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Charmless weak B_s decays in the relativistic quark model

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The form factors of the weak B_s transitions to ground-state and orbitally excited strange mesons are calculated in the framework of the QCD-motivated relativistic quark model based on the quasipotential approach. These form factors are expressed through the overlap integrals of meson wave functions found in their mass spectrum evaluations. The momentum dependence of the form factors is determined in the whole accessible kinematical range without any additional assumptions and extrapolations. Relativistic effects, including the wave function transformation from rest to a moving reference frame as well as the contributions of the intermediate negative-energy states, are consistently taken into account. The calculated form factors are used for the evaluation of the charmless semileptonic decay rates and two-body nonleptonic B_s decays in the factorization approximation. The obtained results are confronted with previous predictions and available experimental data.

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I. INTRODUCTION

In the last few years the study of the properties of B_s mesons and especially their weak decays has attracted the attention of both theorists and experimenters. Indeed, the investigation of such decays is important for the independent determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, studying the *CP* violation, and testing the standard model and "new physics" models. New precise data are coming from the experiments on the Large Hadron Collider, which has significantly extended the number of observed B_s decay channels [1,2].

In a recent paper [3] we considered the weak B_s decays to the ground and excited states of charmed mesons. Such decays are the dominant decay channels of the B_s meson. We calculated the weak decay form factors in the framework of the relativistic quark model with the QCD-motivated interquark potential. Good agreement of our predictions for the semileptonic and nonleptonic decay branching fractions with experimental data has been found. Here we extend our analysis to the consideration of the CKM suppressed B_s decays to strange mesons. First we calculate the corresponding weak decay form factors. The characteristic features of the heavy-to-light $B_s \to K$ transitions are the presence of the light K meson in the final state and a very broad accessible kinematical region. Therefore it is important to consistently take into account the relativistic effects, which give substantial contributions, and to reliably determine the momentum dependence of the form factors. It is necessary to point out that most of the available theoretical approaches either permit evaluation of the form factors at some fixed point (of zero or maximum recoil of a final light meson) or determine the momentum dependence of the form factors in a restricted kinematical range. As a result, they require *ad hoc* assumptions about the q^2 dependence of form factors or their extrapolations, which also rely on specific models. The advantage of our model consists in its ability to explicitly determine the corresponding decay form factors as the overlap integrals of meson wave functions in the whole kinematical range without any additional assumptions and extrapolations.

In our calculations of the decay form factors we use the wave functions of the B_s and K mesons obtained previously in their mass spectra evaluations [4,5]. In Table I we compare our predictions for the masses of ground-state and lowest orbitally excited strange mesons [5] with available experimental data [2]. We find satisfactory agreement of our predictions with data. Note that in our mass spectrum calculations all relativistic effects, including the spin-dependent and spin-independent contributions to the potential, were treated nonperturbatively in v^2/c^2. From Table I we see that our model predicts that all orbitally excited $K_J^{(*)}$ mesons have masses heavier than 1 GeV. The scalar $K_0^*(800)$ (or κ) meson is predicted in our model to be a scalar tetraquark [6], while some other theoretical approaches assume it to be the scalar quark-antiquark (1^3P_0) state. It is clear that the form factors of the weak $B_s \to K_0^*$ transition are significantly different if the $K_0^*(1430)$ meson is the 1^3P_0 or 2^3P_0 state. Therefore the resulting rates of the semileptonic and nonleptonic B_s decays to the $K_0^*(1430)$ meson strongly vary depending on its structure and quantum numbers. Thus the study of the weak B_s decays to the K_0^* meson can help to reveal the nature of light scalar mesons. In the following, we denote the $K_0^*(1430)$ meson by K_0^*.

The calculated form factors are used for evaluation of the rates of the charmless semileptonic decays to both ground-state and orbitally excited strange mesons. The two-body tree-dominated nonleptonic B_s decays are considered within the factorization approximation. The charmless B_s decay rates are evaluated and compared with previous calculations and available experimental data.

TABLE I. Masses of the ground-state and first orbitally excited strange mesons calculated in our model (in MeV).

$n^{2S+1}L_J$	J^P	Meson	Theory [5]	Experiment [2]
1^1S_0	0^-	K	482	493.677(16)
1^3S_1	1^-	$K^*(892)$	897	891.66(26)
1^3P_0	0^+	$K_0^*(1430)$	1362	1425(50)
$1P_1$	1^+	$K_1(1270)$	1294	1272(7)
$1P_1$	1^+	$K_1(1400)$	1412	1403(7)
1^3P_2	2^+	$K_2^*(1430)$	1424	1425.6(15)

II. RELATIVISTIC QUARK MODEL

The employed relativistic quark model is based on the quasipotential approach in quantum chromodynamics (QCD). Hadrons are considered as the bound states of constituent quarks which are described by the single-time wave functions satisfying the three-dimensional relativistically invariant Schrödinger-like equation with the QCD-motivated interquark potential [7]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R=\frac{M^4-(m_1^2-m_2^2)^2}{4M^3}, \tag{2}$$

M is the meson mass, $m_{1,2}$ are the quark masses, and $\mathbf{p}$ is their relative momentum. In the center-of-mass system, the relative momentum squared on the mass shell $b^2(M)$ is expressed through the meson and quark masses:

$$b^2(M)=\frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}. \tag{3}$$

The kernel of this equation is the interquark quasipotential $V(\mathbf{p},\mathbf{q};M)$, which consists of the perturbative one-gluon exchange and the nonperturbative confining parts [7]:

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q), \tag{4}$$

with

$$\mathcal{V}(\mathbf{p},\mathbf{q};M)=\frac{4}{3}\alpha_sD_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu+V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(\mathbf{k})+V^S_{\rm conf}(\mathbf{k}),\qquad \mathbf{k}=\mathbf{p}-\mathbf{q},$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, and γ_μ and $u(p)$ are the Dirac matrices and spinors, respectively. The Lorentz structure of the confining part includes the scalar and vector linearly rising interactions which in the nonrelativistic limit reduce to

$$V_{\rm conf}(r)=V^S_{\rm conf}(r)+V^V_{\rm conf}(r)=Ar+B, \tag{5}$$

with

$$V^V_{\rm conf}(r)=(1-\varepsilon)(Ar+B),\qquad V^S_{\rm conf}(r)=\varepsilon(Ar+B), \tag{6}$$

where ε is the mixing coefficient. Its value $\varepsilon=-1$ has been obtained from the consideration of the heavy quark expansion for the semileptonic $B\to D$ decays [8] and charmonium radiative decays [7].

The long-range vector vertex

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu \tag{7}$$

contains the Pauli term with anomalous chromomagnetic quark moment κ. The value $\kappa=-1$, fixed in our model from the analysis of the fine splitting of heavy quarkonia ^3P_J states [7] and the heavy quark expansion for semileptonic decays of heavy mesons [8] and baryons [9], enables vanishing of the spin-dependent chromomagnetic interaction, proportional to $(1+\kappa)$, in accord with the flux tube model.

Other parameters of our model were determined from the previous analysis of meson spectroscopy [7]. The constituent quark masses are $m_b=4.88$ GeV, $m_c=1.55$ GeV, $m_s=0.5$ GeV, $m_{u,d}=0.33$ GeV, and the parameters of the linear potential are $A=0.18$ GeV^2 and $B=-0.30$ GeV.

For the consideration of the meson weak decays it is necessary to calculate the matrix element of the weak current between meson states. In the quasipotential approach, such a matrix element between a B_s meson with mass M_{B_s} and momentum p_{B_s} and a final K meson with mass M_K and momentum p_K is given by [10]

$$\langle K(p_K)|J^W_\mu|B_s(p_{B_s})\rangle=\int\frac{d^3p\,d^3q}{(2\pi)^6}\bar{\Psi}_{K\mathbf{p}_K}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{B_s\mathbf{p}_{B_s}}(\mathbf{q}), \tag{8}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function, $\Psi_{M\mathbf{p}_M}(\mathbf{p})$ are the meson ($M=B_s$, K) wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame with momentum $\mathbf{p}_M$, and $\mathbf{p}$, $\mathbf{q}$ are relative quark momenta.

It is convenient to carry calculations in the B_s meson rest frame ($\mathbf{p}_{B_s}=0$). Then the final meson is moving with the recoil momentum $\mathbf{\Delta}$. The wave function of the moving meson $\Psi_{K\mathbf{\Delta}}$ is connected with the wave function in the rest frame $\Psi_{K\mathbf{0}}\equiv\Psi_K$ by the transformation [10]

$$\Psi_{K\mathbf{\Delta}}(\mathbf{p})=D_u^{1/2}(R^W_{L_{\mathbf{\Delta}}})D_s^{1/2}(R^W_{L_{\mathbf{\Delta}}})\Psi_{K\mathbf{0}}(\mathbf{p}), \tag{9}$$

where R^W is the Wigner rotation, $L_{\mathbf{\Delta}}$ is the Lorentz boost from the meson rest frame to a moving one and $D^{1/2}(R)$ is the spin rotation matrix.

The wave function of a final $^{2S+1}K_J$ meson at rest is given by

$$\Psi_K(\mathbf{p})\equiv\Psi^{JLS\mathcal{M}}_{^{2S+1}K_J}(\mathbf{p})=\mathcal{Y}^{JLS\mathcal{M}}\psi_{^{2S+1}K_J}(\mathbf{p}), \tag{10}$$

where J and $\mathcal{M}$ are the total meson angular momentum and its projection, and L is the orbital momentum, while $S = 0, 1$ is the total spin. $\psi_{^{2S+1}K_J}(\mathbf{p})$ is the radial part of the wave function. The spin—angular momentum part $\mathcal{Y}^{JLS\mathcal{M}}$ is defined by

$$\mathcal{Y}^{JLS\mathcal{M}} = \sum_{\sigma_1\sigma_2} \langle L\,\mathcal{M}-\sigma_1-\sigma_2, S\sigma_1+\sigma_2|J\mathcal{M}\rangle \times \left\langle \frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2|S\sigma_1+\sigma_2\right\rangle Y_L^{\mathcal{M}-\sigma_1-\sigma_2}\chi_1(\sigma_1)\chi_2(\sigma_2), \tag{11}$$

where $\langle j_1 m_1, j_2 m_2|J\mathcal{M}\rangle$ are the Clebsch-Gordan coefficients, Y_l^m are the spherical harmonics, and $\chi(\sigma)$ (where $\sigma = \pm 1/2$) are the spin wave functions.

The explicit expression for the vertex function $\Gamma_\mu(\mathbf{p}, \mathbf{q})$ can be found in Ref. [3]. It contains contributions both from the leading-order spectator diagram (see Fig. 1) and from subleading-order diagrams accounting for the contributions of the negative-energy intermediate states. The leading-order contribution

$$\Gamma_\mu^{(1)}(\mathbf{p}, \mathbf{q}) = \bar{u}_c(p_c)\gamma_\mu(1-\gamma^5)u_b(q_b)(2\pi)^3\delta(\mathbf{p}_{\bar{s}} - \mathbf{q}_{\bar{s}}) \tag{12}$$

contains the δ function, which allows us to take one of the integrals in the matrix element [Eq. (8)]. Calculation of the subleading-order contribution is more complicated due to the dependence on the relative momentum in the energies of the initial heavy and final light quarks. For the energy of the heavy quark we use heavy quark expansion. For the light quark such expansion is not applicable. However, the final light K meson has a large (compared to its mass) recoil momentum $[|\mathbf{\Delta}_{\max}| = (M_{B_s}^2 - M_K^2)/(2M_{B_s}) \sim 2.6$ GeV] in almost the whole kinematical range except the small region near $q^2 = q^2_{\max}$ ($|\mathbf{\Delta}| = 0$). This also means that the recoil momentum of the final meson is large with respect to the mean relative quark momentum $|\mathbf{p}|$ in the meson (~ 0.5 GeV). Thus, one can neglect $|\mathbf{p}|$ compared to $|\mathbf{\Delta}|$ in the light quark energies $\epsilon_q(p+\Delta) \equiv \sqrt{m_q^2 + (\mathbf{p}+\mathbf{\Delta})^2}$, replacing it with $\epsilon_q(\Delta) \equiv \sqrt{m_q^2 + \mathbf{\Delta}^2}$ in expressions for the subleading contribution. Such replacement removes the relative momentum dependence in the energies of quarks and thus permits the performance of one of the integrations in the subleading contribution using the quasipotential equation. Since the subleading contributions are suppressed, the uncertainty introduced by such a procedure is small. As a result, the weak decay matrix element is expressed through the usual overlap integral of initial and final meson wave functions, and its momentum dependence can be determined in the whole accessible kinematical range without additional assumptions.

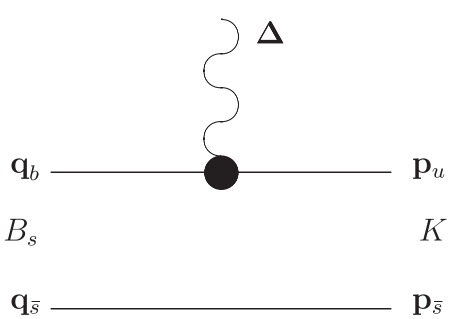

FIG. 1. Leading-order vertex function $\Gamma^{(1)}(\mathbf{p}, \mathbf{q})$ contributing to the current matrix element [Eq. (8)].

III. FORM FACTORS OF THE WEAK TRANSITIONS OF B_s TO K MESONS

The matrix elements of the vector, axial vector and tensor weak currents between B_s and $K^{(*)}$ meson states are parametrized by the following set of form factors:

$$\langle K(p_K)|\bar{u}\gamma^\mu b|B_s(p_{B_s})\rangle = f_+(q^2)\left[p_{B_s}^\mu + p_K^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_{B_s}^2 - M_K^2}{q^2}q^\mu, \tag{13}$$

$$\langle K(p_K)|\bar{u}\gamma^\mu\gamma_5 b|B_s(p_{B_s})\rangle = 0, \tag{14}$$

$$\langle K(p_K)|\bar{u}\sigma^{\mu\nu}q_\nu b|B_s(p_{B_s})\rangle = \frac{if_T(q^2)}{M_{B_s}+M_K}[q^2(p_{B_s}^\mu + p_K^\mu) - (M_{B_s}^2 - M_K^2)q^\mu], \tag{15}$$

$$\langle K^*(p_{K^*})|\bar{u}\gamma^\mu b|B_s(p_{B_s})\rangle = \frac{2iV(q^2)}{M_{B_s}+M_{K^*}}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{B_s\rho}p_{K^*\sigma}, \tag{16}$$

$$\langle K^*(p_{K^*})|\bar{u}\gamma^\mu\gamma_5 b|B_s(p_{B_s})\rangle = 2M_{K^*}A_0(q^2)\frac{\epsilon^*\cdot q}{q^2}q^\mu + (M_{B_s}+M_{K^*})A_1(q^2) \times\left(\epsilon^{*\mu} - \frac{\epsilon^*\cdot q}{q^2}q^\mu\right) - A_2(q^2)\frac{\epsilon^*\cdot q}{M_{B_s}+M_{K^*}} \times\left[p_{B_s}^\mu + p_{K^*}^\mu - \frac{M_{B_s}^2 - M_{K^*}^2}{q^2}q^\mu\right], \tag{17}$$

$$\langle K^*(p_{K^*})|\bar{u}i\sigma^{\mu\nu}q_\nu b|B_s(p_{B_s})\rangle = 2T_1(q^2)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{K^*\rho}p_{B_s\sigma}, \tag{18}$$

$$\langle K^*(p_{K^*})|\bar{u}i\sigma^{\mu\nu}\gamma_5 q_\nu b|B_s(p_{B_s})\rangle = T_2(q^2)[(M_{B_s}^2 - M_{K^*}^2)\epsilon^{*\mu} - (\epsilon^*\cdot q)(p_{B_s}^\mu + p_{K^*}^\mu)] + T_3(q^2)(\epsilon^*\cdot q)\left[q^\mu - \frac{q^2}{M_{B_s}^2 - M_{K^*}^2}(p_{B_s}^\mu + p_{K^*}^\mu)\right]. \tag{19}$$

Here $q = p_{B_s} - p_{K^{(*)}}$ and $M_{B,K^{(*)}}$ are the masses of the B meson and final $K^{(*)}$ meson, respectively, while ϵ_μ is the polarization vector of the final vector K^* meson.

At the maximum recoil point ($q^2 = 0$) these form factors satisfy the following conditions:

$$f_+(0) = f_0(0),$$

$$A_0(0) = \frac{M_{B_s} + M_{K^*}}{2M_{K^*}} A_1(0) - \frac{M_{B_s} - M_{K^*}}{2M_{K^*}} A_2(0),$$

$$T_1(0) = T_2(0).$$

Comparing these decompositions with the results of the calculations of the weak current matrix element in our model, as described in the previous section, we determine the form factors in the whole accessible kinematical range through the overlap integrals of the meson wave functions. The explicit expressions are given in Refs. [11,12]. For the numerical evaluations of the corresponding overlap integrals we use the quasipotential wave functions of B_s and $K^{(*)}$ mesons obtained in their mass spectra calculations [4,5].

We find that the weak $B_s \to K^{(*)}$ transition form factors can be approximated with good accuracy by the following expressions [13,14]:

(a) $F(q^2) = \{f_+(q^2), f_T(q^2), V(q^2), A_0(q^2), T_1(q^2)\}$

$$F(q^2) = \frac{F(0)}{\left(1 - \frac{q^2}{M^2}\right)\left(1 - \sigma_1 \frac{q^2}{M^2_{B^*}} + \sigma_2 \frac{q^4}{M^4_{B^*}}\right)}, \qquad (20)$$

(b) $F(q^2) = \{f_0(q^2), A_1(q^2), A_2(q^2), T_2(q^2), T_3(q^2)\}$

$$F(q^2) = \frac{F(0)}{\left(1 - \sigma_1 \frac{q^2}{M^2_{B^*}} + \sigma_2 \frac{q^4}{M^4_{B^*}}\right)}, \qquad (21)$$

where $M = M_{B^*}$ for the form factors $f_+(q^2)$, $f_T(q^2)$, $V(q^2)$, $T_1(q^2)$, and $M = M_B$ for the form factor $A_0(q^2)$. The obtained values $F(0)$ and $\sigma_{1,2}$ are given in Table II.

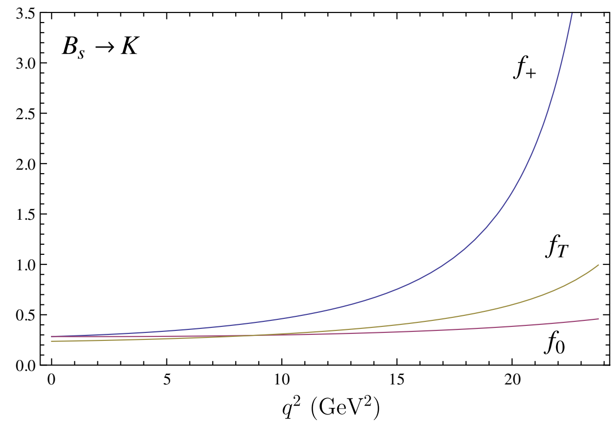

FIG. 2 (color online). Form factors of the weak $B_s \to K$ transition.

The difference between fitted and calculated form factors is less than 1%. We can roughly estimate the total uncertainty of the form factors within our model to be less than 5%. It mainly originates from the subleading contributions to the decay matrix elements in the region of small recoils. These form factors are plotted in Figs. 2 and 3.

In Table III we compare our predictions for the form factors of weak B_s decays at maximum recoil $q^2 = 0$ with results of other calculations [13,15–21]. Light-cone sum rules, including one-loop radiative corrections to twist-2 and twist-3 contributions, and leading-order twist-4 corrections are used in Ref. [15]. A perturbative QCD approach is applied in Refs. [16,17]. Calculations based on the quark model and relativistic dispersion approach are given in Ref. [13], while consideration in Ref. [18] is performed in the light-cone quark model utilizing the soft collinear effective theory. The authors of Ref. [19] employ light-cone sum rules in the framework of heavy quark effective theory. In Ref. [20] the weak transition form factors are evaluated in the six-quark effective Hamiltonian approach. A perturbative QCD factorization approach with the inclusion of the next-to-leading-order corrections is used in Ref. [21]. We find a reasonable agreement between the values of the weak $B_s \to K^{(*)}$ transition form factors at $q^2 = 0$ calculated in significantly different approaches.

TABLE II. Calculated form factors of weak $B_s \to K^{(*)}$ transitions. Form factors $f_+(q^2)$, $f_T(q^2)$, $V(q^2)$, $A_0(q^2)$, $T_1(q^2)$ are fitted by Eq. (20), and form factors $f_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $T_2(q^2)$, $T_3(q^2)$ are fitted by Eq. (21).

	$B_s \to K$			$B_s \to K^*$						
	f_+	f_0	f_T	V	A_0	A_1	A_2	T_1	T_2	T_3
$F(0)$	0.284	0.284	0.236	0.291	0.289	0.287	0.286	0.238	0.238	0.122
$F(q^2_{\max})$	5.42	0.459	0.993	3.06	2.10	0.581	0.953	1.28	0.570	0.362
σ_1	−0.370	−0.072	−0.442	−0.516	−0.383	0	1.05	−1.20	0.241	0.521
σ_2	−1.41	−0.651	0.082	−2.10	−1.58	−1.06	0.074	−2.44	−0.857	−0.613

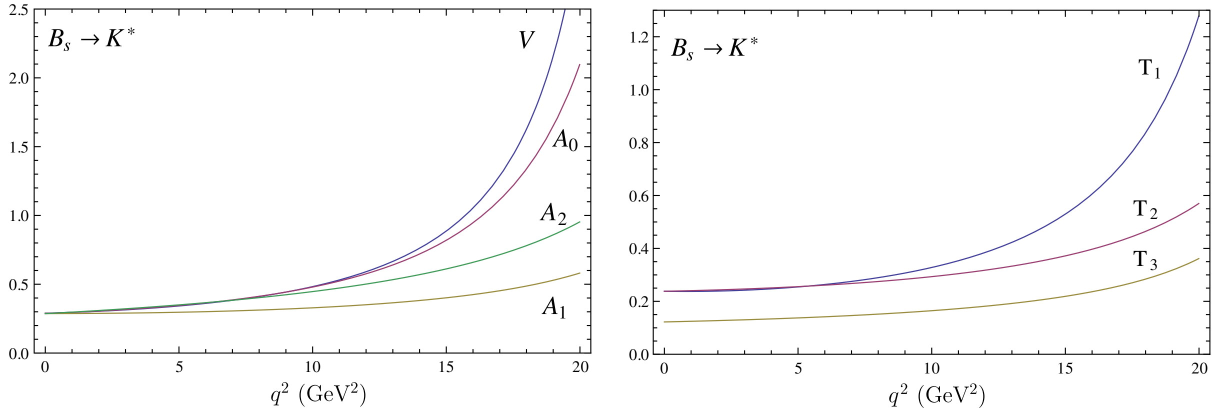

FIG. 3 (color online). Form factors of the weak $B_s \to K^*$ transition.

IV. FORM FACTORS OF WEAK TRANSITIONS OF B_s MESONS TO ORBITALLY EXCITED $K_J^{(*)}$ MESONS

Now we apply the same approach for the calculation of the form factors of the weak B_s decays to orbitally excited $K_J^{(*)}$ mesons. The matrix elements of the weak current $J_\mu^W = \bar{u}\gamma_\mu(1-\gamma_5)b$ for B_s decays to orbitally excited P-wave $K_J^{(*)}$ mesons can be parametrized by the following set of invariant form factors:

$$\langle K_0^*(p_{K_0})|\bar{u}\gamma^\mu b|B_s(p_{B_s})\rangle = 0, \tag{22}$$

$$\langle K_0^*(p_{K_0})|\bar{u}\gamma^\mu\gamma_5 b|B_s(p_{B_s})\rangle = r_+(q^2)(p_{B_s}^\mu + p_{K_0}^\mu) + r_-(q^2)(p_{B_s}^\mu - p_{K_0}^\mu), \tag{23}$$

$$\langle K_1(p_{K_1})|\bar{u}\gamma^\mu b|B_s(p_{B_s})\rangle = (M_{B_s}+M_{K_1})h_{V_1}(q^2)\epsilon^{*\mu} + [h_{V_2}(q^2)p_{B_s}^\mu + h_{V_3}(q^2)p_{K_1}^\mu]\frac{\epsilon^*\cdot q}{M_{B_s}}, \tag{24}$$

$$\langle K_1(p_{K_1})|\bar{u}\gamma^\mu\gamma_5 b|B_s(p_{B_s})\rangle = \frac{2ih_A(q^2)}{M_{B_s}+M_{K_1}}\epsilon^{\mu\nu\rho\sigma}\epsilon_\nu^* p_{B_s\rho}p_{K_1\sigma}, \tag{25}$$

$$\langle K_2^*(p_{K_2})|\bar{u}\gamma^\mu b|B_s(p_{B_s})\rangle = \frac{2it_V(q^2)}{M_{B_s}+M_{K_2}}\epsilon^{\mu\nu\rho\sigma}\epsilon_{\nu\alpha}^*\frac{p_{B_s}^\alpha}{M_{B_s}}p_{B_s\rho}p_{K_2\sigma}, \tag{26}$$

$$\langle K_2^*(p_{K_2})|\bar{u}\gamma^\mu\gamma_5 b|B_s(p_{B_s})\rangle = (M_{B_s}+M_{K_2})t_{A_1}(q^2)\epsilon^{*\mu\alpha}\frac{p_{B_s\alpha}}{M_{B_s}} + [t_{A_2}(q^2)p_{B_s}^\mu + t_{A_3}(q^2)p_{K_2}^\mu]\epsilon_{\alpha\beta}^*\frac{p_{B_s}^\alpha p_{B_s}^\beta}{M_{B_s}^2}, \tag{27}$$

where $q = p_{B_s} - p_{K_J}$, M_{K_J} are P-wave K-meson masses, and ϵ^μ and $\epsilon^{\mu\nu}$ are the polarization vector and tensor of the vector $K_1 \equiv K_1(1270)$ and tensor K_2^* mesons, respectively. The matrix elements of the weak current for B_s decays to the axial vector $K_1(1400)$ meson are obtained from

TABLE III. Comparison of theoretical predictions for the form factors of weak $B_s \to K^{(*)}$ transitions at maximum recoil point $q^2 = 0$.

	$f_+(0)$	$f_T(0)$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$T_1(0)$	$T_3(0)$
This paper	0.284 ± 0.014	0.236 ± 0.012	0.291 ± 0.015	0.289 ± 0.015	0.287 ± 0.015	0.286 ± 0.015	0.238 ± 0.012	0.122 ± 0.006
[15]	0.30 ± 0.04		0.311 ± 0.026	0.360 ± 0.034	0.233 ± 0.022	0.181 ± 0.025	0.260 ± 0.024	0.136 ± 0.016
[16]	0.24 ± 0.05		0.21 ± 0.04	0.25 ± 0.05	0.16 ± 0.04			
[13]	0.31	0.31	0.38	0.37	0.29	0.26	0.32	0.23
[17]			0.20 ± 0.05	$0.24^{+0.07}_{-0.05}$	$0.15^{+0.04}_{-0.03}$	0.11 ± 0.02	0.18 ± 0.05	0.16 ± 0.03
[18]	0.290	0.317	0.323	0.279	0.232	0.210	0.271	0.165
[19]	0.296 ± 0.018	0.288 ± 0.018	0.285 ± 0.013	0.222 ± 0.011	0.227 ± 0.011	0.183 ± 0.010	0.251 ± 0.012	0.169 ± 0.008
[20]	$0.260^{+0.055}_{-0.032}$		$0.227^{+0.064}_{-0.037}$	$0.280^{+0.090}_{-0.045}$	$0.178^{+0.047}_{-0.027}$			
[21]	$0.26^{+0.05}_{-0.04}$	0.28 ± 0.05						

Eqs. (24) and (25) by the replacement of the set of form factors $h_i(q^2)$ with $g_i(q^2)$ ($i = V_1, V_2, V_3, A$).

The P-wave K-meson states with $J = L = 1$ are the mixtures of spin-triplet ($^3P_1$) and spin-singlet ($^1P_1$) states:

$$|K_1(1270)\rangle = |K(^1P_1)\rangle\cos\varphi + |K(^3P_1)\rangle\sin\varphi,$$
$$|K_1(1400)\rangle = -|K(^1P_1)\rangle\sin\varphi + |K(^3P_1)\rangle\cos\varphi, \quad (28)$$

where φ is a mixing angle. Such mixing occurs due to the nondiagonal spin-orbit and tensor terms in the spin-dependent part of the relativistic quasipotential. The masses of physical states are obtained by diagonalizing the mixing terms. The found value of the mixing angle, $\varphi = 43.8°$ [5], implies that physical K_1 mesons are nearly equal mixtures of the spin-singlet $K(^1P_1)$ and spin-triplet $K(^3P_1)$ states in accord with the experimental data [2].

We calculate the matrix elements of the weak current between the initial B_s meson and final orbitally excited P-wave $K_J^{(*)}$ meson using the procedure described above and compare the result with the invariant decomposition Eqs. (22)–(27). In this way we obtain expressions for the decay form factors in terms of the overlap integrals of the initial and final meson wave functions which are valid in the whole accessible kinematical range. Note that these form factors account for the relativistic transformations of the meson wave functions from rest to a moving reference frame [Eq. (9)] and relativistic contributions from the intermediate negative-energy states. The explicit expressions for these form factors can be obtained from the corresponding formulas in the Appendix of Ref. [11] with obvious replacements.

For the numerical evaluations we again use the meson wave functions obtained in their mass spectra calculations [4,5]. The calculated values of B_s to orbitally excited $K_J^{(*)}$ transition form factors at maximum and zero recoil are given in Table IV. The momentum dependence of these form factors is shown in Fig. 4. We can estimate the errors in the calculated form factors to be less than 10%. They mainly originate from the evaluation of the subleading contribution to the vertex function and uncertainties in excited meson wave functions.

In Tables V, VI, and VII we compare our results for form factors of the weak B_s decays to scalar K_0^*, axial vector K_1, and tensor K_2^* mesons at the maximum recoil point $q^2 = 0$ with previous theoretical calculations [17,22–29]. Predictions are mostly available for the weak B_s decays to scalar K_0^* mesons. Such transitions were studied in the light-cone sum rules [22,24], QCD sum rules [23,26] and perturbative QCD approach [25,27]. We find that our result for the form factor $r_+(0)$ is consistent with Ref. [23], but lower than in other calculations [22,24–27]. In contrast, we predict a larger absolute value for the form factor $r_-(0)$ than Refs. [22,24,26]. Predictions for form factors of the weak B_s decays to axial vector K_1 mesons are compared in Table VI. The light-cone sum rule approach is used in Ref. [28], while Ref. [17] employs perturbative QCD. From this table we find that our approach and these theoretical approaches predict significantly different values of form factors at maximum recoil. One of the origins of the discrepancy could be the adopted values of the mixing angle φ, defined in Eq. (28), of axial vector K_1 mesons. Note that in our model this angle φ is explicitly calculated by diagonalizing the mass matrix [5], while in Refs. [17,28] different phenomenologically motivated values are used. The form factors of the weak B_s decays to tensor K_2 mesons were calculated in Ref. [29] in the perturbative QCD approach. The obtained values of the form factors $t_V(0)$ and $t_{A_1}(0)$ are slightly lower than in our model.

V. CHARMLESS SEMILEPTONIC B_s DECAYS

We can now apply the calculated form factors for the evaluation of the semileptonic B_s decays to ground-state and orbitally excited K mesons. The differential decay rate of the B_s meson to a K [$K^{(*)}$ or $K_J^{(*)}$] meson can be expressed in the following form [30]:

$$\frac{d\Gamma(B_s\to Kl\bar\nu)}{dq^2} = \frac{G_F^2}{(2\pi)^3}|V_{ub}|^2\frac{\lambda^{1/2}(q^2-m_l^2)^2}{24M_{B_s}^3q^2}\times\left[(H_+H_+^\dagger + H_-H_-^\dagger + H_0H_0^\dagger)\times\left(1+\frac{m_l^2}{2q^2}\right)+\frac{3m_l^2}{2q^2}H_tH_t^\dagger\right], \quad (29)$$

where G_F is the Fermi constant, V_{ub} is the CKM matrix element, $\lambda \equiv \lambda(M_B^2, M_F^2, q^2) = M_B^4 + M_F^4 + q^4 - 2(M_B^2M_F^2 + M_F^2q^2 + M_B^2q^2)$, and m_l is the lepton mass. The transverse $H_\pm$, longitudinal H_0 and time H_t helicity components of the hadronic tensor are defined through the transition form factors calculated in the previous sections. The corresponding relations for decays to ground and orbitally excited

TABLE IV. Form factors of the weak B_s decays to the P-wave $K_J^{(*)}$ mesons at $q^2 = 0$ and $q^2 = q^2_{\max} \equiv (M_{B_s} - M_{K_J})^2$.

	$B_s\to K_0^*$		$B_s\to K_1(1400)$				$B_s\to K_1(1270)$				$B_s\to K_2^*$			
q^2	r_+	r_-	g_A	g_{V_1}	g_{V_2}	g_{V_3}	h_A	h_{V_1}	h_{V_2}	h_{V_3}	t_V	t_{A_1}	t_{A_2}	t_{A_3}
0	0.27	−0.62	−0.33	−0.08	−0.16	−0.05	0.29	0.08	−0.14	0.42	−0.34	−0.17	−0.01	−0.01
$q^2_{\max}$	0.69	−1.59	−0.78	0.25	−0.75	0.02	0.98	−0.16	−0.03	1.17	−1.48	−0.48	−0.42	0.01

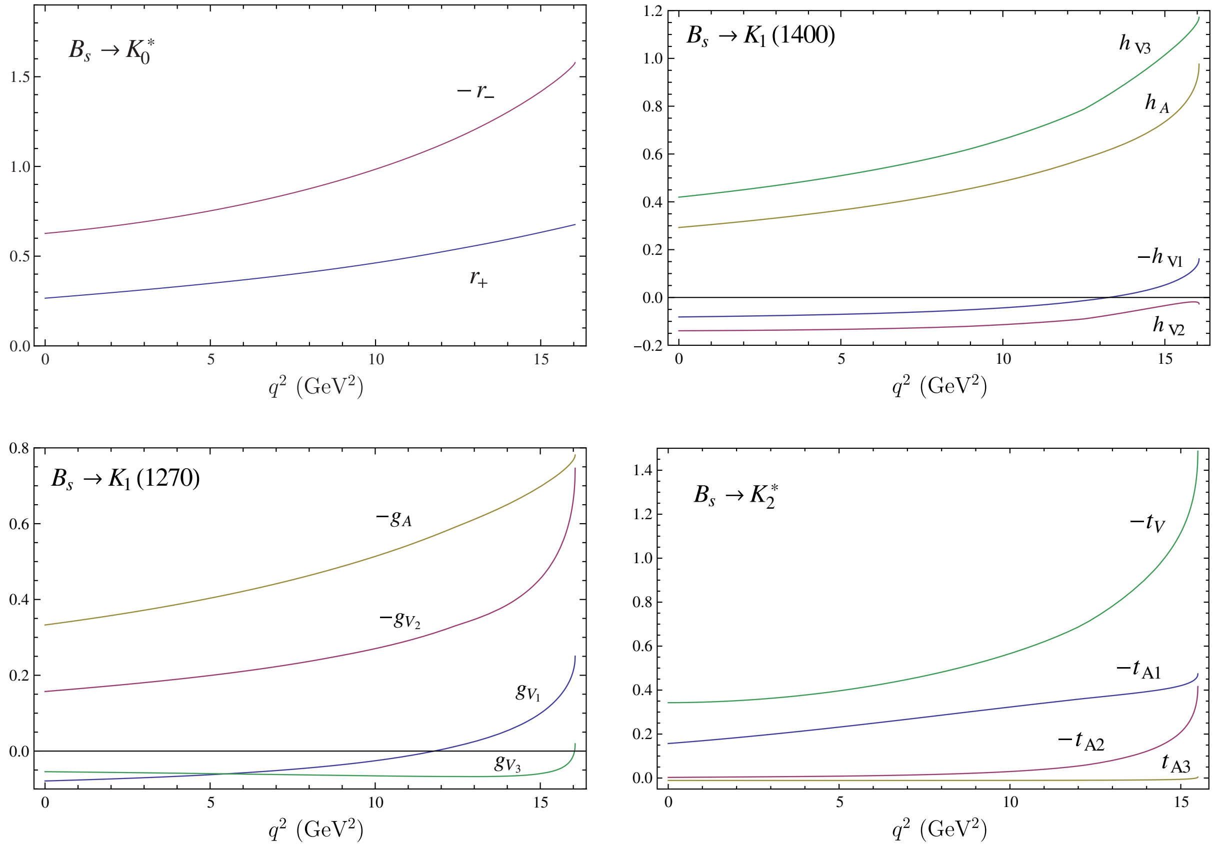

FIG. 4 (color online). Form factors of the B_s decays to the P—wave $K_J^{(*)}$ mesons.

mesons are given in Appendixes C and E of Ref. [3], respectively.

A. Semileptonic B_s decays to ground-state $K^{(*)}$ mesons

First, we calculate the rates of the semileptonic B_s decays to ground-state $K^{(*)}$ mesons. Substituting the form factors obtained in Sec. III into Eq. (29), we get the corresponding differential decay rates. They are plotted in Fig. 5 for decays involving both electron e and τ leptons. Integrating these differential decay rates over q^2, we find the total decay rates. In calculations we use the value of the CKM matrix element $|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}$ found previously from the comparison of the predictions of our model [14,31] with measured semileptonic $B \to \pi(\rho) l \nu_l$ decay rates. The kinematical range accessible in the heavy-to-light $B_s \to K^{(*)}$ transitions is very broad, making knowledge of the q^2 dependence of the form factors an important issue. Therefore, the explicit determination of the momentum dependence of the weak decay form factors in the whole q^2 range without any additional assumptions is an important advantage of our model. The calculated branching fractions of the semileptonic $B_s \to K^{(*)} l \nu_l$ decays are presented in Table VIII in comparison with other theoretical predictions [19,21]. The perturbative QCD factorization approach is used in Ref. [21], while in Ref. [19] light-cone sum rules are employed. From the comparison in Table VIII we see that all theoretical predictions for the B_s semileptonic branching fractions agree within uncertainties. This is not surprising, since these significantly different approaches predict close values of the corresponding weak form factors (see Table III).

We can use the calculated values of the semileptonic B_s decay branching fractions to obtain predictions for the ratios of such decay involving τ leptons and electrons or muons: $R(K) \equiv \mathrm{Br}(B_s \to K \tau \nu_\tau)/\mathrm{Br}(B_s \to K e \nu_e) = 0.59 \pm 0.05$ and

TABLE V. Comparison of theoretical predictions for the form factors of the weak $B_s \to K_0^*$ transitions at the maximum recoil point $q^2 = 0$.

$F(0)$	This paper	[22]	[23]	[24]	[25]	[26]	[27]
$r_+(0)$	0.27 ± 0.03	0.44	0.24 ± 0.10	$0.41^{+0.13}_{-0.07}$	$0.56^{+0.16}_{-0.13}$	0.39 ± 0.04	$0.56^{+0.07}_{-0.10}$
$r_-(0)$	-0.62 ± 0.06	-0.44		$-0.34^{+0.14}_{-0.09}$		-0.25 ± 0.05	

TABLE VI. Same as in Table V, but for the $B_s \to K_1$ transitions.

$F(0)$	This paper	[28]	[17]
$g_A(0)$	-0.33 ± 0.03	$-0.15^{+0.09}_{-0.07}$	0.03 ± 0.01
$g_{V_1}(0)$	-0.08 ± 0.01	$-0.11^{+0.07}_{-0.04}$	0.11 ± 0.07
$h_A(0)$	0.29 ± 0.03	0.49 ± 0.08	0.20 ± 0.05
$h_{V_1}(0)$	0.08 ± 0.01	0.38 ± 0.06	0.87 ± 0.25

TABLE VII. Same as in Table V, but for the $B_s \to K_2^*$ transition.

$F(0)$	This paper	[29]
$t_V(0)$	-0.34 ± 0.03	$-0.18^{+0.05}_{-0.04}$
$t_{A_1}(0)$	-0.17 ± 0.02	$-0.11^{+0.03}_{-0.02}$

TABLE VIII. Comparison of theoretical predictions for the branching fractions of semileptonic decays $B_s \to K^{(*)} l \nu_l$ (in 10^{-4}).

Decay	This paper	[21]	[19]
$B_s \to K e \nu_e$	1.64 ± 0.17	$1.27^{+0.49}_{-0.30}$	1.47 ± 0.15
$B_s \to K \tau \nu_\tau$	0.96 ± 0.10	$0.778^{+0.268}_{-0.201}$	1.02 ± 0.11
$B_s \to K^* e \nu_e$	3.47 ± 0.35		2.91 ± 0.26
$B_s \to K^* \tau \nu_\tau$	1.67 ± 0.17		1.58 ± 0.13

$R(K^*) \equiv \mathrm{Br}(B_s \to K^* \tau \nu_\tau)/\mathrm{Br}(B_s \to K^* e \nu_e) = 0.48 \pm 0.04$. The interest in such ratios is stimulated by recently found deviations of experimental data from theoretical predictions for the similar ratios for the semileptonic B decays to D mesons $R(D^{(*)}) = \mathrm{Br}(B \to D^{(*)} \tau \nu_\tau)/\mathrm{Br}(B \to D^{(*)} e \nu_e)$ (see, e.g., discussion in Ref. [31] and references therein).

Summing up different contributions listed in Table VIII, we obtain the prediction for the total semileptonic B_s decay branching fraction to the ground-state K mesons to be $\mathrm{Br}(B_s \to K^{(*)} e \nu_e) = (5.11 \pm 0.51) \times 10^{-4}$ and $\mathrm{Br}(B_s \to K^{(*)} \tau \nu_\tau) = (2.63 \pm 0.26) \times 10^{-4}$.

B. Semileptonic B_s decays to orbitally excited $K_J^{(*)}$ mesons

Now we calculate the branching fractions of the semileptonic B_s decays to orbitally excited $K_J^{(*)}$ mesons. We substitute the form factors obtained in Sec. IV into the expression for the differential decay rate [Eq. (29)]. The resulting decay rates are plotted in Fig. 6. Integration over q^2 gives the total semileptonic $B_s \to K_J^{(*)} l \nu_l$ branching fractions which are given in Table IX. We see that our model predicts close values (about 1×10^{-4}) for all semileptonic B_s branching fractions to the first orbitally excited $K_J^{(*)}$ mesons. Indeed, the difference between branching fractions is less than a factor of 2. This result is in contradiction to the dominance of specific modes (by more than a factor of 4) in the heavy-to-heavy semileptonic $B \to D_J^{(*)} l \nu_l$ and $B_s \to D_{sJ}^{(*)} l \nu_l$ decays [3,32], but it is consistent with predictions for the corresponding heavy-to-light semileptonic B decays to orbitally excited light mesons [33]. The above mentioned suppression of some heavy-to-heavy decay channels to orbitally excited heavy mesons was mostly pronounced in the heavy quark limit and then slightly reduced by the heavy quark mass corrections which are found to be large [32]. Thus, our result once again indicates that the s quark cannot be treated as a heavy one and should be considered to be light instead, as we always did in our calculations.

In Table IX we compare our predictions for the semileptonic B_s branching fractions to orbitally excited $K_J^{(*)}$ mesons with previous calculations [17,23–25,28,29]. The consideration in Ref. [23] is based on QCD sum rules. The light-cone sum rules are used in Refs. [24,28], while Refs. [17,25,29] employ the perturbative QCD approach. Reasonable agreement between our results and other predictions [23,24,29] is observed for the semileptonic B_s decays to the scalar and tensor K mesons. The values of Ref. [25] are almost a factor of 3 higher. For the semileptonic B_s decays to axial vector K mesons, predictions are significantly different even within rather large errors.

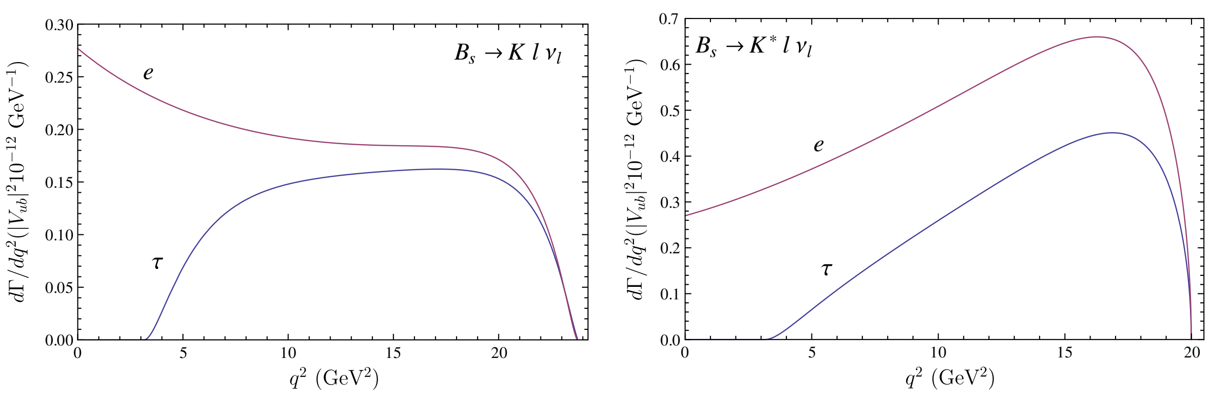

FIG. 5 (color online). Predictions for the differential decay rates of the semileptonic $B_s \to K^{(*)} l \nu_l$ decays.

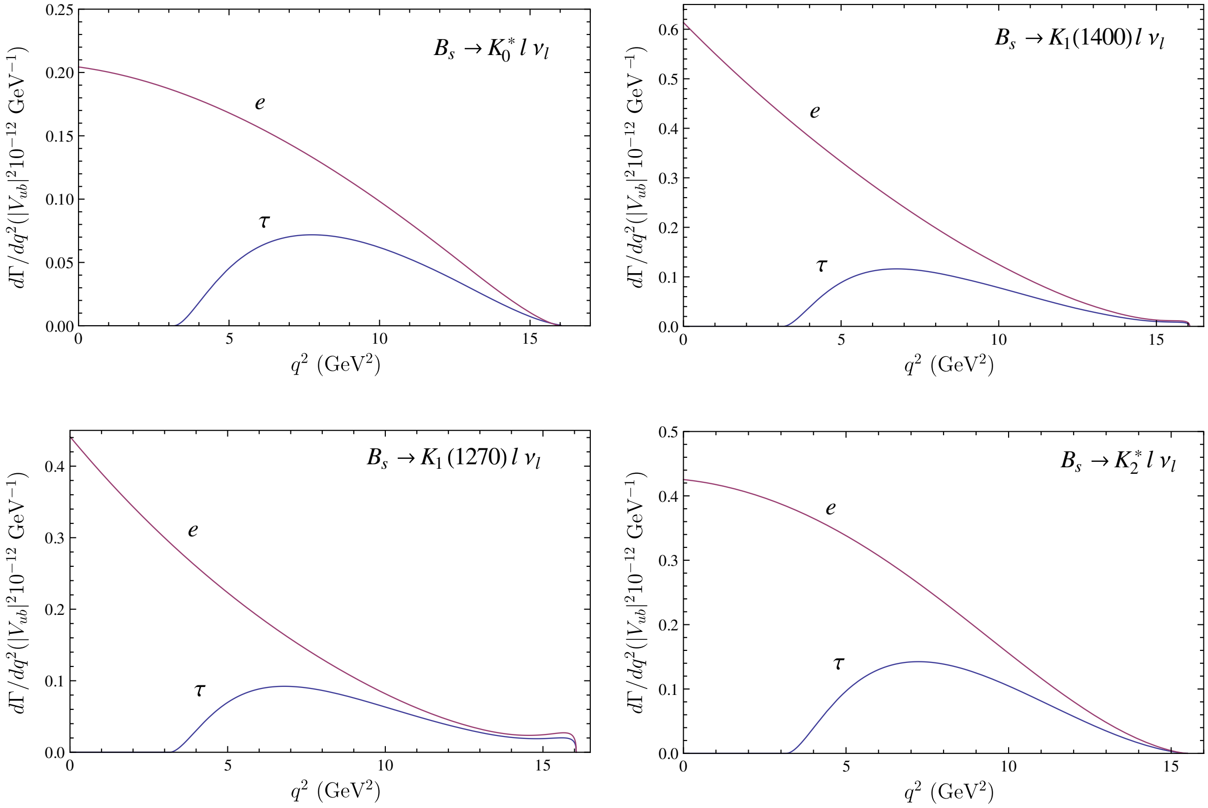

FIG. 6 (color online). Predictions for the differential decay rates of the semileptonic $B \to K_J^{(*)} l \nu_l$ decays.

Therefore experimental measurement of these decay branching fractions can help to discriminate between theoretical approaches.

For the total semileptonic B_s decays to first orbital excitations of K mesons, we get $\mathrm{Br}(B_s \to K_J^{(*)} e \nu_e) = (4.4 \pm 0.9) \times 10^{-4}$ and $\mathrm{Br}(B_s \to K_J^{(*)} \tau \nu_\tau) = (1.1 \pm 0.2) \times 10^{-4}$. These values are close to the ones found for the semileptonic B_s decays to ground-state K mesons in Sec. VA. The similar pattern of the branching fraction dependence on the excitation of a final meson was previously found for the heavy-to-light semileptonic B decays [33]. On the other hand, for the heavy-to-heavy semileptonic $B \to D$ and $B_s \to D_s$ decays, a pronounced hierarchy, where the decay branching fractions rapidly decrease with the growing excitation of the final heavy (D or D_s) meson, is observed [3,32].

VI. CHARMLESS NONLEPTONIC DECAYS OF B_s MESONS

We further apply calculated B_s decay form factors to the evaluation of the charmless nonleptonic decays of the B_s meson. In the discussion we closely follow our previous consideration of nonleptonic decays of B mesons to ground and orbitally excited states of light mesons in the factorization approximation [33,34]. To simplify the problem we limit our analysis to the calculation of the decay processes dominated by the tree diagrams. For such decays the matrix elements of the effective weak Hamiltonian $H_{\rm eff}$, governing nonleptonic decays $B_s \to K^{-,0} M^{+,0}$, where M is a light (π or ρ) meson, can be approximated by the product of one-particle transition amplitudes

$$\langle K^- M^+ | H_{\rm eff} | B_s \rangle \approx \frac{G_F}{\sqrt{2}} V_{ub}^* V_{ud} a_1^{\rm eff} \langle K^- | (\bar{b}u)_{V-A} | B_s \rangle \times \langle M^+ | (\bar{d}u)_{V-A} | 0 \rangle,$$
$$\langle K^0 M^0 | H_{\rm eff} | B_s \rangle \approx \frac{G_F}{\sqrt{2}} V_{ub}^* V_{ud} a_2^{\rm eff} \langle K^0 | (\bar{b}d)_{V-A} | B_s \rangle \times \langle M^0 | (\bar{u}u)_{V-A} | 0 \rangle, \quad (30)$$

with

$$a_1^{\rm eff} = a_1 - \frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} [a_4 + a_{10} + r_q(a_6 + a_8)],$$
$$a_2^{\rm eff} = a_2 - \frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \Big[-a_4 \mp \frac{3}{2} a_7 + \frac{3}{2} a_9 + \frac{1}{2} a_{10} - r_q(2a_6 - a_8) \Big],$$

where terms in square brackets result from the penguin contributions, which are small numerically; minus and plus correspond to $M^0 = \pi^0$ and $M^0 = \rho^0$, respectively. The quantities $a_{2n-1} = c_{2n-1} + c_{2n}/N_c$ and $a_{2n} = c_{2n} + c_{2n-1}/N_c$ ($n = 1, 2 \ldots$ with N_c being the

TABLE IX. Comparison of theoretical predictions for the branching fractions of semileptonic decays $B_s \to K_J^{(*)} l \nu_l$ (in 10^{-4}).

Decay	This paper	[23]	[24]	[25]	[28]	[17]	[29]
$B_s \to K_0^* e \nu_e$	0.71 ± 0.14	$0.36^{+0.38}_{-0.24}$	$1.3^{+1.3}_{-0.4}$	$2.45^{+1.77}_{-1.05}$			
$B_s \to K_0^* \tau \nu_\tau$	0.21 ± 0.04		$0.52^{+0.57}_{-0.18}$	$1.09^{+0.82}_{-0.47}$			
$B_s \to K_1(1270) e \nu_e$	1.41 ± 0.28				$4.53^{+1.67}_{-2.05}$	$5.75^{+3.49}_{-2.89}$	
$B_s \to K_1(1270) \tau \nu_\tau$	0.30 ± 0.06					$2.62^{+1.58}_{-1.31}$	
$B_s \to K_1(1400) e \nu_e$	0.97 ± 0.20				$3.86^{+1.43}_{-1.75}$	$0.03^{+0.05}_{-0.02}$	
$B_s \to K_1(1400) \tau \nu_\tau$	0.25 ± 0.05					$0.01^{+0.02}_{-0.01}$	
$B_s \to K_2^* e \nu_e$	1.33 ± 0.27						$0.73^{+0.48}_{-0.33}$
$B_s \to K_2^* \tau \nu_\tau$	0.36 ± 0.07						$0.25^{+0.17}_{-0.12}$

number of colors) are combinations of the Wilson coefficients c_i, which we take from Ref. [35], and r_q can be found, e.g., in Ref. [36].

The matrix element of the weak current J_μ^W between vacuum and a final pseudoscalar (P) or vector (V) meson can be parametrized by the decay constants $f_{P,V}$

$$\langle P|\bar{q}_1 \gamma^\mu \gamma_5 q_2|0\rangle = i f_P p_P^\mu, \qquad \langle V|\bar{q}_1 \gamma_\mu q_2|0\rangle = \epsilon_\mu M_V f_V. \tag{31}$$

As a result the corresponding nonleptonic matrix element factorizes in the product of the weak $B_s \to K$ decay form factors, which were calculated in the previous sections, and decay constants. The pseudoscalar f_P and vector f_V decay constants of light and heavy mesons were calculated within our model in Ref. [37]. Their values are in agreement with the available experimental data [2]. For the calculations we use the following values of the decay constants: $f_\pi = 0.131$ GeV, $f_\rho = 0.208$ GeV. The relevant CKM matrix elements are $|V_{ud}| = 0.975$, $|V_{td}| = 0.0087$, $|V_{tb}| = 0.999$ [2].

In Table X we present the predictions for the branching fractions of the charmless nonleptonic B_s decays to ground-state K mesons obtained in the factorization approximation with our model form factors. There we also give results of other theoretical approaches [16,20,38–40] and available experimental data [2]. Calculations in Ref. [16] are done in a perturbative QCD approach. QCD factorization is employed in Refs. [38,40]. The authors of Ref. [39] use soft collinear effective theory, while considerations in Ref. [20] are based on an approximate six-quark effective Hamiltonian. Experimentally only the $B_s \to K^- \pi^+$ branching fraction was measured [2] and is available for the $B_s \to K^{*0} \rho^0$ decay upper limit. All theoretical predictions agree with data. In fact, all theoretical results for decays into charged $K^{(*)-}$ and $\pi^+(\rho^+)$ mesons are consistent within rather large error bars, while for decays involving neutral mesons deviations are larger.

We apply the same factorization approach for the calculation of the nonleptonic decays to orbitally excited $K_J^{(*)}$ mesons. Using form factors obtained in Sec. IV, we get predictions for the nonleptonic branching fractions and present them in Table XI in comparison with previous estimates [27,41]. A few predictions are available for selected modes. In Ref. [27] the decay $B_s \to K_0^{*-} \rho^+$ was considered within the perturbative QCD factorization approach. The obtained central value of this decay branching fraction is almost a factor of 4 larger than our result. This is the consequence of a form factor $r_+(0)$ in Ref. [27] 2 times larger than in our model (see Table V). Decays involving the tensor K_2^{*-} meson were considered in Ref. [41] within the Isgur-Scora-Grinstein-Wise II model. The predictions are approximately a factor of 2 lower than our central values of branching fractions.

TABLE X. Comparison of various predictions for the branching fractions of the charmless nonleptonic B_s decays to ground-state K mesons with experiment (in 10^{-6}).

Decay	This paper	[16]	[38]	[39]	[40]	[20]	Experiment [2]
$B_s \to K^- \pi^+$	8.7 ± 2.7	$7.6^{+3.3}_{-2.5}$	$10.2^{+6.0}_{-5.2}$	4.9 ± 1.8	$5.3^{+0.5}_{-0.9}$	$7.1^{+3.3}_{-1.8}$	5.3 ± 1.0
$B_s \to K^- \rho^+$	24.0 ± 7.2	$17.8^{+7.9}_{-5.9}$	$24.5^{+15.2}_{-12.9}$	10.2 ± 1.0	$14.7^{+1.7}_{-2.3}$	$17.6^{+8.2}_{-4.6}$	
$B_s \to K^{*-} \pi^+$	8.6 ± 2.6	$7.6^{+3.0}_{-2.3}$	$8.7^{+5.9}_{-4.9}$	6.6 ± 0.7	$7.8^{+0.6}_{-0.9}$	$7.2^{+5.6}_{-2.3}$	
$B_s \to K^{*-} \rho^+$	25.4 ± 7.6	$20.9^{+8.4}_{-6.5}$	$25.2^{+4.9}_{-3.5}$		$21.6^{+1.6}_{-3.2}$	$21.0^{+13.5}_{-6.5}$	
$B_s \to K^0 \pi^0$	0.25 ± 0.08	$0.16^{+0.11}_{-0.06}$	$0.49^{+0.63}_{-0.35}$	0.76 ± 0.41	$1.7^{+2.7}_{-0.9}$	$1.1^{+0.7}_{-0.3}$	
$B_s \to K^0 \rho^0$	0.67 ± 0.20	$0.08^{+0.07}_{-0.04}$	$0.61^{+1.26}_{-0.60}$	0.81 ± 0.09	$1.9^{+3.2}_{-1.1}$	$0.6^{+0.3}_{-0.2}$	
$B_s \to K^{*0} \pi^0$	0.24 ± 0.07	$0.07^{+0.05}_{-0.03}$	$0.25^{+0.46}_{-0.22}$	1.07 ± 0.19	$0.89^{+1.16}_{-0.49}$	$0.3^{+0.2}_{-0.2}$	
$B_s \to K^{*0} \rho^0$	0.71 ± 0.21	$0.33^{+0.17}_{-0.11}$	$1.5^{+3.3}_{-1.5}$		$1.3^{+2.6}_{-0.7}$	$1.0^{+0.4}_{-0.3}$	<767

TABLE XI. Branching fractions of the nonleptonic B_s decays to orbitally excited $K_J^{(*)}$ mesons (in 10^{-6}).

Decay	This paper	[27]	[41]
$B_s \to K_0^{*-}\pi^+$	9.6 ± 3.8		
$B_s \to K_0^{*-}\rho^+$	27 ± 10	108^{+34}_{-31}	
$B_s \to K_1(1270)\pi^+$	29 ± 12		
$B_s \to K_1(1270)\rho^+$	76 ± 30		
$B_s \to K_1^-(1400)\pi^+$	21 ± 8		
$B_s \to K_1^-(1400)\rho^+$	54 ± 21		
$B_s \to K_2^{*-}\pi^+$	17 ± 6		7.8
$B_s \to K_2^{*-}\rho^+$	47 ± 18		23

VII. CONCLUSIONS

The form factors of the B_s weak decays to strange mesons were calculated in the framework of the relativistic quark model based on the quasipotential approach, which proved to be an efficient tool for studying the hadron weak transition form factors with the consistent account of relativistic kinematical and dynamical effects. Decays both to the ground $K^{(*)}$ and to orbitally excited $K_J^{(*)}$ mesons were considered. The form factors were determined as the overlap integrals of the related meson wave functions, found in Refs. [4,5], in the whole broad kinematical range without additional assumptions about their q^2 dependence. The important relativistic effects, such as transformations of the meson wave functions from rest to a moving reference frame and contributions of the intermediate negative-energy states, were consistently taken into account.

We used these form factors for the evaluation of the charmless semileptonic B_s decay rates to ground-state and orbitally excited K mesons. It was found that total branching fractions of semileptonic B_s decays to ground and first orbitally excited K mesons have close values of about 5×10^{-4}. Summing up these contributions, we get $(9.5 \pm 1.0) \times 10^{-4}$. This value is almost 2 orders of magnitude lower than our prediction for the corresponding sum of branching fractions of the semileptonic B_s to D_s mesons [3] as it was expected from the ratio of CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$. Therefore, the total semileptonic B_s decay branching fraction is dominated by the decays to D_s mesons and in our model is equal to $(10.3 \pm 1.0)\%$ in good agreement with the experimental value $\mathrm{Br}(B_s \to X e \nu_e)_{\mathrm{Exp}} = (9.5 \pm 2.7)\%$ [2]. The obtained predictions for charmless semileptonic B_s decays were compared with previous calculations within quark models, light-cone sum rules, QCD sum rules and a perturbative QCD approach. It was found that different theoretical approaches yield close values of branching fractions for decays to ground-state $K^{(*)}$ mesons agreeing within uncertainties, while the ones for the decays to orbitally excited $K_J^{(*)}$ mesons differ significantly from each other. The latter observation can help to discriminate between theoretical models.

Charmless two-body nonleptonic B_s decays dominated by tree diagrams were considered. The factorization approximation was used, which allowed us to express the decay matrix elements as the product of weak transition matrix elements and decay constants.[1] The branching fractions of the nonleptonic decays to a ground-state $K^{(*)}$ or orbitally excited $K_J^{(*)}$ meson and a pion or ρ meson, both charged and neutral, were calculated. The obtained results were confronted with previous theoretical predictions and experimental data, which are available for only few of the considered decays. Good agreement with data and other evaluations is found for the nonleptonic decays to the ground-state $K^{(*)}$ meson and a light meson, while again for decays involving the orbitally excited $K_J^{(*)}$ mesons significant disagreement between predictions of different approaches is observed.

ACKNOWLEDGMENTS

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[1]Note that the nonfactorizable contributions are suppressed for the considered decays, but for many other decay modes they are not negligible and violate the factorization approximation.

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