot \boldsymbol{A}_\perp = 0$ for the transverse (or physical) component of $\boldsymbol{A}$ given in a noncovariant form. Naturally, a certain substitute for this condition is necessary to uniquely fix the physical component of $A^\mu_{\rm phys}$ in the above-mentioned decomposition given in a (seemingly) covariant form. This fundamental fact of gauge theory is missed in the community, and conflicting views have rapidly spread around.

*wakamatu@phys.sci.osaka-u.ac.jp

On the one hand, Lorcé claims that the above decomposition is not unique because of the presence of what he called the Stueckelberg symmetry, which alters both $A^\mu_{\rm phys}$ and $A^\mu_{\rm pure}$ while keeping their sum unchanged [30,31]. This misapprehension comes from the oversight of the importance of the transversality condition that should be imposed on the physical component. On the other, another argument against the uniqueness of the above-mentioned decomposition is advocated by Ji *et al.* [32–34]. According to them, the Chen decomposition is a gauge-invariant extension (GIE) of the Jaffe-Manohar decomposition based on the Coulomb gauge, while the Bashinsky-Jaffe decomposition is a GIE of the Jaffe-Manohar decomposition based on the light-cone gauge. They claim that—since the GIE with the use of a path-dependent Wilson line is not unique at all—there is no reason that the above two decompositions should give the same physical predictions. This made Ji reopen his longstanding claim that the gluon spin ΔG in the nucleon is not a gauge-invariant quantity in a *true* or *traditional* sense, although it is a measurable quantity in polarized deep inelastic scatterings [40,41]. One should recognize a self-contradiction inherent in this claim. In fact, one should first remember the fundamental proposition of physics, which states that "Observables must be gauge invariant." (Note that we are using the word "observables" in a strict sense. That is, they must be quantities, which can be extracted purely experimentally, i.e., without recourse to particular theoretical schemes or models.) The contraposition of this proposition (note that it is always correct if the original proposition is correct) is "Gauge-variant quantities cannot be observables." This dictates that, if ΔG is claimed to be observable, it must also be gauge invariant in the *traditional* sense.

In view of the above-explained frustrated status, we believe it is urgent to correct the widespread misunderstanding regarding the meaning of *true* or *traditional* gauge invariance in the problem of nucleon spin decomposition. This paper is then organized as follows. In Sec. II, we first clarify the fact that—at least in the case of Abelian gauge theory—the decomposition of the gauge field into the physical and pure-gauge components is nothing but the well-known transverse-longitudinal decomposition of the vector potential. It is a well-established fact that this decomposition is unique as long as we are working in a prescribed Lorentz frame. We also point out a hidden problem of the gauge-invariant extension approach, i.e., the *path dependence*, through a concise pedagogical review of the gauge-invariant formulation of the electromagnetism with the use of the nonlocal gauge link. Next, in Sec. III we give an explicit form of the physical component of the gluon field based on a geometrical formulation of the non-Abelian gauge theory, which also uses a path-dependent Wilson line. After clarifying an inseparable connection between the choice of path contained in the Wilson line and the choice of gauge, we consider a special class of paths, i.e., infinitely long straight-line paths, the direction of which is characterized by a constant four-vector n^μ. This particular choice of path is known to be equivalent to taking the so-called general axial gauge, which contains in it three popular gauges, i.e., the temporal, the light-cone, and the spatial axial gauges. Based on this general axial gauge specified by the four-vector n^μ, we shall calculate the one-loop evolution matrix for the quark and gluon longitudinal gluon spins in the nucleon, in order to check whether the answer depends on the choice of n^μ, which characterizes simultaneously the gauge choices within the general axial gauge and the direction of the straight-line path in the geometric formulation. Concluding remarks will then be given in Sec. IV.

II. CRITIQUES ON THE IDEA OF STUECKELBERG SYMMETRY AND THE GAUGE-INVARIANT-EXTENSION APPROACH

In a series of papers [16–19], we have shown that there are two physically inequivalent decompositions of the nucleon spin, which we call decompositions (I) and (II). The QCD angular momentum tensor in the decomposition (I) is given as follows:

$$M^{\mu\nu\lambda} = M^{\mu\nu\lambda}_{q\text{-spin}} + M^{\mu\nu\lambda}_{q\text{-OAM}} + M^{\mu\nu\lambda}_{G\text{-spin}} + M^{\mu\nu\lambda}_{G\text{-OAM}} + M^{\mu\nu\lambda}_{\rm boost}, \tag{1}$$

with

$$M^{\mu\nu\lambda}_{q\text{-spin}} = \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi, \tag{2}$$

$$M^{\mu\nu\lambda}_{q\text{-OAM}} = \bar{\psi}\gamma^\mu(x^\nu iD^\lambda - x^\lambda iD^\nu)\psi, \tag{3}$$

$$M^{\mu\nu\lambda}_{G\text{-spin}} = 2\,{\rm Tr}[F^{\mu\lambda}A^\nu_{\rm phys} - F^{\mu\nu}A^\lambda_{\rm phys}], \tag{4}$$

$$M^{\mu\nu\lambda}_{G\text{-OAM}} = -2\,{\rm Tr}[F^{\mu\alpha}(x^\nu D^\lambda_{\rm pure} - x^\lambda D^\nu_{\rm pure})A^{\rm phys}_\alpha] + 2\,{\rm Tr}[(D_\alpha F^{\alpha\mu})(x^\nu A^\lambda_{\rm phys} - x^\lambda A^\nu_{\rm phys})], \tag{5}$$

and

$$M'^{\mu\nu\lambda}_{\rm boost} = -\frac{1}{2}\,{\rm Tr}F^2(x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}). \tag{6}$$

On the other hand, the QCD angular momentum tensor in the decomposition (II) is given as follows:

$$M^{\mu\nu\lambda} = M'^{\mu\nu\lambda}_{q\text{-spin}} + M'^{\mu\nu\lambda}_{q\text{-OAM}} + M'^{\mu\nu\lambda}_{G\text{-spin}} + M'^{\mu\nu\lambda}_{G\text{-OAM}} + M'^{\mu\nu\lambda}_{\rm boost}, \tag{7}$$

with

$$M'^{\mu\nu\lambda}_{q\text{-spin}} = M^{\mu\nu\lambda}_{q\text{-spin}}, \tag{8}$$

$$M'^{\mu\nu\lambda}_{q\text{-OAM}} = \bar{\psi}\gamma^{\mu}(x^{\nu}iD^{\lambda}_{\text{pure}} - x^{\lambda}iD^{\nu}_{\text{pure}})\psi, \tag{9}$$

$$M'^{\mu\nu\lambda}_{G\text{-spin}} = M^{\mu\nu\lambda}_{G\text{-spin}}, \tag{10}$$

$$M'^{\mu\nu\lambda}_{G\text{-OAM}} = -2\,\mathrm{Tr}[F^{\mu\alpha}(x^{\nu}D^{\lambda}_{\text{pure}} - x^{\lambda}D^{\nu}_{\text{pure}})A^{\text{phys}}_{\alpha}], \tag{11}$$

$$M'^{\mu\nu\lambda}_{\text{boost}} = M^{\mu\nu\lambda}. \tag{12}$$

In these two decompositions, the quark and gluon intrinsic spin parts are just common, and the difference lies only in the orbital parts. The difference is given as follows:

$$\begin{aligned} M^{\mu\nu\lambda}_{q\text{-OAM}} - M'^{\mu\nu\lambda}_{q\text{-OAM}} &= -(M^{\mu\nu\lambda}_{G\text{-OAM}} - M'^{\mu\nu\lambda}_{G\text{-OAM}}) \\ &= 2\,\mathrm{Tr}[(D_{\alpha}F^{\alpha\mu})(x^{\nu}A^{\lambda}_{\text{phys}} - x^{\lambda}A^{\nu}_{\text{phys}})]. \end{aligned} \tag{13}$$

The quantity characterizing the difference between the two kinds of orbital angular momenta of quarks and gluons, i.e., the quantity appearing on the rhs of the above relation, is a covariant generalization of the following quantity:

$$\boldsymbol{L}_{\text{pot}} = \int \rho^{a}(\boldsymbol{r} \times \boldsymbol{A}^{a})d^{3}r, \tag{14}$$

which we called the *potential angular momentum* in Ref. [16]. The reason is that this just corresponds to the angular momentum carried by the electromagnetic field or potential appearing in Feynman's famous paradox of classical electrodynamics [37]. (For an interesting phenomenological implication concerning the difference between these two physically inequivalent decompositions of the nucleon spin, we refer to Refs. [42–47].)

The whole argument above is based on the decomposition of the gluon field A^{μ} into the physical component and the pure-gauge component as

$$A^{\mu} = A^{\mu}_{\text{phys}} + A^{\mu}_{\text{pure}}, \tag{15}$$

satisfying the following general conditions, i.e., the pure-gauge condition for A^{μ}_{pure},

$$F^{\mu\nu}_{\text{pure}} \equiv \partial^{\mu}A^{\nu}_{\text{pure}} - \partial^{\nu}A_{\text{pure}} - ig[A^{\mu}_{\text{pure}}, A^{\nu}_{\text{pure}}] = 0, \tag{16}$$

supplemented with the gauge-transformation properties for A^{μ}_{phys} and A^{μ}_{pure},

$$A^{\mu}_{\text{phys}}(x) \to U(x)A^{\mu}_{\text{phys}}(x)U^{\dagger}(x), \tag{17}$$

$$A^{\mu}_{\text{pure}}(x) \to U(x)\left(A^{\mu}_{\text{pure}}(x) + \frac{i}{g}\partial^{\mu}\right)U^{\dagger}(x), \tag{18}$$

under an arbitrary gauge transformation $U(x)$ of QCD.

In recent papers [30,31], Lorce criticized that the pure-gauge condition $F^{\mu\nu}_{\text{pure}} = 0$ is insufficient to uniquely determine the decomposition $A^{\mu} = A^{\mu}_{\text{phys}} + A^{\mu}_{\text{pure}}$. According to him, there exists a hidden symmetry, which he calls a Stueckelberg symmetry. In the simpler case of Abelian gauge theory, the proposed Stueckelberg transformation is given by

$$A^{\mu}_{\text{phys}}(x) \to A^{\mu}_{\text{phys},g}(x) = A^{\mu}_{\text{phys}}(x) - \partial^{\mu}C(x), \tag{19}$$

$$A^{\mu}_{\text{pure}}(x) \to A^{\mu}_{\text{pure},g}(x) = A^{\mu}_{\text{pure}}(x) + \partial^{\mu}C(x), \tag{20}$$

with $C(x)$ being an arbitrary function of space and time. Certainly, this transformation changes both A^{μ}_{phys} and A^{μ}_{pure}, but their sum remains intact. It was then claimed that this hidden symmetry dictates the existence of infinitely many decompositions of the gauge field into the physical and pure-gauge components, thereby leading to the conclusion that there are in principle *infinitely many decompositions* of the nucleon spin.

It is certainly true that the pure-gauge condition together with the homogeneous and inhomogeneous transformation properties of A^{μ}_{phys} and A^{μ}_{pure} are not sufficient to determine the decomposition $A^{\mu} = A^{\mu}_{\text{phys}} + A^{\mu}_{\text{pure}}$ uniquely. However, one should remember the original motivation of this decomposition. In the QED case with the noncovariant treatment by Chen *et al.* [8,9], this decomposition is nothing more than the standard decomposition of the vector potential $\boldsymbol{A}$ of the photon field into the transverse and longitudinal components,

$$\boldsymbol{A}(x) = \boldsymbol{A}_{\perp}(x) + \boldsymbol{A}_{\parallel}(x), \tag{21}$$

where the transverse component and the longitudinal component are respectively required to obey the divergence-free and irrotational conditions [38,39],

$$\nabla \cdot \boldsymbol{A}_{\perp} = 0, \qquad \nabla \times \boldsymbol{A}_{\parallel} = 0. \tag{22}$$

For the sake of later discussion, we also recall the fact that the transverse-longitudinal decomposition can be made explicit with the use of the corresponding projection operators as follows:

$$A^{i}(x) = A^{i}_{\perp}(x) + A^{i}_{\parallel}(x) = (P^{ij}_{T} + P^{ij}_{L})A^{j}(x), \tag{23}$$

with

$$P^{ij}_{T} = \delta^{ij} - \frac{\nabla^{i}\nabla^{j}}{\nabla^{2}}, \tag{24}$$

$$P^{ij}_{L} = \frac{\nabla^{i}\nabla^{j}}{\nabla^{2}}. \tag{25}$$

As is well known, these two components transform as

$$\boldsymbol{A}_{\perp}(x) \to \boldsymbol{A}'_{\perp}(x) = \boldsymbol{A}_{\perp}(x), \tag{26}$$

$$\boldsymbol{A}_{\parallel}(x) \to \boldsymbol{A}'_{\parallel}(x) = \boldsymbol{A}_{\parallel}(x) - \nabla\Lambda(x) \tag{27}$$

under a general Abelian gauge transformation. This means that $\boldsymbol{A}_{\parallel}$ carries unphysical gauge degrees of freedom, while $\boldsymbol{A}_{\perp}$ is absolutely intact under an arbitrary gauge transformation. Besides, it is a well-established fact that this decomposition is *unique*, once the Lorentz frame of reference is specified [39]. (To be more precise, the uniqueness is guaranteed by a supplemental condition that $\boldsymbol{A}$ falls off faster than $1/r^2$ at the spatial infinity, which is satisfied in the usual circumstances that happen in electromagnetism.) This uniqueness of the decomposition indicates that, in QED, there exists no Stueckelberg symmetry as suggested by Lorcé. In fact, within the above-mentioned noncovariant framework, the Stueckelberg transformation *à la* Lorcé reduces to

$$\boldsymbol{A}_\perp(x) \to \boldsymbol{A}^g_\perp(x) = \boldsymbol{A}_\perp(x) + \nabla C(x), \tag{28}$$

$$\boldsymbol{A}_\parallel(x) \to \boldsymbol{A}^g_\parallel(x) = \boldsymbol{A}_\parallel(x) - \nabla C(x). \tag{29}$$

One can see that the transformed longitudinal component $\boldsymbol{A}^g_\parallel(x)$ retains the irrotational property,

$$\nabla \times \boldsymbol{A}^g_\parallel = \nabla \times (\boldsymbol{A}_\parallel - \nabla C(x)) = \nabla \times \boldsymbol{A}_\parallel = 0. \tag{30}$$

(This is simply a reflection of the fact the standard gauge transformation for $\boldsymbol{A}_\parallel$ keeps the magnetic field $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ intact.) However, one finds that the transformed component $\boldsymbol{A}^g_\perp(x)$ does not satisfy the desired divergence-free (or transversality) condition $\nabla \cdot \boldsymbol{A}^g_\perp = 0$ any more, since

$$\nabla \cdot \boldsymbol{A}^g_\perp = \nabla \cdot (\boldsymbol{A}_\perp + \nabla C(x)) = \Delta C(x) \neq 0 \tag{31}$$

unless $\Delta C(x) = 0$. (As a matter of course—different from the Stueckelberg transformation—there is no such problem in the standard gauge transformation (26) and (27), because $\boldsymbol{A}_\perp$ is intact under a general gauge transformation.) The condition $\Delta C(x) = 0$ means that $C(x)$ is a harmonic function in three spatial dimensions. If it is required to vanish at the spatial infinity, it must be identically zero owing to the Helmholtz theorem. As is clear from the discussion above, the Stueckelberg-like transformation does not generally preserve the transversality condition of the transverse or physical component of $\boldsymbol{A}$. In other words, the Stueckelberg symmetry does not actually exist and/or it has nothing to do with a physical symmetry of QED. Let us repeat again the well-founded fact in QED: the transverse-longitudinal decomposition is unique once the Lorentz frame of reference is fixed.

Still, a bothersome problem here is that the transverse-longitudinal decomposition is not a relativistically invariant manipulation. A vector field that appears transverse in a certain Lorentz frame is not necessarily transverse in another Lorentz frame. An immediate question is then what meaning one can give to the seemingly covariant decomposition of a gauge field like $A^\mu = A^\mu_{\mathrm{phys}} + A^\mu_{\mathrm{pure}}$. Putting it another way—in view of the fact that the transverse-longitudinal decomposition can be made only at the sacrifice of breaking the Lorentz covariance—how can we get an explicit form of this decomposition, which is usable in a desired Lorentz frame? Leaving this nontrivial question aside, we want to make some general remarks on the treatment of gauge theories. In a covariant treatment of gauge theories, we start with the gauge field A^μ with four components ($\mu = 0, 1, 2, 3$). However, we know that the massless gauge field has only two independent dynamical degrees of freedom, i.e., two transverse components, say A^1 and A^2. The other two components, i.e., the scalar component A^0 and the longitudinal component A^3, are not independent dynamical degrees of freedom. For quantizing a gauge theory, we need a procedure of gauge fixing. A gauge-fixing procedure is essentially an operation which eliminates the unphysical degrees of freedom so as to pick out the two transverse components. In this sense, the transverse-longitudinal decomposition and the gauge-fixing procedure are closely interrelated operations (one might say that they are almost *synonymous*), even though they are not absolutely identical operations.

Another argument against the uniqueness of the nucleon spin decomposition is based on the idea of the gauge-invariant extension with the use of a path-dependent Wilson line [32–34]. The idea of a gauge link in gauge theories is of more general concern and has a long history. DeWitt once tried to formulate the quantum electrodynamics in a gauge-invariant way, i.e., without introducing a gauge-dependent potential [48]. However, it was recognized soon that—although the framework is manifestly gauge-invariant by construction-it *does* depend on the choice of path defining the gauge-invariant potential [49–52]. Since the problem seems to be intimately connected with the one we are confronted with, we think it instructive to briefly review this framework by paying attention to its delicate point.

According to DeWitt, once given an appropriate set of electron and photon fields $[\psi(x), A_\mu(x)]$, the gauge-invariant set of the electron and photon fields $[\psi'(x), A'_\mu(x)]$ can be constructed as

$$\psi'(x) \equiv e^{ie\Lambda(x)}\psi(x), \tag{32}$$

$$A'_\mu(x) \equiv A_\mu(x) + \partial_\mu \Lambda(x), \tag{33}$$

with

$$\Lambda(x) = -\int_{-\infty}^{0} A_\sigma(z)\frac{\partial z^\sigma}{\partial \xi} d\xi, \tag{34}$$

where $z^\mu(x, \xi)$ is a point on the line toward x, with ξ being a parameter chosen in such a way that

$$z^\mu(x, 0) = x^\mu, \quad \text{and} \quad z^\mu(x, -\infty) = \text{spatial infinity}. \tag{35}$$

Note here that $\partial z^\mu/\partial x^\lambda = \delta^\mu_\lambda$ at $\xi = 0$.

One can easily see that the new electron and photon fields defined by Eqs. (32) and (33) are in fact gauge invariant. In fact, under an arbitrary gauge transformation

$$\psi(x) \rightarrow e^{ie\omega(x)} \psi(x), \tag{36}$$

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \omega(x), \tag{37}$$

the function $\Lambda(x)$ transforms as

$$\Lambda(x) \rightarrow -\int_{-\infty}^0 (A_\sigma(z) + \partial_\sigma \omega(z)) \frac{\partial z^\sigma}{\partial \xi} d\xi = -\int_{-\infty}^0 A_\sigma(z) \frac{\partial z^\sigma}{\partial \xi} d\xi - \int_{-\infty}^0 \frac{\partial \omega(z)}{\partial \xi} d\xi = \Lambda(x) - \omega(x). \tag{38}$$

This means that $\psi'(x)$ transforms as

$$\psi'(x) \rightarrow e^{ie(\Lambda(x) - \omega(x))} e^{ie\omega(x)} \psi(x) = e^{ie\Lambda(x)} \psi(x) = \psi'(x), \tag{39}$$

that is, $\psi'(x)$ is gauge invariant. The gauge invariance of $A'_\mu(x)$ can also be easily verified. For instructive purposes, we reproduce here the proof. The manipulation goes as follows:

$$\begin{aligned} A'_\mu(x) &= A_\mu(x) + \partial_\mu \Lambda(x) \\ &= A_\mu - \partial_\mu \int_{-\infty}^0 A_\sigma(z) \frac{\partial z^\sigma}{\partial \xi} d\xi \\ &= A_\mu - \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial x^\mu} \frac{\partial z^\sigma}{\partial \xi} d\xi - \int_{-\infty}^0 A_\sigma(z) \frac{\partial}{\partial \xi} \left(\frac{\partial z^\sigma}{\partial x^\mu} \right) d\xi \\ &= A_\mu - \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial x^\mu} \frac{\partial z^\sigma}{\partial \xi} d\xi + \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial \xi} \frac{\partial z^\sigma}{\partial x^\mu} d\xi - A_\sigma(z) \frac{\partial z^\sigma}{\partial x^\mu} \Big|_{\xi=-\infty}^{\xi=0} \\ &= A_\mu - \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial x^\mu} \frac{\partial z^\sigma}{\partial \xi} d\xi + \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial \xi} \frac{\partial z^\sigma}{\partial x^\mu} d\xi - A_\sigma(x) \delta_\mu{}^\sigma \\ &= -\int_{-\infty}^0 (\partial_\nu A_\sigma - \partial_\sigma A_\nu) \frac{\partial z^\nu}{\partial x^\mu} \frac{\partial z^\sigma}{\partial \xi} d\xi. \end{aligned} \tag{40}$$

We thus find the key relation

$$A'_\mu(x) = -\int_{-\infty}^0 F_{\nu\sigma}(z) \frac{\partial z^\nu}{\partial x^\mu} \frac{\partial z^\sigma}{\partial \xi} d\xi. \tag{41}$$

Since the rhs of the above relation is expressed only in terms of the gauge-invariant field-strength tensor, the gauge invariance of $A'_\mu(x)$ is obvious. This is the essence of the gauge-invariant formulation of QED by DeWitt. There is a catch, however. Although the rhs of Eq. (41) is certainly gauge invariant, it generally depends on the path connecting the point x and spatial infinity. To see this most transparently, let us take constant-time paths in a given Lorentz frame, with the property $\partial z^0 / \partial \xi = 0$. In this case, Eq. (34) reduces to

$$\Lambda(x) = -\int_{-\infty}^x \boldsymbol{A}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z}. \tag{42}$$

Let us now consider two spacelike (or constant-time) paths L_1 and L_2 connecting x and spatial infinity [49]. The corresponding gauge-invariant electron fields are given by

$$\psi'(x; L_1) = \exp\left[-ie \int_{L_1}^x \boldsymbol{A}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi(x), \tag{43}$$

$$\psi'(x; L_2) = \exp\left[-ie \int_{L_2}^x \boldsymbol{A}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi(x). \tag{44}$$

These two gauge-invariant electron fields are related through

$$\psi'(x; L_1) = \exp\left[ie \left(\int_{L_1}^x - \int_{L_2}^x \right) \boldsymbol{A}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi'(x; L_2). \tag{45}$$

Closing the path of integration to a loop L by a connection at spatial infinity, where all fields and potentials are assumed to vanish, we obtain

$$\begin{aligned} \psi'(x; L_1) &= \exp\left[ie \oint_L \boldsymbol{A}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi'(x; L_2) \\ &= \exp\left[ie \iint_S (\nabla_z \times \boldsymbol{A}(x^0, \boldsymbol{z})) \cdot d\boldsymbol{z} \right] \psi'(x; L_2) \\ &= \exp\left[ie \iint_S \boldsymbol{B}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi'(x; L_2). \end{aligned} \tag{46}$$

Since the magnetic flux does not vanish in general, $\psi'(x; L_1)$ and $\psi'(x; L_2)$ do not coincide, which means that $\psi'(x)$ is generally *path dependent*.

Very interestingly, there is one particular choice of $\Lambda(x)$ which enables us to construct $\psi'(x)$ and $A'_\mu(x)$, which are path independent as well as gauge invariant [53,54]. The choice corresponds to taking

$$\Lambda(x) = -\int_{-\infty}^x \boldsymbol{A}_\parallel(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z}, \tag{47}$$

where $\boldsymbol{A}_\parallel(x)$ is the longitudinal component in the decomposition $\boldsymbol{A}(x) = \boldsymbol{A}_\perp(x) + \boldsymbol{A}_\parallel(x)$, with the important properties $\nabla \cdot \boldsymbol{A}_\perp = 0$, $\nabla \times \boldsymbol{A}_\parallel = 0$. Interestingly, since

$$\oint_L \boldsymbol{A}_\parallel(x^0, z) \cdot d\boldsymbol{z} = \iint_S (\nabla_z \times \boldsymbol{A}_\parallel(x^0, z)) \cdot d\boldsymbol{S} = 0 \quad (48)$$

due to the irrotational property of $\boldsymbol{A}_\parallel(x)$, the electron wave function defined by

$$\psi'(x) = \exp\left[-ie \int_{-\infty}^{x} \boldsymbol{A}_\parallel(x^0, z) \cdot d\boldsymbol{z}\right] \psi(x) \quad (49)$$

is not only gauge invariant but also path independent. We also recall the fact that the transverse and longitudinal components of $\boldsymbol{A}$ can be expressed as

$$\begin{aligned} \boldsymbol{A}_\perp(x) &= \boldsymbol{A}(x) - \nabla \frac{1}{\nabla^2} \nabla \cdot \boldsymbol{A}(x), \\ \boldsymbol{A}_\parallel(x) &= \nabla \frac{1}{\nabla^2} \nabla \cdot \boldsymbol{A}(x). \end{aligned} \quad (50)$$

Therefore, $\psi'(x)$ can be reduced to the following form:

$$\begin{aligned} \psi'(x) &= \exp\left[-ie \int_{-\infty}^{x} \left(\nabla_z \frac{1}{\nabla_z^2} \nabla_z \cdot \boldsymbol{A}(x^0, z)\right) \cdot d\boldsymbol{z}\right] \psi(x) \\ &= \exp\left[-ie \frac{\nabla \cdot \boldsymbol{A}}{\nabla^2}(x)\right] \psi(x). \end{aligned} \quad (51)$$

Note that, in this form, the path independence of $\psi'(x)$ is self-evident. We recall that this quantity is nothing but the gauge-invariant *physical electron* introduced by Dirac [55]. (For more discussion about this, we recommend Refs. [56,57].) Using the same function $\Lambda(x)$, the gauge-invariant potential $A'_\mu(x)$ can also be readily found as

$$\boldsymbol{A}'(x) = \boldsymbol{A}_\perp(x), \quad (52)$$

$$A'^0(x) = A^0(x) + \int_{-\infty}^{x} \dot{\boldsymbol{A}}_\parallel(x^0, z) \cdot d\boldsymbol{z}. \quad (53)$$

In this way, one reconfirms that the spatial component of the *gauge-invariant potential* $A'_\mu(x)$ is nothing but the transverse component of $\boldsymbol{A}(x)$.

We can show another interesting example in which we can define gauge-invariant electron and photon fields, which are also path independent at least formally. The construction begins with introducing a constant four-vector n^μ. By using it, we introduce the following decomposition of the photon field:

$$A_\mu = A_\mu^{\mathrm{phys}}(x) + A_\mu^{\mathrm{pure}}(x) \equiv (P_{\mu\nu} + Q_{\mu\nu}) A^\nu(x), \quad (54)$$

where

$$P_{\mu\nu} = g_{\mu\nu} - \frac{\partial_\mu n_\nu}{n \cdot \partial}, \quad (55)$$

$$Q_{\mu\nu} = \frac{\partial_\mu n_\nu}{n \cdot \partial}. \quad (56)$$

One can verify that the projection operators $P^{\mu\nu}$ and $Q^{\mu\nu}$ satisfy the identities

$$P_{\mu\lambda} P^\lambda{}_\nu = P_{\mu\nu}, \quad (57)$$

$$P_{\mu\lambda} Q^\lambda{}_\nu = Q_{\mu\lambda} P^\lambda{}_\nu = 0, \quad (58)$$

$$Q_{\mu\lambda} Q^\lambda{}_\nu = Q_{\mu\nu}. \quad (59)$$

The two components of the above decomposition satisfy the important properties

$$n^\mu A_\mu^{\mathrm{phys}}(x) = 0, \quad (60)$$

$$\partial_\mu A_\nu^{\mathrm{pure}}(x) - \partial_\nu A_\mu^{\mathrm{pure}}(x) = 0. \quad (61)$$

As can be easily verified, under a general Abelian gauge transformation $A_\mu(x) \to A_\mu(x) + \partial_\mu \omega(x)$, these two components respectively transform as

$$A_\mu^{\mathrm{phys}}(x) \to A_\mu^{\mathrm{phys}}(x), \quad (62)$$

$$A_\mu^{\mathrm{pure}}(x) \to A_\mu^{\mathrm{pure}}(x) + \partial_\mu \omega(x). \quad (63)$$

Now we propose taking

$$\Lambda(x) = -\int_{-\infty}^{0} A_\sigma^{\mathrm{pure}}(z) \frac{\partial z^\sigma}{\partial \xi} d\xi = -\int_{-\infty}^{x} A_\mu^{\mathrm{pure}}(z) dz^\mu, \quad (64)$$

and define the new electron and photon fields by Eqs. (32) and (33). Very interestingly, we can show that the line integral in the equation above is actually path independent. In fact, let us recall the Stokes' theorem in four-dimensional spacetime expressed as

$$\oint A_\mu(z) dz^\mu = \frac{1}{2} \int_S (\partial_\mu A_\nu - \partial_\nu A_\mu) d\sigma^{\mu\nu}, \quad (65)$$

where $d\sigma^{\mu\nu}$ is an infinitesimal area element tensor. Owing to the property (61), it holds that

$$\oint A_\mu^{\mathrm{pure}}(z) dz^\mu = 0. \quad (66)$$

Because of this fact, $\Lambda(x)$ defined by Eq. (64) is formally path independent, and can be expressed as

$$\begin{aligned} \Lambda(x) &= -\int_{-\infty}^{x} \frac{\partial_\mu^z n_\nu}{n \cdot \partial^z} A^\nu(z) dz^\mu \\ &= -\int_{-\infty}^{x} \partial_\mu^z \left\{\frac{n \cdot A(z)}{n \cdot \partial^z}\right\} dz^\mu = \frac{n \cdot A(x)}{n \cdot \partial}, \end{aligned} \quad (67)$$

where $\partial_\mu^z \equiv \frac{\partial}{\partial z^\mu}$, while $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$. The gauge-invariant electron and photon fields are therefore given by

$$\psi'(x) = e^{ie\Lambda(x)} \psi(x) = e^{ie\frac{n \cdot A(x)}{n \cdot \partial}} \psi(x), \quad (68)$$

$$A'_\mu(x) = A_\mu(x) + \partial_\mu \Lambda(x) = \left(g_{\mu\mu} - \frac{\partial_\mu n_\nu}{n \cdot \partial}\right) A^\nu(x)$$
$$= A^{\rm phys}_\mu(x). \tag{69}$$

One notices that the condition $n^\mu A^{\rm phys}_\mu = 0$ is nothing but the gauge-fixing condition projecting out the physical component of the gauge field in the framework of the general axial gauge.

Several remarks are in order here. The familiar gauge-fixing condition $n^\mu A_\mu = 0$ does not completely fix the gauge; that is, there still remain residual gauge degrees of freedom. The singular nature of the operator $1/(n \cdot \partial)$ is related to these residual degrees of freedom. How to treat this singularity is connected with what boundary condition is imposed for the gauge field at the infinity. Another concern is a generalization to the non-Abelian case. In the abelian case, we have seen that $A^{\rm phys}_\mu(x)$ and $A^{\rm pure}_\mu(x)$ defined by Eq. (54) supplemented with Eqs. (55) and (56) satisfy the desired gauge transformation properties. Unfortunately, the matter is not so simple in the non-Abelian gauge case. In this case, we need a more sophisticated method for projecting out the physical component of the gauge field, as discussed in the next section.

As is clear from the discussion above, except for some fortunate choices of $\Lambda(x)$, the fields $\psi'(x)$ and $A'_\mu(x)$ defined by Eqs. (32) and (33) supplemented with Eq. (34) are by construction gauge invariant but generally path dependent. How should we interpret this path dependence? Soon after the paper by DeWitt appeared [48], Belinfante conjectured that a "path" is just a "gauge" [49]. He showed that, by averaging over path-dependent potential over the directions of all straight lines at constant time converging to the point where the potential is to be calculated, one is led to the potential in the Coulomb gauge [49]. On the other hand, Rohrlich and Strocchi applied a similar averaging procedure over a covariant path and they obtained the potential in the Lorentz gauge [51]. It was also demonstrated by Yang that, for a simple quantum-mechanical system, the path dependence is eventually a reflection of the gauge dependence [52]. All these investigations appear to indicate that, if a quantity in question is seemingly gauge invariant but path dependent, it is not a gauge-invariant quantity in a *true* or *traditional* sense, which in turn indicates that it may not correspond to *genuine* observables. Clearly, the GIE approach is equivalent to the standard treatment of gauge theory only when its extension by means of a gauge link is path independent. By the standard treatment of the gauge theory, we mean the following. Start with a gauge-invariant quantity or expression. Fix the gauge according to the needs of the practical calculation. The answer should be independent of the gauge choice.

Now we come back to our original question. We are asking whether or not the gluon spin part in the longitudinal nucleon spin sum rule is a gauge-invariant quantity in a traditional. In principle, there are two ways to answer this question. The first is to show that the gauge-invariant longitudinal gluon spin operator can be constructed without recourse to the notion of "path." The second possibility is to adopt a gauge-invariant but generally path-dependent formulation at the beginning and then to show that the quantity of interest is actually path independent. In the following analysis, we take the second route and try to show the traditional gauge invariance of the evolution equation of the quark and gluon longitudinal spins in the nucleon.

III. GAUGE AND PATH INDEPENDENCE OF THE EVOLUTION MATRIX FOR QUARK AND GLUON LONGITUDINAL SPINS IN THE NUCLEON

A primary question we want to address in this section is whether the gluon spin term appearing in the longitudinal nucleon spin sum rule is a gauge-invariant quantity in a traditional sense or whether it is a quantity that has a meaning only in the light-cone gauge or in the gauge-invariant extension based on the light-cone gauge. We have already pointed out that—even for the Abelian case—the choice of gauge, the choice of Lorentz frame, and the transverse-longitudinal decomposition are all intrinsically intertwined. Moreover, an additional complexity arises in the case of non-Abelian gauge theory. The past studies have shown that—different from the Abelian gauge theory—even within the noncovariant treatment, the transverse component cannot be expressed in a closed form; that is, it can be given only in the form of a perturbation series in the gauge coupling constant [11,58]. Still, it remains true that the independent dynamical degrees of freedom of the massless vector field are two transverse components. In broad terms, one might say that the physics is contained in the transverse part of the gauge field. In the past, tremendous efforts have been made to figure out these physical components of the gauge field. DeWitt's formulation of the electrodynamics explained before is one typical example [48]. We realized that what is especially useful for our purpose is a slightly more sophisticated formulation proposed in the papers by Ivanov, Korchemsky, and Radyushkin [59,60]. It is based on the geometric interpretation of the gauge field actualized as a fiber-bundle formulation of gauge theories. (See Ref. [31] for a recent concise review on this geometrical formulation.) In this approach, the gauge field is identified with the connection of the principle fiber bundle $M(R^4, G)$ with the four-dimensional spacetime R^4 being its base space and with the fiber being the gauge group G. For the gauge field $A_\mu(x)$ and each element $g(x)$ of the fiber $G(x)$, one can define the gauge field configuration $A^g(x)$ by

$$A^g(x) = g^{-1}(x)\left(A_\mu(x) + \frac{i}{g}\partial_\mu\right) g(x). \tag{70}$$

Then, the set $\{A^g_\mu(x)\}$ for all $g(x)$ forms the gauge-equivalent field configurations called the orbits. For the quantization, one must choose a unique gauge orbit from infinitely many gauge-equivalent orbits. The most popular way of doing this is to impose an appropriate gauge-fixing condition $f(A^g, g) = 0$ by hand. However, the gauge-fixing condition $f(A^g, g) = 0$ sometimes does not have a unique solution beyond the perturbative regime [61]. A new method was then proposed, which is in principle free from the constraints of the perturbative gauge-fixing procedure. In this framework, the gauge function $g(x)$ is fixed as a solution of the parallel transport equation in the fiber-bundle space,

$$\frac{\partial z^\mu}{\partial s} D_\mu[A] g(z(s)) = 0, \tag{71}$$

where $D_\mu = \partial_\mu - ig A_\mu(z(s))$ is the covariant derivative, while $z(s)$ is a path C in the four-dimensional base space R^4 with the following boundary conditions:

$$z^\mu(s=1) = x^\mu, \qquad z^\mu(s=0) = x^\mu_0. \tag{72}$$

The solution to this equation is well known. It is expressed in terms of the Wilson line as

$$g(x) = W_C(x, x_0) g(x_0), \tag{73}$$

with

$$W_C(x, x_0) = P \exp\left[ig \int_{x_0}^{x} dz^\mu A_\mu(z) \right]. \tag{74}$$

Once $g(x)$ is given, $A^g_\mu(x)$ defined by Eq. (70) is uniquely specified. However, one should clearly keep in mind the fact that $A^g_\mu(x)$ so determined is generally dependent on the choice of the path C connecting x and x^0 (the starting point of the path). By substituting Eq. (74) into Eq. (70) and by using the derivative formula for the Wilson line, together with the identity $W_C^{-1}(x, y) = W_C(y, x)$, $A^g_\mu(x)$ can be expressed as

$$A^g_\mu(x) = A_\nu(x_0) \frac{\partial x^\nu_0}{\partial x^\mu} - \int_{x_0}^{x} dz^\nu \frac{\partial z^\rho}{\partial x^\mu} W_C(x_0, z) F_{\rho\nu}(z; A) W_C(z, x_0), \tag{75}$$

where $F_{\nu\rho}(z; A) \equiv \partial_\nu A_\rho(z) - \partial_\rho A_\nu(z) - ig[A_\nu(z), A_\rho(z)]$. The rhs of the above equation depends on the original gauge field A_μ, and on the starting point x_0 of the path, which in principle can depend on x. In the following, we take x_0 to be a unique point for all contours C, so that $\partial x^\nu_0 / \partial x^\mu = 0$.

With some natural constraints on the choice of the contours C, it was shown in Refs. [59,60] (see also Ref. [62]) that the above way of fixing the gauge is equivalent to taking gauges satisfying the condition

$$W_C(x, x_0) = P \exp\left[ig \int_{x_0}^{x} dz^\mu A^g_\mu(z) \right] = 1. \tag{76}$$

This class of gauges are called the *contour gauges*. An attractive feature of the contour gauge is that they are ghost-free. As specific examples of contour gauges, three gauges were briefly discussed. They are the Fock-Schwinger gauge, the Hamilton gauge, and the general axial gauge. Especially useful for our purpose here is the general axial gauge. The reason is that this is the most convenient gauge among the three for perturbative calculations. In the context of the geometrical approach, the axial gauge corresponds to taking the following infinitely long straight-line path:

$$z^\mu(s) = x^\mu + s n^\mu, \tag{77}$$

with $0 < s < \infty$, where n^μ is a constant four-vector characterizing the direction of the path. Substituting this form of $z^\mu(s)$ into Eq. (75), one obtains the relation between the transformed and original gauge fields as

$$A^g_\mu(x) = n^\nu \int_0^\infty ds W_C^\dagger(x + ns, \infty) \times F_{\mu\nu}(x + ns; A) W_C(x + ns, \infty), \tag{78}$$

with

$$W_C(x, \infty) = P \exp\left(ig \int_0^\infty ds n^\mu A_\mu(x + ns) \right). \tag{79}$$

Taking account of the antisymmetry of the field-strength tensor, $F_{\nu\mu} = -F_{\mu\nu}$, one can easily see that A^g_μ satisfies the identity

$$n^\mu A^g_\mu = 0. \tag{80}$$

Note that this is nothing but the gauge-fixing condition in the general axial gauge. Since n^μ is an arbitrary constant four-vector, this class of gauge contains several popular gauges. For instance, by choosing $n^\mu = (1, 0, 0, 0)$, $n^\mu = (1, 0, 0, 1)/\sqrt{2}$, and $n^\mu = (0, 0, 0, 1)$, we can cover any of the temporal gauge, the light-cone gauge, and the spatial axial gauge. Furthermore, using the property of the Wilson line

$$W_C^\dagger(x + ns, \infty) F_{\mu\nu}(x + ns; A) W_C(x + ns, \infty) = F_{\mu\nu}(x + ns; A^g), \tag{81}$$

Eq. (78) can also be expressed in an equivalent but simpler form as

$$A^g_\mu(x) = n^\nu \int_0^\infty ds F_{\mu\nu}(x + ns; A^g). \tag{82}$$

This identity represents the fact that, in the general axial gauge, the gauge potential A_μ can be expressed in terms of the field-strength tensor [25,63]. (Undoubtedly, Ivanov *et al.* correctly recognized the fact that the choice of a path in the geometrical formulation just corresponds to a gauge-fixing procedure. Note that this understanding is nothing different from the conclusion of Belinfante pointed out before that a "path" is just a "gauge.") With the identification $A_\mu^{\rm phys}(x) \equiv A_\mu^g(x)$, the above equation can then be thought of as a defining equation of the physical component $A_\mu^{\rm phys}(x)$ of the gluon field based on the general axial gauge. We emphasize that this defining equation itself is free from perturbation theory in the gauge coupling constant.

Since the main purpose of our present study is to show the perturbative gauge invariance of the gluon spin—or, more concretely, the traditional gauge invariance of the evolution equation of the quark and gluon longitudinal spins in the nucleon—let us look into the perturbative contents of the above equality (78), which can be interpreted as an equation projecting out the physical component of the gluon field $A_\mu^{\rm phys} \equiv A_\mu^g(x)$ from $A_\mu(x)$. At the lowest order in the gauge coupling constant, this gives

$$A_\mu^{\rm phys}(x) \simeq n^\nu \int_0^\infty ds(\partial_\mu A_\nu(x+ns) - \partial_\nu A_\mu(x+ns)). \tag{83}$$

Introducing the Fourier transform of $A_\mu(x)$,

$$\tilde{A}_\mu(k) = \int d^4x e^{-ikx} A_\mu(x), \tag{84}$$

we therefore get

$$\begin{aligned} A_\mu^{\rm phys}(x) &\simeq n^\nu \int_0^\infty ds \int \frac{d^4k}{(2\pi)^4} e^{ik(x+ns)}(ik_\mu \tilde{A}_\nu(k) - ik_\nu \tilde{A}_\mu(k)) \\ &= \int \frac{d^4k}{(2\pi)^4} e^{ikx}\left(g_{\mu\nu} - \frac{k_\mu n_\nu}{k\cdot n}\right)\tilde{A}^\nu(k) \\ &= \left(g_{\mu\nu} - \frac{\partial_\mu n_\nu}{n\cdot\partial}\right)A^\nu(x). \end{aligned} \tag{85}$$

Note that, although this is simply the lowest-order expression for the physical component for $A_\mu(x)$ in the case of non-Abelian gauge theory, it reproduces the exact one (69) in the Abelian case, discussed in the previous section. One can easily verify that this gives the lowest-order expression for the physical propagator of the gluon as

$$\langle T(A_{\mu,a}^{\rm phys}(x)A_{\nu,b}^{\rm phys}(x))\rangle^{(0)} = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{-i\delta_{ab}}{k^2+i\epsilon} P_{\mu\nu}(k), \tag{86}$$

with

$$P_{\mu\nu}(k) = g_{\mu\nu} - \frac{k_\mu n_\nu + n_\mu k_\nu}{k\cdot n} + \frac{n^2 k_\mu k_\nu}{(k\cdot n)^2}. \tag{87}$$

As anticipated, it just coincides with the free gluon propagator in the general axial gauge.

In this way, one finds that the path dependence or direction dependence of the constant four-vector n^μ in the geometrical formulation is replaced by the gauge dependence within the class of gauges called the general axial gauge. In this setting, then, the gluon spin operator reduces to $M_{G\text{-spin}}^{\lambda\mu\nu} = 2\,{\rm Tr}[F^{\lambda\nu}A^\mu - F^{\lambda\mu}A^\nu]$, where A_μ in this equation should be regarded as the gluon field satisfying the general axial gauge condition $n^\mu A_\mu = 0$.

Our strategy should be clear by now. We want to investigate the one-loop anomalous dimension for the quark and gluon longitudinal spin operators in the nucleon within the general axial gauge characterized by the four-vector n^μ. Since the general axial gauge falls into the category of the so-called noncovariant gauges, one must be careful about the fact that the choice of gauge and the choice of Lorentz frame are intrinsically intertwined. To understand this subtlety, it is instructive to remember the basis of the longitudinal momentum sum rule of the nucleon. The momentum sum rule of the nucleon is derived based on the following covariant relation:

$$\langle Ps|T_{\mu\nu}(0)|Ps\rangle = 2P_\mu P_\nu, \tag{88}$$

where $T_{\mu\nu}$ is the (symmetric) QCD energy-momentum tensor, while $|Ps\rangle$ is a nucleon state with momentum P and spin s. A useful technique for obtaining the momentum sum rule is to introduce a lightlike constant vector n^μ with $n^2 = 0$. By contracting Eq. (88) with n^μ and n^ν, we have

$$\frac{\langle Ps|n^\mu T_{\mu\nu}(0)n^\nu|Ps\rangle}{2(P^+)^2} = 1, \tag{89}$$

which provides us with a convenient basis for obtaining a concrete form of the momentum sum rule of QCD. However, since Eq. (88) itself is relativistically covariant, the above choice of n^μ is not the only choice. With the choice of arbitrary constant four-vector n^μ with $n^2 \neq 0$, we would have a more general relation,

$$\frac{\langle Ps|n^\mu T_{\mu\nu}(0)n^\nu - \frac{1}{4}n^2 T^\mu{}_\mu(0)|Ps\rangle}{2(P\cdot n)^2} = 1. \tag{90}$$

Here, since $n^2 \neq 0$, the subtraction of the trace term is obligatory.

Similarly, the starting point for obtaining the longitudinal nucleon spin sum rule is the following covariant relation:

$$\begin{aligned} &\langle Ps|M^{\lambda\mu\nu}(0)|Ps\rangle \\ &\quad = J_N \frac{P_\rho s_\sigma}{M_N^2}[2P^\lambda \epsilon^{\nu\mu\rho\sigma} - P^\mu \epsilon^{\lambda\nu\rho\sigma} - P^\nu \epsilon^{\mu\lambda\rho\sigma}], \end{aligned} \tag{91}$$

where $M^{\lambda\mu\nu}$ is the angular momentum tensor of QCD, while

$$P^2 = M_N^2, \qquad s^2 = -M_N^2, \qquad s\cdot P = 0, \tag{92}$$

with s_μ being a covariant spin-vector of the nucleon. Note that, without loss of generality, we can take

$P^\mu = (P^0, 0, 0, P^3)$ and $s^\mu = (P^3, 0, 0, P^0)$ with $P^0 = \sqrt{(P^3)^2 + M_N^2}$. The longitudinal nucleon spin sum rule can be obtained by setting $\mu = 1$, $\nu = 2$, which gives

$$\langle Ps|M^{\lambda 12}(0)|Ps\rangle = -2J_N \frac{1}{M_N^2} P^\lambda \epsilon^{12\rho\sigma} P_\rho s_\sigma = 2J_N P^\lambda. \tag{93}$$

Contracting this relation with an arbitrary constant four-vector n_λ, we therefore arrive at the basic equation of the longitudinal nucleon spin sum rule [1],

$$J_N = \frac{1}{2} = \frac{\langle Ps|n_\lambda M^{\lambda 12}(0)|Ps\rangle}{2(P\cdot n)}. \tag{94}$$

An important fact here is that the relations (90) and (94) are not covariant any more. The four-vector n^ν appearing in these equations should therefore be identified with the four-vector that characterizes the Lorentz frame, in which the gauge-fixing condition $n^\mu A_\mu = 0$ is imposed [64].

In the following, we shall confine ourselves to the intrinsic spin parts of quarks and gluons appearing in the nucleon spin decompositions [we recall the fact that they are just common in both decompositions (I) and (II)],

$$M^{\lambda\mu\nu}_{q\text{-spin}} = \frac{1}{2}\epsilon^{\lambda\mu\nu\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi, \tag{95}$$

$$M^{\lambda\mu\nu}_{G\text{-spin}} = 2\,\mathrm{Tr}[F^{\lambda\nu}A^\mu - F^{\lambda\mu}A^\nu]. \tag{96}$$

Here, the gauge fields appearing in $M^{\lambda\mu\nu}_{G\text{-spin}}$ should be regarded as the physical gluon field satisfying the general axial gauge condition $n^\mu A_\mu = 0$.

Generally, the gluon spin operator appearing in Eq. (93) consists of three pieces,

$$M^{\lambda 12}_{G\text{-spin}} = 2\,\mathrm{Tr}[F^{\lambda 1}A^2 - F^{\lambda 2}A^1] = V^\lambda_A + V^\lambda_B + V^\lambda_C, \tag{97}$$

where

$$V^\lambda_A = (\partial^\lambda A^1_a)A^2_a - (\partial^\lambda A^2_a)A^1_a, \tag{98}$$

$$V^\lambda_B = -[(\partial^1 A^\lambda_a A^2_a - (\partial^2 A^\lambda_a)A^1_a], \tag{99}$$

$$V^\lambda_C = g f_{abc} A^\lambda_b [A^1_c A^2_a - A^2_c A^1_a]. \tag{100}$$

Note, however, that the V^λ_B and V^λ_C terms do not contribute to the longitudinal nucleon spin sum rule (94), since $n_\lambda V^\lambda_B = n_\lambda V^\lambda_C = 0$, due to the gauge-fixing condition $n_\lambda A^\lambda = 0$. As a consequence, in the general axial gauge, only the V^λ_A term contributes to the longitudinal spin sum rule.

The momentum-space vertex for the gluon spin therefore reduces to the following simple form supplemented with the diagram illustrated in Fig. 1:

$$V^\lambda_A = 2ik^\lambda(g^{\mu 1}g^{\nu 2} - g^{\mu 2}g^{\nu 1})\delta_{ab}. \tag{101}$$

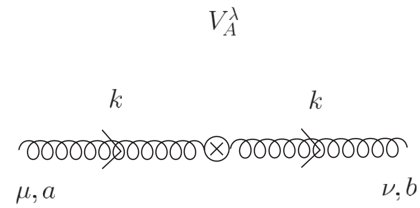

FIG. 1. Momentum-space vertices for the gluon spin.

Now we are ready to investigate the anomalous-dimension matrix for the longitudinal quark and gluon spins in the nucleon,

$$\Delta\gamma = \begin{pmatrix} \Delta\gamma_{qq} & \Delta\gamma_{qG} \\ \Delta\gamma_{Gq} & \Delta\gamma_{GG} \end{pmatrix}, \tag{102}$$

which controls the scale evolution of the quark and gluon spins. We start with the quark spin operator $M^{\lambda 12}_{q\text{-spin}}$, although there is no known problem in this part. The reason is that we want to see the independence of the final result regarding the choice of the constant four-vector n^μ, which specifies the Lorentz frame in which the gauge-fixing condition necessary for the quantization of the gluon field is imposed.

The anomalous dimension $\Delta\gamma_{qq}$ can be obtained by evaluating the matrix element of

$$2n_\lambda M^{\lambda 12}_{q\text{-spin}} = n_\lambda \epsilon^{\lambda 12\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi \tag{103}$$

in a longitudinally polarized quark state $|ps\rangle$ with $s = \pm 1$. The corresponding one-loop diagram is shown in Fig. 2. This gives

$$\begin{aligned} T_{qq} = \frac{1}{2p\cdot n}\int \frac{d^4k}{(2\pi)^4}\bar{u}(ps)ig\gamma^\nu t^a \frac{i(\not{p}-\not{k})}{(p-k)^2+i\varepsilon} \\ \times n_\lambda \epsilon^{\lambda 12\sigma}\gamma_\sigma\gamma_5 \frac{i(\not{p}-\not{k})}{(p-k)^2+i\varepsilon} ig\gamma^\mu t^b u(ps) \\ \times \delta_{ab} D_{\mu\nu}(k), \end{aligned} \tag{104}$$

where

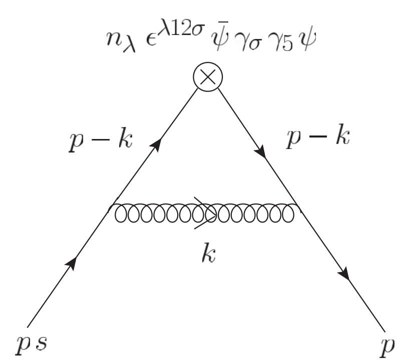

FIG. 2. The Feynman diagram contributing to $\Delta\gamma_{qq}$.

$$D_{\mu\nu}(k) = \frac{-i}{k^2 + i\varepsilon} P_{\mu\nu}(k), \tag{105}$$

with

$$P_{\mu\nu}(k) \equiv P^{\text{axial}}_{\mu\nu}(k) = g_{\mu\nu} - \frac{k_\mu n_\nu + n_\mu k_\nu}{k\cdot n} + \frac{n^2 k_\mu k_\nu}{(k\cdot n)^2}. \tag{106}$$

As is well known, this gluon propagator in the general axial gauge contains a spurious simple pole and also a double pole. In the following, let us evaluate the contributions of the three terms in $P^{\text{axial}}_{\mu\nu}(k)$ separately. The calculation of the part containing $g_{\mu\nu}$ is straightforward. After some Dirac algebra, we get

$$T_{qq}(g_{\mu\nu}) = -i\frac{g^2 C_F}{2p\cdot n}\times\left\{-8 n_\alpha p_\beta \int \frac{d^4k}{(2\pi)^4} \times \frac{k^\alpha k^\beta}{[(p-k)^2 + i\varepsilon]^2 (k^2 + i\varepsilon)} + 4p\cdot n \int \frac{d^4k}{(2\pi)^4}\frac{1}{[(p-k)^2+i\varepsilon]^2}\right\}. \tag{107}$$

Using the standard dimensional regularization with $D \equiv 2\omega$ spacetime dimensions, the divergent parts of the necessary integral are given by

$$\text{div}\int \frac{d^4k}{(2\pi)^4}\frac{k^\alpha k^\beta}{[(p-k)^2+i\varepsilon]^2(k^2+i\varepsilon)} = \frac{1}{4} g^{\alpha\beta}\bar{I}, \tag{108}$$

$$\text{div}\int \frac{d^4k}{(2\pi)^4}\frac{1}{[(p-k)^2+i\varepsilon]^2} = \bar{I}, \tag{109}$$

where

$$\bar{I} = \frac{i\pi^2}{2-\omega}. \tag{110}$$

We therefore obtain

$$T_{qq}(g_{\mu\nu}) = \frac{\alpha_S}{2\pi}\frac{1}{2} C_F \frac{1}{2-\omega}. \tag{111}$$

Next, we evaluate the term containing a simple spurious pole $1/(k\cdot n)$. After some algebra, we get

$$T_{qq}(1/(k\cdot n)) = -i\frac{g^2 C_F}{2p\cdot n}\left\{-p\cdot n \int\frac{d^4k}{(2\pi)^4} \times \frac{1}{[(p-k)^2+i\varepsilon]^2} + n^2 p_\beta \int \frac{d^4k}{(2\pi)^4}\frac{k^\beta}{[(p-k)^2+i\varepsilon]^2 k\cdot n}\right\}. \tag{112}$$

Now we encounter a Feynman integral containing a spurious pole. A consistent method for handling such Feynman integrals was first proposed by Mandelstam [65] and independently by Leibbrandt [66] in the light-cone gauge corresponding to the choice $n^2 = 0$. It is given as

$$\frac{1}{k\cdot n} \to \frac{1}{[k\cdot n]} \equiv \lim_{\varepsilon\to 0}\frac{k\cdot n^*}{k\cdot n k\cdot n^* + i\varepsilon}, \qquad (\varepsilon > 0), \tag{113}$$

where $n^*_\mu = (n_0, -\boldsymbol{n})$ is a dual four-vector to the four-vector $n_\mu = (n_0, \boldsymbol{n})$ with $n^2 = 0$ and $n^{*2} = 0$. [Practically, we can take $n^\mu = (n^0, 0, 0, n^3)$ and $n^{*\mu} = (n^0, 0, 0, -n^3)$ without loss of generality.] Later, Gaigg *et al.* showed that this prescription can be generalized to a more general case of $n^2 \neq 0$ and $n^{*2} \neq 0$ [67,68]. (For review, see Refs. [69,70].) In this generalized n^*_μ prescription, the divergent part of the above integral is given by

$$\int d^{2\omega}k \frac{k^\beta}{[(p-k)^2+i\varepsilon]^2[k\cdot n]} = \frac{1}{D}\left(n^{*\beta} - \frac{n^{*2}}{n^*\cdot n + D} n^\beta\right)\bar{I}, \tag{114}$$

where D is defined by

$$D \equiv \sqrt{(n^*\cdot n)^2 - n^{*2} n^2}. \tag{115}$$

By using this result, the divergent part of $T_{qq}(1/(k\cdot n))$ becomes

$$T_{qq}(1/(k\cdot n)) = \frac{\alpha_S}{4\pi} C_F \frac{1}{2-\omega}\left\{-4 + 2\frac{n^2}{[p\cdot n]} \times \frac{1}{D}\left(p\cdot n^* - \frac{n^{*2}}{n^*\cdot n + D} p\cdot n\right)\right\}. \tag{116}$$

The contribution of the part containing a spurious double-pole structure $1/(k\cdot n)^2$ can similarly be calculated. We get

$$T_{qq}(1/(k\cdot n)^2) = -i g^2 C_F n^2 \int \frac{d^4k}{(2\pi)^4} \times \frac{k^2}{[(p-k)^2+i\varepsilon]^2 (k\cdot n)^2}. \tag{117}$$

Using the generalized n^*_μ prescription again, the divergent part of the relevant integral is given by

$$\text{div}\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{[(p-k)^2+i\varepsilon]^2[k\cdot n]^2} = \frac{2}{D}\frac{n^{*2}}{n^*\cdot n + D}\bar{I}. \tag{118}$$

We therefore obtain

$$T_{qq}(1/(k\cdot n)^2) = -2\frac{\alpha_S}{4\pi} C_F\left(1 - \frac{n\cdot n^*}{D}\right)\frac{1}{2-\omega}. \tag{119}$$

Here, use has been made of the identity

$$\frac{n^2 n^{*2}}{n^*\cdot n + D} = n\cdot n^* - D. \tag{120}$$

By summing the three terms, we arrive at

$$T_{qq} = -\frac{\alpha_S}{4\pi} C_F \frac{1}{2-\omega} - \frac{\alpha_S}{4\pi} 2C_F \frac{1}{[p\cdot n]} \frac{1}{D} [p\cdot n n\cdot n^* - p\cdot n^* n^2] \frac{1}{2-\omega} - \frac{\alpha_S}{4\pi} 2C_F \left(1 - \frac{n\cdot n^*}{D}\right) \frac{1}{2-\omega}. \quad (121)$$

At this stage, it is instructive to consider several special choices of n^μ. The light-cone gauge choice corresponds to taking $n^0 = n^3 = 1/\sqrt{2}$. In this case, we have

$$p\cdot n = p^+, \quad p\cdot n^* = p^-, \quad n\cdot n^* = 1, \quad n^2 = 0, \quad (122)$$

and

$$D = 1, \quad (123)$$

so that we find

$$T_{qq}(LC) = -\frac{\alpha_S}{2\pi} \frac{3}{2} C_F \frac{1}{2-\omega}. \quad (124)$$

This legitimately reproduces the answer first obtained by Ji, Tang, and Hoodbhoy in the light-cone gauge [40].

Another interesting choice is the temporal-gauge limit specified by $n^0 = 1$ and $n^3 = 0$. In this limit, we have

$$p\cdot n = p\cdot n^* = p^0, \quad n\cdot n^* = 1, \quad n^2 = 1, \quad (125)$$

and

$$D = 0. \quad (126)$$

We therefore find that the coefficients of $1/(2-\omega)$ in the second and third term of T_{qq} *diverge*. The temporal-gauge limit is *singular* in this respect. However, for obtaining the anomalous dimension $\Delta\gamma_{qq}$, we must also take account of the self-energy insertion in the external quark lines. The contribution of these diagrams can be easily obtained by using the known result for the one-loop quark self-energy in the general axial gauge. (See, for instance, Ref. [69]). We get

$$T_{qq}^{\text{Self}} = \frac{\alpha_S}{4\pi} C_F \frac{1}{2-\omega} + \frac{\alpha_S}{4\pi} 2C_F \frac{1}{[p\cdot n]} \frac{1}{D} [p\cdot n n\cdot n^* - p\cdot n^* n^2] \frac{1}{2-\omega} + \frac{\alpha_S}{4\pi} 2C_F \left(1 - \frac{n\cdot n^*}{D}\right) \frac{1}{2-\omega}. \quad (127)$$

As anticipated, this exactly cancels T_{qq} obtained above, thereby leading to the standardly known answer, i.e.,

$$\Delta\gamma_{qq} = 0. \quad (128)$$

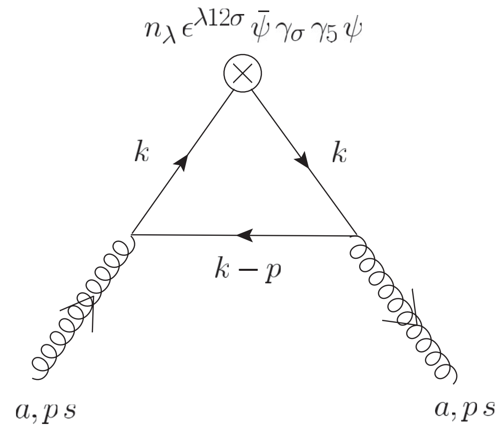

FIG. 3. The Feynman diagram contributing to $\Delta\gamma_{qG}$.

It is important to recognize that this final result is obtained totally independently of the choice of the four-vector n^μ.

The relevant Feynman diagram contributing to the anomalous dimension $\Delta\gamma_{qG}$ is illustrated in Fig. 3. Since no internal gluon propagator appears in this diagram, we do not need to repeat the standard manipulation. One can easily verify that

$$\Delta\gamma_{qG} = 0. \quad (129)$$

Next, we turn to the anomalous dimension $\Delta\gamma_{Gq}$. The relevant one-loop Feynman diagram is shown in Fig. 4. The contribution of the vertex V_A in the gluon spin operator is given by

$$T_{Gq}^A = \frac{1}{2p\cdot n} \int \frac{d^4k}{(2\pi)^4} \bar{u}(ps) i g \gamma^{\nu'} t^d \frac{i(\not{p} - \not{k})}{(p-k)^2 + i\varepsilon} \times \delta^{de} 2i(k\cdot n)[g^{\mu 1} g^{\nu 2} - g^{\mu 2} g^{\nu 1}] \times \delta^{bc} i g \gamma^{\mu'} t^e u(ps) \delta^{bd} D_{\mu\mu'}(k) \delta^{ce} D_{\nu\nu'}(k), \quad (130)$$

where $D_{\mu\mu'}(k)$ and $D_{\nu\nu'}(k)$ are gluon propagators in the general axial gauge excluding trivial color-dependent parts. This gives

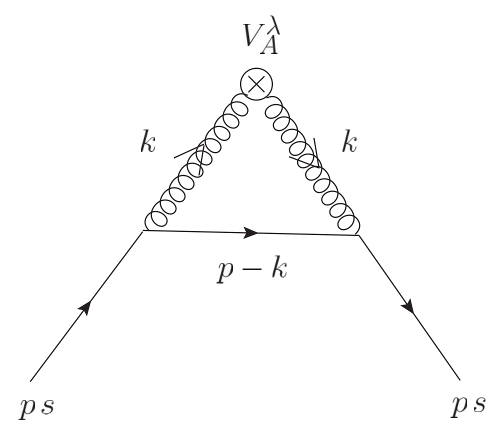

FIG. 4. The Feynman diagram contributing to $\Delta\gamma_{Gq}$.

$$
\begin{aligned}
T^A_{Gq} &= -\frac{g^2 C_F}{p\cdot n}\int \frac{d^4k}{(2\pi)^4}\frac{k\cdot n}{(k^2+i\varepsilon)^2[(k-p)^2+i\varepsilon]} \\
&\quad\times \bar{u}(ps)\gamma^{\nu'}(\not{p}-\not{k})\gamma^{\mu'}u(ps)(g^{\mu 1}g^{\nu 2}-g^{\mu 2}g^{\nu 1}) \\
&\quad\times P_{\mu\mu'}(k)P_{\nu\nu'}(k) \\
&= 2i\frac{g^2 C_F}{p\cdot n}\int \frac{d^4k}{(2\pi)^4}\frac{k\cdot n}{(k^2+i\varepsilon)^2[(k-p)^2+i\varepsilon]} \\
&\quad\times (g^{\mu 1}g^{\nu 2}-g^{\mu 2}g^{\nu 1})\epsilon^{\mu'\nu'\alpha\beta}k_\alpha p_\beta P_{\mu\mu'}(k)P_{\nu\nu'}(k).
\end{aligned}
\tag{131}
$$

After some algebra, we obtain

$$
\begin{aligned}
T^A_{Gq} = -4i\frac{g^2C_F}{p\cdot n}\bigg\{ & p_\mu n_\nu \int\frac{d^4k}{(2\pi)^4}\frac{k^\mu k^\nu}{(k^2+i\varepsilon)^2[(k-p)^2+i\varepsilon]} \\
& - p\cdot n\int\frac{d^4k}{(2\pi)^4}\frac{k_\perp^2}{(k^2+i\varepsilon)^2[(k-p)^2+i\varepsilon]}\bigg\},
\end{aligned}
\tag{132}
$$

with $k_\perp^2 \equiv k_1^2 + k_2^2$. Evaluating its divergent part by dimensional regularization, we get

$$
T^A_{Gq} = \frac{\alpha_S}{2\pi}\frac{3}{2}C_F\frac{1}{2-\omega}. \tag{133}
$$

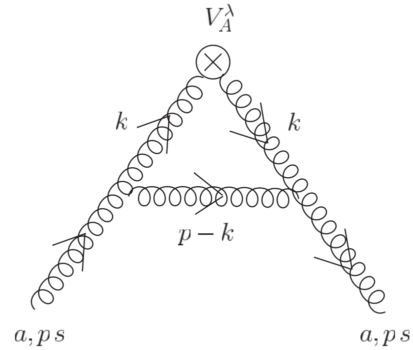

FIG. 5. The Feynman diagrams contributing to $\Delta\gamma_{GG}$.

In this way, we arrive at the standardly known answer for $\Delta\gamma_{Gq}$ given by

$$
\Delta\gamma_{Gq} = \frac{\alpha_S}{2\pi}\frac{3}{2}C_F. \tag{134}
$$

Now we are in a position to investigate the most nontrivial part of our analysis, i.e., the anomalous dimension $\Delta\gamma_{GG}$. The contribution of the vertex V_A is given by the Feynman diagram illustrated in Fig. 5. This gives

$$
\begin{aligned}
T^A_{GG} = \frac{1}{2p\cdot n}\int & \frac{d^4k}{(2\pi)^4}\epsilon^{\lambda *}(ps)\epsilon^\rho(ps) \\
&\times g f^{ac'e}[-(p+k)_\sigma g_{\lambda\nu'} + (2k-p)_\lambda g_{\sigma\nu'} + (2p-k)_{\nu'}g_{\lambda\sigma}] \\
&\times 2i(k\cdot n)[g^{\mu 1}\delta^{\nu 2}(k) - g^{\mu 2}\delta^{\nu 1}(k)]\delta^{bc} \\
&\times g f^{ab'd}[(p+k)_\tau g_{\rho\mu'} + (p-2k)_\rho g_{\mu'\tau} + (k-2p)_{\mu'}g_{\rho\tau}] \\
&\times \delta^{cc'}D^{\nu\nu'}(k)\delta^{bb'}D^{\mu\mu'}(k)\delta^{de}D^{\tau\sigma}(p-k).
\end{aligned}
\tag{135}
$$

This can be rewritten in the form

$$
\begin{aligned}
T^A_{GG} = +\frac{g^2C_A}{p\cdot n}\int & \frac{d^4k}{(2\pi)^4}\frac{k\cdot n}{(k^2+i\varepsilon)^2[(p-k)^2+i\varepsilon]}\epsilon^{\lambda *}(ps)\epsilon^\rho(ps) \\
&\times [(p+k)_\sigma g_{\lambda\nu'} + (p-2k)_\lambda g_{\sigma\nu'} + (k-2p)_{\nu'}g_{\lambda\sigma}] \\
&\times [(p+k)_\tau g_{\rho\mu'} + (p-2k)_\rho g_{\mu'\tau} + (k-2p)_{\mu'}g_{\rho\tau}] \\
&\times (g^{\mu 1}g^{\nu 2} - g^{\mu 2}g^{\nu 1})P_\nu{}^{\nu'}(k)P_\mu{}^{\mu'}(k)P^{\tau\sigma}(p-k),
\end{aligned}
\tag{136}
$$

where $C_A = f^{abc}f^{abc} = 3$ is the standard color factor. After tedious but straightforward algebra, T^A_{GG} can further be rewritten in the form

$$
\begin{aligned}
T^A_{GG} = +C_A\frac{g^2}{p\cdot n}\int & \frac{d^4k}{(2\pi)^4}\frac{k\cdot n}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon]}\epsilon^{\lambda *}(ps)\epsilon^\rho(ps) \\
\times\bigg\{ & (g^{\mu' 1}g^{\nu' 2} - g^{\mu' 2}g^{\nu' 1}) - (k^1g^{\nu' 2} - k^2g^{\nu' 1})\frac{n^{\mu'}}{k\cdot n} + (k^1g^{\mu' 2} - k^2g^{\mu' 1})\frac{n^{\nu'}}{k\cdot n} + (k^1g^{\nu' 2} - k^2g^{\nu' 1})\frac{n^2k^{\mu'}}{(k\cdot n)^2} \\
& - (k^1g^{\mu' 2} - k^2g^{\mu' 1})\frac{n^2k^{\nu'}}{(k\cdot n)^2}\bigg\}[\epsilon^*_{\nu'}(p+k)_\sigma - 2\epsilon^*\cdot k g_{\sigma\nu'} + \epsilon^*_\sigma(k-2p)_{\nu'}] \\
\times & [\epsilon_{\mu'}(p+k)_\tau - 2\epsilon\cdot k g_{\tau\mu'} + \epsilon_\tau(k-2p)_{\mu'}]\bigg[g^{\tau\sigma} - \frac{(k-p)^\tau n^\sigma + n^\tau(k-p)^\sigma}{(k-p)\cdot n} + \frac{n^2(k-p)^\tau(k-p)^\sigma}{[(k-p)\cdot n]^2]}\bigg].
\end{aligned}
\tag{137}
$$

We shall again calculate the three contributions from $P^{\tau\sigma}(k)$ separately. The part containing the tensor $g^{\tau\sigma}$ reduces to

$$T^{A}_{GG}(g^{\tau\sigma}) = T^{A_1}_{GG}(g^{\tau\sigma}) + T^{A_2}_{GG}(g^{\tau\sigma}), \qquad (138)$$

where

$$T^{A_1}_{GG}(g^{\tau\sigma}) = -iC_A \frac{g^2}{p\cdot n}\int \frac{d^4k}{(2\pi)^4} \times \frac{k\cdot n(p+k)^2 - 8p\cdot nk_\perp^2}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon]}, \qquad (139)$$

and

$$T^{A_2}_{GG}(g^{\tau\sigma}) = 2iC_A \frac{g^2}{p\cdot n} n^2 \int \frac{d^4k}{(2\pi)^4} \times \frac{k_\perp^2(k^2 - 3p\cdot k)}{(k^2+i\varepsilon)^2[(k-p)^2+i\varepsilon](k\cdot n)}. \qquad (140)$$

The first part, which does not contain the $1/(k\cdot n)$-type spurious singularity, can be calculated in a standard manner, which gives

$$T^{A_1}_{GG}(g^{\tau\sigma}) = +\frac{\alpha_S}{2\pi}\frac{5}{2}C_A\frac{1}{2-\omega}. \qquad (141)$$

The second part can be evaluated by using the formulas

$$\mathrm{div}\int d^{2\omega}k \frac{k_\perp^2}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon][k\cdot n]} = -\frac{1}{D}\left(p\cdot n^* - \frac{n^{*2}}{n^*\cdot n + D}p\cdot n\right)\bar{I}, \qquad (142)$$

$$\mathrm{div}\int d^{2\omega}k \frac{k^\mu k_\perp^2}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon][k\cdot n]} = -\frac{1}{2D}\left(n^{*\mu} - \frac{n^{*2}}{n^*\cdot n + D}n^\mu\right)\bar{I}. \qquad (143)$$

The answer is given as

$$T^{A_2}_{GG}(g^{\tau\sigma}) = -\frac{\alpha_S}{2\pi}\frac{1}{2}C_A\frac{n^2}{D}\frac{1}{[p\cdot n]} \times \left(p\cdot n^* - \frac{n^{*2}}{n^*\cdot n + D}p\cdot n\right)\frac{1}{2-\omega}. \qquad (144)$$

Collecting the two pieces, we thus arrive at

$$T^{A}_{GG}(g^{\tau\sigma}) = +\frac{\alpha_S}{2\pi}\frac{5}{2}C_A\frac{1}{2-\omega} - \frac{\alpha_S}{2\pi}\frac{1}{4}C_A\frac{n^2}{D}\frac{1}{[p\cdot n]} \times \left(p\cdot n^* - \frac{n^{*2}}{n^*\cdot n + D}p\cdot n\right)\frac{1}{2-\omega}. \qquad (145)$$

Next, we evaluate the term containing the spurious singularity of $1/(k-p)\cdot n$ in $P^{\tau\sigma}(k-p)$. After lengthy algebra, we obtain

$$\begin{aligned} T^{A}_{GG}(1/(k-p)\cdot n) &= -iC_A\frac{g^2}{p\cdot n}\Big\{-2\int\frac{d^4k}{(2,\pi)^4}\frac{k\cdot n}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon]} \\ &\quad - 4p\cdot n\int\frac{d^4k}{(2,\pi)^4}\frac{k\cdot n}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon](k-p)\cdot n} \\ &\quad + 2n^2\int\frac{d^4k}{(2,\pi)^4}\frac{k_\perp^2}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon](k-p)\cdot n} \\ &\quad - n^2\int\frac{d^4k}{(2,\pi)^4}\frac{k_\perp^2}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon]k\cdot n}\Big\}. \end{aligned} \qquad (146)$$

Using the known integral formulas

$$\mathrm{div}\int d^{2\omega}k\frac{k\cdot n}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon][(k-p)\cdot n]} = \bar{I}, \qquad (147)$$

$$\mathrm{div}\int d^{2\omega}k\frac{k_\perp^2}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon][(k-p)\cdot n]} = \frac{1}{D}\left(p\cdot n^* - \frac{n^{*2}}{n^*\cdot n + D}p\cdot n\right)\bar{I}, \qquad (148)$$

$$\mathrm{div}\int d^{2\omega}k\frac{k_\perp^2}{(k^2+i\varepsilon)[(k-p)^2+i\varepsilon][k\cdot n]} = -\frac{1}{D}\left(p\cdot n^* - \frac{n^{*2}}{n^*\cdot n + D}p\cdot n\right)\bar{I}, \qquad (149)$$

we find that

$$T^{A}_{GG}(1/(k-p)\cdot n) = \frac{\alpha_S}{2\pi}\left(-\frac{5}{2}C_A\right)\frac{1}{2-\omega} + \frac{\alpha_S}{2\pi}\frac{3}{2}C_A\frac{n^2}{D}\frac{1}{[p\cdot n]} \times \left(p\cdot n^* - \frac{n^{*2}}{n^*\cdot n + D}p\cdot n\right)\frac{1}{2-\omega}. \qquad (150)$$

Finally, we evaluate the contribution of the spurious double-pole term $1/[(k-p)\cdot n]^2$ in $P^{\tau\sigma}(k-p)$. After some algebra, we obtain

$$T^{A}_{GG}(1/[(k-p)\cdot n]^2) = -iC_A\frac{g^2}{p\cdot n}n^2\int\frac{d^4k}{(2\pi)^4} \times \frac{k\cdot n}{[(k-p)^2+i\varepsilon][(k-p)\cdot n]^2}. \qquad (151)$$

Now, by using the integral formula

$$\mathrm{div}\int d^{2\omega}k\frac{k^\mu}{[(k-p)^2+i\varepsilon][(k-p)\cdot n]^2} = p^\mu\frac{2}{D}\frac{n^{*2}}{n^*\cdot n + D}\bar{I}, \qquad (152)$$

we obtain

$$T_{GG}^{A}(1/[(k-p)\cdot n]^{2})=\frac{\alpha_{S}}{2\pi}C_{A}\frac{1}{D}\frac{n^{2}n^{*2}}{n^{*}\cdot n+D}\frac{1}{2-\omega}. \tag{153}$$

Summing up the three contributions, we finally arrive at

$$T_{GG}^{A}=\frac{\alpha_{S}}{2\pi}C_{A}\frac{n^{2}}{D}\frac{p\cdot n^{*}}{[p\cdot n]}\frac{1}{2-\omega}. \tag{154}$$

Again, it is instructive to consider several limiting cases. In the light-cone limit with $n^0 = n^3 = 1/\sqrt{2}$ and $n^2 = 0$, one sees that T_{GG}^{A} above vanishes. This is consistent with the direct calculation in the light-cone gauge [40]. On the other hand, in the temporal limit with $n^0 = 1, n^3 = 0$, and $n^2 = 1$, the coefficient of $1/(2-\omega)$ *diverges*, since $D \to 0$ in this limit. However, for obtaining the anomalous dimension $\Delta\gamma_{GG}$, we must also take account of the self-energy insertion in the external gluon lines. The contribution of these diagrams turn out to be (see, for instance, Refs. [67,69])

$$T_{GG}^{\rm Self}=\frac{\alpha_{S}}{2\pi}\left(\frac{11}{6}C_{A}-\frac{1}{3}n_{f}\right)\frac{1}{2-\omega} -\frac{\alpha_{S}}{2\pi}C_{A}\frac{n^{2}}{D}\frac{p\cdot n^{*}}{[p\cdot n]}\frac{1}{2-\omega}. \tag{155}$$

One finds that the dangerous terms in T_{GG} and $T_{GG}^{\rm Self}$ cancel exactly, thereby leading to

$$T_{GG}^{A}+T_{GG}^{\rm Self}=\frac{\alpha_{S}}{2\pi}\left(\frac{11}{6}C_{A}-\frac{1}{3}n_{f}\right)\frac{1}{2-\omega}, \tag{156}$$

which gives

$$\Delta\gamma_{GG}=\frac{\alpha_{S}}{2\pi}\left(\frac{11}{6}C_{A}-\frac{1}{3}n_{f}\right). \tag{157}$$

In this way, we have succeeded in reproducing the well-known answer completely independently of the choice of the four-vector n^μ, which is interpreted to characterize the Lorentz frame in which the gauge-fixing condition is imposed. The flexibility of our treatment regarding the choice of the four-vector n^μ enables us to handle several interesting cases in a unified way with the help of the generalized n^*_μ prescription. They include the temporal-gauge limit with $n^2 = 1$, the light-cone gauge limit with $n^2 = 0$, and the spatial axial-gauge limit with $n^2 = -1$. We have shown that the temporal-gauge limit should be treated with special care, because singular terms appear in the course of manipulation. Nevertheless, after summing up all the relevant contributions, dangerous singular terms cancel among themselves and the final answer is shown to be the same in all the cases. As we have shown, since these three different gauges belonging to the general axial gauge can also be connected with different choices of path in the geometric formulation, what we have shown is also interpreted as the path independence of the longitudinal gluon spin, although within a restricted class of path choices. Undoubtedly, this is a gauge invariance in a traditional sense.

Before ending this section, we make several supplementary remarks on the significance of our finding above. In the previous paper [17], we gave a formal proof that the quark and gluon dynamical OAMs appearing in our nucleon spin decomposition (I) can be related to the difference between the second moment of the unpolarized generalized parton distributions (GPDs) and the first moment of the longitudinally polarized parton distribution functions (PDFs) as

$$\begin{aligned} L_q &= \langle ps|n_\lambda M^{\lambda 12}_{q\text{-OAM}}|ps\rangle/(n\cdot p) \\ &= \frac{1}{2}\int x[H^q(x,0,0)+E^q(x,0,0)]dx-\frac{1}{2}\int\Delta q(x)dx \end{aligned} \tag{158}$$

and

$$\begin{aligned} L_G &= \langle ps|n_\lambda M^{\lambda 12}_{G\text{-OAM}}|ps\rangle/(n\cdot p) \\ &= \frac{1}{2}\int x[H^g(x,0,0)+E^g(x,0,0)]dx-\int\Delta g(x)dx. \end{aligned} \tag{159}$$

As is widely known, Eq. (158) was first derived by Ji. The relation (159) was also written down by Ji, but as an *ad hoc* definition of the gluon orbital angular momentum. This is because his viewpoint was that the decomposition (159) is not a *truely* gauge-invariant one. It would be instructive to reconsider these relations in the context of the gauge-invariant-extension approach using the gauge link or Wilson line. It is widely accepted that the gauge-invariant definitions of the GPDs as well as the polarized PDFs necessarily require the gauge link connecting two different spacetime points. However, the quantities appearing on the rhs of the above relations are not GPDs and PDFs themselves but their lower moments. In fact, the above relations can also be expressed as [4]

$$L_q=\frac{1}{2}[A_{20}^q(0)+B_{20}^q(0)]-\frac{1}{2}a^q(0), \tag{160}$$

$$L_G=\frac{1}{2}[A_{20}^G(0)+B_{20}^G(0)]-a^G(0). \tag{161}$$

Here, $A_{20}^q(0)$, $B_{20}^q(0)$, $A_{20}^G(0)$, and $B_{20}^G(0)$ are the forward limit ($t \to 0$) of the gravitational form factors $A_{20}^q(t)$, $B_{20}^q(t)$, $A_{20}^G(t)$, and $B_{20}^G(t)$, while $a^q(0)$ and $a^G(0)$ are the axial charges of quarks and gluons corresponding to the forward limits of the axial form factors $a^q(t)$ and $a^G(t)$. [We recall that the quark and gluon axial charges are identified with the quark and gluon intrinsic spins in the gauge-invariant $\overline{\rm MS}$ regularization scheme, i.e., $a^q(0) = \Delta\Sigma$ and $a^G(0) = \Delta G$.] Note that, to extract the form factors, deep inelastic scattering measurements are not mandatory. For example, the gravitational form factors can in principle be extracted from graviton-nucleon elastic scattering just as the electromagnetic form factors can be extracted from electron-nucleon elastic scatterings, even though this is just a Gedanken experiment. This means that, at least for these quantities, i.e., for the form factors, we do not need to stick to such an idea that the path of the gauge link has a physical meaning as claimed in the gauge-invariant-extension approach. In fact, we have explicitly demonstrated the path independence of the evolution matrix for the quark and gluon spins, although within a restricted class of choices called the general axial gauges specified by the direction of the infinitely long path. This indicates that at least the above relations (158) and (159) are not affected by a continuous deformation of the path of Wilson lines used in the definitions of the GPDs and the polarized PDFs.

Also worth remembering is the following well-known but sometimes unregarded fact. Why does one not need to pay much attention to the notion of the path dependence of the Wilson line in the case of the standard collinear PDFs? For clarity, let us first consider the simplest leading-twist PDF, i.e., the unpolarized PDF. The modern way of defining the unpolarized quark distribution function is to use the bilinear quark operator with light-cone separation. The nonlocal Wilson line is necessary here to ensure the gauge invariance of the bilocal quark operator. However, this definition of the PDF is known to be completely equivalent to the one based on the operator product expansion (OPE). That is, the bilinear and bilocal quark field with a Wilson line is equivalent to the infinite tower of local and gauge-invariant operators with higher covariant derivatives. Since these infinite towers of gauge-invariant operators are just local operators (although with higher derivatives), they are free from the notion of path, i.e., they are independent of a particular direction in space and time. The situation is simply the same for other PDFs. Within the framework of the OPE, the gluon distribution can also be defined in terms of infinite towers of local and gauge-invariant operators. Namely, within the framework of the OPE, they can be defined without using the notion of paths. (Only one important exception is the gluon spin operator corresponding to the first moment of the longitudinally polarized gluon distribution functions discussed in the present paper. The long-known worrying fact was that—as long as one sticks to the locality—there is no twist-two spin-one gauge-invariant gluon operator.) For this reason, the notion of the path dependence of the Wilson line has seldom been an issue, at least in the case of collinear PDFs. The same can also be said for the GPDs appearing in the sum rules (158) and (159).

Unfortunately, such a simplification cannot be expected for the transverse-momentum-dependent PDFs or more general Wigner distributions. This is the reason why the status for another gauge-invariant decomposition (II) is still unclear. In fact, a very interesting relation between the OAMs and Wigner distributions was first suggested by Lorcé and Pasquini [71]. However, the gauge-invariant definition of the Wigner distribution requires a gauge link or Wilson line, which is generally path dependent. Hatta showed that the light-cone-like path choice gives "canonical" OAM [29]. On the other hand, Ji, Xiong, and Yuan argued that the straight path connecting the relevant two spacetime points gives "dynamical" OAM [33]. Assuming that both are correct, one might be led to two possible scenarios. The first possibility is that, because there are infinitely many paths connecting the two relevant spacetime points appearing in the gauge-invariant definition of the Wigner distribution, there are infinitely many Wigner distributions and consequently infinitely many quark and gluon OAMs. The second possibility is that the Wigner distributions with infinitely many paths of gauge link are classified into some discrete pieces or equivalent classes, which cannot be continuously deformable into each other. The recent consideration by Burkardt may be thought of as an indication of this second possibility [72]. At any rate, it would be fair to say that, at least up to now, we do not have any convincing answer to the question of the real observability of the nucleon spin decomposition (II).

IV. CONCLUSION

We have investigated the uniqueness or nonuniqueness problem of the decomposition of the gluon field into the physical and pure-gauge components, which is the basis of the recently proposed two physically inequivalent gauge-invariant decompositions of the nucleon spin. It was emphasized that the physical motivation of this decomposition is the familiar transverse-longitudinal decomposition in QED, which is known to be unique once the Lorentz frame of reference is fixed. In the case of non-Abelian gauge theory, this transverse-longitudinal decomposition becomes a little more nontrivial even in the noncovariant treatment. In fact, past studies have revealed the fact that the transverse component of the non-Abelian gauge field can be expressed only in a perturbation series in the gauge coupling constant. Nevertheless, it is very important to recognize the fact that to project out the physical component of the gauge field is essentially equivalent to the process of gauge fixing. In fact, in the geometrical formulation of the non-Abelian gauge theory, a closed form of the physical component of the gauge field is known, although it requires the nonlocal Wilson line, depending on a path in the four-dimensional spacetime. It is also known that a choice of path is inseparably connected with a choice of gauge. An especially useful choice for our purpose of defining a gauge-invariant gluon spin operator is an infinitely long straight-line path connecting the spacetime point of the gauge field and the spacetime infinity, the direction of which is characterized by a constant four-vector. This particular choice of path is known to be equivalent to taking the so-called general axial gauge, which contains three popular gauges, i.e., the temporal, the light-cone, and the spatial axial gauges. Based on this general axial gauge, characterized by the constant four-vector n^μ, we have calculated the one-loop anomalous-dimension matrix for the quark and gluon longitudinal spins in the nucleon. We then found that the final answer is exactly the same independent of the choice of n^μ, which amounts to proving the gauge independence and the path independence simultaneously. After all, what we have explicitly shown is only the perturbative gauge and path independence of the gluon spin. Nevertheless, our general argument offers strong counterevidence to the idea that there are infinitely many decompositions of the nucleon spin. It also gives support to our claim that the total angular momentum of the gluon can be gauge-invariantly decomposed into the orbital and intrinsic spin parts as long as the longitudinal spin sum rule of the nucleon is concerned. This means that the dynamical OAMs of quarks and gluons appearing in our decomposition (I) can be thought of as genuine observables, in the sense that there is no contradiction between this decomposition and the general gauge principle of physics.

On the other hand, the observability of the OAM appearing in the decomposition (II), i.e., the generalized "canonical" OAM, is not completely clear yet. This is because, although the relation between the "canonical" OAM and a Wigner distribution is suggested, its path dependence or path independence should be clarified more convincingly. Moreover, once quantum loop effects are included, the very existence of transverse-momentum-dependent PDFs as well as Wigner distributions satisfying gauge invariance and factorization (or universality) at the same time is under debate. (See Ref. [73] and references therein.) Is process-independent extraction of "canonical" OAM possible? This is still a challenging open question.

ACKNOWLEDGMENTS

The author greatly acknowledges many stimulating discussions with Cedric Lorcé, although our opinions still differ in several essential points. He also greatly appreciates useful discussion with Takahiro Kubota.

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