

Implication of possible observation of enhanced $B_d^0 \rightarrow \mu^+ \mu^-$ decayWei-Shu Hou,¹ Masaya Kohda,¹ and Fanrong Xu²¹*Department of Physics, National Taiwan University, Taipei 10617, Taiwan*²*Department of Physics, National Tsing Hua University, Hsinchu 30013, Taiwan*

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The very rare $B_d^0 \rightarrow \mu^+ \mu^-$ decay may be the last chance for new physics in the flavor sector at the LHC, before the 13 TeV run in 2015. Partially motivated by the known tension in $\sin 2\beta/\phi_1$, enhancement beyond $(3-4) \times 10^{-10}$ would likely imply the effect of a fourth generation of quarks. If observed at this level, the 126 GeV boson may not be the actual Higgs boson, while the $b \rightarrow d$ quadrangle (modulo $m_{t'}$) would jump out. The 2011–2012 data is likely not sensitive to values below 3×10^{-10} , and the mode should continue to be pursued with the 13 TeV run.

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I. INTRODUCTION

Despite the discovery [1,2] of a 126 GeV boson by the Large Hadron Collider (LHC) in 2012, the LHC has so far been a disappointment: no new physics beyond the Standard Model (SM) has been seen, and even the new boson appears Higgs-like, i.e., as prescribed by SM.

Surveying the terrain, there seems one last hope for discovering new physics, namely $B_d^0 \rightarrow \mu^+ \mu^-$. There is some motivation for enhancement, from the well-known [3,4] mild (of order 2σ) but lingering tension between direct measurement of the CP violation (CPV) phase of \bar{B}_d-B_d mixing versus extraction by indirect means. If an enhanced $B_d^0 \rightarrow \mu^+ \mu^-$ rate is discovered with 2011–2012 LHC data, the likely explanation would be a fourth generation of quarks. This would then cast doubt on the Higgs boson interpretation of the 126 GeV boson.

The $B_s^0 \rightarrow \mu^+ \mu^-$ decay has been a highlight pursuit since Tevatron times, and only recently surpassed [5] in sensitivity by the LHC. The drive has been the possibly huge enhancement by exotic scalar effects inspired by supersymmetry, but now excluded by the first evidence for SM-like rates by the LHCb experiment [6]. In contrast, the search for $B_d^0 \rightarrow \mu^+ \mu^-$ has not shared the limelight. This is because the SM prediction itself is 30 times lower than $B_s^0 \rightarrow \mu^+ \mu^-$. However, the combined LHC bound is now within [5] a factor of 8 of the SM prediction, and one may ask whether this mode could be anywhere enhanced up to this order.

As pictorialized by the ‘‘Straub plot’’ [7] and discussed recently by Stone [8], most models of enhancement for $B_d^0 \rightarrow \mu^+ \mu^-$ have now been eliminated by the SM-like $B_s^0 \rightarrow \mu^+ \mu^-$ rate measured by LHCb, with two exceptions. One is an old, purely left-handed supersymmetric model [9]. However, the region allowed by current data is but a corner of the parameter space, hence not plausible. The other would be [10] the 4th generation (4G), where $B_d^0 \rightarrow \mu^+ \mu^-$ and $B_s^0 \rightarrow \mu^+ \mu^-$ decays are modulated by different Cabibbo-Kobayashi-Maskawa (CKM) products $V_{t'd}^* V_{t'b}$ and $V_{t's}^* V_{t'b}$, allowing $B_d^0 \rightarrow \mu^+ \mu^-$ to be enhanced

up to the current bound, even if $B_s^0 \rightarrow \mu^+ \mu^-$ is SM-like. Stone has followed conventional wisdom to argue [8] that 4G has been ‘‘eliminated by the Higgs discovery,’’ because it ‘‘would cause the Higgs production cross-section to be nine times larger . . .’’ [11]. In fact, a comprehensive analysis [12] including electroweak and flavor observables plus earlier Higgs production data already ruled out 4G in SM framework. There are two catches in this pessimism, however. First of all, it is not yet established that the observed 126 GeV object is the Higgs boson of the SM. For example, a dilaton might mimic [13] the Higgs with current data. Second, the Higgs boson of the SM does not enter into the $B_d^0 \rightarrow \mu^+ \mu^-$ process (the same holds for the B_d box diagram and $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ processes we consider). To assume indirect arguments in the flavor pursuit is self defeating, especially when there is still room for large enhancement; it actually highlights the potential impact of a discovery.

It was shown [14] recently, through an empirical gap equation [15], that dynamical electroweak symmetry breaking (DEWSB) could occur through strong Yukawa coupling of 4G quarks. Although there is no account for how a dilaton actually emerges, the scale invariance of this gap equation allows for a dilaton to appear. The dilaton possibility can be checked experimentally through the absence, or suppression, of vector boson fusion (VBF) and associated production (VH) processes, which requires more data than currently available. The very large Yukawa coupling needed for DEWSB is consistent with not finding the 4G quarks so far, where the current bounds [16] are already above the nominal [17] unitarity bound (UB). Thus, the numerical study we present below is only meant as an illustration.

In the following, we review input parameters and constraints, then present our numerical study. We indeed find enhancement beyond 4×10^{-10} (4 times the SM) is possible [18] within the parameter space indicated by the known tension in $\sin 2\Phi_{B_d} \equiv \sin 2\phi_1/\beta$. We give an assessment of immediate and longer term prospects.

II. CONSTRAINTS AND INPUT PARAMETERS

There is no indication for new physics in $b \rightarrow s$ transitions at present. The best probe is the $\sin 2\Phi_{B_s}$ measurement pursued by LHCb, where Φ_{B_s} is defined as the CPV phase in the $\bar{B}_s \rightarrow B_s$ mixing amplitude (hence $\sin 2\Phi_{B_s} \equiv \sin \phi_s$). This definition is consistent with $\sin 2\beta/\phi_1 \equiv \sin 2\Phi_{B_d}$ used by the B factories. The 4G t' quark could have easily affected many $b \rightarrow s$ processes [10,19]. However, all of these, including $s \rightarrow d$ transition effects, can be tuned away or softened by a small $|V_{t's}^* V_{t'b}|$ strength, which is demanded by $\sin 2\Phi_{B_s}$ being consistent with SM expectations and is yet to be measured. As illustrated by the Straub plot [7,10], $B_s \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow \mu^+ \mu^-$ can vary independently from each other, i.e., through $V_{t'd}^* V_{t'b}$ and $V_{t's}^* V_{t'b}$, subject to constraint from kaon physics (affected by $V_{t'd}^* V_{t's}$).

It is well known [3,4], however, that there is some tension between the directly measured value [20] of

$$\sin 2\beta/\phi_1 = 0.679 \pm 0.020, \quad (1)$$

and SM expectation via $\beta/\phi_1 \equiv \arg \lambda_t^{\text{SM}}$, where [21]

$$\lambda_t^{\text{SM}} = -\lambda_u - \lambda_c \simeq -|V_{ud}| |V_{ub}| e^{-i\phi_3} + |V_{cd}| |V_{cb}|, \quad (2)$$

with $\lambda_i \equiv V_{id}^* V_{ib}$. The terms on the right-hand side of Eq. (2) can be measured at the tree level. Currently [20],

$$\phi_3 = (68_{-10}^{+11})^\circ, \quad (3)$$

and we take the central values $|V_{ud}| = 0.974$, $|V_{cd}| = 0.23$ and $|V_{cb}| = 0.041$ [20]. Variations in these values are not central to our discussion.

In contrast, $|V_{ub}|$ also has some tension in the measured values. Extraction via inclusive or exclusive semi-leptonic B decays yield approximately 4.41×10^{-3} and 3.23×10^{-3} [20], respectively, with the average value of 4.15×10^{-3} (the inclusive approach has better statistics). We use central values, as our purpose is only for illustration, hence we will treat the average (which is close to inclusive) and exclusive cases separately.

Although the strength of $|\lambda_t^{\text{SM}}| \simeq 0.0088$ is not sensitive to $|V_{ub}|$, the phase is sensitive to its value,

$$\sin 2\beta/\phi_1 = \begin{cases} 0.76 & \text{for } |V_{ub}|^{\text{ave}} \\ 0.63 & \text{for } |V_{ub}|^{\text{excl}}, \end{cases} \quad (4)$$

which both deviate from Eq. (1) by more than 2σ (the inclusive value of 0.81 deviates even more). This deviation offers some motivation for new physics in $b \rightarrow d$ transitions. It could easily be due to the 4G quark t' , where one simply augments Eq. (2) by

$$\lambda_t = \lambda_t^{\text{SM}} - \lambda_{t'}, \quad (5)$$

and the $b \rightarrow d$ triangle becomes a quadrangle,

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \quad (6)$$

In our following study, we parameterize [22]

$$\lambda_{t'} = r_{db} e^{i\phi_{db}}. \quad (7)$$

In our phase convention, $\lambda_c = V_{cd}^* V_{cb}$ is practically real, while $\lambda_u = V_{ud}^* V_{ub}$ is basically the same as in the SM.

To study $\sin 2\Phi_{B_d}$ and $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ in the $r_{db}-\phi_{db}$ plane, other constraints should be considered:

- (i) Radiative $b \rightarrow d\gamma$ processes (including $B \rightarrow \rho\gamma$) are ineffective because it is hard to separate from $b \rightarrow s\gamma$, difficult to study with LHCb, and in any case insensitive to virtual 4G effects.
- (ii) $B \rightarrow \pi\pi$ decays, while quite well studied, suffers from hadronic effects (even $B \rightarrow K\pi$ suffers from hadronic effects), and do not provide good constraints.
- (iii) The well-measured Δm_{B_d} provides a constraint through uncertainties in $f_{B_d}^2 \hat{B}_{B_d}$.
- (iv) Only very recently was the electroweak penguin $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay measured [23], in contrast to electroweak $b \rightarrow s$ penguins.

Although it may be a little surprising, there are not many observables that provide sound constraints on $\lambda_{t'}$. We collect below the relevant formulas for our study.

The t' effect to B_d mixing

$$\Delta m_{B_d} \simeq \frac{G_F^2 M_W^2}{6\pi^2} m_{B_d} \hat{B}_{B_d} f_{B_d}^2 \eta_B |\Delta_{12}^d|, \quad (8)$$

$$\sin 2\Phi_{B_d} \simeq \sin(\arg \Delta_{12}^d),$$

is (explicit forms can be found in Ref. [24])

$$\Delta_{12}^d \equiv (\lambda_t^{\text{SM}})^2 S_0(x_t) + 2\lambda_t^{\text{SM}} \lambda_{t'} \Delta S_0^{(1)} + \lambda_{t'}^2 \Delta S_0^{(2)}, \quad (9)$$

$$\Delta S_0^{(1)} \equiv \tilde{S}_0(x_t, x_{t'}) - S_0(x_t), \quad (10)$$

$$\Delta S_0^{(2)} \equiv S_0(x_{t'}) - 2\tilde{S}_0(x_t, x_{t'}) + S_0(x_t), \quad (11)$$

where $x_i = m_i^2/M_W^2$. Besides 4G parameters, the main uncertainty is in Ref. [25]

$$f_{B_d} \hat{B}_{B_d}^{1/2} = (227 \pm 19) \text{ MeV}. \quad (12)$$

For the current bound [5] of

$$\mathcal{B}(B_d \rightarrow \mu^+ \mu^-) < 8.1 \times 10^{-10}, \quad (13)$$

our purpose is to illustrate whether, and how, it could get enhanced to such values by the 4G effect. Here, we use the usual trick [26] of ‘‘normalizing’’ the branching ratio,

$$\begin{aligned} \hat{\mathcal{B}}(B_d \rightarrow \mu^+ \mu^-) &\equiv \frac{\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)}{\Delta m_{B_d}} \Delta m_{B_d}^{\text{exp}} \\ &= C \frac{\tau_{B_d} \Delta m_{B_d}^{\text{exp}}}{\hat{B}_{B_d}} \frac{\eta_Y^2 |\lambda_t^{\text{SM}} Y_0(x_t) + \lambda_{t'} \Delta Y_0|^2}{\eta_B |\Delta_{12}^d|}, \end{aligned} \quad (14)$$

where $\Delta Y_0 = Y_0(x_{t'}) - Y_0(x_t)$ with $Y_0(x)$ given in Ref. [10], and

$$C = 6\pi \left(\frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 \frac{m_\mu^2}{M_W^2}. \quad (15)$$

Through the ratio of Eq. (14), one not only eliminates the hadronic parameter f_{B_d} , but the λ_t^{SM} factor also cancels in the SM case, and one recovers the SM result of 1.1×10^{-10} , with little sensitivity to $|V_{ub}|$.

The treatment of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ would be given in the next section.

III. PHENOMENOLOGICAL STUDY WITH HEAVY t'

We plot in Fig. 1 for $m_{t'} = 700$ GeV the 2σ range in the $r_{db} - \phi_{db}$ plane, for $\sin 2\Phi_{B_d}$ (narrow green shaded region) allowed [27] by experimental measurement of Eq. (1), Δm_{B_d} (broad pink shaded region) allowed by lattice error in Eq. (12), and the bound on $B_d \rightarrow \mu^+ \mu^-$ (semitransparent gray exclusion) according to Eq. (13). We include labeled contours of 0.5, 1, 2, 4, 8 for $10^{10} \mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$. Figures 1(a) and 1(b) are for taking $|V_{ub}|$ to be the central values of 4.15×10^{-3} and 3.23×10^{-3} , respectively, for the mean (between inclusive and exclusive) and exclusive values from semileptonic B decay studies.

Consider Fig. 1(a), i.e., for $|V_{ub}| = 4.15 \times 10^{-3}$, the average between inclusive and exclusive measurements (the inclusive case is qualitatively similar). The well-measured CP phase $\sin 2\Phi_{B_d}$ is sensitive to t' effects, but free from hadronic uncertainties, hence the narrow (green) contour bands. In contrast, Δm_{B_d} is less sensitive to ϕ_{db} , and more accommodating because of hadronic uncertainty in $f_{B_d} \hat{B}_{B_d}^{1/2}$. The broad (pink) contours show the 1 and 2σ allowed region by Eq. (12), and rules out a branch of the $\sin 2\Phi_{B_d}$ contour (for ϕ_{db} between -10° to 15°), due to coherent enhancement of Δm_{B_d} from t' effects.

Consider now the gray excluded region from the combined LHC bound on $B_d \rightarrow \mu^+ \mu^-$, Eq. (13). It is seen that there are two slivers of parameter space, around $(r_{db}, \phi_{db}) \sim (0.0025, 180^\circ)$ (region A) and $(0.002, 252^\circ)$ (region B), where $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ could be above 4×10^{-10} , or enhanced by 4 times over SM, which are discovery zones for 2011–2012 LHC data. Near region B, $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ quickly drops below 4×10^{-10} as r_{db} becomes weaker than 0.002. For $\phi_{db} \sim 245^\circ$ and r_{db} varying from 0.0008 to 0.0015, $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ hovers at $(1-2) \times 10^{-10}$, while for $r_{db} \sim 0.0004$ to 0.0008 and ϕ_{db} varying from 240° to 330° , $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ hovers at $(0.5-2) \times 10^{-10}$, i.e., within a factor of two of SM expectations. These regions, combining to a broad crescent shape which we refer to as “region C,” would likely need much more data to probe.

The LHCb experiment has recently measured [23]

$$\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-) = (2.3 \pm 0.6 \pm 0.1) \times 10^{-8}, \quad (16)$$

which is the rarest B decay observed to date. The result is consistent with SM expectations, but interpretation depends on form factor models. To reduce form factor dependence, we take the ratio

$$R_{\pi\mu\mu} \equiv \frac{\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)_{4G}}{\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)_{\text{SM}}}, \quad (17)$$

where both 4G and SM results are integrated from $q^2 = (1, 6)$ GeV², which is under better numerical control [28,29]. Since this does not match what the LHCb does, we draw contours in Fig. 2 (red dashed), and view $R_{\pi\mu\mu} \sim 2-3$ as the range beyond which the LHCb would have found inconsistency with SM expectations. Thus, we are interpreting LHCb’s statement of consistency with the SM, allowing for form factor uncertainties. It is clear that this approach is not as good as the zero crossing point q_0^2 for $A_{\text{FB}}(B \rightarrow K^* \mu \mu)$, but this is the first observation of rare $b \rightarrow d\ell\ell$ decays, compared to the decade-long exploration of $b \rightarrow s\ell\ell$ processes. For numerics, we combine Wilson coefficients at next-to-leading order with leading order

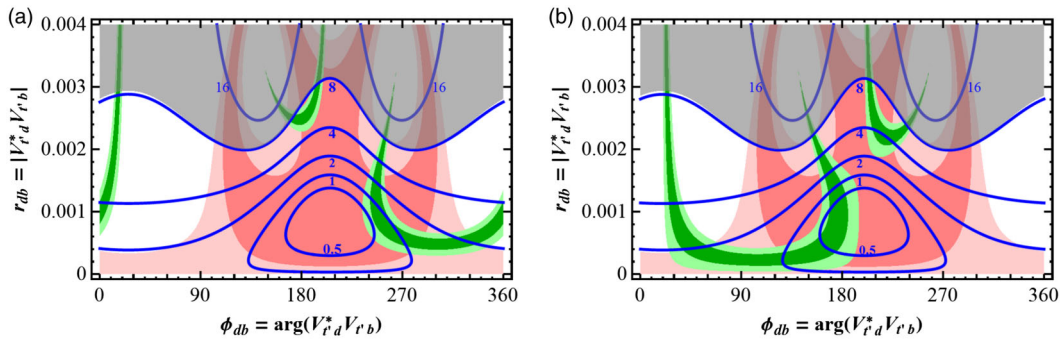


FIG. 1 (color online). Allowed region in $|V_{t'd}^* V_{t'b}| - \arg V_{t'd}^* V_{t'b}$ (i.e., $r_{db} - \phi_{db}$) plane for (a) average (b) exclusive $|V_{ub}|$ values, for $m_{t'} = 700$ GeV. The solid blue lines are labeled $10^{10} \mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ contours, where above the value of 8 (semitransparent gray) is excluded by the combined result of LHC experiments. The dark (light) narrow green shaded contours correspond to the $1(2)\sigma$ regions of $\sin 2\Phi_{B_d}$ [Eq. (1)], while the broad pink shaded contours correspond to the $1(2)\sigma$ regions of Δm_{B_d} allowed by Eq. (12).

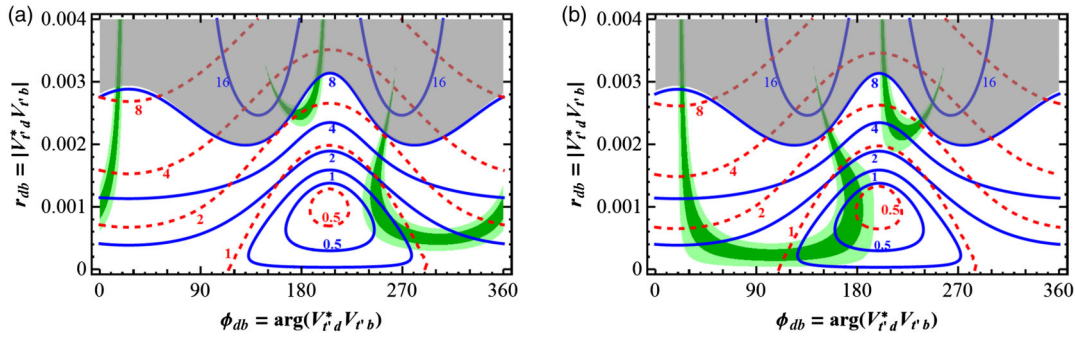


FIG. 2 (color online). Same as Fig. 1, but with Δm_{B_d} allowed regions replaced by the contours (red dashed) of ratio of 4G over SM branching ratios for $B^+ \rightarrow \pi^+ \mu^+ \mu^-$, integrated over the q^2 range of 1–6 GeV².

decay amplitude based on the QCD factorization (QCDF) approach [28,29]. For dealing with new physics, and as we take a ratio, this should suffice for our purpose.

If we now compared Fig. 1(a) with Fig. 2(a), we see that Δm_{B_d} is more powerful than $\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$ in excluding the $\sin 2\Phi_{B_d}$ -allowed branch near $\phi_{db} \sim 0$. This is reasonable, since $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ is only recently observed and prone to hadronic form factor uncertainties, while Δm_{B_d} has been measured for 25 years, with hadronic uncertainty narrowed down to $f_{B_d} \hat{B}_{B_d}^{1/2}$, which itself has been subject to intense lattice studies for years. It is, however, comforting to see that for region A, $R_{\pi\mu\mu}$ is not more than 2 (except the upper reach near $\phi_{db} \sim 190^\circ$), hence should be easy to accommodate by form factors, while for region B and especially region C, $R_{\pi\mu\mu}$ is even less than 2 and closer to 1. Thus, the newly measured $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ does provide a sanity check.

Turning to the case of exclusive $|V_{ub}|$ value, Figs. 1(b) and 2(b), we find that regions A and B basically switch roles. This is because for $|V_{ub}| \sim 3.23 \times 10^{-3}$, the expected $\sin 2\Phi_{B_d}$ value in the SM falls below that of direct measurement, as seen in comparing Eq. (4) to Eq. (1). Calling it region A', the sliver of region around $(r_{db}, \phi_{db}) \sim (0.002, 160^\circ)$ could enhance $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ more than 4 times above the SM, and observable with present LHC data. Region A' extends to the broad crescent region C', where even r_{db} values as low as 0.0002 could account for the measured $\sin 2\Phi_{B_d}$, but $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ can be

probed only beyond 2015. Again, Δm_{B_d} excludes the $\sin 2\Phi_{B_d}$ -allowed branch around $\phi_{db} \sim 30^\circ$. Region B' is now a considerably broader region in parameter space that allows enhancement of $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ above 4×10^{-10} . For example, for r_{db} above 0.0023 and ϕ_{db} above 230° , $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ can be greater than 6×10^{-10} , $f_{B_d} \hat{B}_{B_d}^{1/2}$ is within 2σ of Eq. (12), while $R_{\pi\mu\mu}$ is not more than 2. We also see that, for region B', $R_{\pi\mu\mu}$ provides good, perhaps better constraint than Δm_{B_d} , disfavoring the region of r_{db} greater than 0.0025 around $\phi_{db} \sim 205^\circ$, that seems perfectly allowed by Δm_{B_d} .

Now let us consider $m_{t'}$ values. The 700 GeV value used so far is just above current experimental limits [16], and correspond to Yukawa coupling strength $y_{t'} \simeq 4$, or $\alpha_{t'} \simeq 1.3$, which is why there is UB violation (UBV). However, we do not quite know what is the true expansion parameter. Furthermore, even if perturbation breaks down, it does not mean there is no t' effect. In fact, perturbation in $\lambda_{t'}$ certainly holds, but the functions $\Delta S_0^{(i)}$ and ΔY_0 in Eqs. (9) and (14) get modified by UBV effects. Though, the overall form of these equations should not change. We, therefore, consider the $m_{t'} = 1000$ GeV case, i.e., $\alpha_{t'} \simeq 2.6$, to illustrate the situation far beyond UBV [17]. Note that Ref. [14] finds DEWSB occurs for λ_Q (the 4G doublet is treated as very close to degenerate) of order 4π , i.e., of order the πNN coupling, implying 4G quark masses no less than 2 TeV.

The plots corresponding to Figs. 1 and 2, but with $m_{t'} = 1000$ GeV, are given in Figs. 3 and 4. We generally see

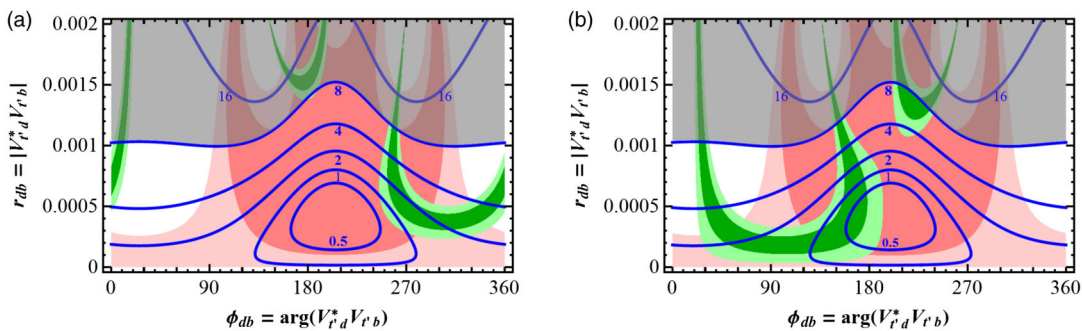
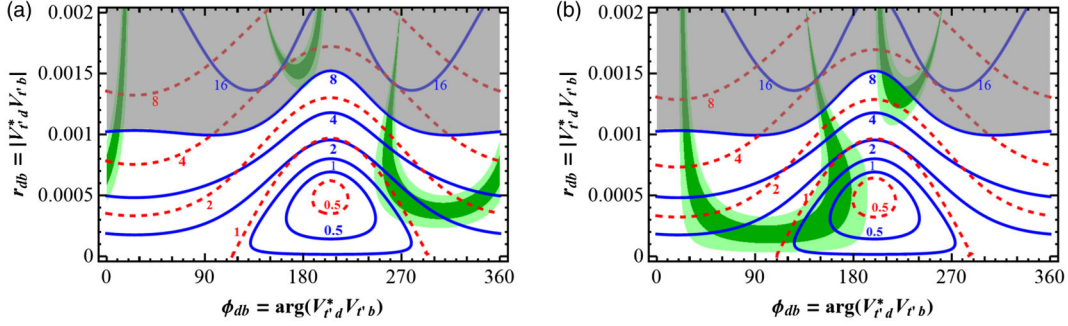


FIG. 3 (color online). Same as Fig. 1, but for $m_{t'} = 1000$ GeV.


 FIG. 4 (color online). Same as Fig. 2, but for $m_{t'} = 1000$ GeV.

reduced r_{db} values. Region A is now excluded, but regions B, A', and B' become more robust in Δm_{B_d} . Values for $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ higher than $(5-6) \times 10^{-10}$ are slightly disfavored by $B^+ \rightarrow \pi^+ \mu^+ \mu^-$. Viewed differently, if enhanced, $B_d \rightarrow \mu^+ \mu^-$ is discovered; one may try to scrutinize whether $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ is also somewhat enhanced beyond SM. Regions C and C' generally stand well, with at best mildly enhanced $B_d \rightarrow \mu^+ \mu^-$.

IV. DISCUSSION AND CONCLUSION

We started with the question of what could still enhance $B_d \rightarrow \mu^+ \mu^-$ decay, when everything at the LHC seems consistent with SM. The answer is that, probably only the 4G t' quark could do the job, even if 4G seems disfavored by the Higgs-like nature of the 126 GeV boson. Admittedly, even if 4G is the explanation for the $\sin 2\Phi_{B_d}$ tension as seen by the B factories, to have $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ within a factor of 2 of the current bound of 8.1×10^{-10} is only a fraction of the allowed parameter space, hence not particularly likely. However, only with such enhancement is there any chance for LHC experiments to make the discovery with 2011–2012 data, and discovery it indeed will be. If discovered—within 2013—then not only would 4G get uplifted, but some doubt would be cast on the SM Higgs nature of the 126 GeV boson, while “impostors” such as dilaton would gain in weight. We have remarked in the Introduction that it would take the establishment of VBF and VH production processes to exclude the dilaton possibility, which cannot be achieved with 2011–2012 data [14].

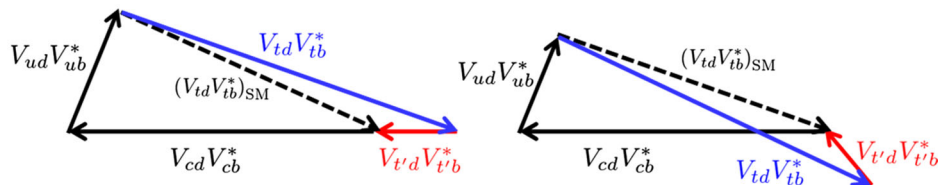
An intriguing outcome of discovering $B_d \rightarrow \mu^+ \mu^-$ decay would be that, all of a sudden, the $b \rightarrow d$ triangle falls into our lap. Let us illustrate. Since $m_{t'} = 1000$ GeV

cases have smaller $r_{db} \equiv |\lambda_{t'}| \equiv |V_{t'd}^* V_{t'b}|$ values, for reasons of plotting, we take two examples from $m_{t'} = 700$ GeV. From region A of Fig. 1(a) (average $|V_{ub}| = 4.15 \times 10^{-3}$), we take $\lambda_{t'} = V_{t'd}^* V_{t'b} = 0.0025e^{i180^\circ}$. From region B of Fig. 1(b) (exclusive $|V_{ub}| = 3.23 \times 10^{-3}$), we take $\lambda_{t'} = V_{t'd}^* V_{t'b} = 0.0023e^{i230^\circ}$.

The quadrangle of Eq. (6) is constructed as follows. To simplify discussions, we normalize to $\lambda_c = V_{cd}^* V_{cb} = -0.0094$, which becomes a unit vector pointing left. Then, $\hat{\lambda}_u = V_{ud}^* V_{ub} / |\lambda_c| = 0.44e^{-i68^\circ}$, $0.34e^{-i68^\circ}$, respectively, for the average and inclusive cases, with corresponding $\hat{\lambda}_{t'} = 0.27e^{i180^\circ}$, $0.24e^{i230^\circ}$. Then $\hat{\lambda}_{t'}$ just connects the tip of $\hat{\lambda}_u$ with the end of $\hat{\lambda}_c$. The two examples for 700 GeV are plotted in Fig. 5 in the form to compare with the usual SM triangle [20]. These are relatively precise quadrangles, and illustrate how 4G accounts for a shift in $\sin 2\Phi_{B_d}$ away from SM expectation, where $\Phi_{B_d}^{\text{SM}}$ is the angle between the dashed line, λ_t^{SM} and the real axis. Since t' is much heavier than t , a smaller $\lambda_{t'}$ could cause the shift.

The sample $b \rightarrow d$ quadrangles are for largest allowed solutions for r_{db} , i.e., regions A (for $|V_{ub}|^{\text{ave}}$) and B' (for $|V_{ub}|^{\text{excl}}$) for $m_{t'} = 700$ GeV, and would be the case if $B_d \rightarrow \mu^+ \mu^-$ is discovered soon. They are relatively extreme, however, since even for $m_{t'} = 700$ GeV, regions C and C' can provide solutions for $\sin 2\Phi_{B_d}$ for much smaller $r_{db} = |V_{t'd}^* V_{t'b}|$ values, with possible phase values extending over a large range. For heavier t' illustrated by 1000 GeV, $|V_{t'd}^* V_{t'b}|$ is smaller by half compared to 700 GeV case, with region A is eliminated.

The quadrangles of Fig. 5 remind us of the possible [30] link to the baryon asymmetry of the Universe (BAU): 4G


 FIG. 5 (color online). Sample $b \rightarrow d$ quadrangles for $\lambda_{t'} = V_{t'd}^* V_{t'b} = 0.0025e^{i180^\circ}$ with average $|V_{ub}| = 4.15 \times 10^{-3}$ (left), and for $\lambda_{t'} = V_{t'd}^* V_{t'b} = 0.0023e^{i230^\circ}$ with exclusive $|V_{ub}| = 3.23 \times 10^{-3}$ (right).

greatly enhances CPV from SM, and is seemingly sufficient for BAU (although a first-order phase transition remains an issue), which boosts the merit of 4G. It does not depend much on the area of the quadrangle, as the enhancement rests in powers of $m_{t'}$ and $m_{b'}$. We note that $\lambda_{t'}$ in Fig. 5, though smaller in strength than λ_t and λ_c , is not that small compared with λ_u . Furthermore, we know that $|V_{t'b}|$ cannot be more than 0.1 [31], especially for our large $m_{t'}$ values. Hence, $|\lambda_{t'}|$ plotted in Fig. 5 corresponds to $|V_{t'd}|$ that is larger than $|V_{td}|^{\text{SM}} \simeq 0.0088$, which does not fit the CKM pattern of trickling off as one goes further off diagonal. One could use this to argue that enhanced $B_d \rightarrow \mu^+ \mu^-$ decay to the level observable with 2011–2012 data is not plausible. However, the issue is best left to experiment.

For $m_{t'} = 1000$ GeV, $|\lambda_{t'}|$ values tend to drop by half, but $|V_{t'd}|$ would still be comparable to $|V_{td}|$. Only if one gives up enhancement would the ratio $|V_{t'd}|/|V_{td}|$ turn “natural.” In fact, for the exclusive value case for V_{ub} , $|\lambda_{t'}|$ (i.e., r_{db}) could be $(1-2) \times 10^{-4}$ and still account for $\sin 2\Phi_{B_d}$ “anomaly.” Such values for $|V_{t'd}|$ would become natural when compared with $|V_{td}|$. However, even if 4G gains support by 2015, this region (C and C') would need a very large data set to explore.

We conclude that 2013 remains a pivotal year where one could discover the very rare $B_d \rightarrow \mu^+ \mu^-$ decay mode at over 4 times SM expectations. The chance is not large, but not zero either, with partial motivation from the (mild) $\sin 2\Phi_{B_d}$ discrepancy. If discovered with the 2011–2012 data set, the implications would be quite huge: uplifting the 4th generation (with prospect of CPV for BAU), casting some doubt on the SM Higgs interpretation of the 126 GeV boson, and perhaps the only new physics (at least in flavor sector) uncovered at the 7 and 8 TeV runs at the LHC. But it is more likely that the LHC would once again push the limits down towards the SM. If such is the case, the fate of the 4G would have to be determined elsewhere. But $B_d \rightarrow \mu^+ \mu^-$ should certainly be pursued further at the 13 TeV run.

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