Meson spectrum in strong magnetic fields

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We study the relativistic quark-antiquark system embedded in a magnetic field (MF). The Hamiltonian containing confinement, one gluon exchange, and spin-spin interaction is derived. We analytically follow the evolution of the lowest meson states as a function of MF strength. Calculating the one gluon exchange interaction energy $\langle V_{\rm OGE}\rangle$ and spin-spin contribution $\langle a_{SS}\rangle$ we have observed that these corrections remain finite at large MF, preventing the vanishing of the total ρ meson mass at some $B_{\rm crit}$, as previously thought. We display the ρ masses as functions of the MF in comparison with recent lattice data.

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I. INTRODUCTION

During the last years we have witnessed impressive progress in the fundamental physics in the ultraintense magnetic field (MF) reaching strength of up to $eB \sim 10^{18}\ \mathrm{G} \sim m_\pi^2$ [1]. Until recently, magnetars [2] were the only physical objects, where this, or a somewhat weaker MF could be realized. Now the MF of the above strength and even stronger is within reach in peripheral heavy ion collisions at RHIC and LHC [3]. High intensity lasers are another prospective tool to achieve MF beyond the Schwinger limit [4]. On the theoretical side, striking progress has been achieved along several lines. It is beyond our scope to discuss these works or even present a list of corresponding references. We mention only two lines of research which have a certain overlap with our work. The first one [5,6] is the behavior of the hydrogen atom and positronium in a very strong MF. The second one [7] is the conjecture of the vacuum reconstruction due to vector meson condensation in a large MF. The relation between the above studies and our work will be clarified in what follows. Our goal is to study from the first principles the spectrum of a meson composed of quark-antiquark material embedded in a MF. Use will be made of the Fock-Feynman-Schwinger representation (see [8] for a review and references) of the quark Green's function with strong (QCD) interaction and MF included. An alternative approach could have been a Bethe-Salpeter type formalism. However, for the confinement originating from the area law of the Wilson loop, the use of the gluon propagator is inadequate. Numerous attempts in this direction failed because of gauge dependence and the vector character of the gluon propagator, while confinement is scalar and gauge invariant. Therefore, it is sensible to use the path integral technique for QCD + QED Green's functions. This method, based on the proper-time formalism, allows us to represent the quark-antiquark Green's function via the Hamiltonian (see [9] for a new derivation), and was used in [10] to construct explicit expressions for meson Hamiltonians without a MF. In this way the spectra of light-light, light-heavy, and heavy-heavy mesons were computed with good accuracy, using the string tension σ, strong coupling constant α_S, and quark current masses as an input [11,12].

In what follows we expand this technique to incorporate the effects of MF on mesons. The latter contains (i) the direct influence of MF on the quark and antiquark, and (ii) the influence on gluonic fields, e.g., on the gluon propagator via $q\bar{q}$ loops and on the gluon field correlators determining the string tension σ. Since the MF acts on charged objects, its influence on the gluonic degrees of freedom enters only via $(N_c)^{-k}$, $k = 1, 2, \ldots$; however, corrections of the second type can be important, as shown in [13]. (iii) As was shown recently in the framework of our method, the MF also changes quark condensate $\langle\bar{q}q\rangle$ and quark decay constants f_π etc., and in this way strongly influences chiral dynamics [14].

The important step in our relativistic formalism is the implementation of the pseudomomentum notion and center-of-mass (c.m.) factorization in the MF, suggested in the nonrelativistic case in [15] for neutral two particle systems. Recently the c.m. factorization was proved for the neutral three-body system in [16], the situation with charged two-body system was clarified and an approximation scheme was suggested in [17].

The plan of the paper is the following. Section II contains a brief pedagogical reminder of how the two-body problem in MF is solved in quantum mechanics. The central point here is the integral of motion ("pseudomomentum") which allows the separation of the center of mass. Here we also show how to diagonalize the spin-dependent interaction.

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In Sec. III we formulate the path integral for a quark-antiquark system with QCD + QED interaction. Then from the Green's function the relativistic Hamiltonian is obtained. Section IV is devoted to the treatment of confining and color Coulomb terms. Here we also present the derivation of the eigenvalue equations for the relativistic Coulomb problem. In Sec. V we discuss the spectrum of the system focusing on the regime of an ultrastrong MF. Section VI contains the discussion of the results, comparison with lattice calculations, drawing further perspectives, and intersections of our results with those of other authors [5–7].

The present work develops the approach presented earlier in [18]. As compared to [18], one will find in what follows novel results, changing the physical picture outlined in [18]. The new point is that the mass of the quark-antiquark system remains nonzero however strong the MF is. This is similar to freezing by radiative corrections of the hydrogen atom ground state in strong magnetic fields [5,6]. A few more remarks on the relation between the two papers will be added in the final section below.

II. PSEUDOMOMENTUM AND WAVE FUNCTION FACTORIZATION

The total momentum of N mutually interacting particles with translation invariant interaction is a constant of motion and the center of mass motions can be separated in the Schrödinger equation. It was shown [15] that a system embedded in a constant MF also possesses a constant of motion—pseudomomentum. As a result for the case of zero total electric charge $Q = 0$ the c.m. motion can be removed from the total Hamiltonian. The simplest example is a two-particle system with equal masses $m_1 = m_2 = m$ and electric charges $e_1 = -e_2 = e$. We define

$$\mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \qquad \boldsymbol{\eta} = \mathbf{r}_1 - \mathbf{r}_2, \qquad \mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2. \tag{1}$$

Straightforward calculation in the London gauge $\mathbf{A} = \frac{1}{2}(\mathbf{B}\times\mathbf{r})$ yields

$$\hat{H} = \frac{1}{4m}\left(\mathbf{P} - \frac{e}{2}(\mathbf{B}\times\boldsymbol{\eta})\right)^2 + \frac{1}{m}\left(-i\frac{\partial}{\partial\boldsymbol{\eta}} - \frac{e}{2}(\mathbf{B}\times\mathbf{R})\right)^2 + V(\eta). \tag{2}$$

One can verify that the following pseudomomentum operator $\mathbf{F}$ commutes with the Hamiltonian (2)

$$\hat{\mathbf{F}} = \mathbf{P} + \frac{e}{2}(\mathbf{B}\times\boldsymbol{\eta}). \tag{3}$$

This immediately leads to the following factorization of the wave function

$$\Psi(\mathbf{R},\boldsymbol{\eta}) = \varphi(\boldsymbol{\eta})\exp\left\{i\mathbf{P}\mathbf{R} - i\frac{e}{2}(\mathbf{B}\times\boldsymbol{\eta})\mathbf{R}\right\}. \tag{4}$$

For the oscillator-type potential $V(\eta)$ the problem reduces to a set of three oscillators, two of them are in a plane perpendicular to the magnetic field and their frequencies are degenerate, while the third one is connected solely with $V(\eta)$.

Next, we briefly elucidate the spin interaction in the presence of a MF. The corresponding part of the Hamiltonian may be written as

$$\hat{H}_s = a_{hf}(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2) - \mu\mathbf{B}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2), \tag{5}$$

where $e_1 = -e_2 = e > 0$ and $\mu > 0$. Diagonalization of $\hat{H}_s$ yields the following four eigenvalues, e.g., for the $u\bar{u}$ system, comprising both ρ and π levels.

$$E^{(s)}_{1,2} = a_{hf}, \qquad E^{(s)}_{3,4} = \pm a_{hf}\left(2\sqrt{1 + \left(\frac{\mu B}{a_{hf}}\right)^2} \mp 1\right), \tag{6}$$

where we assume that $\mathbf{B}$ is aligned along the positive z axis and $B = |\mathbf{B}|$. In a strong MF when $\mu B > a_{hf}$ spin-spin interaction becomes unimportant and $E^{(s)}_{3,4} \simeq \pm 2\mu B$. For the lowest level $E^{(s)}_4$ this corresponds to a configuration $|+-\rangle$ when the spin of a negatively charged particle is aligned antiparallel to $\mathbf{B}$, and the spin of the positively charged one is aligned parallel to $\mathbf{B}$. This means that the spin (and isospin) are no longer good quantum numbers and eigenvalues (6) correspond to the mixture of spin 1 and spin 0 states. As a result, the $q\bar{q}$ state will split into 4 states (two of them coinciding as $E^{(s)}_1 = E^{(s)}_2$). Until now we treated a nonrelativistic system, to incorporate relativistic effects we shall exploit the path integral form of relativistic Green's functions [8,9].

III. RELATIVISTIC $q\bar{q}$ GREEN'S FUNCTION AND EFFECTIVE HAMILTONIAN

The derivation of the relativistic Hamiltonian of the $q\bar{q}$ system in a MF consist of several steps. The first one is the $4d$ relativistic path integral for the $q\bar{q}$ Green's function. The starting point is the Fock-Feynmann-Schwinger (world-line) representation of the quark Green's function [8]. The role of the "time" parameter along the path $z^{(i)}_\mu(s_i)$ of the ith quark is played by the Fock-Schwinger proper time s_i, $i = 1, 2$. Consider a quark with a charge e_i in a gluonic field A_μ and the electromagnetic vector potential $A^{(e)}_\mu$, corresponding to a constant magnetic field $\mathbf{B}$. Then the quark propagator in the Euclidean space-time is

$$S_i(x,y) = (m_i + \hat{\partial} - ig\hat{A} - ie_i\hat{A}^{(e)})^{-1}_{xy} \equiv (m_i + \hat{D}^{(i)})^{-1}_{xy}. \tag{7}$$

The path-integral representation for S_i [8] is

$$S_i(x,y) = (m_i - \hat{D}^{(i)})\int_0^\infty ds_i (D^4 z)_{xy} e^{-K_i}\Phi^{(i)}_\sigma(x,y) \equiv (m_i - \hat{D}^{(i)})G_i(x,y), \tag{8}$$

where

$$K_i = m_i^2 s_i + \frac{1}{4}\int_0^{s_i} d\tau_i \left(\frac{dz_\mu^{(i)}}{d\tau_i}\right)^2, \tag{9}$$

$$\Phi_\sigma^{(i)}(x,y) = P_A P_F \exp\left(ig\int_y^x A_\mu dz_\mu^{(i)} + ie_i\int_y^x A_\mu^{(e)} dz_\mu^{(i)}\right) \times \exp\left(\int_0^{s_i} d\tau_i \sigma_{\mu\nu}(gF_{\mu\nu} + e_i B_{\mu\nu})\right). \tag{10}$$

Here, $F_{\mu\nu}$ and $B_{\mu\nu}$ are correspondingly gluon and MF tensors, P_A, P_F are ordering operators, $\sigma_{\mu\nu} = \frac{1}{4i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. Equations (7)–(10) hold for the quark, $i = 1$, while for the antiquark one should reverse the signs of e_i and g. In explicit form one writes

$$\sigma_{\mu\nu}F_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{H} & \boldsymbol{\sigma}\mathbf{E} \\ \boldsymbol{\sigma}\mathbf{E} & \boldsymbol{\sigma}\mathbf{H} \end{pmatrix}, \qquad \sigma_{\mu\nu}B_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{B} & 0 \\ 0 & \boldsymbol{\sigma}\mathbf{B} \end{pmatrix}. \tag{11}$$

Next, we consider the $q_1\bar{q}_2$ system born at the point x with the current $j_{\Gamma_1}(x) = \bar{q}_1(x)\Gamma_1 q_2(x)$ and annihilated at the point y with the current $j_{\Gamma_2}(y)$. Here, x and y denote the sets of initial and final coordinates of quark and antiquark. Using the non-Abelian Stokes theorem and cluster expansion for the gluon field (see [11] for reviews) and leaving the MF term intact, we can write

$$G_{q_1\bar{q}_2}(x,y) = \int_0^\infty ds_1 \int_0^\infty ds_2 (D^4 z^{(1)})_{xy} (D^4 z^{(2)})_{xy} e^{-K_1-K_2} \operatorname{tr}\langle \hat{T} W_\sigma(A)\rangle_A \exp\left(ie_1\int_y^x A_\mu^{(e)} dz_\mu^{(1)} - ie_2\int_y^x A_\mu^{(e)} dz_\mu^{(2)} + e_1\int_0^{s_1} d\tau_1(\boldsymbol{\sigma}\mathbf{B}) - e_2\int_0^{s_2} d\tau_2(\boldsymbol{\sigma}\mathbf{B})\right), \tag{12}$$

where

$$\hat{T} = \Gamma_1(m_1 - \hat{D}_1)\Gamma_2(m_2 - \hat{D}_2), \tag{13}$$

and $\Gamma_1 = \gamma_\mu$, $\Gamma_2 = \gamma_\nu$ for vector currents, $\Gamma_i = \gamma_5$ for pseudoscalar currents, while

$$\langle W_\sigma(A)\rangle_A = \exp\left(-\frac{g^2}{2}\int d\pi_{\mu\nu}(1) d\pi_{\lambda\sigma}(2)\langle F_{\mu\nu}(1)F_{\lambda\sigma}(2)\rangle + \mathcal{O}(\langle FFF\rangle)\right), \tag{14}$$

where $d\pi_{\mu\nu} \equiv ds_{\mu\nu} + \sigma_{\mu\nu}^{(1)} d\tau_1 - \sigma_{\mu\nu}^{(2)} d\tau_2$, and $ds_{\mu\nu}$ is an area element of the minimal surface, which can be constructed using straight lines, connecting the points $z_i^{(1)}(t)$ and $z_j^{(2)}(t)$ on the paths of q_1 and $\bar{q}_2$ at the same time t [8,10]. Note, that operator $\hat{T}$ actually does not participate in the field averaging procedure: as was shown in [19], the following replacement is valid, $m - \hat{D} \to m - i\hat{p}$, $p_\mu = \frac{1}{2}(\frac{dz_\mu}{d\tau})_{\tau=s}$.

As a result of the first step the $q\bar{q}$ Green's function is represented as a $4d$ path integral (including Euclidean time paths) and, in addition, also integrates over proper times s_1, s_2. In the second step one introduces monotonic Euclidean time $t_E(\tau) = x_4 + \frac{\tau}{s}T$, where $T \equiv |x_4 - y_4|$, so that $z_4(\tau) = t_E(\tau) + \Delta z_4(\tau)$, where $\Delta z_4(\tau)$ is fluctuation of time trajectory around $t_E(\tau)$. This new variable t_E is an ordering parameter for trajectories $\mathbf{z}^{(1)}(t_E)$, $\mathbf{z}^{(2)}(t_E)$, and proper times transform into physical parameters—virtual q and $\bar{q}$ energies $\omega_i \equiv \frac{T}{2s_i}$, so that $ds_i = -\frac{T}{2\omega_i^2}d\omega_i$.

Combining for simplicity all fields into one Wilson loop $W(A, A^{(e)})$, one can rewrite the Green's function in new variables as

$$G_{q_1\bar{q}_2}(x,y) = \frac{T}{8\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\frac{d\omega_2}{\omega_2^{3/2}}(D^3 z^{(1)} D^3 z^{(2)})_{\mathbf{xy}} \times e^{-K_1(\omega_1)-K_2(\omega_2)}\langle\langle \hat{T} W_F\rangle\rangle_{\Delta z_4}, \tag{15}$$

(see [9] for details of derivation). Here, $K_1(\omega_1)$, $K_2(\omega_2)$ are obtained from K_i in (9) by the same replacement $\frac{dz_i}{d\tau_i} = 2\omega_i\frac{dz_i}{dt_E}$,

$$K_1(\omega_1) + K_2(\omega_2) = \left(\frac{m_1^2+\omega_1^2}{2\omega_1} + \frac{m_2^2+\omega_2^2}{2\omega_2}\right)T + \int_0^T dt_E\left[\frac{\omega_1}{2}\left(\frac{d\mathbf{z}^{(1)}}{dt_E}\right)^2 + \frac{\omega_2}{2}\left(\frac{d\mathbf{z}^{(2)}}{dt_E}\right)^2\right]. \tag{16}$$

The final step is the use of the Wilson loop dynamics to express all dynamics in terms of instantaneous interaction. Indeed, the quadratic field correlator in (14) is represented through two scalar functions $D(z)$ and $D_1(z)$ (see, e.g., [11,20] for details), the first of them is responsible for confinement, while the second one gives one gluon exchange (OGE) potential. So, for the case of zero quark orbital momenta with the minimal surface discussed above, integrating over relative time $\nu = t_E^1 - t_E^2$ in $D(\nu, \mathbf{z}_1 - \mathbf{z}_2)$, $D_1(\nu, \mathbf{z}_1 - \mathbf{z}_2)$, one obtains a simple instantaneous answer for spin-independent (SI) part of $\langle W_\sigma(A)\rangle_A$,

$$\langle W_\sigma(A)\rangle_A^{\rm SI} = \exp\left(-\int_0^T dt^E\left[\sigma|\mathbf{z}^{(1)} - \mathbf{z}^{(2)}| - \frac{4}{3}\frac{\alpha_s}{|\mathbf{z}^{(1)} - \mathbf{z}^{(2)}|}\right]\right), \tag{17}$$

containing $V_{\rm conf}(r) = \sigma r$ and $V_{\rm OGE}(r) = -\frac{4\alpha_s}{3r}$. Here σ is the QCD string tension, $\sigma = 0.18\ \mathrm{GeV}^2$ in our calculations.

First, we need to find the Hamiltonian $H_{q_1\bar{q}_2}$ of the system at $t_1^E = t_2^E = t^E$. To this end we define the Euclidean Lagrangian $L_{q_1\bar{q}_2}^E$. We write $\frac{dz^{(i)}}{d\tau_i} = 2\omega_i\frac{dz_k^{(i)}}{dt^E} = 2\omega_i\dot{z}_k$, $k = 1, 2, 3$. Then all terms in the exponents in (12), (14), and (17) can be represented as $\exp(-\int dt^E L^E_{q_1\bar{q}_2})$ and thus we arrive at the following representation:

$$G_{q_1\bar{q}_2}(x,y) = \frac{T}{8\pi}\int_0^\infty\int_0^\infty \frac{d\omega_1 d\omega_2}{(\omega_1\omega_2)^{3/2}}(D^3z^{(1)}D^3z^{(2)})_{\mathbf{xy}}\,\mathrm{tr}(e^{-S^E_{q_1\bar{q}_2}}\hat{T}) \tag{18}$$

with the action

$$S^E_{q_1\bar{q}_2} = \int_0^T dt^E\Big[\sum_i\Big(\frac{\omega_i}{2}(\dot{z}^{(i)}_k)^2 - ie_iA^{(e)}_k\dot{z}^{(i)}_k\Big) + \frac{\omega_1+\omega_2}{2} + \frac{m_1^2}{2\omega_1} + \frac{m_2^2}{2\omega_2} + e_1\frac{\boldsymbol{\sigma}_1\mathbf{B}}{2\omega_1} + e_2\frac{\boldsymbol{\sigma}_2\mathbf{B}}{2\omega_2} + \sigma|\mathbf{z}^{(1)}-\mathbf{z}^{(2)}| - \frac{4}{3}\frac{\alpha_s}{|\mathbf{z}^{(1)}-\mathbf{z}^{(2)}|}\Big]. \tag{19}$$

Here, $A^{(e)}_k$ is the kth component of the QED vector potential. The next step is the transition to the Minkowski metric. This is easy, since confinement is already expressed in terms of string tension. We have $\exp(-\int L^E dt_E) \to \exp(i\int L^M dt_M)$, $t_E \to it_M$, and

$$p^{(i)}_k = \frac{\partial L^M}{\partial \dot{z}^{(i)}_k} = \omega_i\dot{z}^{(i)}_k + e_iA^{(e)}_k, \qquad H_{q_1\bar{q}_2} = \sum_i \dot{z}^{(i)}_k p^{(i)}_k - L_M. \tag{20}$$

The explicit expression for the Hamiltonian without spin-dependent terms is

$$H_{q_1\bar{q}_2} = \sum_{i=1,2}\frac{(\mathbf{p}^{(i)} - e_i\mathbf{A}(\mathbf{z}^{(i)}))^2 + m_i^2 + \omega_i^2 - e_i\boldsymbol{\sigma}^{(i)}\mathbf{B}}{2\omega_i} + \sigma|\mathbf{z}^{(1)}-\mathbf{z}^{(2)}| - \frac{4}{3}\frac{\alpha_s}{|\mathbf{z}^{(1)}-\mathbf{z}^{(2)}|}. \tag{21}$$

The $q\bar{q}$ Green's function (15) takes the "heat–kernel" form when going back to Euclidean time with Hamiltonian (21)

$$G_{q_1\bar{q}_2}(x,y) = \frac{T}{8\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty\frac{d\omega_2}{\omega_2^{3/2}}\langle\mathbf{x}|\mathrm{tr}(\hat{T}e^{-H_{q_1\bar{q}_2}T})|\mathbf{y}\rangle. \tag{22}$$

The c.m. projection of the Green's function yields

$$\int G_{q_1\bar{q}_2}(x,y)d^3(x-y) = \frac{T}{8\pi}\int_0^\infty\frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty\frac{d\omega_2}{\omega_2^{3/2}}\sum_{n=0}^\infty \varphi_n^2(0)\langle\mathrm{tr}(\hat{T})\rangle e^{-M_n(\omega_1,\omega_2)T}, \tag{23}$$

where φ_n and M_n are eigenfunctions and eigenvalues of the Hamiltonian $H_{q_1\bar{q}_2}$. At large T the integral over ω_1, ω_2 can be taken by the stationary point method, and hence the effective energies ω_i are to be found from the minimum of the total mass $M_n(\omega_1,\omega_2)$, as was suggested in [10]. To introduce the minimization procedure and to check its accuracy we shall begin by the calculation of the eigenvalues of one and two quarks in the MF, and the energy of the ground state of a relativistic charge in the atom in the next section, reproducing the known exact results.

We have the following equations defining ω_i from the total mass $M(\omega_i)$

$$\hat{H}\psi = M_n(\omega_1,\omega_2)\psi, \qquad \frac{\partial M_n(\omega_1,\omega_2)}{\partial\omega_i} = 0. \tag{24}$$

For a single quark in MF the first of the above equations gives

$$M_n(\omega) = \frac{p_z^2 + m_q^2 + |eB|(2n+1) - eB\sigma_z}{2\omega} + \frac{\omega}{2}. \tag{25}$$

Then the minimization over ω yields the correct answer

$$\bar{M}_n = (p_z^2 + m_q^2 + |eB|(2n+1) - eB\sigma_z)^{1/2}. \tag{26}$$

Now we turn to the case of $q_1\bar{q}_2$ system and introduce the coordinates which are the generalization of (1):

$$\mathbf{R} = \frac{\omega_1\mathbf{z}^{(1)} + \omega_2\mathbf{z}^{(2)}}{\omega_1+\omega_2}, \qquad \boldsymbol{\eta} = \mathbf{z}^{(1)} - \mathbf{z}^{(2)}, \tag{27}$$

$$\mathbf{P} = -i\frac{\partial}{\partial\mathbf{R}}, \qquad \mathbf{p} = -i\frac{\partial}{\partial\boldsymbol{\eta}}. \tag{28}$$

It is convenient to introduce the following two additional parameters

$$\tilde{\omega} = \frac{\omega_1\omega_2}{\omega_1+\omega_2}, \qquad s = \frac{\omega_1-\omega_2}{\omega_1+\omega_2}. \tag{29}$$

Let us consider the case of the neutral meson, so that $e_1 = -e_2 = e$. Then the total Hamiltonian may be written as

$$H_{q_1\bar{q}_2} = H_B + H_\sigma + W, \tag{30}$$

where

$$\begin{aligned} H_B &= \frac{1}{2\omega_1}\Big[\frac{\tilde{\omega}}{\omega_2}\mathbf{P} + \boldsymbol{\pi} - \frac{e}{2}\mathbf{B}\times\Big(\mathbf{R} + \frac{\tilde{\omega}}{\omega_1}\boldsymbol{\eta}\Big)\Big]^2 + \frac{1}{2\omega_2}\Big[\frac{\tilde{\omega}}{\omega_1}\mathbf{P} - \boldsymbol{\pi} + \frac{e}{2}\mathbf{B}\times\Big(\mathbf{R} - \frac{\tilde{\omega}}{\omega_2}\boldsymbol{\eta}\Big)\Big]^2 \\ &= \frac{1}{2\tilde{\omega}}\Big(\boldsymbol{\pi} - \frac{e}{2}\mathbf{B}\times\mathbf{R} + s\frac{e}{2}\mathbf{B}\times\boldsymbol{\eta}\Big)^2 + \frac{1}{2(\omega_1+\omega_2)}\Big(\mathbf{P} - \frac{e}{2}\mathbf{B}\times\boldsymbol{\eta}\Big)^2. \end{aligned} \tag{31}$$

Equation (31) is an obvious generalization of (2). The two other terms in (31) read

$$H_\sigma = \frac{m_1^2 + \omega_1^2 - e\boldsymbol{\sigma}_1\mathbf{B}}{2\omega_1} + \frac{m_2^2 + \omega_2^2 + e\boldsymbol{\sigma}_2\mathbf{B}}{2\omega_2}, \tag{32}$$

$$W = V_{\rm conf} + V_{\rm OGE} + \Delta W = \sigma\eta - \frac{4}{3}\frac{\alpha_s(\eta)}{\eta} + \Delta W, \quad (33)$$

and ΔW contains self-energy and spin–spin contributions, which come from the unaccounted spin-dependent terms of $\langle W_\sigma(A)\rangle$. One can verify that the pseudomomentum operator in (3), introduced in Sec. II, commutes with H_B and hence we can again separate the c.m. motion according to the ansatz (4):

$$H_B\Psi(\mathbf{R},\boldsymbol{\eta}) = \exp\left\{i\mathbf{PR} - i\frac{e}{2}(\mathbf{B}\times\boldsymbol{\eta})\mathbf{R}\right\}\tilde{H}_B\varphi(\boldsymbol{\eta}). \quad (34)$$

Then the problem reduces to the eigenvalue problem for $\varphi(\boldsymbol{\eta})$ with the Hamiltonian $\tilde{H}_B$ having the following form:

$$\tilde{H}_B = \frac{1}{2\tilde{\omega}}\left(-i\frac{\partial}{\partial\boldsymbol{\eta}} + s\frac{e}{2}\mathbf{B}\times\boldsymbol{\eta}\right)^2 + \frac{1}{2(\omega_1+\omega_2)}(\mathbf{P} - e\mathbf{B}\times\boldsymbol{\eta})^2. \quad (35)$$

For $\mathbf{P}\times\mathbf{B} = 0$ the system has a rotational symmetry and the c.m. is freely moving along the z axis. Here we shall consider a state with zero orbital momentum $(\mathbf{L}_\eta)_z = [\boldsymbol{\eta}\times\frac{\partial}{i\partial\boldsymbol{\eta}}]_z = 0$. As a result, $\tilde{H}_B$ is replaced by a purely internal space operator

$$H_0 = \frac{1}{2\tilde{\omega}}\left(-\frac{\partial^2}{\partial\boldsymbol{\eta}2} + \frac{e^2}{4}(\mathbf{B}\times\boldsymbol{\eta})^2\right), \quad (36)$$

To test our method we put $W = 0$ and arrive at the equation

$$(H_0 + h_\sigma)\varphi = M(\omega_1,\omega_2)\varphi. \quad (37)$$

Consequent minimization of $M(\omega_1,\omega_2)$ in ω_1, ω_2, as in (26), yields the expected answer for the two independent quarks,

$$M = \sqrt{m_1^2 + eB(2n_1+1) - e\boldsymbol{\sigma}_1\mathbf{B}} + \sqrt{m_2^2 + eB(2n_2+1) + e\boldsymbol{\sigma}_2\mathbf{B}}. \quad (38)$$

We turn now to the particular case of a charged two-body system in the MF $e_1 = e_2 = e$ and also $m_1 = m_2$, when exact factorization of $\mathbf{R}$ and $\boldsymbol{\eta}$ can be done. In this case, for $\omega_1 = \omega_2 = \omega$ and $\mathbf{P}\times\mathbf{B} = 0$, the Hamiltonian has the following form [9]:

$$H_{q_1\bar{q}_2} = \frac{\mathbf{P}^2}{4\omega} + \frac{e^2}{4\omega}(\mathbf{B}\times\mathbf{R})^2 + \frac{\boldsymbol{\pi}^2}{\omega} + \frac{e^2}{16\omega}(\mathbf{B}\times\boldsymbol{\eta})^2 + \frac{2m^2 + 2\omega^2 - e(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\mathbf{B}}{2\omega} + \frac{\sigma}{2}\left(\frac{\eta^2}{\gamma}+\gamma\right) + V_{\rm OGE} + V_{SS} + \Delta M_{SE}. \quad (39)$$

IV. TREATING CONFINEMENT AND GLUON EXCHANGE TERMS. THE ABSENCE OF THE MAGNETIC QCD COLLAPSE

From (33) and (36) it is clear that inclusion of $V_{\rm conf}$ and $V_{\rm OGE}$ in $H_0 + W$ leads to a differential equation in variables $\eta_\perp$, η_z, which can be solved numerically. However, in order to obtain a clear physical picture, we shall represent $V_{\rm conf}$ in a quadratic form. This will allow us to get an exact analytic solution in terms of oscillator functions with eigenvalue accuracy of the order of 5%. The OGE contribution will be estimated as an average $\langle\varphi|V_{\rm OGE}|\varphi\rangle$, thus yielding an upper limit for the total mass.

For $V_{\rm conf}$ we choose the form

$$V_{\rm conf} \to \tilde{V}_{\rm conf} = \frac{\sigma}{2}\left(\frac{\eta^2}{\gamma}+\gamma\right). \quad (40)$$

Here, γ is a positive variational parameter; minimizing $\tilde{V}_{\rm conf}$ with respect to γ, one returns to $V_{\rm conf}$. We shall determine $M(\omega_1\omega_2,\gamma)$ corresponding to $\tilde{V}_{\rm conf}$, and to define γ an additional condition

$$\frac{\partial M(\omega_1,\omega_2,\gamma)}{\partial\gamma}\bigg|_{\gamma=\gamma_0} = 0 \quad (41)$$

will be added to (24). As a result, $M(\omega_1^{(0)},\omega_2^{(0)},\gamma_0)$ will be the final answer for the mass of the system, neglecting the ΔW contribution. The difference of the exact numerical solution from that obtained with the genuine potential $V_{\rm conf}$ does not exceed 5%. The solution of the equation $(H_0 + \tilde{V}_{\rm conf})\varphi = M(\omega_1,\omega_2,\gamma_0)\varphi$ for the ground state is

$$\psi(\boldsymbol{\eta}) = \frac{1}{\sqrt{\pi^{3/2}r_\perp^2 r_0}}\exp\left(-\frac{\eta_\perp^2}{2r_\perp^2} - \frac{\eta_z^2}{2r_0^2}\right), \quad (42)$$

where $r_\perp = \sqrt{\frac{2}{eB}}(1 + \frac{4\sigma\tilde{\omega}}{\gamma e^2B^2})^{-1/4}$, $r_0 = (\frac{\gamma}{\sigma\tilde{\omega}})^{1/4}$. As we shall see below, for the lowest mass eigenvalue with $eB \gg \sigma$, one has $r_\perp \approx \sqrt{\frac{2}{eB}}$, $r_0 \approx \frac{1}{\sqrt{\sigma}}$ and the $(q_1\bar{q}_2)$ system acquires the form of an elongated ellipsoid. A similar quasi-one-dimensional picture was observed before for the hydrogen-like atoms in strong MF [5,6]. In such geometrical configuration $V_{\rm OGE}$ manifests itself in a peculiar way, again similar to what happens in hydrogen, or positronium atoms; and, as was shown in [13], in QCD the outcome is also similar to the case of QED, with the screening of the diverging effects.

We turn now to the OGE term to treat it in our formalism. As a starting point we present another check of our approach; namely, we shall obtain the ground state energy of two relativistic particles with opposite charges without MF interacting via the Coulomb potential. The corresponding Hamiltonian reads $H = H_0 + H_\sigma - \frac{\alpha}{\eta}$, then $H\phi = M\phi$, and for $eB = 0$ we have

$$M = -\frac{\tilde{\omega}\alpha^2}{2} + \frac{m_1^2+\omega_1^2}{2\omega_1} + \frac{m_2^2+\omega_2^2}{2\omega_2}. \quad (43)$$

Minimizing in ω_1 in the limit $m_2 \gg m_1$ (the hydrogen atom), one obtains

$$M = m_1\sqrt{1-\alpha^2} + m_2, \tag{44}$$

which coincides with the known eigenvalue of the Dirac equation.

In our $(q_1\bar{q}_2)$ case one can calculate the expectation value of $V_{\rm OGE} = -\frac{4}{3}\frac{\alpha_s(\eta)}{\eta}$ with the asymptotic freedom and IR saturation behavior in $\mathbf{p}$ space (see [21] for a derivation and a short review)

$$\alpha_s(q) = \frac{4\pi}{\beta_0 \ln\left(\frac{q^2+M_B^2}{\Lambda_{\rm QCD}^2}\right)}, \tag{45}$$

where $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$, M_B is proportional to $\sqrt{\sigma}$, $M_B \approx 1$ GeV [21]. With the wave function (42) the average value of $V_{\rm OGE}$ takes the form

$$\Delta M_{\rm OGE} \equiv \int V_{\rm OGE}(q)\tilde{\psi}^2(\mathbf{q})\frac{d^3q}{(2\pi)^3} = -\frac{4}{3\pi}\int_0^\infty \alpha_s(q)dq e^{-\frac{q^2r_\perp^2}{4}} I\left[\frac{q^2(r_0^2-r_\perp^2)}{4}\right], \tag{46}$$

where $\psi^2(\mathbf{q})$ is the Fourier transform of squared wave function $\psi^2(\boldsymbol{\eta})$ and $I(a^2) = \int_{-1}^{+1} dx e^{-a^2x^2}$. Estimating the integral in (46), for $eB \gg \sigma$, i.e., for $r_0 \gg r_\perp$ one obtains for massless quarks

$$\Delta M_{\rm OGE} \approx -\frac{16\sqrt{\pi}}{3r_0\beta_0}\ln\ln\frac{r_0^2}{r_\perp^2} \approx -\sqrt{\sigma}\ln\ln\frac{eB}{\sigma}. \tag{47}$$

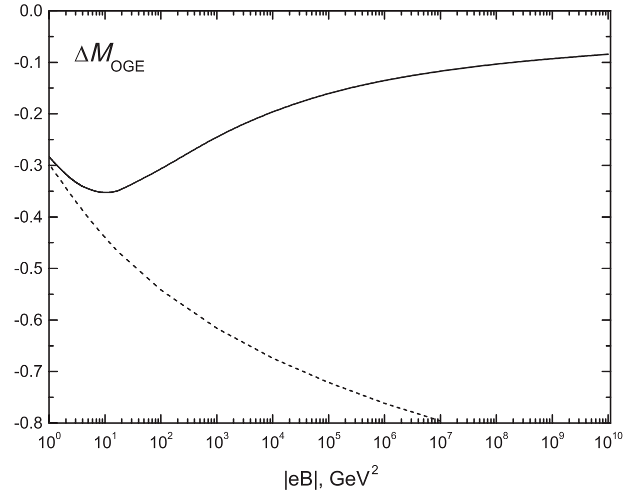

FIG. 1. One gluon exchange correction to the meson mass in GeV as a function of the magnetic field with (solid line) (49) and without (broken line) (46) account of the quark loops' contributions.

With eB increasing the upper bound for the $q\bar{q}$ mass is boundlessly decreasing. The exact eigenvalue should lie even lower.

This situation is similar to the hydrogen atom case, where $\Delta M_{\rm Coul}$ diverges as $-\ln^2 eB$, and in this case e^+e^- loop contribution to the photon line stabilizes the result (the screening effect, [6,7]). In our case the $q\bar{q}$ loop contribution to the OGE term can by written in a similar way, adding to the gluon loop also the lowest Landau level of the $q\bar{q}$ in the MF [13],

$$\tilde{V}_{\rm OGE}(Q) = -\frac{16\pi\alpha_s^{(0)}}{3[Q^2(1+\frac{\alpha_s^{(0)}}{4\pi}\frac{11}{3}N_c\ln\frac{Q^2+M_B^2}{\mu_0^2}) + \frac{\alpha_s^{(0)}n_f|e_qB|}{\pi}\exp\left(\frac{-q_\perp^2}{2|e_qB|}\right)T\left(\frac{q_3^2}{4\sigma}\right)]}, \tag{48}$$

where $T(z) = -\frac{\ln(\sqrt{1+z}+\sqrt{z})}{\sqrt{z(z+1)}} + 1$. Calculating now the average value of (48),

$$\Delta M_{\rm OGE} = \langle\tilde{V}_{\rm OGE}\rangle, \tag{49}$$

one obtains saturation of $\Delta M_{\rm OGE}$ at large eB, as shown in Fig. 1, eliminating in this way the possible "color Coulomb catastrophe", discussed in the first version of this paper [18].

V. MESON MASSES IN THE MAGNETIC FIELD

Our next task is to calculate analytically the mass $M_n(\omega_1, \omega_2, \gamma)$ of a $(q_1\bar{q}_2)$ meson. We have to solve the equation

$$(H_0 + H_\sigma + W)\Psi_n(\eta) = M_n(\omega_1, \omega_2, \gamma)\Psi_n(\eta), \tag{50}$$

where H_0, H_σ, W are given in (32), (33), and (36), and the total Hamiltonian for the charged meson is given in (39).

The resulting mass for neutral meson without spin-dependent contribution from ΔW is

$$M_n(\omega_1,\omega_2,\gamma) = \varepsilon_{n_\perp,n_z} + \Delta M_{\rm OGE} + \frac{m_1^2+\omega_1^2 - e\mathbf{B}\boldsymbol{\sigma}_1}{2\omega_1} + \frac{m_2^2+\omega_2^2+e\mathbf{B}\boldsymbol{\sigma}_2}{2\omega_2} \equiv \bar{M}_n(\omega_1,\omega_2,\gamma) - \frac{e\mathbf{B}\boldsymbol{\sigma}_1}{2\omega_1} + \frac{e\mathbf{B}\boldsymbol{\sigma}_2}{2\omega_2}, \tag{51}$$

where

$$\varepsilon_{n_\perp,n_z} = \frac{1}{2\tilde{\omega}}\left[\sqrt{e^2B^2 + \frac{4\sigma\tilde{\omega}}{\gamma}(2n_\perp+1)} + \sqrt{\frac{4\sigma\tilde{\omega}}{\gamma}}\left(n_z+\frac{1}{2}\right)\right] + \frac{\gamma\sigma}{2}, \tag{52}$$

$\Delta M_{\rm OGE}$ is given by (48) and (49). So, for fixed n we have four states for different quark spin orientations, $|+\,+\rangle$, $|+\,-\rangle$, $|-\,+\rangle$, and $|-\,-\rangle$, where $+/-$ are the up and down directions of individual quark spins, with corresponding masses

$$M_n^{++} = \bar{M}_n - eB\left(\frac{1}{2\omega_1} - \frac{1}{2\omega_2}\right), \quad (53)$$

$$M_n^{--} = \bar{M}_n + eB\left(\frac{1}{2\omega_1} - \frac{1}{2\omega_2}\right), \quad (54)$$

$$M_n^{+-} = \bar{M}_n - eB\left(\frac{1}{2\omega_1} + \frac{1}{2\omega_2}\right), \quad (55)$$

$$M_n^{-+} = \bar{M}_n + eB\left(\frac{1}{2\omega_1} + \frac{1}{2\omega_2}\right). \quad (56)$$

The spin-dependent part ΔW contains self-energy V_{SE} and spin-spin V_{SS} contributions. As was shown in [22], the mass correction, corresponding to V_{SE}, is given by

$$\Delta M_{SE} = -\frac{3\sigma}{4\pi\omega_1}\left(1 + \eta\left(\lambda\sqrt{2eB + m_1^2}\right)\right) - \frac{3\sigma}{4\pi\omega_2}\left(1 + \eta\left(\lambda\sqrt{2eB + m_2^2}\right)\right), \quad (57)$$

where $\eta(t) = t\int_0^\infty z^2 K_1(tz)e^{-z}dz$ and $\lambda \sim 1\ \mathrm{GeV}^{-1}$ is vacuum correlation lengths.

Let us introduce now the spin-spin interaction. It has nondiagonal structure

$$V_{SS} = \frac{8\pi\alpha_s}{9\omega_1\omega_2}\delta^{(3)}(\mathbf{r})\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 \equiv a_{SS}\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2, \quad (58)$$

so we should diagonalize the Hamiltonian with respect to spin variables. This results in new four states, two of them are a mixture of $|+-\rangle$ and $|-+\rangle$ states, corresponding to π^0 and ρ^0 with zero spin projection $s_z = 0$, the other two states $|++\rangle$ and $|--\rangle$ correspond to ρ^0 states with $s_z = 1$ and $s_z = -1$ (we consider the ground state $n = 0$). We note that both V_{SS} and ΔM_{SE} are to be considered as corrections and contain $\omega_1^{(0)}$, $\omega_2^{(0)}$, obtained from minimization of the remaining part of the meson mass. Note also, that masses M_n^{+-} and M_n^{-+} are symmetric with respect to $\omega_1 \leftrightarrow \omega_2$ for equal quark masses, so in this case we have in fact only two variables in the minimization procedure ω and γ.

The masses of the first two states are

$$E_{1,2} = \frac{1}{2}(M_{11} + M_{22}) \pm \sqrt{\left(\frac{M_{22} - M_{11}}{2}\right)^2 + 4a_{12}a_{21}}, \quad (59)$$

where

$$M_{11} = (M_0^{+-} + \Delta M_{SE} - \langle a_{SS}\rangle)|_{\omega_1^{(0)}=\omega_2^{(0)}=\omega_\pm}, \quad M_{22} = (M_0^{-+} + \Delta M_{SE} - \langle a_{SS}\rangle)|_{\omega_1^{(0)}=\omega_2^{(0)}=\omega_\mp}, \quad (60)$$

$a_{12} = a_{21} = \langle a_{SS}\rangle|_{\omega_1^{(0)}=\omega_\pm, \omega_2^{(0)}=\omega_\mp}$ and $\langle a_{SS}\rangle$ is the averaging with the wave function (42) (see [22] for derivation). The parameters $\omega_\pm$ and $\omega_\mp$ are obtained by minimizing of corresponding diagonal eigenvalues M_0^{+-} and M_0^{-+}, the parameter γ_0 [see (35)] for the ground state is defined from the condition

$$\frac{\partial M_0(\omega_1, \omega_2, \gamma)}{\partial\gamma}\bigg|_{\gamma=\gamma_0} = \frac{\partial\varepsilon_{0,0}}{\partial\gamma}\bigg|_{\gamma=\gamma_0} = 0. \quad (61)$$

It is easy to see, that at large eB masses $E_{1,2}$ tend to diagonal values

$$E_1(eB \to \infty) \to M_{22}, \qquad E_2(eB \to \infty) \to M_{11}. \quad (62)$$

The remaining two states have masses

$$E_3 = M_0^{++} + \Delta M_{SE} + \langle a_{SS}\rangle, \quad E_4 = M_0^{--} + \Delta M_{SE} + \langle a_{SS}\rangle, \quad (63)$$

taken in point $(\omega_1^{(0)}, \omega_2^{(0)}, \gamma^0)$ in accordance with minimization conditions (24) and (61).

It should be noted that actually we have eight states instead of four, since $q\bar{q}$ systems with different quark charges behave differently in the MF, as we see from our Hamiltonians. Isospin is not conserved now and each neutral state splits into two states with different quark content $u\bar{u}$ and $d\bar{d}$.

Let us consider also the particular case of the charged meson with Hamiltonian (39) in a state with $s_z = 1$ ($|++\rangle$-state, corresponding to ρ^+). The eigenvalue, corresponding to this state is given by the following expression

$$M_n(\omega, \gamma) = \frac{eB}{2\omega}(2N_\perp + 1) + \sqrt{\left(\frac{eB}{2\omega}\right)^2 + \frac{2\sigma}{\omega\gamma}(2n_\perp + 1)} + \sqrt{\frac{2\sigma}{\omega\gamma}}\left(n_\parallel + \frac{1}{2}\right) - \frac{eB}{\omega} + \frac{\sigma\gamma}{2} + \frac{m^2 + \omega^2}{\omega} + \Delta M_{OGE} + \Delta M_{SE} + \langle a_{SS}\rangle. \quad (64)$$

Among the considered states, the mass of the charged meson ground state (ρ^+ with $s_z = 1$) and E_2, corresponding to π^0, tend to finite value at large MFs due to cancellation of linearly growing terms in $\varepsilon_{n_\perp,n_z}$ and in H_σ, while other masses grow with eB. This is true, provided that the spin-spin contribution $\langle a_{SS}\rangle$ remains finite at large MF. However, it contains the factor $\psi^2(0) \sim eB$, which leads to unbounded decrease of E_2. As was shown in [22], this situation is not physical, the total mass eigenvalues should be positive, and the reason for this decrease is the unlawful use of the perturbation theory for the potential $c\delta^{(3)}(\mathbf{r})$. One should replace a_{SS} by a smeared out version, e.g.,

$$\delta^{(3)}(\mathbf{r}) \to \tilde{\delta}^{(3)}(\mathbf{r}) = \left(\frac{1}{\lambda\sqrt{\pi}}\right)^3 e^{-\mathbf{r}^2/\lambda^2}, \qquad \lambda \sim 1\ \mathrm{GeV}^{-1}. \quad (65)$$

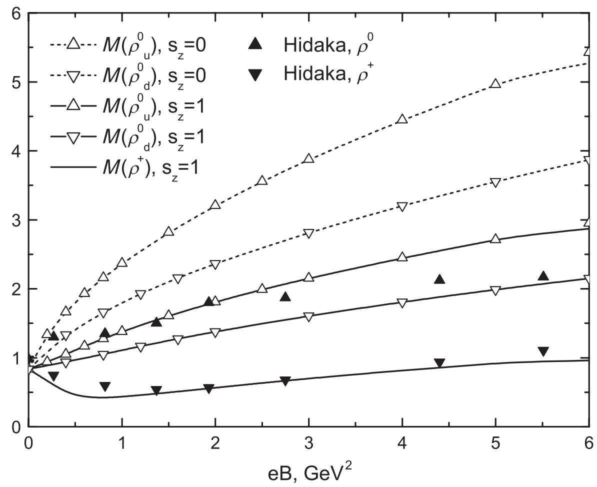

FIG. 2. The masses of the systems in GeV as a functions of eB. See the text for explanations.

Using the wave function (42), one obtains for $\langle a_{SS}\rangle$

$$\langle a_{SS}\rangle = \frac{c}{\pi^{3/2}\sqrt{\lambda^2 + r_0^2}(\lambda^2 + r_\perp^2)}, \qquad c = \frac{8\pi\alpha_s}{9\omega_1\omega_2}. \tag{66}$$

The smearing length λ on the lattice corresponds to the lattice unit a ($\lambda \sim a$), in a physical situation the relativistic smearing is connected with the gluelamp mass parameters in $D(z)$ and $D_1(z)$; see [20] for details.

In Fig. 2 we plot the masses of some selected systems as a function of eB (e is the ρ^+ charge, not the charge of individual quarks). Calculations were performed according to (59), (63), and (64) and the minimization procedure. The dashed curves correspond to the ρ^0 state with $s_z = 0$ (eigenvalue E_1), the solid-symbol lines describe the ρ^0 state with $s_z = 1$, the lower solid curve refers to the state of the charged meson ρ^+ with $s_z = 1$. The black triangles are from lattice calculations [23]. One can see that the masses of the first states are increasing, while the last one tends to the finite limit in accordance with the discussion above (note that the results plotted in Fig. 2 were obtained for massless quarks).

VI. DISCUSSION AND CONCLUSIONS

In our treatment of the relativistic quark-aniquark system embedded in a MF we relied on pseudomomentum factorization of the wave function and the relativistic Hamiltonian technique. The Hamiltonian for mesons in the MF, containing confinement, one gluon exchange, and spin interaction was derived. Using a suitable approximation for the confining force we were able to calculate analytically meson masses as functions of the MF. In this paper, to simplify things, we started with ρ^0 meson states at $B = 0$, taking γ_i in place of Γ_1 and Γ_2 in (13). In this way we essentially left aside the complicated problem of chiral dynamics and the pseudo-Goldstone spectrum. In this oversimplified picture the lowest neutral state with $s_z = 0$ is a mixture of the ρ^0 and π^0, as can be seen from its spin and isospin structure. Indeed, the $u\bar{u}$ or $d\bar{d}$ system under consideration is a mixture of isospin $I = 0$ and $I = 1$ states, and at large MF it has a spin structure $|u\uparrow, \bar{u}\downarrow\rangle$, which is a mixture of $S = 0$ and $S = 1$ states. We have calculated the mass of the higher state of this mixture, which we call $\rho^0(eB)$, while the lower state, associated with $\pi^0(eB)$, can be subject to chiral corrections. These states have negative corrections from one-gluon exchange and spin-spin interactions. As was shown, these corrections (and the total mass) stay finite at large B, preventing the so called ‘‘magnetic collapse in QCD’’, discussed earlier in [18].

As shown in Fig. 2, our analytical results are in agreement with lattice calculations [23], both for the ρ^0 and ρ^+ states.

Here, we add a few more remarks concerning the relation between the present work and the preceding one [18]. The amendments include (a) screening of the color Coulomb interaction, which leads to the nonvanishing ρ meson mass at large eB, in contrast with the results and Fig. 1 of [18], (b) separate treatment of the $u\bar{u}$ and $d\bar{d}$ states, (c) inclusion of the ρ^+ state, (d) account of the MF in self-energy, and (e) smearing of the singular δ function in spin-spin interaction. The main conclusion from (a) and (e) stressed here which was absent in [18] is that ‘‘fall to the center’’ in a MF does not occur. We believe this is a general conclusion which holds true independently of the technical approach to the problem.

Another system which can be treated using the same technique is the neutral three-body system, like the neutron. The results might be important for neutron star physics.

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