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Correlation functions of the Aharony-Bergman-Jafferis-Maldacena model

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In the Aharony-Bergman-Jafferis-Maldacena model, we study the three-point function of two heavy operators and an (ir)relevant one. Following the AdS/CFT correspondence, the structure constant in the large 't Hooft coupling limit can be factorized into two parts. One is the structure constant with a marginal operator, which is fully determined by the physical quantities of heavy operators and gives rise to a result that is consistent with the renormalization-group analysis. The other can be expressed as the universal form depending only on the conformal dimension of an (ir)relevant operator. We also investigate the new size effect of a circular string dual to a certain closed spin chain.

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I. INTRODUCTION

The application of the AdS/CFT correspondence to a strongly interacting system is one of the active research areas in theoretical physics. In order to understand the duality in depth and the gauge theory in the strong-coupling regime, we need to clarify the underlying structure of the AdS/CFT correspondence. A good example that helps one to understand the AdS/CFT correspondence is the four-dimensional $\mathcal{N}=4$ super-Yang-Mills theory dual to the string theory or supergravity in the $AdS_5 \times S^5$ space-time, in which the conformal symmetry and the integrability play a crucial role in figuring out the physics of the strongly interacting system [1–6]. Recently, such works have been generalized to other dimensions. For example, in order to account for the worldvolume theory of an M-brane, the three-dimensional $\mathcal{N}=8$ Bagger-Lambert-Gustavsson model and the Aharony-Bergman-Jafferis-Maldacena (ABJM) model for the $\mathcal{N}=6$ Chern-Simons gauge theory have been widely investigated [7–11]. Moreover, it was shown that the ABJM model has a dual gravity description in the $AdS_4 \times CP^3$ background and is integrable at least up to the two-loop level [12], and in the $SU(2) \times SU(2)$ subsector even at the four-loop level [13]. In this paper, following Refs. [14–21], we will further investigate the AdS/CFT correspondence of the ABJM model by calculating the three-point function with an (ir)relevant operator.

In the conformal field theory (CFT), if we know the two- and three-point correlation functions, we can use them to determine the higher-point functions. In general, the coordinate dependence of two- and three-point functions is unambiguously fixed by the global conformal symmetry,

$$\begin{aligned} \langle \mathcal{O}_A(x)\mathcal{O}_B(y)\rangle &= \frac{\delta_{AB}}{|x-y|^{2\Delta}}, \\ \langle \mathcal{O}_A(x)\mathcal{O}_B(y)\mathcal{O}_C(z)\rangle &= \frac{a_{ABC}}{|x-y|^{\Delta_A+\Delta_B-\Delta_C}|x-z|^{\Delta_A+\Delta_C-\Delta_B}|y-z|^{\Delta_B+\Delta_C-\Delta_A}}, \end{aligned} \tag{1}$$

where Δ_A and a_{ABC} are the conformal dimension and the structure constant, respectively. Actually, since the structure constant is not constrained by the global conformal symmetry, we should determine it by other means. In particular, in the strong-coupling regime it is almost impossible to fix the structure constant except in those cases in which it is determined by further symmetries [22]. Another exception is the case that includes a marginal deformation caused by the Lagrangian density itself. Since such a marginal deformation modifies the coupling constant only, the structure constant can be determined by a renormalization group (RG) analysis, even on the gauge theory side [15].

In this paper, we will investigate the three-point function with an (ir)relevant operator. Although the RG analysis does not work anymore, the AdS/CFT correspondence can give a clue about the three-point function in the large 't Hooft coupling limit. On the string theory side, the three-point function of two heavy operators and an (ir)relevant one can be described by a leading interaction between a solitonic string and a massive dilaton field propagating on the AdS_4 space. The string theory calculation shows that the resulting three-point function has the coordinate dependence that is expected by the global

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conformal symmetry and its structure constant is closely related to that with a marginal operator. Finally, we suggest a new circular string dual to a closed spin chain. Its two- and three-point functions show that the size effect of the closed spin chain is suppressed linearly by $(\bar{J}_1 - J_1)$, while the open spin chain has an exponentially suppressed finite-size effect [6,10,20,23].

The rest of the paper is organized as follows. In Sec. II, we briefly summarize the results of the RG analysis with a marginal deformation [15]. In Sec. III, after evaluating the three-point function of two heavy operators and an (ir)relevant one, we show that the ratio between the structure constants has a universal form independent of the details of the heavy operator. Moreover, we find that the closed spin chain can have a linearly suppressed size correction. Finally, we finish our work with some concluding remarks.

II. MARGINAL DEFORMATION OF THE CONFORMAL FIELD THEORY

Many authors have recently calculated the three-point correlation functions of two heavy operators $\mathcal{O}_H$ and a light one $\mathcal{O}_L$ in the $AdS_5 \times S^5$ background by using the AdS/CFT correspondence [14–16]. For an N-point function with only two heavy operators, it can be rewritten in a factorized form as a product of two- and three-point functions in the large 't Hooft coupling limit [17]. So it is important to know the conformal dimensions of various primary operators and the structure constants for understanding the CFT in the strong-coupling regime. However, little is known about the structure constant except for Bogomol'nyi-Prasad-Sommerfield operators, whereas the conformal dimensions of heavy operators have been widely studied [18]. The goal of this paper is to obtain more insights about the structure constant.

Let us start with summarizing the known results. Assuming a heavy operator $\mathcal{O}_H$ with a conformal dimension Δ_H, its two-point function is exactly determined by the global conformal symmetry up to the normalization

$$\langle \mathcal{O}_H(x)\mathcal{O}_H(y)\rangle = \frac{1}{|x-y|^{2\Delta_H}}, \tag{2}$$

where we set the normalization constant to 1. Similar to the two-point function, the global conformal symmetry also fixes the coordinate dependence of the three-point function with another operator $\mathcal{O}_L$,

$$\langle \mathcal{O}_H(x)\mathcal{O}_H(y)\mathcal{O}_L(z)\rangle = \frac{a_{HHL}}{|x-y|^{2\Delta_H-\Delta_L}|x-z|^{\Delta_L}|y-z|^{\Delta_L}}, \tag{3}$$

where Δ_L denotes the conformal dimension of $\mathcal{O}_L$. Note that since the structure constant a_{HHL} is not constrained by the conformal symmetry, it should be determined by other methods. If we take account of a marginal Lagrangian density operator $\mathcal{O}_{\mathcal{L}}$, we can determine the structure constant through the RG analysis. For an Euclidean four-dimensional CFT, the structure constant is associated with the conformal dimension of the heavy operator [15],

$$-g^2\frac{\partial}{\partial g^2}\Delta_H = 2\pi^2 a_{HH\mathcal{L}}, \tag{4}$$

where g denotes a coupling constant and $2\pi^2$ corresponds to the solid angle of S^3. Since this relation should be satisfied in all coupling regimes, we can test the AdS/CFT correspondence in the strong-coupling limit. To do so, we first must know the operator that corresponds to the spectrum. Following the AdS/CFT correspondence, a heavy operator usually corresponds to a solitonic string moving in the dual geometry, whereas a light one is matched with a supergravity mode. In particular, a marginal Lagrangian-density operator is dual to a massless dilaton field. It was shown by many authors that solitonic strings moving in the $AdS_5 \times S^5$ background satisfy the above relation [15,20].

One can easily generalize the relation (4) to the d-dimensional CFT case,

$$-g^2\frac{\partial}{\partial g^2}\Delta_H = \frac{2\pi^{d/2}}{\Gamma(d/2)} a_{HH\mathcal{L}}, \tag{5}$$

where the multiplication factor implies the solid angle of S^{d-1}. In the string theory, there exists another interesting superconformal theory—the so-called ABJM model—which describes a three-dimensional $\mathcal{N} = 6$ Chern-Simons theory [8]. Its dual is the supergravity theory in the $AdS_4 \times CP^3$ background. Since the ABJM model is also conformal, one can easily expect that the ABJM model also satisfies (5) in the form

$$-g^2\frac{\partial}{\partial g^2}\Delta_H = 4\pi a_{HH\mathcal{L}}. \tag{6}$$

In Ref. [19], various solitonic string solutions moving in the $AdS_4 \times CP^3$ background were investigated, and it was shown that the RG analysis (6) is really working in the ABJM model as expected.

III. THREE-POINT FUNCTION WITH AN (IR)RELEVANT OPERATOR

In the three-point function of two heavy operators and a marginal one, the structure constant can be exactly determined by the RG analysis on the CFT side. On the other hand, the same result can also be reproduced on the gravity side by evaluating the semiclassical partition function with an interaction between a solitonic string and a dual supergravity mode. This result is one piece of evidence of the AdS/CFT correspondence. Can we generalize such a calculation to the more general cases? More specifically, what is the three-point function with an (ir)relevant operator instead of a marginal one? When evaluating the three-point function with an (ir)relevant operator in the strong-coupling regime, the RG analysis and the perturbative calculation are not valid. However, the AdS/CFT correspondence can give the answer. In this section, we will discuss the three-point functions of various combinations of two heavy operators and an (ir)relevant one in the large 't Hooft coupling limit.

A. Point-like string in AdS_4

Let us first consider a point particle propagating only on the Euclidean AdS_4 space, whose metric in the Poincare patch reads

$$ds^2 = \frac{dz^2 + \delta_{ij} dx^i dx^j}{z^2}, \tag{7}$$

where z and x^i correspond to the radial and boundary coordinates, respectively. The worldline action of a particle is given by the following Polyakov-type action:

$$S_P = \frac{1}{2} \int_{-s/2}^{s/2} d\tau \left(\frac{\dot{x}^i \dot{x}_i + \dot{z}^2}{z^2} - m^2 \right), \tag{8}$$

where the mass of the particle m is very large and s denotes the modular parameter. The solution satisfying the equation of motion becomes

$$x(\tau) = R \tanh \kappa\tau + x_0, \qquad z(\tau) = \frac{R}{\cosh \kappa\tau}. \tag{9}$$

Under the boundary conditions

$$\begin{aligned} \{x(-s/2), z(-s/2)\} &= \{0, \epsilon\} \quad \text{and} \\ \{x(s/2), z(s/2)\} &= \{x_f, \epsilon\}, \end{aligned} \tag{10}$$

and at an appropriate UV cutoff ϵ ($\epsilon \to 0$), the parameters are related to each other as

$$\kappa \approx \frac{2}{s} \log \frac{x_f}{\epsilon} \quad \text{and} \quad x_f \approx 2R \approx 2x_0, \tag{11}$$

where higher-order corrections are ignored. After regarding the convolution with the relevant wave function [14,15], the saddle point $\bar{s}$ is given by

$$\bar{s} = -\frac{2i}{m} \log \frac{x_f}{\epsilon}. \tag{12}$$

At this saddle point, the semiclassical partition function reduces to

$$e^{iS_P} = \left(\frac{\epsilon}{x_f}\right)^{2\Delta_H}, \quad \text{with} \quad \Delta_H = m, \tag{13}$$

where Δ_H corresponds to the energy of a massive particle. Following the AdS/CFT correspondence, Δ_H is reinterpreted as the conformal dimension of the dual heavy operator, and the semiclassical partition function is associated with its two-point function.

In order to evaluate the three-point function with an (ir)relevant operator, we first introduce a massive dilaton field propagating on the AdS_4 space. If its mass is denoted by m_ϕ ($\ll m$), the conformal dimension of the dual light operator $\mathcal{O}_\phi$ is given by[1]

$$h = \frac{3}{2} + \frac{\sqrt{9 + 4m_\phi^2}}{2}. \tag{14}$$

Note that a dilaton field is allowed to have a negative mass squared in the anti–de Sitter space, $m_\phi^2 \geq -\frac{9}{4}$, where the lower limit corresponds to the Breitenlohner-Freedman bound [24]. The operator dual to a dilaton with a negative or positive mass is relevant or irrelevant, respectively. A massless dilaton corresponds to the Lagrangian-density operator $\mathcal{O}_\mathcal{L}$ studied in the previous section. The bulk-boundary propagator of a massive dilaton in the AdS_4 space is given by [25–27]

$$\mathcal{D}_\phi(z, x; 0, y) = \frac{\Gamma(h)}{\pi^{3/2}\Gamma(h - 3/2)} \left(\frac{z}{z^2 + (x - y)^2} \right)^h, \tag{15}$$

where a dilaton propagates from the boundary $\{0, y\}$ to the bulk $\{z, x\}$. Then, the three-point function can be expressed by

$$\langle \mathcal{O}_H(x_f) \mathcal{O}_H(0) \mathcal{O}_\phi(y) \rangle = \frac{I}{x_f^{2\Delta_H}}, \tag{16}$$

with

$$\begin{aligned} I &= \frac{i\Gamma(h)}{8\pi^{3/2}\Gamma(h - 3/2)} \int_{-\bar{s}/2}^{\bar{s}/2} d\tau \left(\frac{\dot{x}^i \dot{x}_i + \dot{z}^2}{z^2} - m^2 \right) \\ &\quad \times \left(\frac{z}{z^2 + (x - y)^2} \right)^h \\ &= -\frac{m}{2^{h+2}\pi} \frac{\Gamma(\frac{h}{2})\Gamma(h)}{\Gamma(\frac{h+1}{2})\Gamma(h - \frac{3}{2})} \frac{1}{x_f^{-h} |x_f - y|^h y^h} + \cdots, \end{aligned} \tag{17}$$

where the solutions in Eq. (9) are used and the ellipsis implies higher-order corrections in the large-$\bar{s}$ limit. In Eq. (16), I implies the interaction between a solitonic string and a massive dilaton field.

For $m_\phi = 0$, the light operator is marginal and the three-point function simply reduces to

$$\langle \mathcal{O}_H(x_f) \mathcal{O}_H(0) \mathcal{O}_\mathcal{L}(y) \rangle = -\frac{m}{16\pi} \frac{1}{x_f^{2\Delta_H - 3} |x_f - y|^3 y^3}, \tag{18}$$

which coincides with the result in Ref. [19]. Assuming that $\Delta_H = m \sim \sqrt{g}$ [15], we can easily check that the structure constant satisfies the result of the RG analysis (6),

$$-g^2 \frac{\partial \Delta_H}{\partial g^2} = -\frac{m}{4} = 4\pi a_{HH\mathcal{L}}. \tag{19}$$

For $m_\phi \neq 0$, the three-point function can be summarized to

[1]In Ref. [19], the three-point functions with marginal operators with $m_\phi = 0$ have been considered.

$$\langle \mathcal{O}_H(x_f)\mathcal{O}_H(0)\mathcal{O}_\phi(y)\rangle = \frac{a_{HH\phi}}{x_f^{2\Delta_H-h}|x_f-y|^h y^h}, \quad (20)$$

with

$$a_{HH\phi} = -\frac{m}{2^{h+2}\pi}\frac{\Gamma(\frac{h}{2})\Gamma(h)}{\Gamma(\frac{h+1}{2})\Gamma(h-\frac{3}{2})}, \quad (21)$$

which shows the coordinate dependence expected by the global conformal symmetry.

B. A circular string wrapped in θ

Now, consider a solitonic string moving in the $AdS_4 \times S^3$ background, which is a subspace of the $AdS_4 \times CP^3$ geometry dual to the ABJM model. Here, S^3 represents the diagonal subspace of CP^3 [10,11],

$$ds^2 = \frac{1}{4}(d\theta^2 + \sin^2\theta d\phi_1^2 + \cos^2\theta d\phi_2^2). \quad (22)$$

Under the ansatz for a circular string extended in θ with rotations in ϕ_1 and ϕ_2,

$$\theta = \sigma, \qquad \phi_1 = \omega_1\tau, \qquad \phi_2 = \omega_2\tau, \quad (23)$$

the Polyakov string action becomes

$$S = \frac{T}{2}\int_{-s/2}^{s/2} d\tau \int_0^{2\pi} d\sigma\left[\frac{\dot{x}^i\dot{x}_i + \dot{z}^2}{z^2} - \theta'^2 + \sin^2\theta\dot{\phi}_1^2 + \cos^2\theta\dot{\phi}_2^2\right], \quad (24)$$

where the dot and the prime represent the derivatives with respect to τ and σ, respectively. In Eq. (24), the first two terms describe the motion of the string in the AdS_4 space. Since all solitonic strings studied in this paper behave like a point particle in the AdS_4 space, their solutions are also given by Eq. (9). Note that the string tension T in the $AdS_4 \times CP^3$ space is associated with the 't Hooft coupling constant λ [10],

$$T = \sqrt{\frac{\lambda}{2}} = 2g, \quad (25)$$

where g is the coupling constant appearing in Sec. II. Following Refs. [14,15], at the saddle point

$$\bar{s} = \frac{2\sqrt{2}}{i\sqrt{2+\omega_1^2+\omega_2^2}}\log\left(\frac{x_f}{\epsilon}\right) \quad (26)$$

the semiclassical partition function becomes

$$e^{iS} = \left(\frac{\epsilon}{x_f}\right)^{2\Delta_H}, \quad (27)$$

where the energy of a circular string Δ_H reads

$$\Delta_H = \sqrt{2}\sqrt{J_1^2+J_2^2+2\pi^2T^2} \quad (28)$$

in terms of the angular momenta

$$J_1 = \pi T\omega_1 \quad \text{and} \quad J_2 = \pi T\omega_2. \quad (29)$$

If a dilaton field is massless, the RG analysis (6) expects that the structure constant will be

$$4\pi a_{AA\mathcal{L}} = -\frac{\sqrt{2}\pi^2T^2}{\sqrt{J_1^2+J_2^2+2\pi^2T^2}}. \quad (30)$$

On the string theory side, the result (30) can also be reproduced from the three-point function with a general light operator. The general form of the three-point function with an (ir)relevant operator is given by

$$\langle \mathcal{O}_H(0)\mathcal{O}_H(x_f)\mathcal{O}_\phi(y)\rangle = \frac{I}{x_f^{2\Delta_H}}, \quad (31)$$

with

$$I = \frac{i\Gamma(h)}{4\pi^{3/2}\Gamma(h-3/2)}\int_{-\bar{s}/2}^{\bar{s}/2} d\tau \times \int_0^{2\pi} d\sigma\left(\frac{\dot{x}^i\dot{x}_i+\dot{z}^2}{z^2} - \theta'^2 + \sin^2\theta\dot{\phi}_1^2 + \cos^2\theta\dot{\phi}_2^2\right) \times \left(\frac{z}{z^2+(x-y)^2}\right)^h. \quad (32)$$

After integrating Eq. (32), the leading term of the three-point function gives rise to

$$\langle \mathcal{O}_H(0)\mathcal{O}_H(x_f)\mathcal{O}_\phi(y)\rangle = -\frac{\pi T^2}{2^{h-1/2}\sqrt{J_1^2+J_2^2+2\pi^2T^2}}\frac{\Gamma(\frac{h}{2})\Gamma(h)}{\Gamma(\frac{h+1}{2})\Gamma(h-\frac{3}{2})} \times \frac{1}{x_f^{2\Delta_H-h}|x_f-y|^h y^h}. \quad (33)$$

Its coordinate dependence is the exact form expected by the CFT. For $m_\phi = 0$, the dual light operator becomes marginal, $h = 3$, and the three-point function reduces to

$$\langle \mathcal{O}_H(0)\mathcal{O}_H(x_f)\mathcal{O}_\mathcal{L}(y)\rangle = -\frac{\sqrt{2}\pi T^2}{4\sqrt{J_1^2+J_2^2+2\pi^2T^2}}\frac{1}{x_f^{2\Delta_H-3}|x_f-y|^3y^3}, \quad (34)$$

where we can see that the structure constant perfectly coincides with Eq. (30) derived by the RG analysis. Comparing the above two structure constants yields the following ratio:

$$\frac{a_{HH\phi}}{a_{HH\mathcal{L}}} = \frac{1}{2^{h-2}}\frac{\Gamma(\frac{h}{2})\Gamma(h)}{\Gamma(\frac{h+1}{2})\Gamma(h-\frac{3}{2})}, \quad (35)$$

in which the result shows a universal form in that it does not contain any information about the heavy operator. This implies that in the large 't Hooft coupling limit the structure constant with an (ir)relevant operator can be factorized into two parts. One is $a_{HH\mathcal{L}}$, which is determined by the details of the heavy operator, and the other depends on the conformal dimension of the light operator only. In the following sections, we will check this new feature of the structure constant with more complicated solitonic strings.

C. A dyonic magnon

In this section, we will take into account a more nontrivial solitonic string called a dyonic magnon. In the $AdS_5 \times S^5$ background dual to the $N = 4$ super-Yang-Mills theory, the three-point function of two dyonic magnons and a marginal operator has been investigated [15,18–20,23]. This work was also generalized to the ABJM model [20]. Here, we will further study the three-point function with an (ir)relevant operator and check the universal behavior of the structure constant ratio.

A dyonic magnon corresponds to a bound state of magnons in the spin-chain model, which can be described on the string theory side by a solitonic string rotating on $S^3 \subset CP^3$. The ansatz for a dyonic string is given by

$$\theta = \theta(y), \qquad \phi_1 = \nu_1 \tau + g_1(y) \quad \text{and} \quad \phi_2 = \nu_2 \tau + g_2(y), \tag{36}$$

with

$$y = a\tau + b\sigma. \tag{37}$$

The rotational symmetries in ϕ_1 and ϕ_2 give rise to

$$g_1' = \frac{1}{b^2 - a^2}\left(a\nu_1 - \frac{c_1}{\sin^2\theta}\right), \quad g_2' = \frac{1}{b^2 - a^2}\left(a\nu_2 - \frac{c_2}{\cos^2\theta}\right), \tag{38}$$

where c_1 and c_2 are integration constants and the prime means a derivative with respect to y. Here, we take $b^2 > a^2$ and $c_2 = 0$ to obtain a dyonic magnon solution [10,20]. Using the Virasoro constraints, the equation of motion for θ can be rewritten as the first-order differential equation

$$\theta'^2 = \frac{b^2(\nu_1^2 - \nu_2^2)}{(b^2 - a^2)^2 \sin^2\theta}(\sin^2\theta_{\max} - \sin^2\theta)(\sin^2\theta - \sin^2\theta_{\min}), \tag{39}$$

with

$$\sin^2\theta_{\max} = \frac{c_1}{a\nu_1}, \tag{40}$$

$$\sin^2\theta_{\min} = \frac{a\nu_1 c_1}{b^2(\nu_1^2 - \nu_2^2)}. \tag{41}$$

From now on, we concentrate on the infinite-size limit ($J_1 \to \infty$), which can be accomplished by setting $c_1 = a\nu_1$ ($\sin\theta_{\max} = 1$). After the convolution, the semiclassical partition function is represented as

$$e^{iS} = \left(\frac{\epsilon}{x_f}\right)^{2\Delta_H}, \tag{42}$$

with

$$\Delta_H = J_1 + \sqrt{J_2^2 + 4T^2 \sin^2\frac{p}{2}}, \tag{43}$$

which was evaluated at the saddle point

$$\bar{s} = -\frac{2i}{\nu_1}\log\frac{x_f}{\epsilon}. \tag{44}$$

In Eq. (43), J_i ($i = 1, 2$) means the angular momentum in the ϕ_i direction and p is the worldsheet momentum, which can be reinterpreted in the target space as the angle difference of two ends of a string [10].

Following the method used in the previous sections, one can finally find the following three-point function of two dyonic magnons and an (ir)relevant operator:

$$\langle \mathcal{O}_H(0)\mathcal{O}_H(x_f)\mathcal{O}_\phi(y)\rangle = \frac{a_{HH\phi}}{x_f^{2\Delta_H - h}|x_f - y|^h y^h}, \tag{45}$$

where the structure constant is given by

$$a_{HH\phi} = -\frac{T^2\sin^2(p/2)}{2^{h-1}\pi\sqrt{J_2^2 + 4T^2\sin^2(p/2)}}\frac{\Gamma(\frac{h}{2})\Gamma(h)}{\Gamma(\frac{h+1}{2})\Gamma(h-\frac{3}{2})}. \tag{46}$$

For $m_\phi = 0$, it reduces to

$$a_{HH\mathcal{L}} = -\frac{T^2\sin^2(p/2)}{2\pi\sqrt{J_2^2 + 4T^2\sin^2(p/2)}}, \tag{47}$$

and satisfies the RG analysis (6). Furthermore, the ratio of the above structure constants shows the same universal form as in Eq. (35).

D. A circular string wrapped in ϕ_1

The motivation of this section is not only to check the universality mentioned before, but also to investigate a new size effect of the closed spin chain. Usually, a circular string corresponds to a closed spin chain in the dual CFT, whereas a magnon is dual to an open spin chain. If the magnon's size J_1 in Eq. (43) is large but finite, there is an additional finite-size effect on the conformal dimension, which is exponentially suppressed like e^{-J_1} [20,23].

What is the size effect of the closed spin chain? In order to answer this question, we think of another circular string that is wrapped in ϕ_1 and rotating in ϕ_1 and ϕ_2. Then, the appropriate ansatz is given by

$$\phi_1 = \omega_1\tau + w\sigma, \qquad \phi_2 = \omega_2\tau \quad \text{and} \quad \theta = \theta_0, \tag{48}$$

where w is the winding number and $0 \leq \sigma < 2\pi$. We assume that the position of the string in θ is fixed to θ_0 and that the two angular velocities ω_1 and ω_2 are finite. For $\theta_0 = \pi/2$, the above ansatz reduces to one wrapping the equator of S^2. From the string action

$$S = \frac{T}{2}\int_{-s/2}^{s/2} d\tau \int_0^{2\pi} d\sigma \Big[\frac{(\dot{x}^i)^2 - (x^{i\prime})^2 + \dot{z}^2 - z^{\prime 2}}{z^2} + \dot{\theta}^2 - \theta^{\prime 2} + \sin^2\theta(\dot{\phi}_1^2 - \phi_1^{\prime 2}) + \cos^2\theta(\dot{\phi}_2^2 - \phi_2^{\prime 2})\Big], \tag{49}$$

and after regarding the convolution contribution and setting $\sin\theta_0 = \sqrt{1-\delta^2}$, we find a saddle point at

$$\bar{s} = \frac{2}{i\sqrt{(w^2+\omega_1^2)(1-\delta^2)+\omega_2^2\delta^2}} \log\frac{x_f}{\epsilon}. \tag{50}$$

At this point, the semiclassical partition function leads to

$$e^{iS} = \left(\frac{\epsilon}{x_f}\right)^{2\Delta_H}, \tag{51}$$

with

$$\Delta_H = 2\pi T\sqrt{\frac{J_1^2}{4\pi^2T^2(1-\delta^2)} + (1-\delta^2)w^2 + \frac{J_2^2}{4\pi^2T^2\delta^2}}, \tag{52}$$

where the two angular momenta are defined by

$$J_1 = 2\pi T\omega_1(1-\delta^2), \qquad J_2 = 2\pi T\omega_2\delta^2. \tag{53}$$

In order to understand the above result in more depth, we first take account of the case $\delta = 0$. In this case, S^3 reduces to S^2 and the conformal dimension of a circular string is given by

$$\bar{\Delta}_H = \sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2}, \tag{54}$$

where the bar symbol means a quantity defined on S^2, like $\bar{J}_1 \equiv 2\pi T\omega_1$.

In the large 't Hooft coupling limit ($T \gg 1$), Δ_H and J_1 are large ($\sim T$) but J_2 is proportional to $T\delta^2$. If we define $\bar{J}_2 \equiv 2\pi T\omega_2$, the conformal dimension in Eq. (52) can be expanded near the equator of S^2 ($\delta \ll 1$) to

$$\Delta_H = \sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2} + \frac{\bar{J}_2^2 - \bar{J}_1^2 - 4\pi^2T^2w^2}{2\sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2}}\delta^2 + \cdots. \tag{55}$$

Here, the first term is nothing but the conformal dimension of the circular string living on the equator of S^2 and the second is the leading size effect caused by the change of the string length. Near the equator of S^2, the string length l can be expanded to

$$l = 2\pi\sqrt{1-\delta^2} \approx 2\pi - \pi\delta^2, \tag{56}$$

where $\pi\delta^2$ parametrizes the deviation of its string length from that of a string wrapping around the equator. Rewriting δ^2 in terms of J_1 and $\bar{J}_1$,

$$\delta^2 = \frac{\bar{J}_1 - J_1}{\bar{J}_1}, \tag{57}$$

the conformal dimension of the circular string becomes

$$\Delta_H = \sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2 + (\bar{J}_2^2 - \bar{J}_1^2 - 4\pi^2T^2w^2)\frac{\bar{J}_1 - J_1}{\bar{J}_1}}. \tag{58}$$

In the dual closed-spin-chain picture, this shows that the leading size effect on the conformal dimension is linearly proportional to $\bar{J}_1 - J_1$, unlike the magnon case.

After some calculations, the three-point function with an (ir)relevant operator finally becomes

$$\langle \mathcal{O}_H(0)\mathcal{O}_H(x_f)\mathcal{O}_\phi(y)\rangle = -\frac{\pi T^2w^2J_1}{2^{h-1}\bar{J}_1\sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2 + (\bar{J}_2^2 - \bar{J}_1^2 - 4\pi^2T^2w^2)\frac{\bar{J}_1 - J_1}{\bar{J}_1}}}\frac{\Gamma(\frac{h}{2})\Gamma(h)}{\Gamma(\frac{h+1}{2})\Gamma(h-\frac{3}{2})}\frac{1}{x_f^{2\Delta-h}|x_f-y|^h y^h}. \tag{59}$$

For $m_\phi = 0$, it simply reduces to the three-point function with a marginal operator with the following structure constant:

$$a_{HH\mathcal{L}} = -\frac{\pi T^2w^2J_1}{2\bar{J}_1\sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2 + (\bar{J}_2^2 - \bar{J}_1^2 - 4\pi^2T^2w^2)\frac{\bar{J}_1 - J_1}{\bar{J}_1}}}, \tag{60}$$

which also satisfies the RG analysis (6). Furthermore, the universality in Eq. (35) is still preserved. For a small δ, the structure constant is also expanded to

$$a_{HH\mathcal{L}} \approx -\frac{\pi T^2w^2}{2\sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2}} + \frac{\pi T^2w^2(\bar{J}_1^2 + \bar{J}_2^2 + 4\pi^2T^2w^2)}{4(\bar{J}_1^2 + 4\pi^2T^2w^2)^{3/2}}\frac{\bar{J}_1 - J_1}{\bar{J}_1}, \tag{61}$$

in which the first term is the structure constant of a circular string living on the equator of S^2 and the second term is the leading size effect caused by the change of the string length.

IV. CONCLUSIONS

In the strong-coupling regime, it is almost impossible to calculate a general three-point function by using the traditional methods of quantum field theory. However, there are several exceptions. If there is an additional symmetry, the three-point function with its current operator can be determined by the Ward identity, even in the strong-coupling regime [22]. Another possible example is the three-point function with a marginal operator, more specifically a Lagrangian-density operator. On the CFT side, the three-point function of two heavy operators and a marginal one can be evaluated by the RG analysis, which shows that the structure constant $a_{HH\mathcal{L}}$ is related to the derivative of the conformal dimension of the heavy operator. Following the AdS/CFT correspondence, we investigated the three-point function of the ABJM model in the strong-coupling regime and showed that the string calculation really leads to a result that is consistent with the RG analysis.

In this paper, we further investigated the three-point function with an (ir)relevant operator,

$$\langle \mathcal{O}_H(0)\mathcal{O}_H(x_f)\mathcal{O}_\phi\rangle = \frac{a_{HH\phi}}{x_f^{2\Delta-h}|x_f-y|^h y^h}, \tag{62}$$

where the structure constant is given by

$$a_{HH\phi} = \frac{1}{2^{h-2}}\frac{\Gamma(\frac{h}{2})\Gamma(h)}{\Gamma(\frac{h+1}{2})\Gamma(h-\frac{3}{2})}a_{HH\mathcal{L}}. \tag{63}$$

This result shows that the coordinate dependence is exactly of the form expected by the conformal symmetry. Interestingly, the above structure constant is closely related to that with a marginal operator and their ratio has a universal feature that does not depend on the details of the heavy operator. We have checked this universality with the various heavy operators corresponding to solitonic strings moving in the $\mathrm{AdS}_4 \times CP^3$ space. These results can easily be generalized to the higher-dimensional cases like the $\mathrm{AdS}_5 \times \mathrm{S}^5$ background, in which the definition of the string tension should be modified [10]. Finally, we found a solitonic string dual to a certain closed spin chain in the dual CFT and studied the new size effect of it. In the large 't Hooft coupling limit, the size effect of the closed spin chain is suppressed linearly by $(\bar{J}_1 - J_1)$, while the open spin chain described by a magnon has the exponential suppression e^{-J_1}.

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