

$q\bar{q}$ potential

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(Received 26 October 2012; published 8 April 2013)

We show how to define and compute in a nonperturbative way the potential between q and \bar{q} color sources in the singlet and octet (adjoint) representations of the color group.

DOI: [10.1103/PhysRevD.87.085014](https://doi.org/10.1103/PhysRevD.87.085014)

PACS numbers: 11.15.-q, 11.15.Ha

I. INTRODUCTION

There is some interest in computing the potential between two pointlike sources with the $q\bar{q}$ color quantum numbers in the octet (adjoint) representation of the color group [1–8]. However, conflicting results have been reported in the literature [6,7] and the question seems to still need some clarification.

In this paper we reanalyze the problem discussing, in the pure Yang-Mills (YM) theory, the structure of energy eigenstates in the presence of color sources and derive explicit formulas expressing the singlet and adjoint potentials in terms of the Feynman propagation kernel (sometimes also called the Schrödinger functional) computed in the temporal ($A_0 = 0$) gauge. The theoretical framework we shall use is the formulation of the temporal gauge developed in Refs. [9–11].

In Sec. II we review the general formalism. In Sec. III we illustrate the constraints imposed by global color rotations on the structure of the functional integral. The solution to the problem of characterizing energy eigenstates belonging to different color representations is given in Sec. IV in the formal continuum theory. In Sec. V we give explicit formulas for extracting the singlet and adjoint $q\bar{q}$ potentials from the knowledge of the Feynman kernel. In Sec. VI we illustrate how to reformulate the previous analysis in the lattice language, suitable for numerical simulations. A number of technical issues are discussed in the appendices. In Appendix A we recall some relevant group theoretical formulas. In Appendix B we construct the projectors necessary to single out states with definite global color transformation properties. In Appendix C we recall the formulas necessary to decompose the $[N_c] \otimes [\bar{N}_c]$ tensor product into the sum of singlet and adjoint representations. For completeness we give in Appendix D the expression that the Feynman kernel in the presence of $q\bar{q}$ sources takes in the Coulomb gauge.

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II. GENERAL FORMALISM

The Feynman kernel in the presence of color pointlike sources belonging to arbitrary color representations takes in the temporal gauge the expression [9–11]

$$K(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1, \{u_1\}; T) = \int_{\mathcal{G}_0} \mathcal{D}\mu(h) \mathcal{R}(U_h)_{\{u_2\}\{u_1\}} \tilde{K}(\mathbf{A}_2^{U_h}, \mathbf{A}_1; T), \quad (1)$$

where $\mathcal{D}\mu(h)$ is the Haar invariant measure over the gauge group, \mathcal{G}_0 , of the topologically trivial, time-independent gauge transformation that tends to the identity at spatial infinity. In Eq. (1) we have defined

$$\tilde{K}(\mathbf{A}_2, \mathbf{A}_1; T) = \int_{\mathbf{A}(\mathbf{x}, T_1) = \mathbf{A}_1(\mathbf{x})}^{\mathbf{A}(\mathbf{x}, T_2) = \mathbf{A}_2(\mathbf{x})} \mathcal{D}\mathbf{A} \exp[-S_{\text{YM}}(\mathbf{A}, A_0 = 0)] \quad (2)$$

where $S_{\text{YM}}(\mathbf{A}, A_0 = 0)$ is the YM action taken at $A_0 = 0$ and

$$\mathcal{D}\mathbf{A} = \prod_{\mathbf{x}, T_1 < t < T_2} d\mathbf{A}(\mathbf{x}, t). \quad (3)$$

In Eq. (1) we have set $T = T_2 - T_1$ and made use of the definitions [12]

$$A_k^{U_h}(\mathbf{x}) = U_h^\dagger A_k U_h(\mathbf{x}) + i U_h^\dagger \partial_k U_h(\mathbf{x}), \quad k = 1, 2, 3, \quad (4)$$

$$U_h(\mathbf{x}) = \exp[i\lambda^a h^a(\mathbf{x})] \in \mathcal{G}_0, \quad (5)$$

where the matrices λ^a ($a = 1, 2, \dots, N_c^2 - 1$) are the Hermitian $\text{SU}(N_c)$ generators in the fundamental representation $[N_c]$, normalized to $\text{tr}[\lambda^a \lambda^b] = \delta^{ab}/2$ and we have indicated with $\bar{\lambda}^a = -(\lambda^a)^*$ the generators in the conjugate fundamental representation $[\bar{N}_c]$. Finally, we have introduced the compact notation

$$\mathcal{R}(U_h)_{\{u_2\}\{u_1\}} = \prod_{j=1}^L \left[\exp i\lambda_a^{[j]} h_a(\mathbf{x}_j) \right]_{u_2^i u_1^i} \quad (6)$$

to represent a set of L sources in the color representations $[j]$ localized at the points \mathbf{x}_j . In Eq. (6) we denoted with

$[\lambda_a^{[j]}]_{u_2^j u_1^j}$ the $SU(N_c)$ generators in the representation $[j]$. The indices u_1^j and u_2^j run over values appropriate for the representation $[j]$.

Since $K(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1, \{u_1\}; T)$ is the matrix element of the (Euclidean) time translation operator $\exp(-\mathcal{H}T)$, we can write its spectral decomposition in the form

$$K(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1, \{u_1\}; T) = \sum_k e^{-E_k T} \psi_k(\mathbf{A}_2, \{u_2\}) \psi_k^*(\mathbf{A}_1, \{u_1\}), \quad (7)$$

where the functional $\psi_k(\mathbf{A}; \{u\})$ is the eigenstate of the Hamiltonian, \mathcal{H} , corresponding to the eigenvalue E_k

$$\mathcal{H} \psi_k(\mathbf{A}, \{u\}) = E_k \psi_k(\mathbf{A}, \{u\}). \quad (8)$$

The state functional $\psi_k(\mathbf{A}; \{u\})$ transforms, under time-independent gauge transformations ($U_w \in \mathcal{G}_0$), as

$$\psi_k(\mathbf{A}^{U_w}, \{u\}) = \sum_{\{u'\}} \mathcal{R}^\dagger(U_w)_{\{u\}\{u'\}} \psi_k(\mathbf{A}, \{u'\}). \quad (9)$$

Equation (9) is equivalent to Gauss' law in the Hilbert space sector of states with L external color sources, as specified above. This can be checked by taking the functional derivative of both sides of the equation with respect to $w_a(\mathbf{x})$ at $w_a(\mathbf{x}) = 0$.

It was shown in Ref. [9] that the kernel $K(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1, \{u_1\}; T)$ and the eigenfunctionals $\psi_k(\mathbf{A}; \{u\})$ are gauge independent in the sense that, had we chosen to quantize the theory in a different canonical gauge, say $F(\mathbf{A}) = 0$ [typically $F(\mathbf{A}) = \nabla \cdot \mathbf{A}$], the corresponding kernel and the related eigenfunctionals in the F gauge would coincide with those in the temporal one, when evaluated on a gauge field \mathbf{A} satisfying $F(\mathbf{A}) = 0$.

Since the quantity $\sum_{\{u\}} \psi^*(\mathbf{A}, \{u\}) \phi(\mathbf{A}, \{u\})$ is invariant under gauge transformations [see Eq. (4)], in order to define a scalar product we must make use of the Faddeev-Popov (FP) procedure. This is done by choosing any spatial gauge-fixing condition, say $F(\mathbf{A}) = 0$, and accordingly defining the scalar product by means of the formula [13]

$$(\psi, \phi) \equiv \int \mathcal{D}\mu_F(\mathbf{A}) \sum_{\{u\}} \psi^*(\mathbf{A}, \{u\}) \phi(\mathbf{A}, \{u\}), \quad (10)$$

where

$$\mathcal{D}\mu_F(\mathbf{A}) = \prod_{\mathbf{x}} \Delta_F(\mathbf{A}) \delta[F(\mathbf{A})] d\mathbf{A}(\mathbf{x}) \quad (11)$$

is the gauge field integration measure. In Eq. (11) $\Delta_F(\mathbf{A})$ is the FP determinant [14] defined by

$$\Delta_F(\mathbf{A}) \int_{\mathcal{G}_0} \mathcal{D}\mu(h) \delta[F(\mathbf{A}^{U_h})] = 1. \quad (12)$$

We recall that the value of the scalar product in Eq. (10) is independent on the gauge function $F(\mathbf{A})$ chosen to define it.

Denoting with d_k the degeneracy of the energy level E_k , we can compute the complete trace of the Feynman propagation kernel (7) obtaining

$$\int \mathcal{D}\mu_F(\mathbf{A}) \sum_{\{u\}} K(\mathbf{A}, \{u\}; \mathbf{A}, \{u\}; T) = \sum_k d_k e^{-E_k T}. \quad (13)$$

The gauge invariance of the lhs of Eq. (13) implies the gauge invariance of the quantities d_k and E_k .

III. GLOBAL COLOR TRANSFORMATIONS

In this section we discuss how global color transformations are implemented in the temporal gauge when external sources are present.

Let us denote by V a global color rotation, i.e., a constant $SU(N_c)$ transformation. The global color invariance of $S_{\text{YM}}(\mathbf{A}, A_0 = 0)$ implies

$$\tilde{K}(\mathbf{A}_2^V, \mathbf{A}_1^V; T) = \tilde{K}(\mathbf{A}_2, \mathbf{A}_1; T). \quad (14)$$

From the definition (1) we then have

$$\begin{aligned} K(\mathbf{A}_2^V, \{u_2\}; \mathbf{A}_1^V, \{u_1\}; T) &= \int_{\mathcal{G}_0} \mathcal{D}\mu(h) \mathcal{R}(U_h)_{\{u_2\}\{u_1\}} \tilde{K}(\mathbf{A}_2^{VU_h}, \mathbf{A}_1^V; T) \\ &= \int_{\mathcal{G}_0} \mathcal{D}\mu(h) \mathcal{R}(U_h)_{\{u_2\}\{u_1\}} \tilde{K}(\mathbf{A}_2^{VU_h V^\dagger}, \mathbf{A}_1; T) \\ &= \int_{\mathcal{G}_0} \mathcal{D}\mu(h) \mathcal{R}(V^\dagger U_h V)_{\{u_2\}\{u_1\}} \tilde{K}(\mathbf{A}_2^{U_h}, \mathbf{A}_1; T) \\ &= \mathcal{R}(V^\dagger)_{\{u_2\}\{u_2'\}} \int_{\mathcal{G}_0} \mathcal{D}\mu(h) \mathcal{R}(U_h)_{\{u_2'\}\{u_1'\}} \tilde{K}(\mathbf{A}_2^{U_h}, \mathbf{A}_1; T) \\ &\quad \times \mathcal{R}(V)_{\{u_1'\}\{u_1\}} \\ &= \mathcal{R}(V^\dagger)_{\{u_2\}\{u_2'\}} K(\mathbf{A}_2, \{u_2'\}; \mathbf{A}_1, \{u_1'\}; T) \mathcal{R}(V)_{\{u_1'\}\{u_1\}}. \end{aligned} \quad (15)$$

Notice that the functional change of variables $VU_h V^\dagger \rightarrow U_h$ in the third equality of Eq. (15) is allowed by the fact that $VU_h V^\dagger$ is a (topologically trivial) gauge transformation that tends to unit at spatial infinity and hence belongs to \mathcal{G}_0 .

Equation (15) can be rewritten in the form

$$\begin{aligned} \mathcal{R}(V)_{\{u_2\}\{u_2'\}} K(\mathbf{A}_2^V, \{u_2'\}; \mathbf{A}_1, \{u_1\}; T) \\ = K(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1^{V^\dagger}, \{u_1'\}; T) \mathcal{R}(V)_{\{u_1'\}\{u_1\}}. \end{aligned} \quad (16)$$

Equation (16) allows us to define the operator $\mathcal{U}(V)$, implementing global color transformations, which acts on wave functionals as

$$[\mathcal{U}(V)\psi](\mathbf{A}, \{u\}) = \mathcal{R}(V)_{\{u\}\{u'\}} \psi(\mathbf{A}^V, \{u'\}). \quad (17)$$

$\mathcal{U}(V)$ provides a unitary representation of the $SU(N_c)$ group in the Hilbert space of state functionals and commutes with the kernel K .

The unitarity of $\mathcal{U}(V)$ follows from the chain

$$\begin{aligned} & (\mathcal{U}^\dagger(V)\psi, \phi) \\ &= (\psi, \mathcal{U}(V)\phi) \\ &= \int \mathcal{D}\mu_F(\mathbf{A}) \psi^*(\mathbf{A}, \{u'\}) \mathcal{R}(V)_{\{u'\}\{u\}} \phi(\mathbf{A}^V, \{u\}) \\ &= \int \mathcal{D}\mu_F(\mathbf{A}) [\mathcal{R}^\dagger(V)_{\{u\}\{u'\}} \psi(\mathbf{A}^{V^\dagger}, \{u'\})]^* \phi(\mathbf{A}, \{u\}). \end{aligned} \quad (18)$$

In fact, Eq. (18) shows that

$$[\mathcal{U}^\dagger(V)\psi](\mathbf{A}, \{u\}) = \mathcal{R}^\dagger(V)_{\{u\}\{u'\}} \psi(\mathbf{A}^{V^\dagger}, \{u'\}), \quad (19)$$

which, together with Eq. (17), gives $\mathcal{U}(V^\dagger) = \mathcal{U}^\dagger(V)$ entailing the unitarity of $\mathcal{U}(V)$.

To prove that the Feynman kernel commutes with \mathcal{U} we take an arbitrary wave functional, $\psi(\mathbf{A}, \{u\})$, in the appropriate source sector [see Eq. (9)], and consider the chain of equalities

$$\begin{aligned} & \mathcal{U}(V) \int \mathcal{D}\mu_F(\mathbf{A}_1) K(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1, \{u_1\}; T) \psi(\mathbf{A}_1, \{u_1\}) \\ &= \mathcal{R}(V)_{\{u_2\}\{u_2'\}} \int \mathcal{D}\mu_F(\mathbf{A}_1) K(\mathbf{A}_2^V, \{u_2'\}; \mathbf{A}_1, \{u_1\}; T) \\ & \quad \times \psi(\mathbf{A}_1, \{u_1\}) \\ &= \int \mathcal{D}\mu_F(\mathbf{A}_1) K(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1^{V^\dagger}, \{u_1'\}; T) \mathcal{R}(V)_{\{u_1'\}\{u_1\}} \\ & \quad \times \psi(\mathbf{A}_1, \{u_1\}) \\ &= \int \mathcal{D}\mu_F(\mathbf{A}_1) K(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1, \{u_1'\}; T) \mathcal{R}(V)_{\{u_1'\}\{u_1\}} \\ & \quad \times \psi(\mathbf{A}_1^V, \{u_1\}) \\ &= \int \mathcal{D}\mu_F(\mathbf{A}_1) K(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1, \{u_1\}; T) [\mathcal{U}(V)\psi](\mathbf{A}_1, \{u_1\}), \end{aligned} \quad (20)$$

where the third equality is a consequence of the invariance of the measure (11) under global color rotations. Owing to the arbitrariness of $\psi(\mathbf{A}, \{u\})$, Eq. (20) implies that $\mathcal{U}(V)$ commutes with the Feynman propagation kernel.

Setting $\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{A}$ in Eq. (15) and integrating over \mathbf{A} with the measure (11), we get the key formula

$$\begin{aligned} K_{\{u\}\{u'\}}(T) &\equiv \int \mathcal{D}\mu_F(\mathbf{A}) K(\mathbf{A}, \{u\}; \mathbf{A}, \{u'\}; T) \\ &= \int \mathcal{D}\mu_F(\mathbf{A}) K(\mathbf{A}^V, \{u\}; \mathbf{A}^V, \{u'\}; T) \\ &= \mathcal{R}(V^\dagger)_{\{u\}\{v\}} K_{\{v\}\{v'\}}(T) \mathcal{R}(V)_{\{v'\}\{u'\}}. \end{aligned} \quad (21)$$

Equation (21) tells us that $K(T)$ commutes with $\mathcal{R}(V)$, so that by Schur's lemma it is a multiple of the identity matrix within any irreducible color source representation. If we are in a sector in which more than one color source is present, $\mathcal{R}(V)$ can be decomposed into the direct sum of

irreducible representations of $SU(N_c)$, and $K(T)$ itself is a direct sum of multiples of the unit matrix, one for each irreducible component. We will make use of this result below.

We end this section with some considerations on what happens if the gauge integration over \mathcal{G}_0 is extended to the set $\bar{\mathcal{G}}_0$ of transformations that also includes global color rotations. In this case the color averaged kernel

$$\begin{aligned} & \bar{K}(\mathbf{A}_2, \{u_2\}; \mathbf{A}_1, \{u_1\}; T) \\ &= \int_{\bar{\mathcal{G}}_0} \mathcal{D}\mu(h) \mathcal{R}(U_h)_{\{u_2\}\{u_1\}} \tilde{K}(\mathbf{A}_2^{U_h}, \mathbf{A}_1; T) \end{aligned} \quad (22)$$

satisfies the property

$$\begin{aligned} & \bar{K}(\mathbf{A}_2^V, \{u_2\}; \mathbf{A}_1, \{u_1\}; T) \\ &= \mathcal{R}(V^\dagger)_{\{u_2\}\{u_2'\}} \bar{K}(\mathbf{A}_2, \{u_2'\}; \mathbf{A}_1, \{u_1\}; T), \end{aligned} \quad (23)$$

that implies for the averaged state functionals, $\bar{\psi}(\mathbf{A}, \{u\})$, appearing in its spectral decomposition, the invariance property

$$[\mathcal{U}(V)\bar{\psi}](\mathbf{A}, \{u\}) = \mathcal{R}(V)_{\{u\}\{u'\}} \bar{\psi}(\mathbf{A}^V, \{u'\}) = \bar{\psi}(\mathbf{A}, \{u\}). \quad (24)$$

Equation (24) means that, for the averaged state functionals, a global color rotation on the gauge field amounts to a rotation $\mathcal{R}(V)$ acting only on source indices, or in other words that any such state functional is a global color singlet.

IV. THE $q\bar{q}$ SOURCE SYSTEM

In this section we specialize the formulation of Secs. II and III to the particular case of two sources with the q and \bar{q} color quantum numbers and show explicitly how to define and extract the $q\bar{q}$ potential in the singlet and adjoint representations.

The Feynman kernel in this particular source sector has the form [9–11]

$$\begin{aligned} & K(\mathbf{A}_2, s_2, r_2; \mathbf{A}_1, s_1, r_1; T) \\ &= \int_{\mathcal{G}_0} \mathcal{D}\mu(h) [\exp[i\lambda^a h^a(\mathbf{x}_q)]]_{s_2 s_1} \\ & \quad \times [\exp[i\bar{\lambda}^a h^a(\mathbf{x}_{\bar{q}})]]_{r_2 r_1} \tilde{K}(\mathbf{A}_2^{U_h}, \mathbf{A}_1; T) \\ &= \int_{\mathcal{G}_0} \mathcal{D}\mu(h) [\exp[i\lambda^a h^a(\mathbf{x}_q)]]_{s_2 s_1} \\ & \quad \times [\exp[-i\lambda^a h^a(\mathbf{x}_{\bar{q}})]]_{r_1 r_2} \tilde{K}(\mathbf{A}_2^{U_h}, \mathbf{A}_1; T). \end{aligned} \quad (25)$$

In order to pick up the energies of the lowest lying states one has to study the large T behavior of the expression (25) while, at the same time, projecting out the desired color structure. To this end one needs to classify the energy eigenfunctionals in terms of their global color transformation properties.

A. Classification of the energy eigenstates

Since, according to Eq. (20), the Feynman kernel commutes with $\mathcal{U}(V)$, the energy eigenstates are classified in terms of the irreducible representations of the color group $SU(N_c)$.

Every energy eigenstate in the $q\bar{q}$ sector is described by a wave functional which can be parametrized as

$$\begin{aligned} \psi(\mathbf{A}; sr) &= [\phi(\mathbf{A})I + \lambda^a \phi_a(\mathbf{A})]_{sr} \equiv \phi(\mathbf{A})I + \lambda^a \phi_a(\mathbf{A}) \\ &\equiv \psi(\mathbf{A}), \end{aligned} \quad (26)$$

in terms of the $1 + (N_c^2 - 1)$ functionals $\phi(\mathbf{A})$ and $\phi_a(\mathbf{A})$. The parametrization given in Eq. (26) holds for any one of the components of an irreducible multiplet. The totality of the members of the multiplet can be reached through global color transformations, according to the formula

$$\begin{aligned} \mathcal{U}(V)\psi(\mathbf{A}) &\equiv \psi^V(\mathbf{A}) = V\psi(\mathbf{A}^V)V^\dagger \\ &= \phi(\mathbf{A}^V)I + V\lambda^a V^\dagger \phi_a(\mathbf{A}^V). \end{aligned} \quad (27)$$

As it can be seen from Eq. (27), the global color transformation of a state is composed of two different contributions:

- (i) the ‘‘color-spin’’ contribution coming from the action of the matrix V on the source indices
- (ii) and the ‘‘orbital’’ contribution coming from the transformation $\mathbf{A} \rightarrow \mathbf{A}^V$.

As we said, any energy eigenstate must belong to an irreducible representation of the color group, then $\psi(\mathbf{A})$ must span a unique irreducible representation for *any* value of \mathbf{A} , when transformed as in Eq. (27). Notice, however, that for $\mathbf{A} = \mathbf{0}$ the orbital contribution is suppressed and Eq. (27) becomes

$$\psi^V(\mathbf{0}) = V\psi(\mathbf{0})V^\dagger = \phi(\mathbf{0})I + V\lambda^a V^\dagger \phi_a(\mathbf{0}). \quad (28)$$

Equation (28) implies that the two terms in the rhs cannot be simultaneously different from zero; otherwise $\psi^V(\mathbf{0})$ would belong to the reducible $I \oplus [N_c^2 - 1]$ representation. This means that we have the following three alternatives:

- (1) $\phi_a(\mathbf{0}) = 0$ and $\phi(\mathbf{0}) \neq 0$
- (2) $\phi(\mathbf{0}) = 0$ and $\phi_a(\mathbf{0}) \neq 0$ (for some a)
- (3) $\phi(\mathbf{0}) = \phi_a(\mathbf{0}) = 0$

in correspondence to three different types of irreducible representations that we are now going to discuss.

(1)–(2) When $\phi(\mathbf{0}) \neq 0$ [or $\phi_a(\mathbf{0}) \neq 0$], $\psi(\mathbf{A})$ must belong to the color-spin singlet (or adjoint) representation, because the representation it belongs to cannot change discontinuously under a smooth variation of the ‘‘parameters’’ \mathbf{A} .

(3) For states such that $\phi(\mathbf{0}) = \phi_a(\mathbf{0}) = 0$ we can have more general assignments of global color quantum numbers.

Recalling that the adjoint representation $R^{[\text{Ad}]}(V)$ is defined by

$$V\lambda^a V^\dagger = R_{ca}^{[\text{Ad}]}(V)\lambda^c, \quad (29)$$

with $R^{[\text{Ad}]}(V)$ real, Eq. (27) can be rewritten in the form

$$\psi^V(\mathbf{A}) = \phi(\mathbf{A}^V)I + R_{ca}^{[\text{Ad}]}(V)\phi_a(\mathbf{A}^V)\lambda^c. \quad (30)$$

The two \mathbf{A} functionals in the rhs of Eq. (30) must transform under the color group and therefore will display ‘‘magnetic’’ quantum numbers. Thus we will denote them as $\phi_m(\mathbf{A})$, $\phi_{ak}(\mathbf{A})$ respectively, with transformation properties under $\mathbf{A} \rightarrow \mathbf{A}^V$ given by

$$\phi_m(\mathbf{A}^V) = R_{mm'}^{[\alpha]}(V)\phi_{m'}(\mathbf{A}), \quad (31)$$

$$\phi_{ak}(\mathbf{A}^V) = R_{kk'}^{[\beta]}(V)\phi_{ak'}(\mathbf{A}), \quad (32)$$

where $R^{[\alpha]}$ and $R^{[\beta]}$ are $SU(N_c)$ representations. Using Eqs. (31) and (32), Eq. (30) becomes

$$\psi^V(\mathbf{A}) = R_{mm'}^{[\alpha]}(V)\phi_{m'}(\mathbf{A})I + R_{ca}^{[\text{Ad}]}(V)R_{kk'}^{[\beta]}(V)\phi_{ak'}(\mathbf{A})\lambda^c, \quad (33)$$

showing explicitly that the ϕ_m 's transform according to representation $[\alpha]$, while the ϕ_{ak} 's belong to the direct product $[\beta] \otimes [N_c^2 - 1]$.

Due to the trace orthogonality of the identity and the λ matrices, the two terms in Eq. (33) must separately belong to irreducible representations.

We conjecture that the two terms in Eq. (33) correspond to two different types of energy eigenfunctionals, although we cannot exclude that, when the two irreducible representations are equivalent, both terms may simultaneously contribute to a given eigenfunctional.

It follows from this analysis that the $[\alpha]$ representation must be irreducible and that, owing to the color invariance of the dynamics, the ‘‘wave function’’ ϕ_{ak} will be such to single out an irreducible representation from the tensor product $[\beta] \otimes [N_c^2 - 1]$.

In summary, depending on the structure of the source indices and the color representation to which the gluon wave function belongs, we can classify the energy eigenstates in four classes.

- (i) Color-spin singlet, orbital singlet states

$$\psi_{[S]}^{[S]}(\mathbf{A}) = \phi(\mathbf{A})I \quad \phi(\mathbf{A}^V) = \phi(\mathbf{A}), \quad (34)$$

with $\phi(\mathbf{0}) \neq 0$.

- (ii) Color-spin adjoint, orbital singlet states

$$\psi_{[\text{Ad}]}^{[S]}(\mathbf{A}) = \lambda^a \phi_a(\mathbf{A}) \quad \phi_a(\mathbf{A}^V) = \phi_a(\mathbf{A}), \quad (35)$$

with $\phi_a(\mathbf{0}) \neq 0$ for some values of a .

- (iii) Color-spin singlet, orbital $[\alpha]$ states with singlet source indices, and belonging to the irreducible color representation $[\alpha]$ [see Eq. (31)]

$$\begin{aligned}\psi_{m[S]}^{[\alpha]}(\mathbf{A}) &= \phi^{[\alpha]}_m(\mathbf{A})I \\ \phi^{[\alpha]}_m(\mathbf{A}^V) &= R_{mm'}^{[\alpha]}(V)\phi^{[\alpha]}_{m'}(\mathbf{A}),\end{aligned}\quad (36)$$

with $\phi^{[\alpha]}_m(\mathbf{0}) = 0$.

- (iv) Color-spin adjoint, orbital $[\beta]$ states giving rise to an irreducible color representation $[\alpha] \in [\beta] \otimes [N_c^2 - 1]$ [see Eq. (32)]

$$\begin{aligned}\psi_{m[\text{Ad}]}^{[\alpha]}(\mathbf{A}) &= \lambda^a \phi_{ak}(\mathbf{A}) \\ \phi_{ak}(\mathbf{A}^V) &= R_{kk'}^{[\beta]}(V)\phi_{ak'}(\mathbf{A}),\end{aligned}\quad (37)$$

with $\phi_{ak}(\mathbf{0}) = 0$. We repeat that in Eq. (37) $\phi_{ak}(\mathbf{A})$ is a tensor in the $[\beta] \otimes [N_c^2 - 1]$ space living in the $[\alpha]$ -invariant subspace with values of a 's and k 's constrained so as to yield the desired value of the index m .

We remark that, since $\mathbf{A}^V = \mathbf{A}$ for $V \in Z_{N_c}$, where Z_{N_c} is the center of $SU(N_c)$, the representation $[\alpha]$ in Eqs. (36) and (37) is actually bound to be a representation of $SU(N_c)/Z_{N_c}$.

The above classification is confirmed by perturbation theory [10,15,16]. States like those in Eqs. (34) and (35) correspond, at zero order in the coupling constant g , to wave functionals which do not contain any gluon component, and are therefore nonvanishing at $\mathbf{A} = \mathbf{0}$. Excited states consist of states with gluons added to external sources. The presence of gluons allows the energy levels to reach any zero N_c -ality (triality for $N_c = 3$) representation. States of this kind, however, contain some power of the gluon fields, so that their wave functional vanishes at $\mathbf{A} = \mathbf{0}$. As a working hypothesis, to be checked by non-perturbative numerical simulations, we conjecture that similar structures also describe the nonperturbative dynamics of external source sectors.

B. The structure of the \mathbf{A} -traced propagation kernel

In this section we determine the way in which the four classes of color multiplets, discussed in Sec. IV A, contribute to the Feynman propagation kernel traced over the field boundary values

$$K_{s_2 r_2; s_1 r_1}(T) \equiv \int \mathcal{D}\mu_F(\mathbf{A}) K(\mathbf{A}, s_2, r_2; \mathbf{A}, s_1, r_1; T). \quad (38)$$

We remark that the gauge fixing in the \mathbf{A} integration in Eq. (38) is needed in order to make all the matrix elements of $K_{s_2 r_2; s_1 r_1}(T)$ finite. In fact, although $K(\mathbf{A}, s_2, r_2; \mathbf{A}, s_1, r_1; T)$ is invariant only under gauge transformations which are equal to the identity at the location of the color sources, the color-traced kernel $\sum_{sr} K(\mathbf{A}, s, r; \mathbf{A}, s, r; T)$ is invariant under *all* the time-independent gauge transformations belonging to \mathcal{G}_0 , and therefore $\sum_{sr} K_{sr; sr}(T)$ would be infinite in the absence of gauge fixing.

The contribution of the states given in Eqs. (34)–(37) to the partial trace (38) can be computed from the expressions of the partial traces of the corresponding projectors given in Appendix B. We find

- (i) Color-spin singlet, orbital singlet states of the form (34)

$$K_{s_2 r_2; s_1 r_1}(T) \Big|_{[S]}^{[S]} = \frac{\delta_{s_2 r_2} \delta_{r_1 s_1}}{N_c} e^{-E^{[S]}T}. \quad (39)$$

- (ii) Color-spin adjoint, orbital singlet states of the form (35)

$$K_{s_2 r_2; s_1 r_1}(T) \Big|_{[\text{Ad}]}^{[S]} = 2 \sum_c \lambda^c_{s_2 r_2} \lambda^c_{r_1 s_1} e^{-E^{[\text{Ad}]T}. \quad (40)$$

- (iii) Color-spin singlet, orbital $[\alpha]$ states of the form (36)

$$K_{s_2 r_2; s_1 r_1}(T) \Big|_{[S]}^{[\alpha]} = \frac{D_{[\alpha]}}{N_c} \delta_{s_2 r_2} \delta_{r_1 s_1} e^{-E^{[\alpha]}T}. \quad (41)$$

- (iv) Color-spin adjoint, orbital $[\beta]$ states of the form (37) with $[\alpha] \in [\beta] \otimes [N_c^2 - 1]$

$$K_{s_2 r_2; s_1 r_1}(T) \Big|_{[\text{Ad}]}^{[\alpha]} = 2 \frac{D_{[\alpha]}}{N_c^2 - 1} \sum_c \lambda^c_{s_2 r_2} \lambda^c_{r_1 s_1} e^{-E^{[\alpha]}T}. \quad (42)$$

C. The structure of the $K(\mathbf{0}, s_2, r_2; \mathbf{0}, s_1, r_1; T)$ kernel

The previous considerations immediately imply that only the states in Eqs. (34) and (35) contribute to $K(\mathbf{0}, s_2, r_2; \mathbf{0}, s_1, r_1; T)$, yielding terms with a tensor structure proportional to that of Eqs. (39) and (40), respectively. In formulas we get [see Eqs. (B2) and (B5)]

$$\begin{aligned}K(\mathbf{0}, s_2, r_2; \mathbf{0}, s_1, r_1; T) &= |\phi(\mathbf{0})|^2 \frac{\delta_{s_2 r_2} \delta_{r_1 s_1}}{N_c} e^{-E^{[S]}T} \\ &+ \sum_a |\phi_a(\mathbf{0})|^2 \sum_b \lambda^b_{s_2 r_2} \lambda^b_{r_1 s_1} e^{-E^{[\text{Ad}]T} + \dots.\end{aligned}\quad (43)$$

From the structure of states summarized in equations from (34) to (37) and the results in perturbation theory [10,15,16], we conjecture that only the first two terms displayed in Eq. (43) are actually present. This statement can be tested in lattice numerical simulations.

If we were only interested in the computation of the $q\bar{q}$ potential in the vacuum, the relevant quantity to study would be $K(\mathbf{0}, s_2, r_2; \mathbf{0}, s_1, r_1; T)$. Instead, in order to explore more complicated situations such as, for instance, the $q\bar{q}$ potential at finite temperature, the relevant quantity to consider is $K_{s_2 r_2; s_1 r_1}(T)$.

V. EXTRACTING SINGLET AND ADJOINT POTENTIAL IN THE CONTINUUM

To extract the lowest energy eigenvalues of the singlet and adjoint channels we have to project Eq. (38) [or Eq. (43)] over the $\delta_{r_2 s_2} \delta_{s_1 r_1} / N_c$ and $2 \sum_a \lambda_{r_2 s_2}^a \lambda_{s_1 r_1}^a$ tensor projector (see Appendix C), respectively.

Referring to the partially traced kernel (38) and using Eqs. (39) and (41), we obtain at large times for the color-spin singlet channels [see Eqs. (34) and (36)]

$$\sum_{s_2 r_2 s_1 r_1} \frac{1}{N_c} \delta_{r_2 s_2} \delta_{s_1 r_1} K_{s_2 r_2; s_1 r_1}(T) \xrightarrow{T \rightarrow \infty} e^{-E^{[S]}T} + \dots + D_{[\alpha]} e^{-E^{[\alpha]}T} + \dots, \quad (44)$$

where the dots stand for exponentially suppressed terms and, as discussed before, $D_{[\alpha]}$ is the dimension of the color representation $[\alpha]$ to which the energy eigenstate belongs.

Similarly, using Eqs. (40) and (42), for the color-spin adjoint channels [see Eqs. (35) and (37)] we find

$$\sum_{s_2 r_2 s_1 r_1} 2 \sum_a \lambda_{r_2 s_2}^a \lambda_{s_1 r_1}^a K_{s_2 r_2; s_1 r_1}(T) \xrightarrow{T \rightarrow \infty} (N_c^2 - 1) e^{-E^{[Ad]}T} + \dots + D_{[\alpha']} e^{-E^{[\alpha']}T} + \dots. \quad (45)$$

In perturbation theory the ‘‘singlet and adjoint $q\bar{q}$ potentials’’ defined in Eqs. (44) and (45) contain divergent self-energy contributions, independent of the relative position of the sources. In perturbation theory these contributions can be renormalized away, dividing the Feynman kernel (38) by the product of the two traces (characters) of the kernels with the insertion of a single q or \bar{q} source. This can be done by computing, at large T , the quantity

$$\Gamma_{s_2 r_2; s_1 r_1}(T) = \frac{K_{s_2 r_2; s_1 r_1}(T)}{K_q(T) K_{\bar{q}}(T)}, \quad (46)$$

where

$$K_q(T) = \int_{\mathcal{G}_0} \mathcal{D}\mu(h) \text{tr}[e^{i\lambda^a h^a(\mathbf{x}_q)}] \int \mathcal{D}\mu_F(\mathbf{A}) \tilde{K}(\mathbf{A}^{U_h}, \mathbf{A}; T), \quad (47)$$

$$K_{\bar{q}}(T) = \int_{\mathcal{G}_0} \mathcal{D}\mu(h) \text{tr}[e^{-i\lambda^a h^a(\mathbf{x}_{\bar{q}})}] \int \mathcal{D}\mu_F(\mathbf{A}) \tilde{K}(\mathbf{A}^{U_h}, \mathbf{A}; T). \quad (48)$$

We stress again that, had we included in the gauge average also the integration over global color rotations, the states in the spectral decomposition of the averaged Feynman kernel corresponding to color nonsinglet states would be missing [see Eq. (24)]. Consequently, in the large T limit, we might not be any more in position to reach the desired lowest energy eigenvalues. Another unwanted consequence is that color averaging makes the Feynman kernel contribution of a state vanish when the tensor product of source representations does not contain the singlet. This is

precisely what would happen to the factors in the denominator of Eq. (46).

In the nonperturbative regime, where color is supposed to be confined, the quantities defined in Eqs. (47) and (48) could be zero anyway. So the need and mode of this particular renormalization should be the object of numerical investigations.

VI. EXTRACTING THE $q\bar{q}$ POTENTIAL FROM LATTICE SIMULATIONS

In this section we discuss how to compute, in lattice simulations, the singlet and adjoint $q\bar{q}$ potentials. Although it is not difficult to translate in lattice language the temporal gauge-fixing procedure presented in the previous sections and provide a discretized version of Eqs. (38) or (43), it turns out that these formulas are not well suited to the structure of practical numerical simulations.

The reason is that it is not easy to implement on the lattice the condition $U_h \in \mathcal{G}_0$ [i.e., $U_h(\mathbf{x}) \xrightarrow{|\mathbf{x}| \rightarrow \infty} I$] on the boundary gauge integration in Eq. (1). The limitation $U_h \in \mathcal{G}_0$ is, however, absolutely crucial, as we have discussed, in order to avoid the cancellation from the color averaged kernel of all the global color nonsinglet states (if not dynamically confined).

The way out of this difficulty is to isolate in the Feynman propagation kernel on the lattice the contribution of states belonging to given representations, averaging with the $SU(N_c)$ characters. This idea is based on the formula

$$\begin{aligned} & \sum_{sr} \int \mathcal{D}\mu_F(\mathbf{A}) K(\mathbf{A}^V, s, r; \mathbf{A}, s, r; T) \\ &= \sum_{sr} \int \mathcal{D}\mu_F(\mathbf{A}) \sum_{k,m} \psi_m^{[\alpha_k]}(\mathbf{A}^V, s, r) (\psi_m^{[\alpha_k]}(\mathbf{A}, s, r))^* \\ & \quad \times e^{-E^{[\alpha_k]}T} \\ &= \sum_{sr} \int \mathcal{D}\mu_F(\mathbf{A}) \sum_{k,m,m'} R_{mm'}^{[\alpha_k]}(V) \psi_{m'}^{[\alpha_k]}(\mathbf{A}, s, r) \\ & \quad \times (\psi_m^{[\alpha_k]}(\mathbf{A}, s, r))^* e^{-E^{[\alpha_k]}T} \\ &= \sum_k \text{tr}[R^{[\alpha_k]}(V)] e^{-E^{[\alpha_k]}T} \\ &\equiv \sum_k \chi^{[\alpha_k]}(V) e^{-E^{[\alpha_k]}T}, \end{aligned} \quad (49)$$

which shows that performing a global color rotation on one of the boundary gauge fields and taking the full trace of the kernel amounts to replacing the multiplicity factor appearing in Eq. (13) with the character of the irreducible representation to which each energy eigenfunctional belongs.

A. Temporal lattice gauge fixing

We start by discussing how the temporal gauge-fixing procedure can be implemented on the lattice. Link variables are generated by some Monte Carlo algorithm with

weight given by (minus) the exponent of the (gauge unfixed) plaquette action, assuming periodic boundary conditions in the four dimensions. Lattice points are denoted by $\{n_0, \mathbf{n}\}$, with $n_0 = 0, 1, \dots, N_T$, $n_k = 1, \dots, N_L$, $k = 1, 2, 3$.

On each configuration the temporal gauge fixing is carried out by subjecting the gauge links to the gauge transformation

$$\Omega(\mathbf{n}, t) = U_0^\dagger(\mathbf{n}, t-1; \mathbf{n}, t) \dots U_0^\dagger(\mathbf{n}, 1; \mathbf{n}, 2) U_0^\dagger(\mathbf{n}, 0; \mathbf{n}, 1),$$

$$t = 1, \dots, N_T, \quad (50)$$

$$\Omega(\mathbf{n}, 0) = I, \quad (51)$$

which sets $U_0(\mathbf{n}, 0; \mathbf{n}, t) = I$ at $t = 1, \dots, N_T - 1$. Under the gauge transformation (50) the spatial links transform as

$$U_k(\mathbf{n}, t; \mathbf{n} + \hat{\mathbf{k}}, t) \rightarrow U'_k(\mathbf{n}, t; \mathbf{n} + \hat{\mathbf{k}}, t) = \Omega^\dagger(\mathbf{n}, t) U_k(\mathbf{n}, t; \mathbf{n} + \hat{\mathbf{k}}, t) \Omega(\mathbf{n} + \hat{\mathbf{k}}, t), \quad t = 1, \dots, N_T. \quad (52)$$

Notice that, owing to Eq. (51), the spatial links at $t = 0$ are untouched.

The time ordered product of the temporal links (open Polyakov lines) that represent [in the unfixed theory, see also Eq. (D1) of Appendix D] the external q and \bar{q} color sources at \mathbf{n}_q and $\mathbf{n}_{\bar{q}}$ become

$$\prod_{t=1}^{N_T} U_0(\mathbf{n}_q, t-1; \mathbf{n}_q, t) = \prod_{t=1}^{N_T} \Omega(\mathbf{n}_q, t-1) U'_0(\mathbf{n}_q, t-1; \mathbf{n}_q, t) \Omega^\dagger(\mathbf{n}_q, t) = \Omega^\dagger(\mathbf{n}_q, N_T) \equiv \Omega^\dagger(\mathbf{n}_q), \quad (53)$$

$$\left[\prod_{t=1}^{N_T} U_0(\mathbf{n}_{\bar{q}}, t-1; \mathbf{n}_{\bar{q}}, t) \right]^\dagger = \left[\prod_{t=1}^{N_T} \Omega(\mathbf{n}_{\bar{q}}, t-1) U'_0(\mathbf{n}_{\bar{q}}, t-1; \mathbf{n}_{\bar{q}}, t) \Omega^\dagger(\mathbf{n}_{\bar{q}}, t) \right]^\dagger = \Omega(\mathbf{n}_{\bar{q}}, N_T) \equiv \Omega(\mathbf{n}_{\bar{q}}). \quad (54)$$

Configuration averages performed at this stage will inevitably also include an average over global color rotations. In the next subsection we describe a method based on character projections which exploits this color average to select the contribution to the Feynman propagation kernel coming from states belonging to a given global color representation.

B. Character projection

As discussed above, we can easily compute on the lattice the color averaged kernel [see Eq. (25)], which in continuum notation reads

$$\bar{K}_{s_2, r_2; s_1, r_1}(T) \equiv \int \mathcal{D}\mu_F(\mathbf{A}) \int_{\bar{\mathcal{G}}_0} \mathcal{D}\mu(h) [e^{i\lambda^a h^a(\mathbf{x}_q)}]_{s_2 s_1} \times [e^{i\lambda^a h^a(\mathbf{x}_{\bar{q}})}]_{r_2 r_1}^* \tilde{K}(\mathbf{A}_2^{U_h}, \mathbf{A}_1; T). \quad (55)$$

Within the $\bar{\mathcal{G}}_0$ integration we can factorize the integration over $\bar{\mathcal{G}}_0$ times the integration over the global $SU(N_c)$ group, getting

$$\bar{K}_{s_2, r_2; s_1, r_1}(T) = \int_{SU(N_c)} \mathcal{D}V V_{s_2 s_3} V_{r_2 r_3}^* \times \int \mathcal{D}\mu_F(\mathbf{A}) K(\mathbf{A}^V, s_3, r_3; \mathbf{A}, s_1, r_1; T), \quad (56)$$

where $K(\mathbf{A}, s_3, r_3; \mathbf{A}, s_1, r_1; T)$ is the original Feynman kernel in the temporal gauge defined in Eq. (25).

We will now show that the information about $q\bar{q}$ potentials can be extracted by computing the character-weighted kernel

$$\bar{K}^{[\gamma]}_{s_2, r_2; s_1, r_1}(T) \equiv \int_{SU(N_c)} \mathcal{D}V (\chi^{[\gamma]}(V))^* V_{s_2 s_3} V_{r_2 r_3}^* \times \int \mathcal{D}\mu_F(\mathbf{A}) K(\mathbf{A}^V, s_3, r_3; \mathbf{A}, s_1, r_1; T), \quad (57)$$

where $\chi^{[\gamma]}(V)$ is the character of the representation $[\gamma]$. In the following we will also be interested in the total trace

$$\bar{K}^{[\gamma]}(T) \equiv \sum_{rs} \bar{K}^{[\gamma]}_{s, r; s, r}(T), \quad (58)$$

for which, by virtue of its $\bar{\mathcal{G}}_0$ invariance, we have the further benefit that there is no need for a lattice gauge fixing while computing the boundary \mathbf{A} integration.

The proof of these facts is based on the orthogonality property of characters

$$\int_{SU(N_c)} \mathcal{D}V (\chi^{[\alpha]}(V))^* \chi^{[\beta]}(V) = \delta_{[\alpha], [\beta]}, \quad (59)$$

where $\chi^{[\alpha]}(V)$ and $\chi^{[\beta]}(V)$ are the characters of the irreducible representations $[\alpha]$ and $[\beta]$.

To proceed we need to study the contribution to $\bar{K}^{[\gamma]}_{s_2, r_2; s_1, r_1}(T)$ and its total trace of the four types of energy eigenstates (34)–(37) discussed in Sec. IV.

- (i) From (34) the color-spin singlet, orbital singlet state contribution to $\bar{K}^{[\gamma]}_{s_2, r_2; s_1, r_1}(T)$ is seen to be

$$\bar{K}^{[\gamma]}_{s_2, r_2; s_1, r_1}(T) \Big|_{[S]}^{[S]} = \delta_{[\gamma], [S]} \frac{\delta_{s_2 r_2} \delta_{r_1 s_1}}{N_c} e^{-E[S]T}. \quad (60)$$

Thus for the fully traced kernel we get

$$\bar{K}^{[\gamma]}(T) \Big|_{[S]}^{[S]} = \delta_{[\gamma],[S]} e^{-E^{[S]}T}. \quad (61)$$

- (ii) From (B6) we find that the contribution of color-spin adjoint, orbital singlet states (35) to the character-weighted kernel is

$$\begin{aligned} \bar{K}^{[\gamma]}_{s_2, r_2; s_1, r_1}(T) \Big|_{[Ad]}^{[S]} &= 2 \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \sum_c (V \lambda^c V^\dagger)_{s_2 r_2} (\lambda_{s_1 r_1}^c)^* \\ &\quad \times e^{-E^{[Ad]}T} \\ &= 2 \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \sum_{ac} R_{ac}^{[N_c^2-1]}(V) \lambda_{s_2 r_2}^a (\lambda_{s_1 r_1}^c)^* \\ &\quad \times e^{-E^{[Ad]}T}. \end{aligned} \quad (62)$$

The rhs of Eq. (62) is different from zero only if $[\gamma] = [N_c^2 - 1]$. Tracing with $\delta_{s_2 s_1} \delta_{r_2 r_1}$ the resulting trace over the λ matrices provides the factor $\delta_{ac}/2$ that brings in the character of the adjoint representation $\sum_a R_{aa}^{[N_c^2-1]}(V) = \chi^{[N_c^2-1]}(V)$. From the orthogonality of inequivalent characters we thus get for the fully traced kernel

$$\begin{aligned} \bar{K}^{[\gamma]}(T) \Big|_{[Ad]}^{[S]} &= \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \chi^{[N_c^2-1]}(V) \\ &\quad \times e^{-E^{[Ad]}T} \\ &= \delta_{[\gamma],[N_c^2-1]} e^{-E^{[Ad]}T}. \end{aligned} \quad (63)$$

- (iii) Recalling the form of the projector (B9), we see that the contribution of the color-spin singlet, orbital $[\alpha]$ states (36) to the character-weighted kernel is

$$\begin{aligned} \bar{K}^{[\gamma]}_{s_2, r_2; s_1, r_1}(T) \Big|_{[S]}^{[\alpha]} &= \delta_{s_2 r_2} \delta_{r_1 s_1} \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \\ &\quad \times \sum_m \int \mathcal{D}\mu_F(\mathbf{A}) \phi_m^{[\alpha]}(\mathbf{A}^V) \phi_m^{[\alpha]*}(\mathbf{A})^* e^{-E^{[\alpha]}T} \\ &= \delta_{s_2 r_2} \delta_{r_1 s_1} \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \sum_{mm'} R_{mm'}^{[\alpha]}(V) \\ &\quad \times \int \mathcal{D}\mu_F(\mathbf{A}) \phi_{m'}^{[\alpha]}(\mathbf{A}) \phi_m^{[\alpha]*}(\mathbf{A})^* e^{-E^{[\alpha]}T} \\ &= \frac{\delta_{s_2 r_2} \delta_{r_1 s_1}}{N_c} \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \chi^{[\alpha]}(V) \\ &= \frac{\delta_{s_2 r_2} \delta_{r_1 s_1}}{N_c} \delta_{[\gamma],[\alpha]} e^{-E^{[\alpha]}T}, \end{aligned} \quad (64)$$

where we have used the orthogonality of the wave functionals $\phi_m^{[\alpha]}(\mathbf{A})$ for different Cartan indices. For the fully traced kernel we get

$$\bar{K}^{[\gamma]}(T) \Big|_{[S]}^{[\alpha]} = \delta_{[\gamma],[\alpha]} e^{-E^{[\alpha]}T}. \quad (65)$$

- (iv) Finally using Eq. (B13) and the orthogonality of $\phi_{am}(\mathbf{A})$ wave functionals for different Cartan indices, we find that the contribution of the color-spin adjoint, orbital $[\beta]$ states [Eq. (37)], composing into the irreducible color representation $[\alpha]$ to the character-weighted kernel is

$$\begin{aligned} \bar{K}^{[\gamma]}_{s_2, r_2; s_1, r_1}(T) \Big|_{[Ad]}^{[\alpha]} &= \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \sum_{akk'bc} R_{bk;ck'}^{[\alpha]}(V) \\ &\quad \times \int \mathcal{D}\mu_F(\mathbf{A}) \phi_{ak'}(\mathbf{A}) \phi_{ak}(\mathbf{A})^* \lambda_{s_2 r_2}^c (\lambda_{s_1 r_1}^b)^* \\ &\quad \times e^{-E^{[\alpha]}T} \\ &= 2 \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \sum_{kbc} R_{bk;ck}^{[\alpha]}(V) \\ &\quad \times \lambda_{s_2 r_2}^c (\lambda_{s_1 r_1}^b)^* e^{-E^{[\alpha]}T} \end{aligned} \quad (66)$$

and is different from zero only if $[\gamma] = [\alpha]$. For the fully traced kernel we get

$$\begin{aligned} \bar{K}^{[\gamma]}(T) \Big|_{[Ad]}^{[\alpha]} &= \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \\ &\quad \times \sum_{bk} R_{bk;bk}^{[\alpha]}(V) e^{-E^{[\alpha]}T} \\ &= \int_{\text{SU}(N_c)} \mathcal{D}V(\chi^{[\gamma]}(V))^* \chi^{[\alpha]}(V) e^{-E^{[\alpha]}T} \\ &= \delta_{[\gamma],[\alpha]} e^{-E^{[\alpha]}T}, \end{aligned} \quad (67)$$

since $\sum_{bk} R_{bk;bk}^{[\alpha]}(V) = \chi^{[\alpha]}(V)$ is the character of the representation $[\alpha]$.

C. Extracting $q\bar{q}$ potential energies from lattice data

To extract the interesting $q\bar{q}$ potential energies from lattice data we propose to use the lattice version of (the large T limit of) Eqs. (61) and (63) for the singlet and adjoint representations, respectively. These formulas simply read

$$\begin{aligned} \bar{K}^{\chi^{[S]}(T)} \Big|_{[S]_{\text{lat}}}^{[S]} &= \frac{1}{N_{\text{conf}}} \sum_{\ell=1}^{N_{\text{conf}}} [\text{tr}[\Omega^\dagger(\mathbf{n}_q)] \text{tr}[\Omega(\mathbf{n}_{\bar{q}})]]^{(\ell)} + \dots \\ &\xrightarrow{T \rightarrow \infty} e^{-E^{[S]}T} + \dots, \end{aligned} \quad (68)$$

$$\begin{aligned} \bar{K}\chi^{[\text{Ad}]}(T) \Big|_{[S]_{\text{lat}}}^{[\text{Ad}]} &= \frac{1}{N_{\text{conf}}} \sum_{\ell=1}^{N_{\text{conf}}} (\chi^{[\text{Ad}]}(\Omega(\mathbf{n}_{\infty}))^*) \\ &\quad \times [\text{tr}[\Omega^{\dagger}(\mathbf{n}_q)]\text{tr}[\Omega(\mathbf{n}_{\bar{q}})]]^{(\ell)} + \dots \\ &\xrightarrow{T \rightarrow \infty} e^{-E^{[\text{Ad}]T} + \dots}, \end{aligned} \quad (69)$$

where $N_{\text{conf}} (\gg 1)$ is the number of gauge configurations that have been generated, and the dots represent corrections due to the finiteness of N_{conf} as well as terms exponentially suppressed in the large T limit. In Eq. (69) \mathbf{n}_{∞} denotes the lattice point at infinity, far from the location of the color sources, and $\Omega(\mathbf{n}_{\infty})$ identifies the global color integration, unavoidable in lattice simulations, mentioned at the end of Sec. III [see also the step from Eqs. (55) to (56) where V is what we are calling here $\Omega(\mathbf{n}_{\infty})$].

Naturally the character of the $[S]$ representation is unit, while for the adjoint character we have $\chi^{[\text{Ad}]}(V) = 2 \sum_a \text{tr}[\lambda^a V \lambda^a V^{\dagger}]$.

For the q and \bar{q} self-energy we find

$$\begin{aligned} \bar{K}^{[q]}(T)|_{\text{lat}} &= \frac{1}{N_{\text{conf}}} \sum_{\ell=1}^{N_{\text{conf}}} (\chi^{[q]}(\Omega(\mathbf{n}_{\infty}))^* \text{tr}[\Omega^{\dagger}(\mathbf{n}_q)]^{(\ell)} + \dots \\ &\xrightarrow{T \rightarrow \infty} e^{-E^{[q]}T} + \dots, \end{aligned} \quad (70)$$

$$\begin{aligned} \bar{K}^{[\bar{q}]}(T)|_{\text{lat}} &= \frac{1}{N_{\text{conf}}} \sum_{\ell=1}^{N_{\text{conf}}} (\chi^{[\bar{q}]}(\Omega(\mathbf{n}_{\infty}))^* \text{tr}[\Omega(\mathbf{n}_{\bar{q}})]^{(\ell)} + \dots \\ &\xrightarrow{T \rightarrow \infty} e^{-E^{[\bar{q}]T} + \dots}, \end{aligned} \quad (71)$$

where, we recall, $\chi^{[q]}(V) = \text{tr}[V]$ and $\chi^{[\bar{q}]}(V) = \text{tr}[V^{\dagger}]$.

VII. CONCLUSIONS AND OUTLOOK

In this paper we have derived explicit expressions for the singlet and the octet (adjoint) potential between two static, pointlike sources with the color quantum numbers of a $q\bar{q}$ pair [see Eqs. (68) and (69)]. They have a particularly transparent form in the temporal gauge. For completeness in Appendix D we provide the formulas valid in the Coulomb gauge.

We have discussed in Sec. III the importance of limiting the gauge integration in Eq. (1) to the gauge transformations that tend to the identity at spatial infinity, showing that otherwise only global color singlet eigenfunctionals would contribute to the color averaged Feynman kernel.

In Sec. VI we have shown how the temporal gauge fixing can be implemented in practical lattice simulations and how it is possible to extract the singlet and adjoint $q\bar{q}$ potential energy from lattice simulations, weighting the fully traced Feynman kernel with the character of the representation one is interested in filtering out.

Explicit numerical lattice simulations are under way [17] to check the validity of the analysis presented in this paper and the viability of the formulas we have derived for extracting singlet and octet $q\bar{q}$ potentials.

ACKNOWLEDGMENTS

We wish to thank K. Jansen for bringing the problem addressed in this paper to our attention and for discussions. G. C. R. thanks the Galileo Galilei Institute for Theoretical Physics for the hospitality during the workshop ‘‘New Frontiers in Lattice Gauge Theories,’’ while this work was initiated.

APPENDIX A: COMPLETENESS AND GROUP INTEGRATION

When the degeneracy of an energy level is due to the existence of a symmetry group, it is possible to describe the completeness sum over the degeneracy subspace in terms of the invariant group integration. The basic idea is easily explained as follows.

Starting from an irreducible representation $R(g)$ of a group G and a basis $|i\rangle$ of the representation space, we have the definition

$$R(g)|i\rangle = \sum_j |j\rangle \langle j|R(g)|i\rangle = \sum_j R_{ji}(g)|j\rangle. \quad (\text{A1})$$

The completeness relation can be expressed as

$$I = \sum_i |i\rangle \langle i| = D_{[R]} \int_G \mathcal{D}g R(g) |\bar{k}\rangle \langle \bar{k}| R^{\dagger}(g), \quad (\text{A2})$$

with $|\bar{k}\rangle$ an arbitrary representation state. In Eq. (A2) $D_{[R]}$ is the dimension of the space spanned by the representation $[R]$. In fact, from (see Ref. [18])

$$\int_G \mathcal{D}g R_{jk}(g) R_{lm}(g)^* = \frac{\delta_{jl} \delta_{km}}{D_{[R]}}, \quad (\text{A3})$$

we get

$$\begin{aligned} D_{[R]} \int_G \mathcal{D}g R(g) |\bar{k}\rangle \langle \bar{k}| R^{\dagger}(g) \\ &= D_{[R]} \sum_{j,l} \int_G \mathcal{D}g R_{j\bar{k}}(g) |j\rangle \langle l| R_{l\bar{k}}(g)^* \\ &= \sum_l |l\rangle \langle l|. \end{aligned} \quad (\text{A4})$$

The interest of Eq. (A2) is that it allows us to write the completeness relation by only knowing a single (arbitrary) representation state, $|\bar{k}\rangle$.

APPENDIX B: COLOR PROJECTORS

We apply the formalism developed in Appendix A to write the color projectors necessary to single out the states listed in Eqs. (34)–(37).

- (i) We start with the normalization condition for the color-spin singlet, orbital singlet states of Eq. (34) which reads

$$\begin{aligned}
1 &= \int \mathcal{D}\mu_F(\mathbf{A}) \text{tr} \left[|\psi_{[S]}^{[S]}(\mathbf{A})^\dagger \psi_{[S]}^{[S]}(\mathbf{A})| \right] \\
&= N_c \int \mathcal{D}\mu_F(\mathbf{A}) |\phi(\mathbf{A})|^2. \tag{B1}
\end{aligned}$$

The projector over a color-spin singlet, orbital singlet state is given by

$$\mathcal{P}_{[S]}^{[S]} = \phi(\mathbf{A}_2) \phi^*(\mathbf{A}_1) \delta_{s_2 r_2} \delta_{r_1 s_1}, \tag{B2}$$

so that it contributes to the partially traced kernel (38) with a term

$$\int \mathcal{D}\mu_F(\mathbf{A}) |\phi(\mathbf{A})|^2 \delta_{s_2 r_2} \delta_{r_1 s_1} e^{-E^{[S]}T} = \frac{\delta_{s_2 r_2} \delta_{r_1 s_1}}{N_c} e^{-E^{[S]}T}, \tag{B3}$$

as it follows from Eq. (B1).

(ii) The normalization condition for the color-spin adjoint, orbital singlet states of Eq. (35) is

$$\begin{aligned}
1 &= \int \mathcal{D}\mu_F(\mathbf{A}) \text{tr} \left[\psi_{[\text{Ad}]}^{[S]}(\mathbf{A})^\dagger \psi_{[\text{Ad}]}^{[S]}(\mathbf{A}) \right] \\
&= \text{tr} [\lambda^a \lambda^b] \int \mathcal{D}\mu_F(\mathbf{A}) \phi_a(\mathbf{A}) \phi_b^*(\mathbf{A}) \\
&= \frac{1}{2} \sum_a \int \mathcal{D}\mu_F(\mathbf{A}) |\phi_a(\mathbf{A})|^2. \tag{B4}
\end{aligned}$$

The projector over the color-spin adjoint, orbital singlet states can be computed with the help of the group integration formula (A3) obtaining [we recall that $R_{ab}^{[\text{Ad}]}(V)$ is a real matrix]

$$\begin{aligned}
\mathcal{P}_{[\text{Ad}]}^{[S]} &= (N_c^2 - 1) \int_{\text{SU}(N_c)} \mathcal{D}V (V \psi_{[\text{Ad}]}^{[S]}(\mathbf{A}_2^V) V^\dagger)_{s_2 r_2} (V \psi_{[\text{Ad}]}^{[S]}(\mathbf{A}_1^V) V^\dagger)_{s_1 r_1}^* \\
&= (N_c^2 - 1) \int_{\text{SU}(N_c)} \mathcal{D}V (V \lambda^a V^\dagger)_{s_2 r_2} (V \lambda^b V^\dagger)_{s_1 r_1}^* \phi_a(\mathbf{A}_2) \phi_b(\mathbf{A}_1)^* \\
&= (N_c^2 - 1) \int_{\text{SU}(N_c)} \mathcal{D}V R_{ca}^{[\text{Ad}]}(V) \lambda_{s_2 r_2}^c R_{db}^{[\text{Ad}]}(V) (\lambda_{s_1 r_1}^d)^* \phi_a(\mathbf{A}_2) \phi_b(\mathbf{A}_1)^* \\
&= \sum_a \phi_a(\mathbf{A}_2) \phi_a(\mathbf{A}_1)^* \sum_c \lambda_{s_2 r_2}^c (\lambda_{s_1 r_1}^c)^*. \tag{B5}
\end{aligned}$$

Using the normalization condition (B4), we conclude that a color-spin adjoint, orbital singlet state contributes to the partially traced kernel of Eq. (21) with a term

$$\sum_a \int \mathcal{D}\mu_F(\mathbf{A}) |\phi_a(\mathbf{A})|^2 \sum_c \lambda_{s_2 r_2}^c (\lambda_{s_1 r_1}^c)^* e^{-E^{[\text{Ad}]}T} = 2 \sum_c \lambda_{s_2 r_2}^c \lambda_{s_1 r_1}^c e^{-E^{[\text{Ad}]}T}. \tag{B6}$$

As a check of Eqs. (B3) and (B6) we can trace them over color source indices obtaining 1 for the singlet and $(N_c^2 - 1)$ for the adjoint state, in agreement with the fact that the trace of a projector is the dimension of the space over which it projects.

(iii) The normalization condition for the color-spin singlet, orbital $[\alpha]$ states of Eq. (36) is (no sum over m)

$$1 = \int \mathcal{D}\mu_F(\mathbf{A}) \text{tr} \left[\psi_{[S]m}^{[\alpha]}(\mathbf{A})^\dagger \psi_{[S]m}^{[\alpha]}(\mathbf{A}) \right] = N_c \int \mathcal{D}\mu_F(\mathbf{A}) |\phi^{[\alpha]}_m(\mathbf{A})|^2. \tag{B7}$$

Equation (A2) gives for the projector over the $\psi_{[S]m}^{[\alpha]}$ multiplet the expression

$$\mathcal{P}_{[S]}^{[\alpha]} = D_{[\alpha]} \int_{\text{SU}(N_c)} \mathcal{D}V \phi^{[\alpha]}_m(\mathbf{A}_2^V) \phi^{[\alpha]}_m(\mathbf{A}_1^V)^* \delta_{s_2 r_2} \delta_{r_1 s_1} \tag{B8}$$

for any fixed value of m . From the transformation properties (36) and using Eq. (A3), we can further elaborate Eq. (B8) with the result

$$\mathcal{P}_{[S]}^{[\alpha]} = D_{[\alpha]} \int_{\text{SU}(N_c)} \mathcal{D}V R_{mm'}^{[\alpha]}(V) \phi^{[\alpha]}_{m'}(\mathbf{A}_2) (R_{mm''}^{[\alpha]}(V) \phi^{[\alpha]}_{m''}(\mathbf{A}_1))^* \delta_{s_2 r_2} \delta_{r_1 s_1} = \sum_m \phi^{[\alpha]}_m(\mathbf{A}_2) \phi^{[\alpha]}_m(\mathbf{A}_1)^* \delta_{s_2 r_2} \delta_{r_1 s_1}. \tag{B9}$$

Starting with Eq. (B8), we now compute the contribution of this state multiplet to the \mathbf{A} partial trace of the kernel, finding

$$D_{[\alpha]} \int_{\text{SU}(N_c)} \mathcal{D}V \int \mathcal{D}\mu_F(\mathbf{A}) \phi^{[\alpha]}_m(\mathbf{A}^V) \phi^{[\alpha]}_m(\mathbf{A}^V)^* \delta_{s_2 r_2} \delta_{r_1 s_1} e^{-E^{[\alpha]}T} = \frac{D_{[\alpha]}}{N_c} \delta_{s_2 r_2} \delta_{r_1 s_1} e^{-E^{[\alpha]}T}. \tag{B10}$$

To derive Eq. (B10) we have exploited the color invariance of the measure $\mathcal{D}\mu_F(\mathbf{A})$ and the normalization of the group measure, $\int_{\text{SU}(N_c)} \mathcal{D}V = 1$. As before, the total trace gives the dimension of the representation space, $D_{[\alpha]}$.

(iv) In order to apply the same procedure to the color-spin adjoint, orbital $[\alpha]$ states of Eq. (37), we start from the normalization condition (no sum over m or k)

$$1 = \int \mathcal{D}\mu_F(\mathbf{A}) \text{tr} \left[\psi_{[\text{Ad}]m}^{[\alpha]}(\mathbf{A})^\dagger \psi_{[\text{Ad}]m}^{[\alpha]}(\mathbf{A}) \right] \\ = \frac{1}{2} \sum_a \int \mathcal{D}\mu_F(\mathbf{A}) |\phi_{ak}(\mathbf{A})|^2, \quad (\text{B11})$$

and we then construct the state projector (no sum over k)

$$\mathcal{P}_{[\text{Ad}]}^{[\alpha]} = D_{[\alpha]} \int_{\text{SU}(N_c)} \mathcal{D}V \phi_{ak}(\mathbf{A}_2^V) \phi_{bk}^*(\mathbf{A}_1^V) R_{ca}^{[\text{Ad}]}(V) \\ \times R_{db}^{[\text{Ad}]}(V) \lambda_{s_2 r_2}^c (\lambda_{s_1 r_1}^d)^*. \quad (\text{B12})$$

This expression can be simplified by recalling the transformation properties $\phi_{ak}(\mathbf{A}^V) = R_{kk'}^{[\beta]}(V) \times \phi_{ak'}(\mathbf{A})$ [see Eq. (37)] and the fact that the wave functions ϕ_{ak} must be such to project out the irreducible representation $[\alpha]$ from the tensor product $[\beta] \otimes [N_c^2 - 1]$. We get from Eq. (B12)

$$\mathcal{P}_{[\text{Ad}]}^{[\alpha]} = D_{[\alpha]} \int_{\text{SU}(N_c)} \mathcal{D}V R_{kk''}^{[\beta]}(V) \phi_{ak''}(\mathbf{A}_2) (R_{kk'}^{[\beta]}(V) \phi_{bk'}(\mathbf{A}_1))^* R_{ca}^{[\text{Ad}]}(V) R_{db}^{[\text{Ad}]}(V) \lambda_{s_2 r_2}^c (\lambda_{s_1 r_1}^d)^* \\ = D_{[\alpha]} \int_{\text{SU}(N_c)} \mathcal{D}V \phi_{ak''}(\mathbf{A}_2) \phi_{bk''}^*(\mathbf{A}_1) R_{ck;ak''}^{[\alpha]}(V) (R_{dk;bk''}^{[\alpha]}(V))^* \lambda_{s_2 r_2}^c (\lambda_{s_1 r_1}^d)^* \\ = \sum_{a,k} \phi_{ak}(\mathbf{A}_2) \phi_{ak}^*(\mathbf{A}_1) \sum_b \lambda_{s_2 r_2}^b (\lambda_{s_1 r_1}^b)^*, \quad (\text{B13})$$

where in the second line the matrix $R_{ck;ak''}^{[\alpha]}(V)$ is the irreducible $[\alpha]$ component in the tensor product $R_{kk'}^{[\beta]}(V) R_{ca}^{[\text{Ad}]}(V)$.

Going back to Eq. (B12), we see that the contribution of this state multiplet to the \mathbf{A} partial trace of the kernel is

$$D_{[\alpha]} \int_{\text{SU}(N_c)} \mathcal{D}V \int \mathcal{D}\mu_F(\mathbf{A}) \phi_{ak}(\mathbf{A}^V) \phi_{bk}(\mathbf{A}^V)^* R_{ca}^{[\text{Ad}]}(V) R_{db}^{[\text{Ad}]}(V) \lambda_{s_2 r_2}^c (\lambda_{s_1 r_1}^d)^* e^{-E^{[\alpha]}T} = \frac{D_{[\alpha]}}{N_c^2 - 1} 2 \sum_c \lambda_{s_2 r_2}^c \lambda_{r_1 s_1}^c e^{-E^{[\alpha]}T}. \quad (\text{B14})$$

One can again check that tracing also over source indices yields precisely $D_{[\alpha]}$.

APPENDIX C: DECOMPOSING THE $[N_c] \otimes [\bar{N}_c]$ TENSOR PRODUCT

The $[N_c] \otimes [\bar{N}_c]$ tensor product representation acts on the space of complex matrices w_{sr} , $s, r = 1, \dots, N_c$, endowed with the scalar product

$$(w_2, w_1) = \sum_{sr} w_{2sr}^* w_{1sr} = \text{tr}[w_2^\dagger w_1]. \quad (\text{C1})$$

From the normalization $\text{tr}[\lambda^a \lambda^b] = \frac{1}{2} \delta_{ab}$, we have the identity

$$w - \frac{I_{N_c}}{N_c} \text{tr}[w] = 2 \sum_a \lambda^a \text{tr}[\lambda^a w], \quad (\text{C2})$$

which implies

$$w = \frac{I_{N_c}}{N_c} \text{tr}[w] + 2 \sum_a \lambda^a \text{tr}[\lambda^a w], \quad (\text{C3})$$

where I_{N_c} is the $N_c \times N_c$ unit matrix. In components Eq. (C3) reads

$$w_{s_2 r_2} = \sum_{s_1 r_1} w_{s_1 r_1} \delta_{s_2 s_1} \delta_{r_2 r_1} \\ = \sum_{s_1 r_1} w_{s_1 r_1} \left(\frac{1}{N_c} \delta_{s_2 r_2} \delta_{s_1 r_1} + 2 \sum_a \lambda_{s_2 r_2}^a \lambda_{r_1 s_1}^a \right). \quad (\text{C4})$$

Equation (C4) is the algebraic identity

$$\delta_{s_2 s_1} \delta_{r_2 r_1} = \frac{1}{N_c} \delta_{s_2 r_2} \delta_{r_1 s_1} + 2 \sum_a \lambda_{s_2 r_2}^a \lambda_{r_1 s_1}^a. \quad (\text{C5})$$

The two tensors in the rhs of Eq. (C5) are precisely the two projectors onto the singlet and the adjoint representations, as it follows from the identities [1]

$$P^{[S]} = \frac{2}{N_c} \sum_a \lambda_{s_2 s_1}^a \lambda_{r_1 r_2}^a + \frac{1}{N_c^2} \delta_{s_2 s_1} \delta_{r_1 r_2} = \frac{1}{N_c} \delta_{s_2 r_2} \delta_{r_1 s_1}, \quad (\text{C6})$$

$$P^{[\text{Ad}]} = -\frac{2}{N_c} \sum_a \lambda_{s_2 s_1}^a \lambda_{r_1 r_2}^a + \frac{N_c^2 - 1}{N_c^2} \delta_{s_2 s_1} \delta_{r_2 r_1} = 2 \sum_a \lambda_{s_2 r_2}^a \lambda_{r_1 s_1}^a. \quad (\text{C7})$$

Notice that within each irreducible representation space the two tensors (C6) and (C7) act as unit operators of appropriate dimension.

APPENDIX D: THE $q\bar{q}$ KERNEL IN THE COULOMB GAUGE

For completeness we give in this appendix the explicit expression of the Feynman kernel in the presence of external sources in the Coulomb gauge. The formulas collected in the previous sections can be straightforwardly rewritten in the Coulomb gauge because, as proved in Ref. [9], not only the energies (that are gauge invariant quantities) but also the eigenstates of the Hamiltonian are exactly the same in the Coulomb and the temporal gauge.

In fact, in going from the Coulomb to the temporal gauge, the Feynman kernel stays the same, as the two gauges are related by a change of variables in the functional integral [9].

Explicitly Eq. (25) takes the form

$$\begin{aligned}
& K(\mathbf{A}_2, s_2, r_2; \mathbf{A}_1, s_1, r_1; T) \\
&= \int_{\mathbf{A}(\mathbf{x}, T_1)=\mathbf{A}_1(\mathbf{x})}^{\mathbf{A}(\mathbf{x}, T_2)=\mathbf{A}_2(\mathbf{x})} \mathcal{D}\mu_C(\mathbf{A}) \prod_{\mathbf{x}, T_1 \leq t \leq T_2} dA_0(\mathbf{x}, t) e^{-S_{\text{YM}}(\mathbf{A}, A_0)} \\
&\quad \times \left[T \exp\left(i \int_{T_1}^{T_2} A_0(\mathbf{x}_{\bar{q}}, \tau) d\tau\right) \right]_{s_2, s_1} \\
&\quad \times \left[T \exp\left(-i \int_{T_1}^{T_2} A_0(\mathbf{x}_{\bar{q}}, \tau) d\tau\right) \right]_{r_1, r_2}, \quad (\text{D1})
\end{aligned}$$

where

$$\mathcal{D}\mu_C(\mathbf{A}) = \prod_{\mathbf{x}, T_1 < t < T_2} \Delta_C(\mathbf{A}) \delta[\nabla \mathbf{A}] d\mathbf{A}(\mathbf{x}, t) \quad (\text{D2})$$

is the Coulomb gauge-fixed integration measure and $\Delta_C(\mathbf{A})$ is the corresponding FP determinant. We note that, unlike the integration over the spatial components of the gauge field, the integration over the temporal one in Eq. (D1) is extended to include also the values at the boundary times. This is related to the fact that the temporal component of the gauge field, A_0 , plays the role of a Lagrange multiplier which enforces Gauss' law at any instant including the initial and final times [19]. This is why the exponentials in Eq. (D1) do not wind up in the time direction.

Setting now as in (38), $\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{A}$ and integrating over \mathbf{A} , again with the Coulomb integration measure, we conclude that the resulting quantity coincides with the expression displayed in the rhs of Eq. (38) from which singlet and adjoint potentials can be extracted.

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