Nonrenormalization and naturalness in a class of scalar-tensor theories

Claudia de Rham,[1] Gregory Gabadadze,[2] Lavinia Heisenberg,[1,3] and David Pirtskhalava[4]

[1]*Department of Physics, Case Western Reserve University, 10900 Euclid Avenue, Cleveland, Ohio 44106, USA*

[2]*Center for Cosmology and Particle Physics, Department of Physics, New York University, New York, New York 10003, USA*

[3]*Département de Physique Théorique and Center for Astroparticle Physics, Université de Genève, 24 Quai Ernest Ansermet, CH-1211 Genève, Switzerland*

[4]*Department of Physics, University of California, San Diego, La Jolla, California 92093, USA*

(Received 29 December 2012; published 10 April 2013)

We study the renormalization of some dimension-4, 7 and 10 operators in a class of nonlinear scalar-tensor theories. These theories are invariant under (a) linear diffeomorphisms which represent an exact symmetry of the full nonlinear action, and (b) global field-space Galilean transformations of the scalar field. The Lagrangian contains a set of nontopological interaction terms of the above-mentioned dimensionality, which we show are not renormalized at any order in perturbation theory. We also discuss the renormalization of other operators, that may be generated by loops and/or receive loop corrections, and identify the regime in which they are subleading with respect to the operators that do not get renormalized. Interestingly, such scalar-tensor theories emerge in a certain high-energy limit of the ghost-free theory of massive gravity. One can use the nonrenormalization properties of the high-energy limit to estimate the magnitude of quantum corrections in the full theory. We show that the quantum corrections to the three free parameters of the model, one of them being the graviton mass, are strongly suppressed. In particular, we show that having an arbitrarily small graviton mass is technically natural.

DOI: 10.1103/PhysRevD.87.085017 PACS numbers: 04.50.Kd

I. MOTIVATION

Arguably, two of the biggest puzzles of modern cosmology remain the origin of the accelerated expansion of the present-day Universe, and the old cosmological constant (CC) problem, arising from a giant mismatch between the theoretically expected magnitude of the vacuum energy and the tiny value of the observed space-time curvature. Although the resolution of these two puzzles may be related to each other, general relativity (GR)—which is a very successful theory otherwise—fails to address the CC problem, while only being able to accommodate cosmic acceleration via the postulated dark energy, offering no insights into its origin.

Theories that extend/modify GR at a large distance scale (say at some scale $m^{-1} \sim H_0^{-1}$, H_0 denoting the present-day value of the Hubble parameter) offer a hope to cancel the vacuum energy by evading S. Weinberg's no-go theorem, at the same time describing the accelerated expansion in terms of a small dimensionful parameter m. Although a satisfactory framework does not yet exist, the resolution seems to be not too far in the future [1,2].[1]

In such a scenario, a natural question arises as to whether the introduced small parameter itself (e.g., the graviton mass m) is subject to strong renormalization by quantum loops, similar to the renormalization of a small cosmological constant.[2] This is one of the questions we will address in the present work.

Technically natural tunings are not uncommon within the Standard Model of particle physics. According to 't Hooft's naturalness argument [3,4], a physical parameter c_i can remain naturally small at any energy scale E if the limit $c_i \to 0$ enhances symmetry of the system. While the electron mass m_e is much smaller than the electroweak scale for instance, it is technically natural, since quantum corrections give rise to a renormalization of m_e proportional to m_e itself, making the mass parameter logarithmically sensitive to the UV scale. The reason for this is simple: taking the $m_e \to 0$ limit implies an additional chiral symmetry, enforcing the separate conservation of left- and right-handed electrons, meaning that in the massless limit the electron mass receives no quantum corrections. For the case of the cosmological constant, λ, on the other hand, there is no symmetry recovered in the limit $\lambda \to 0$, and any particle of mass M is expected to contribute to the CC by terms among which are $M^2\Lambda_{\rm UV}^2/M_{\rm Pl}^2$, $\Lambda_{\rm UV}^4/M_{\rm Pl}^2$, with $\Lambda_{\rm UV}$ denoting the UV cutoff. The smallness of the CC is thus unnatural in the 't Hooft sense.

We will show in the present work that the introduction of the small graviton mass m is technically natural, in sharp distinction from the small CC scenario.

[1]An interesting feature of the cosmological solutions obtained in these references is that they break spatial translation invariance in a way that is still in an apparent agreement with cosmological observations, at the same time providing a way to evade the no-go theorem.

[2]To be clear, by strong renormalization of a parameter we mean an additive renormalization which is proportional to positive powers of the UV cutoff, as opposed to a multiplicative renormalization that is only logarithmically sensitive to it; see more on this below.

A Lorentz-invariant modification of GR, such as massive gravity, introduces extra propagating degrees of freedom, as well as a strong coupling energy scale Λ_3 associated with these degrees of freedom. Usually, the energy scale Λ_3 is much smaller than the Planck mass, while significantly exceeding m. For example, on flat space $\Lambda_3 = (M_{\rm Pl} m^2)^{1/3}$; however it can be much higher on nontrivial backgrounds. On the one hand, the presence of strong coupling is typically required to hide unneeded extra forces from observations via the Vainshtein mechanism [5,6].[3] On the other hand, the strongly coupled behavior calls for important questions on calculability, quantum consistency, and in this particular case on superluminality of the massive theory. The first two are the questions that we will address below, arguing that it is precisely the classical strong coupling, supplemented by ghost freedom of the theory that extends its predictivity to distances parametrically lower than Λ_3^{-1}. The question of superluminal propagation is tied to that of potential UV extensions of these theories above the scale Λ_3 [7,8], and will be addressed elsewhere.

II. THE EFFECTIVE FIELD THEORY

We will consider a theory of a massless spin-2 field $h_{\mu\nu}$, and a scalar π, which couple to each other via some dimension 4, 7, and 10 operators; the latter two will be suppressed by powers of a dimensionful scale Λ_3. The interactions become strong at the energy scale $E \sim \Lambda_3$. Nevertheless, we will show that the special structure of interactions in this theory guarantees that the operators presented in the tree-level Lagrangian do not get renormalized at any order in perturbation theory.

The (noncanonically normalized) Lagrangian of the above-described theory reads as follows [9]:

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu} \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}(\Pi), \quad (1)$$

where $\mathcal{E}^{\alpha\beta}_{\mu\nu}$ is the Einstein operator, so that the first term denotes the quadratic Einstein-Hilbert contribution. The dimensionless coefficients a_n are tree-level free parameters (we will fix $a_1 = -1/2$ below for a definite normalization of the scalar kinetic term) and the three X's are explicitly given by the following expressions in terms of $\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$ and the Levi-Civita symbol $\varepsilon^{\mu\nu\alpha\beta}$:

$$\begin{aligned} X^{(1)}_{\mu\nu}(\Pi) &= \varepsilon_\mu{}^{\alpha\rho\sigma} \varepsilon_\nu{}^\beta{}_{\rho\sigma} \Pi_{\alpha\beta}, \\ X^{(2)}_{\mu\nu}(\Pi) &= \varepsilon_\mu{}^{\alpha\rho\gamma} \varepsilon_\nu{}^{\beta\sigma}{}_\gamma \Pi_{\alpha\beta} \Pi_{\rho\sigma}, \\ X^{(3)}_{\mu\nu}(\Pi) &= \varepsilon_\mu{}^{\alpha\rho\gamma} \varepsilon_\nu{}^{\beta\sigma\delta} \Pi_{\alpha\beta} \Pi_{\rho\sigma} \Pi_{\gamma\delta}. \end{aligned} \quad (2)$$

$X^{(1,2,3)}$ are respectively linear, quadratic and cubic in $\partial^2 \pi$, so that the action involves operators up to quartic order in the fields.

The symmetries of the theory include (a) linearized diffeomorphisms, $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu} \xi_{\nu)}$, which represent an exact symmetry of the full nonlinear action (i.e., including the interactions $h^{\mu\nu} X^{(n)}_{\mu\nu}$), and (b) (global) field-space Galilean transformations, $\pi \to \pi + b_\mu x^\mu + b$. The first of these is a symmetry up to a total derivative.

Although the interactions involve two derivatives on the scalar field π, the theory defined by (1) is ghost free [9,10]: it propagates exactly two polarizations of the massless tensor field and exactly one massless scalar; it thus represents a nontrivial example of a model with nontopologically interacting spin-2 and spin-0 fields.

We will show in the next section that the operators of this theory remain protected against quantum corrections to all orders in perturbation theory, despite the existence of nontrivial interactions governed by the scale Λ_3. Technically, this nonrenormalization is due to the specific structure of the interaction vertices: they contain two derivatives per scalar line, all contracted by the epsilon tensors. Then, it is not too difficult to show, as done in the next section, that the loop diagrams cannot induce any renormalization of the tree-level terms in (1). Conceptually, the nonrenormalization appears because the tree-level interactions in the Lagrangian are diffeomorphism invariant up to total derivatives only; on the other hand, the variations of the Lagrangian with respect to fields in this theory are exactly diffeomorphism invariant; therefore, no Feynman diagram can generate operators that would not be diffeomorphism invariant, and the original operators that are diffeomorphism invariant only up to total derivatives stay unrenormalized.[4]

In a conventional approach that would regard (1) as an effective field theory below the scale Λ_3, there would be new terms induced by quantum loops, in addition to the nonrenormalizable terms already present in (1). Let us consider one-loop terms in the 1-particle irreducible (1PI) action. These are produced by an infinite number of one-loop diagrams with external h and/or π lines. The diagrams contain power-divergent terms, the log-divergent pieces, and finite terms. The power-divergent terms are arbitrary, and cannot be fixed without the knowledge of the UV completion. For instance, dimensional regularization would set these terms to zero. Alternatively, one could use any other regularization, but perform subsequent subtraction so that the net result in the 1PI action is zero.

In contrast, the log-divergent terms are uniquely determined: they give rise to nonzero imaginary parts of various amplitudes, such as the one depicted in Fig. 1; the latter

[3] Note, however, that a different mechanism of hiding the extra degrees of freedom has been found in Ref. [1], which does not rely on the Vainshtein effect, but is perturbative. Its virtues will be discussed later.

[4] This is similar to nonrenormalization of the Galileon operators [11–13], with diffeomorphism invariance replaced by Galilean invariance; we thank Kurt Hinterbichler for useful discussions on these points.

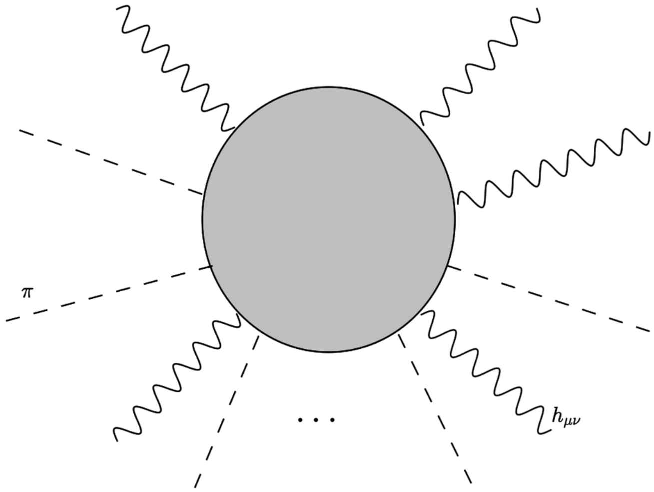

FIG. 1. An arbitrary 1PI diagram with gravitational degrees of freedom in the loop.

determine the forward scattering cross sections through the optical theorem. Therefore, these pieces would have to be included in the 1PI action.

All the induced terms in the 1PI action would appear suppressed by the scale Λ_3, since the latter is the only scale in the effective field theory approach (including the scale of the UV cutoff). Moreover, due to the same specific structure of the interaction vertices that guarantees nonrenormalization of (1), the induced terms will have to have more derivatives per field than the unrenormalized terms. Therefore at low energies, formally defined by the condition $(\partial/\Lambda_3) \ll 1$, the tree-level terms will dominate over the induced terms with the same number of fields, as well as over the induced terms with a greater number of fields and derivatives. This property clearly separates the unrenormalized terms from the induced ones, and shows that the theory (1) is a good effective field theory below the scale Λ_3.

We now move to the discussion of how classical sources enter the above picture. As we will show below, there are similarities to DGP [14] and Galileon [13] theories, but there is also an additional important ingredient that is specific to the present theory (1). We will present the discussion for the simplest case $a_3 = 0$, when the Lagrangian can be explicitly diagonalized [9], and will show in the next section that setting $a_3 = 0$ is technically natural. Some of the novel qualitative features readily apply even when $a_3 \neq 0$; however, in this case there are differences too, as we will briefly discuss below.

To this end, it is helpful to perform the field redefinition $h_{\mu\nu} \to h_{\mu\nu} + \pi\eta_{\mu\nu}$ and rewrite the two nonlinear interactions of the $a_3 = 0$ theory as follows:

$$\frac{a_2}{\Lambda_3^3}(2G^{\mu\nu}(h)\partial_\mu\pi\partial_\nu\pi + 3\Box\pi(\partial_\mu\pi)^2). \tag{3}$$

The second term in (3) is what appears in DGP (and hence in the cubic Galileon theory). Since in this basis the π field couples to the trace of the stress-tensor as $\pi T/M_{\rm Pl}$, the nonlinear term $\Box\pi(\partial\pi)^2$ gives rise to the conventional Vainshtein mechanism: for a source of mass $M_s \gg M_{\rm Pl}$, below the Vainshtein radius, $r_* = (M_s/M_{\rm Pl})^{1/3}\Lambda_3^{-1}$, the classical value of the π field is severely suppressed (as compared to its value in the linear theory) due to the fact that in this regime $\partial^2\pi/\Lambda_3^3 \gg 1$.[5] The novelty here is the first term in (3): this term dominates over the second one both inside and outside the source (but still inside the Vainshtein radius). Outside the source it amounts of having a quartic Galileon in the theory, and since its phenomenology is well known [13], we will not discuss it in detail here. However, inside the source we get $G_{\mu\nu}(h) \simeq T_{\mu\nu}/M_{\rm Pl}^2$, with a good accuracy. Hence, in this region the π field gets an additional kinetic term, and the full quadratic term for it can be written as follows:

$$-\partial^\mu\pi\partial^\nu\pi\left(\eta_{\mu\nu} - 2a_2\frac{T_{\mu\nu}}{\Lambda_3^3 M_{\rm Pl}}\right). \tag{4}$$

Thus, for a negative value of a_2 one gets a classical renormalization of the π kinetic term. To appreciate how big this renormalization is we note that the scale in the denominator of the second term in (4) is $M_{\rm Pl}^2 m^2$; for a graviton mass comparable with the Hubble parameter H_0, this is of the order of the critical density of the present day Universe. For instance, taking the Earth's atmosphere as a source, we get for the kinetic and gradient terms

$$(1 + |a_2|10^{26})(\partial_0\pi)^2 - (1 - |a_2|10^{14})(\partial_j\pi)^2. \tag{5}$$

For higher density/pressure sources, such as the Earth itself, or for any Earthly measuring device, we get even higher factors of the order 10^{30} and 10^{18}, respectively, for the kinetic and gradient terms.

Thanks to these new couplings, the strength of the interactions of the π fluctuations above the classical source, $\delta\pi = \pi - \pi_{\rm cl}$, changes qualitatively. Recall that in the DGP and the standard Galileon theories, the regime of validity of the classical solutions can be meaningfully established in the full quantum effective theory due to the strong classical renormalization of the scalar kinetic terms via the Vainshtein mechanism [12].[6] Here we get an additional strong classical renormalization of the kinetic term for the fluctuations $\delta\pi$

$$-\partial^\mu\delta\pi\partial^\nu\delta\pi(Z^V_{\mu\nu} + |a_2|Z^T_{\mu\nu}), \tag{6}$$

[5]The same applies to time-dependent sources, for which an additional scale due to the time dependence enters the Vainshtein radius [15].

[6]The right procedure is to first solve for a classical scalar profile in the presence of a source, and then calculate quantum corrections. Of course the opposite order should also give the same result once done correctly; however in the latter case one would have to resum quantum corrections enhanced by large classical terms.

where the first term in (6) is due to the Vainshtein mechanism, which gives rise to a large M_s-dependent factor $Z^V \sim a_2(r_*/R_{\rm Earth})$, while the second one is due to the above-mentioned novel coupling [the first term in (3)].

Furthermore, following Ref. [12], the 1PI action can be organized (using some reasonable assumptions about the UV theory) so that the local strong coupling scale determining the interactions of the fluctuations $\delta\pi$ schematically reads as follows:

$$\Lambda_{\rm eff}(x) \equiv (Z^V + |a_2|Z^T)^{1/3}\Lambda_3.$$

Very often $Z^T \gg Z^V$ [16], and therefore Z^T (although localized in the source) should be taken into account when and if bounds are imposed on the graviton mass from the existence of this strong scale. For instance, as argued in Ref. [13] for the quartic Galileon, the angular part of the quadratic term for the fluctuations is not enhanced by Z^V, presenting a challenge; luckily, the enhancement due to Z^T removes this issue in the theory at hand. Moreover, the Z^T enhancement is present irrespective of whether a_3 is chosen to be zero or not—it is solely defined by a nonzero a_2. Regretfully, this effect has not been taken into account in Ref. [17], and the bounds on the graviton mass obtained there will have to be reconsidered [16].

In addition to what we discussed above, there are additional subtleties when $a_3 \neq 0$. In this case the Lagrangian (1) cannot be diagonalized by any local field redefinition [9]. Hence, the nonlinear mixing term $h^{\mu\nu}X^{(3)}_{\mu\nu}$ will be present, no matter what. Insertions of this vertex into quantum loops will generate higher powers and/or derivatives of the Riemann tensor $R_{\mu\alpha\nu\beta}$, as well as mixed terms between the Riemann tensor (with or without derivatives) and derivatives of π. In a theory without sources all these terms will be suppressed by Λ_3, again representing a good effective field theory below this scale.

However, with classical sources included, there should appear a Z-factor suppression of the terms containing $R_{\mu\alpha\nu\beta}$, due to the fact that on nontrivial backgrounds of classical sources there will be a large quadratic mixing between fluctuations of h and π, and the latter has a large kinetic term due to the Z factor as discussed above. All this will be discussed in Ref. [16].

Last but not least, we note that the Lagrangian (1) is not a garden-variety nonrenormalizable model, as is clear from the above discussions, and there may be a diagram resummation approach to the strong coupling issue, or perhaps a dual formulation along the lines of Ref. [18]. However, in the present work we adopted a conventional low-energy effective field theory approach.

A. Relation to massive gravity

Interestingly, the action given in (1) appears in a certain limit of a recently proposed class of theories of massive gravity, free of the Boulware-Deser (BD) ghost [19].

A two-parameter family of such theories has been proposed in Refs. [9,10]. The theory has been shown to be free of the BD ghost perturbatively in Ref. [10], at the full nonlinear level in the Hamiltonian formalism in Refs. [20,21], and covariantly around any background in Ref. [22] (see also Refs. [23–25] for a complementary analysis in the Stückelberg and helicity languages, and [26] for a proof in the first order formalism).

This class of theories provides a promising framework for tackling the cosmological constant problem, given that the graviton mass m can be tuned to be around the Hubble scale today. Such a tuning of m with respect to the theoretically expected vacuum energy is of the same order as that of the conventional cosmological constant, $m^2 \lesssim 10^{-120}M_{\rm Pl}^2$; however, unlike the tuning of the cosmological constant, it is anticipated to be technically natural. The reason for this lies in the fact that in the $m \to 0$ limit we recover general relativity, which, being a gauge theory, is fully protected from a quantum-mechanically induced graviton mass.

There is however a possible loophole in this reasoning: the $m \to 0$ limit is obviously discontinuous in the number of gravitational degrees of freedom and the presence of these extra polarizations for $m \neq 0$ deserves a special treatment in the context of naturalness.[7]

In particular, the theory (1) emerges as the leading part of the ghost-free massive gravity action describing the interactions of the helicity-2 and helicity-0 polarizations of the graviton in the limit

$$m \to 0, \qquad M_{\rm Pl} \to \infty, \qquad \Lambda_3 \equiv (M_{\rm Pl}m^2)^{1/3} = \text{finite}. \tag{7}$$

Beyond this limit, the free parameters of the theory are expected to be renormalized, albeit by an amount that should vanish in the limit (7). As a result, quantum corrections to the three defining parameters of the full theory (namely the mass m and the two free coefficients $a_{2,3}$) are strongly suppressed. In particular, the graviton mass receives a correction proportional to itself [with a coefficient that goes as $\delta m^2/m^2 \sim (m/M_{\rm Pl})^{2/3}$], thus establishing the technical naturalness of the theory.

One should stress at this point that technical naturalness is not an exclusive property of ghost-free massive gravity. Even theories with the BD ghost can be technically natural, satisfying the $\delta m^2 \propto m^2$ property [32]. Besides the fact that the latter theories are unacceptable, there are two important distinctions between the theories with and without BD ghosts. These crucial distinctions can be

[7]Nevertheless, this does not mean that the physical predictions of the theory are discontinuous. As mentioned above, the presence of the Vainshtein mechanism in this model [27–30], as well as general [31] extensions of the Fierz-Pauli theory, makes most of the physical predictions identical to that of GR in the massless limit.

formulated in the decoupling limit, which occurs at a much lower energy scale, $\Lambda_5 = (m^4 M_{\rm Pl})^{1/4} \ll \Lambda_3$, if the theory propagates a BD ghost. In the latter case the classical part of the decoupling limit is not protected by a nonrenormalization theorem. As a consequence (a) quantum corrections in ghost-free theories are significantly suppressed with respect to those in the theories with the BD ghost, and (b) unlike a generic massive gravity, the nonrenormalization guarantees that *any* relative tuning of the parameters in the ghost-free theories, that is m, a_2, a_3, is technically natural. The latter property makes any relation between the free coefficients of the theory stable under quantum corrections.[8]

The rest of the paper is organized as follows. We show in Sec. III that the interactions of the scalar-tensor theory defined in (1) do not receive quantum corrections to any order in perturbation theory. Identifying the latter theory with the decoupling limit of massive gravity, we discuss the implications of such renormalization group (RG) invariance of relevant parameters in the given limit for the full theory, showing explicitly in Sec. IV that quantum corrections to the graviton mass and the two free parameters of the potential are significantly suppressed. Finally, we conclude in Sec. V.

III. THE NONRENORMALIZATION THEOREM

In this section we present the nonrenormalization argument for a class of scalar-tensor theories, defined by the Lagrangian (1). In particular, we will show that the two parameters $a_{2,3}$ do not get renormalized, and that there is no wave function renormalization for the spin-2 field $h_{\mu\nu}$.

Using the antisymmetric structure of these interactions, we can follow roughly the same arguments as for Galileon theories to show the RG invariance of these parameters [11]. The only possible difference may emerge due to the gauge invariance $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$, and consequently the necessity of gauge fixing for the tensor field. Working in e.g., the de Donder gauge, the relevant modification of the arguments is trivial: gauge invariance is Abelian, so the corresponding Faddeev-Popov ghosts are free and do not affect the argument in any way. Moreover, the gauge fixing term changes the graviton propagator, but as we shall see below, all the arguments that follow solely depend on the special structure of vertices and are hence independent of the exact structure of the propagator. With these arguments in mind, one can thus proceed with the proof of the nonrenormalization of the theory without being affected by gauge invariance.

[8]For example, a particular ghost-free theory with the decoupling limit, characterized by the vanishing of all interactions in (1), has been studied due to its simplicity (e.g., see Refs. [33,34] for one-loop divergences in that model). The nonrenormalization of ghost-free massive gravity in this case guarantees that such a vanishing of the classical scalar-tensor interactions holds in the full quantum theory as well.

The scalar π only appears within interactions/mixings with the spin-2 field in (1). In order to associate a propagator with it, we have to diagonalize the quadratic Lagrangian by eliminating the $h^{\mu\nu}X^{(1)}_{\mu\nu}(\Pi)$ term. Such a diagonalization gives rise to a kinetic term for π, as well as additional scalar self-interactions of the Galileon form [9],

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + \frac{3}{2}\pi\Box\pi + (h^{\mu\nu} + \pi\eta^{\mu\nu}) \times \sum_{n=2}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}(\Pi) \tag{8}$$

(here the interactions of the form $\pi X^{(n)}(\Pi)$ are nothing else but the cubic and quartic Galileons).

In the special case when the parameter a_3 vanishes, all scalar-tensor interactions are redundant and equivalent to pure scalar Galileon self-interactions. This can be seen through the field redefinition (under which the S matrix is invariant) $h_{\mu\nu} = \tilde{h}_{\mu\nu} + \pi\eta_{\mu\nu} - \frac{2a_2}{\Lambda_3^3}\partial_\mu\pi\partial_\nu\pi$. We then recover a decoupled spin-2 field, supplemented by the Galileon theory for the scalar of the form

$$\mathcal{L}_{\rm Gal} = -\frac{1}{2}\sum_{n=0}^{2}\frac{b_n}{\Lambda_3^{3n}}X^{(n)}_{\mu\nu}(\Pi)\partial^\mu\pi\partial^\nu\pi, \tag{9}$$

where the Galileon coefficients b_n are in one-to-one correspondence with a_n and $X^{(0)}_{\mu\nu} \equiv \eta_{\mu\nu}$. The nonrenormalization of the theory (1) then directly follows from the analogous property of the Galileons. For $a_3 \neq 0$, such a redefinition is however impossible [9].[9]

We will now show that, similarly to what happens in the pure Galileon theories, any external particle comes along with at least two derivatives acting on it in the 1PI action, hence establishing the nonrenormalization of the operators present in (1). Of course, we keep in mind that these operators are merely the leading piece of the full 1PI action, which features an infinite number of additional higher derivative terms. They however are responsible for most of the phenomenology that the theories at hand lead to, making the nonrenormalization property essential.

Consider an arbitrary 1PI diagram, such as the one depicted in Fig. 1. All vertices in (8) have one field without a derivative, while all the rest come with two derivatives acting on them. Any external leg, contracted with a field with two derivatives in a vertex, obviously contributes to an operator with two derivatives on the field in the 1PI effective interaction, so if all the external legs were of that kind, this would lead to an operator of the form $\partial^{2j}(\partial^2\pi)^k \times (\partial^2 h_{\mu\nu})^\ell$, with $j, k, \ell > 0$. The only possibility of generating an operator with fewer derivatives on some of the fields

[9]This can be understood by noting that the $h^{\mu\nu}X^{(3)}_{\mu\nu}$ coupling encodes information about the linearized Riemann tensor for $h_{\mu\nu}$, which cannot be expressed through π on the basis of the lower-order equations of motion [29].

comes from contracting fields without derivatives in vertices with external states. For example, in the interaction $V = h^{\mu\nu}X^{(2)}_{\mu\nu}(\Pi) \sim h^{\mu\nu}\varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma}{}_\gamma\Pi_{\alpha\beta}\Pi_{\rho\sigma}$, the spin-2 field comes without derivatives, so let us look at an external $h_{\mu\nu}$ leg coming out of this vertex in an arbitrary 1PI graph, while letting the other two π-particles from this vertex run in the loop [of course, all of this reasoning will equivalently apply to any other vertex, such as $h^{\mu\nu}X^{(3)}_{\mu\nu}(\Pi)$, or $\pi X^{(2,3)}(\Pi)$]. Let us denote the external, spin-2 momentum by p_μ, while the momenta corresponding to the two π-particles in the loop are k_μ and $(p+k)_\mu$, respectively. The contribution of this vertex to the graph is given as follows:

$$i\mathcal{M} \propto i\int \frac{d^4k}{(2\pi)^4}\mathcal{G}_k\mathcal{G}_{k+p}\epsilon^{*\mu\nu}\varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma}{}_\gamma k_\alpha k_\beta(p+k)_\rho \times (p+k)_\sigma \ldots, \tag{10}$$

where the Feynman propagator is denoted by $\mathcal{G}_k \equiv \frac{i}{k^2-i\epsilon}$ and $\epsilon^{*\mu\nu}$ is the spin-2 polarization tensor, while the ellipses encode information about the rest of the diagram. Now, the key observation is that the term independent of the external momentum p and the term linear in it both cancel due to the antisymmetric structure of the vertex. Hence, the only non-vanishing term involves two powers of the external spin-2 momentum $p_\rho p_\sigma$

$$i\mathcal{M} \propto i\epsilon^{*\mu\nu}\varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma}{}_\gamma p_\rho p_\sigma\int\frac{d^4k}{(2\pi)^4}\mathcal{G}_k\mathcal{G}_{k+p}k_\alpha k_\beta \ldots, \tag{11}$$

yielding at least two derivatives on the external helicity-2 mode in the position-space. Thus any external leg coming out of the $h^{\mu\nu}X^{(2)}_{\mu\nu}$ vertex will necessarily have two or more derivatives on the corresponding field in the effective action. The same is trivially true for the $\pi X^{(2)}_{\mu\nu}$ vertex (as one can simply substitute $h_{\mu\nu}$ by π in the above discussion).

Similarly, if the external leg is contracted with the derivative-free field in vertices $h^{\mu\nu}X^{(3)}_{\mu\nu}$ and $\pi X^{(3)}_{\mu\nu}$, their contribution will always involve the external momentum p_μ and the loop momenta k_μ and k'_μ with the following structure:

$$\begin{aligned} i\mathcal{M} &\propto i\int\frac{d^4k d^4k'}{(2\pi)^8}\mathcal{G}_k\mathcal{G}_{k'}\mathcal{G}_{k+k'+p}f^{\mu\nu}\varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma\delta}k_\alpha k_\beta k'_\rho k'_\sigma \\ &\quad\times(p+k+k')_\gamma(p+k+k')_\delta\ldots \\ &\propto if^{\mu\nu}\varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma\delta}p_\gamma p_\delta\int\frac{d^4k d^4k'}{(2\pi)^8} \\ &\quad\times\mathcal{G}_k\mathcal{G}_{k'}\mathcal{G}_{k+k'+p}k_\alpha k_\beta k'_\rho k'_\sigma\ldots, \end{aligned} \tag{12}$$

where the contraction on the Levi-Civita symbols is performed with either the graviton polarization tensor $f^{\mu\nu} = \epsilon^{*\mu\nu}$ or with $f^{\mu\nu} = \eta^{\mu\nu}$ depending on whether we are dealing with the vertex $h^{\mu\nu}X^{(3)}_{\mu\nu}$ or $\pi X^{(3)\mu}{}_\mu$. Similar arguments, as can be straightforwardly checked, lead to the same conclusion regarding the minimal number of derivatives on external fields for cases in which there are two external states coming out of these vertices (with the other two consequently running in the loops).

This completes the proof of the absence of quantum corrections to the two parameters $a_{2,3}$, as well as to the spin-2 kinetic term and the scalar-tensor kinetic mixing in the theory defined by (1).

IV. MASSIVE GRAVITY AND ITS DECOUPLING LIMIT

For massive gravity the action is a functional of the metric $g_{\mu\nu}(x)$ and four spurious scalar fields $\phi^a(x)$, $a = 0, 1, 2, 3$; the latter are introduced to give a manifestly diffeomorphism-invariant description [32,35]. One defines a covariant tensor $H_{\mu\nu}$ as follows:

$$g_{\mu\nu} = \partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab} + H_{\mu\nu}, \tag{13}$$

where $\eta_{ab} = \mathrm{diag}(-1,1,1,1)$ is the *field* space metric.

In this formulation, the tensor $H_{\mu\nu}$ propagates on Minkowski space. In the unitary gauge all the four scalars $\phi^a(x)$ are frozen and equal to the corresponding space-time coordinates, $\phi^a(x) = x^\mu\delta^a_\mu$ and the tensor $H_{\mu\nu}$ coincides with the metric perturbation, $H_{\mu\nu} = h_{\mu\nu}$. However, often it is helpful to use a nonunitary gauge in which the $\phi^a(x)$'s are allowed to fluctuate.

A covariant Lagrangian density for massive gravity can be written as follows:

$$\mathcal{L} = \frac{M^2_{\rm Pl}}{2}\sqrt{-g}\left(R - \frac{m^2}{4}\mathcal{U}(g,H)\right), \tag{14}$$

where $\mathcal{U}$ includes the mass and nonderivative interaction terms for $H_{\mu\nu}$ and $g_{\mu\nu}$, while R denotes the scalar curvature associated with the metric $g_{\mu\nu}$.

A necessary condition for the theory to be ghost free in the decoupling limit (DL) is that the potential $\sqrt{-g}\,\mathcal{U}(g,H)$ be a total derivative upon the field substitution $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu} = 0$, $\phi^a = \delta^a_\mu x^\mu - \eta^{a\mu}\partial_\mu\pi$ [9]. With this substitution, the potential becomes a function of $\Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\pi$ and its various contractions with respect to the flat metric $\eta_{\mu\nu}$. The relevant terms can be constructed straightforwardly by using the procedure outlined in Ref. [10].

In any dimension there are only a finite number of total derivative combinations, made out of Π [13]. They are all captured by the recurrence relation [9]:

$$\mathcal{L}^{(n)}_{\rm der} = -\sum_{m=1}^{n}(-1)^m\frac{(n-1)!}{(n-m)!}[\Pi^m]\mathcal{L}^{(n-m)}_{\rm der}, \tag{15}$$

with $\mathcal{L}^{(0)}_{\rm der} = 1$ and $\mathcal{L}^{(1)}_{\rm der} = [\Pi]$. This also guarantees that the sequence terminates, i.e., $\mathcal{L}^{(n)}_{\rm der} \equiv 0$, for any $n \geq 5$ in four dimensions. The list of all nonzero total derivative terms starting with the quadratic one reads as

$$\mathcal{L}^{(2)}_{\rm der}(\Pi) = [\Pi]^2 - [\Pi^2], \tag{16}$$

$$\mathcal{L}^{(3)}_{\rm der}(\Pi) = [\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3], \tag{17}$$

$$\mathcal{L}^{(4)}_{\rm der}(\Pi) = [\Pi]^4 - 6[\Pi^2][\Pi]^2 + 8[\Pi^3][\Pi] + 3[\Pi^2]^2 - 6[\Pi^4], \tag{18}$$

where we use the notation: $[\Pi] \equiv {\rm Tr}\Pi^\mu_\nu$, $[\Pi]^2 \equiv ({\rm Tr}\Pi^\mu_\nu)^2$, while $[\Pi^2] \equiv {\rm Tr}\Pi^\mu_\nu\Pi^\nu_\alpha$, with an obvious generalization to terms of higher order in nonlinearity.

Then, as argued in Ref. [10], the Lagrangian for massive gravity that is automatically ghost free to all orders in the DL is obtained by replacing the matrix elements Π^μ_ν in the total derivative terms (16)–(18) by the matrix elements $\mathcal{K}^\mu{}_\nu$, defined as follows:

$$\mathcal{K}^\mu{}_\nu(g,H) = \delta^\mu_\nu - \sqrt{\partial^\mu\phi^a\partial_\nu\phi^b\eta_{ab}} = \sqrt{\delta^\mu_\nu - H^\mu{}_\nu}. \tag{19}$$

Here, and everywhere below, the indices on $\mathcal{K}$ should be lowered and raised with $g_{\mu\nu}$ and its inverse, respectively.

This procedure defines the mass term, along with the interaction potential in the Lagrangian density of massive gravity [10]:

$$\begin{aligned}\mathcal{L} &= \frac{M_{\rm Pl}^2}{2}\sqrt{-g}[R + m^2(\mathcal{L}^{(2)}_{\rm der}(\mathcal{K}) + \alpha_3\mathcal{L}^{(3)}_{\rm der}(\mathcal{K}) \\ &\quad + \alpha_4\mathcal{L}^{(4)}_{\rm der}(\mathcal{K}))] \\ &= \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\left[R - \frac{m^2}{4}(H^{\mu\nu}H_{\mu\nu} - H^2 + \cdots)\right]. \end{aligned}\tag{20}$$

Since all terms in (15) with $n \geq 5$ vanish identically, by construction all terms $\mathcal{L}^{(n)}_{\rm der}$ with $n \geq 5$ in (20) are also zero. Hence, the most general Lagrangian density (20) has three free parameters, m, α_3 and α_4.

As is straightforward to see, Minkowski space $g_{\mu\nu} = \eta_{\mu\nu}$ with $\phi^a = x^a$ is a vacuum solution, and the spectrum of the theory (20) contains a graviton of mass m; the graviton also has additional nonlinear interactions specified by the action at hand.

The high-energy dynamics of the system is best displayed in the DL, defined by (7). Being a direct analog of the nonlinear sigma model description of the high-energy limit of massive spin-1 theories, the decoupling limit of massive gravity features the five polarization states of the graviton, represented by separate helicity states 0, ± 1, ± 2. The helicity-2 mode $h_{\mu\nu}$ enters linearly in the decoupling limit, while the helicity-0 mode π is fully nonlinear[10] (we will for the moment ignore the vector polarization and will comment on it below). The resulting DL theory takes precisely the form (7). It fully captures the most important features of massive gravity, such as the absence of the Boulware-Deser ghost [9] (which has been shown to generalize beyond the DL [22]), the existence of self-accelerating and screening solutions [1,2,27,36], etc. Moreover, the DL carries all the interactions that become relevant within the massless limit and provides a simple illustration of how the helicity-0 mode decouples from the rest of the gravitational sector for $m \to 0$ as an explicit realization of the Vainshtein mechanism. The limit at hand thus represents a powerful tool to study generic physical properties of the theory. Above we have uncovered a further interesting property of this limit: the nonrenormalization theorem protecting the leading operators; we will see that this allows us to extract information on quantum corrections to the full theory from simple DL arguments [32].

A. Helicity-1 modes

The scalar-tensor action given in (1) does not include the DL interactions involving the helicity-1 modes A_μ of the massive graviton, defined through

$$\phi^a = \delta^a_\mu x^\mu - \eta^{a\mu}\left(\frac{A_\mu}{M_{\rm Pl}m} + \frac{\partial_\mu\pi}{M_{\rm Pl}m^2}\right), \tag{21}$$

in (13). So far, their precise form has only been found perturbatively (see for instance Refs. [22,37,38]). Schematically, to all orders they are given as follows [22]:

$$\mathcal{L}_A = -\frac{1}{4}F^2 + FF\sum_{n>0}\frac{d_n}{\Lambda_3^{3n}}\Pi^n, \tag{22}$$

where F denotes the field strength for A_μ and d_n are constant coefficients. These vertices can contribute to effective operators involving the helicity-0 mode, π. However, from the explicit form of these interactions it is manifest that every external π originating from such a vertex will have at least two derivatives on it, in complete analogy to the case considered above. Taking into account the vector-scalar interactions of the form (22) therefore does not change the nonrenormalization properties of the scalar-tensor part of the action given in (8).

B. Implications for the full theory

In this subsection we will comment on the implications of the above emergent DL nonrenormalization property for the full theory. Below we will continue to treat massive gravity as an effective field theory with a cutoff $\Lambda_3 \gg m$.

We have established previously that in the DL the leading scalar-tensor part of the action does not receive quantum corrections in massive gravity: all operators generated by quantum corrections in the effective action have at least two extra derivatives compared to the leading terms, making the coefficients a_i invariant under the renormalization group flow. This in particular implies the absence of wave-function renormalization for the helicity-2 and helicity-0

[10]Here we denote the canonically normalized fields, obtained by $h_{\mu\nu} \to \frac{h_{\mu\nu}}{M_{\rm Pl}}$ and $\pi \to \frac{\pi}{M_{\rm Pl}m^2}$ by the same symbols.

fields in the DL. Moreover, the coupling with external matter fields goes as $\frac{1}{M_{\rm Pl}} h_{\mu\nu} T^{\mu\nu}$ and thus vanishes as $M_{\rm Pl} \to \infty$. The nonrenormalization theorem is thus unaffected by external quantum matter fields.

The DL analysis of the effective action, much like the analogous nonlinear sigma models of non-Abelian spin-1 theories [39], provides an important advantage over the full treatment (see Ref. [32] for a discussion of these matters). In addition to being significantly simpler, the DL explicitly displays the relevant degrees of freedom and their (most important) interactions. In fact, as we will see below, we will be able to draw important conclusions regarding the magnitude of quantum corrections to the full theory based on the DL power counting analysis alone.

Now, whatever the renormalization of the specific coefficients α_i (and more generally, of any relative coefficient between terms of the form $[H^{\ell_1}]\ldots[H^{\ell_n}]$ in the graviton potential) is in the full theory (20), it has to vanish in the DL, since α_i are in one-to-one correspondence with the unrenormalized DL parameters a_i. Let us work in the unitary gauge, in which $H_{\mu\nu} = h_{\mu\nu}$, and for example look at quadratic terms in the graviton potential. We start with an action, the relevant part of which (in terms of the so-far dimensionless $h_{\mu\nu}$) is

$$\mathcal{L} \supset -\frac{1}{4} M_{\rm Pl}^2 m^2 ((1 + c_1) h_{\mu\nu}^2 - (1 + c_2) h^2 + \cdots), \qquad (23)$$

where c_1 and c_2 are generated by quantum corrections after integrating out a small Euclidean shell of momenta and indices are assumed to be contracted with the flat metric.[11] There is of course no guarantee that the two constants $c_{1,2}$ are equal, so they could lead to a detuning of the Fierz-Pauli structure and consequently to a ghost below the cutoff, unless sufficiently suppressed. Returning to the Stückelberg formalism, in terms of the canonically normalized fields

$$h_{\mu\nu} \to \frac{h_{\mu\nu}}{M_{\rm Pl}}, \qquad \pi \to \frac{\pi}{M_{\rm Pl} m^2} \qquad (24)$$

the tree-level part (i.e., the one without c_1 and c_2) of the above Lagrangian would lead to the following scalar-tensor kinetic mixing in the DL (1):

$$\mathcal{L} \supset -h^{\mu\nu}(\partial_\mu \partial_\nu \pi - \eta_{\mu\nu} \Box \pi) + \cdots. \qquad (25)$$

Now, from the DL analysis, we know that this mixing does not get renormalized. What does this imply for the renormalization of the graviton mass and the parameters of the potential in the full theory?

One immediate consequence of such nonrenormalization is that in the decoupling limit, c_1 and c_2 both vanish. To infer the scaling of these parameters with $M_{\rm Pl}$, let us look at the scalar-tensor interactions that arise beyond the DL. They are of the following schematic form[12]:

$$\mathcal{L} = \sum_{n\geq 1, \ell \geq 0} \frac{f_{n,\ell}}{\Lambda_3^{3(\ell-1)}} h (\partial^2 \pi)^\ell \left(\frac{h}{M_{\rm Pl}}\right)^n, \qquad (26)$$

i.e., they are all suppressed by an *integer* power of $M_{\rm Pl}^{-1}$ compared to vertices arising in the DL. Then, judging from the structure of these interactions, generically the nonrenormalization theorem for the classical scalar-tensor action should no longer be expected to hold beyond the DL.

This implies that c_1 and c_2 generated by quantum corrections are of the form

$$c_{1,2} \sim \left(\frac{\Lambda_3}{M_{\rm Pl}}\right)^k, \qquad (27)$$

with k some positive integer $k \geq 1$, if the loops are to be cut off at the Λ_3 scale[13] (the fact that k needs to be an integer relies on the fact that the theory remains analytic beyond the DL). Taking the worst possible case (i.e., $k = 1$), one can directly read off the magnitude of the coefficients $c_{1,2}$,

$$c_{1,2} \lesssim \left(\frac{\Lambda_3}{M_{\rm Pl}}\right) \sim \left(\frac{m}{M_{\rm Pl}}\right)^{2/3}. \qquad (28)$$

In terms of the quantum correction to the graviton mass itself, this implies

$$\delta m^2 \lesssim m^2 \left(\frac{m}{M_{\rm Pl}}\right)^{2/3}, \qquad (29)$$

providing an explicit realization of technical naturalness for massive gravity.

One can extend these arguments to an arbitrary interaction in the effective potential. Consider a generic term of the following schematic form in the unitary gauge involving ℓ factors of the (dimensionless) metric perturbation:

$$\mathcal{L} \supset M_{\rm Pl}^2 m^2 \sqrt{-g} (\bar{c} + c) h^\ell. \qquad (30)$$

Here indices are contracted with the full metric, $\bar{c}$ denotes the "classical" coefficient of the given term obtained from (20), and c is its quantum correction. Our task is to estimate the magnitude of c based on the nonrenormalization of the DL scalar-tensor Lagrangian. Introducing back the Stückelberg fields through the replacement $h_{\mu\nu} \to H_{\mu\nu}$, and recalling the definition of different helicities (21), the quantum correction to the given interaction can be

[11]We could as well assume that the full nonlinear metric contracts indices, since the two cases are indistinguishable at the quadratic level.

[12]We are omitting here the part containing the helicity-1 interactions, which can uniquely be restored due to diffeomorphism invariance of the helicity-2 + helicity-1 system, and the $U(1)$ invariance of the helicity-1 + helicity-0 system.

[13]In this analysis, the graviton mass m is completely absorbed into Λ_3, and nothing special happens at the scale m as far as the strong coupling is concerned.

schematically written in terms of the various canonically normalized helicities as follows:

$$\left(1+\frac{h}{M_{\rm Pl}}+\cdots\right)^{1+\ell}\left(\frac{h}{M_{\rm Pl}}+\frac{\partial A}{M_{\rm Pl}m}+\frac{\partial^2\pi}{\Lambda_3^3}+\frac{\partial A\partial^2\pi}{M_{\rm Pl}m\Lambda_3^3}+\frac{(\partial A)^2}{M_{\rm Pl}^2m^2}+\frac{(\partial^2\pi)^2}{\Lambda_3^6}\right)^{\ell}.$$

The first parentheses denote a schematic product of $\sqrt{-g}$ and ℓ factors of the inverse metric needed to contract the indices. In the classical ghost-free massive gravity, the pure scalar self-interactions are carefully tuned to collect into total derivatives, projecting out the BD ghost. From the DL arguments, we know that quantum corrections do produce such operators, e.g., of the form $(\partial^2\pi)^\ell$, suppressed by the powers of Λ_3. This immediately bounds the magnitude of the coefficient c to be the same as for the $\ell = 2$ case

$$c \lesssim \left(\frac{\Lambda_3}{M_{\rm Pl}}\right) \sim \left(\frac{m}{M_{\rm Pl}}\right)^{2/3}. \tag{31}$$

Indeed, for c given by (31), we get $M_{\rm Pl}^2 m^2 c \sim \Lambda_3^4$ and the upper bound on c is the same as that coming from the mass term renormalization.

V. DISCUSSION AND CONCLUSIONS

We have presented a nonrenormalization theorem in a special class of scalar-tensor theories, relevant for infrared modifications of gravity.

Although these theories feature irrelevant, nontopological interactions of a spin-2 field with a scalar, the couplings corresponding to these interactions do not get renormalized to any order in perturbation theory. This provides an interesting example of nonrenormalization in nonsupersymmetric theories with dimensionful couplings.

The scalar-tensor theories of this kind arise in the DL of the recently proposed models of ghost-free massive gravity. The emergent DL nonrenormalization property, as we have seen, allows one to estimate the magnitude of quantum corrections to various parameters defining the full theory beyond any limit. In particular, one can show that setting an arbitrarily small graviton mass is technically natural. The significance of the DL theory is hard to overestimate: it unambiguously determines all the physical dynamics of the theory at distances $\Lambda_{\rm eff}^{-1} \lesssim r \lesssim m^{-1}$, essentially capturing all physics at astrophysical and cosmological scales (see e.g., Ref. [40] for a detailed discussion of local cosmology as a perturbation over Minkowski space). Technical naturalness, along with a yet stronger nonrenormalization theorem, provides perfect predictivity of the theory, enforcing quantum corrections to play essentially no role at these scales. Moreover, dictated by various physical considerations, one frequently chooses to set certain relations between the two free parameters of the theory, α_3 and α_4. Nonrenormalization in this case means that such relative tunings of parameters, along with any physical consequences that these tunings may have, are not subject to destabilization via quantum corrections.

In this work, we have not made any assumptions regarding the UV completion of the ghost-free massive gravity. The special structure of the graviton potential might lead to a resummation of the infinite number of loop diagrams allowing us to stay in the weakly coupled regime without the need for invoking new dynamics. The properties of the DL, including nonrenormalization, might be pointing towards such a simplification of the S matrix at the apparent strong coupling scale Λ_3, which might become transparent in a certain alternative field basis [18] (for other proposals for UV behavior, see Ref. [41]).

We have not addressed the latter questions here and have presented a standard effective field theory interpretation of massive gravity. Given the effective theory at the scale Λ_3, we have shown that the couplings of the leading (decoupling limit) action are RG invariant as the theory flows towards the infrared. Since it is precisely this part of the action that is responsible for most of the relevant physics at the astrophysical/cosmological scales, one arrives at rather powerful predictivity properties of the theory: (a) all the defining parameters of the theory are technically natural; (b) moreover, any choice of relations among them is also technically natural: if one sets a relation at the scale Λ_3, it remains unchanged at any other lower scale. This leads to the possibility to study the predictions of the classical theory at the scales at hand without ever worrying about quantum corrections.

We expect similar nonrenormalization properties to hold in the recently proposed theory of quasidilaton massive gravity [42].

ACKNOWLEDGMENTS

We would like to thank L. Berezhiani, G. Chkareuli, K. Hinterbichler, R. Rosen and A. J. Tolley for useful discussions. G. G. is supported by NSF Grant No. PHY-0758032. D. P. is supported by the U.S. Department of Energy under Contract No. DOE-FG03-97ER40546 and would like to thank the Center for Cosmology and Particle Physics at New York University for hospitality during the final stages of completion of the paper.

[1] C. de Rham, G. Gabadadze, L. Heisenberg, and D. Pirtskhalava, Phys. Rev. D **83**, 103516 (2011).
[2] G. D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, and A. J. Tolley, Phys. Rev. D **84**, 124046 (2011).
[3] G. 't Hooft, NATO Advanced Study Institutes, Ser. B, Vol. 59 (1980), p. 135.
[4] S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979).
[5] A. Vainshtein, Phys. Lett. **39B**, 393 (1972).
[6] C. Deffayet, G. Dvali, G. Gabadadze, and A. I. Vainshtein, Phys. Rev. D **65**, 044026 (2002).
[7] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, J. High Energy Phys. 10 (2006) 014.
[8] C. Burrage, C. de Rham, L. Heisenberg, and A. J. Tolley, J. Cosmol. Astropart. Phys. 07 (2012) 004.
[9] C. de Rham and G. Gabadadze, Phys. Rev. D **82**, 044020 (2010).
[10] C. de Rham, G. Gabadadze, and A. J. Tolley, Phys. Rev. Lett. **106**, 231101 (2011).
[11] M. A. Luty, M. Porrati, and R. Rattazzi, J. High Energy Phys. 09 (2003) 029.
[12] A. Nicolis and R. Rattazzi, J. High Energy Phys. 06 (2004) 059.
[13] A. Nicolis, R. Rattazzi, and E. Trincherini, Phys. Rev. D **79**, 064036 (2009).
[14] G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **485**, 208 (2000).
[15] C. de Rham, A. J. Tolley, and D. H. Wesley, Phys. Rev. D **87**, 044025 (2013).
[16] L. Berezhiani, G. Chkareuli, and G. Gabadadze, arXiv:1302.0549.
[17] C. Burrage, N. Kaloper, and A. Padilla, arXiv:1211.6001.
[18] G. Gabadadze, K. Hinterbichler, and D. Pirtskhalava, Phys. Rev. D **85**, 125007 (2012).
[19] D. Boulware and S. Deser, Phys. Rev. D **6**, 3368 (1972).
[20] S. Hassan and R. A. Rosen, Phys. Rev. Lett. **108**, 041101 (2012).
[21] S. Hassan and R. A. Rosen, J. High Energy Phys. 04 (2012) 123.
[22] M. Mirbabayi, Phys. Rev. D **86**, 084006 (2012).
[23] C. de Rham, G. Gabadadze, and A. J. Tolley, Phys. Lett. B **711**, 190 (2012).
[24] C. de Rham, G. Gabadadze, and A. J. Tolley, J. High Energy Phys. 11 (2011) 093.
[25] S. Hassan, A. Schmidt-May, and M. von Strauss, Phys. Lett. B **715**, 335 (2012).
[26] K. Hinterbichler and R. A. Rosen, J. High Energy Phys. 07 (2012) 047.
[27] K. Koyama, G. Niz, and G. Tasinato, Phys. Rev. Lett. **107**, 131101 (2011).
[28] K. Koyama, G. Niz, and G. Tasinato, Phys. Rev. D **84**, 064033 (2011).
[29] G. Chkareuli and D. Pirtskhalava, Phys. Lett. B **713**, 99 (2012).
[30] F. Sbisa, G. Niz, K. Koyama, and G. Tasinato, Phys. Rev. D **86**, 024033 (2012).
[31] E. Babichev, C. Deffayet, and R. Ziour, Phys. Rev. D **82**, 104008 (2010).
[32] N. Arkani-Hamed, H. Georgi, and M. D. Schwartz, Ann. Phys. (Amsterdam) **305**, 96 (2003).
[33] M. Park, Classical Quantum Gravity **28**, 105012 (2011).
[34] I. Buchbinder, D. Pereira, and I. Shapiro, Phys. Lett. B **712**, 104 (2012).
[35] W. Siegel, Phys. Rev. D **49**, 4144 (1994).
[36] A. E. Gumrukcuoglu, C. Lin, and S. Mukohyama, J. Cosmol. Astropart. Phys. 11 (2011) 030.
[37] C. de Rham and G. Gabadadze, Phys. Lett. B **693**, 334 (2010).
[38] G. Tasinato, K. Koyama, and G. Niz, arXiv:1210.3627.
[39] A. Vainshtein and I. Khriplovich, Yad. Fiz. **13**, 198 (1971) [Nucl. Phys. **13**, 198 (1971)].
[40] D. Baumann, A. Nicolis, L. Senatore, and M. Zaldarriaga, J. Cosmol. Astropart. Phys. 07 (2012) 051.
[41] G. Dvali, G. F. Giudice, C. Gomez, and A. Kehagias, J. High Energy Phys. 08 (2011) 108; G. Dvali and D. Pirtskhalava, Phys. Lett. B **699**, 78 (2011).
[42] G. D'Amico, G. Gabadadze, L. Hui, and D. Pirtskhalava, arXiv:1206.4253.