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Noise kernel for the self-similar Tolman-Bondi metric: Fluctuations on the Cauchy horizon

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We attempt to calculate the point separated noise kernel for the self-similar Tolman-Bondi metric, using a method similar to that developed by Eftekharzadeh *et al.* for ultrastatic spacetimes, referring to the work by Page. In the case of formation of a naked singularity, the noise kernel thus obtained is found to be regular except on the Cauchy horizon, where it diverges. The behavior of the noise in the case of the formation of a covered singularity is found to be regular. This result seemingly renders backreaction non-negligible, which causes us to question the stability of the results obtained from the semiclassical treatment of the self-similar Tolman-Bondi metric.

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I. INTRODUCTION

Cosmic censorship, ever since its proposition in 1969, has been an issue studied mostly within the framework of classical relativity. This includes important conjectures like the hoop conjecture, studies on stability of scalar fields on spherically symmetric collapse metrics, as well as particular examples of gravitational collapse metrics. Some studies have addressed gravitational collapse at the semiclassical level, though most of the work concerns $1 + 1$-dimensional metrics. In the semiclassical theory [1,2], the quantum expectation of the energy-momentum tensor (also referred to as RSET or the renormalized stress energy tensor) is central to studying quantum fields on spacetimes. One line of investigation in applying this to gravitational collapse is to question features such as trapped surfaces that arise earlier and the final fate of the collapse [3,4]. Another line of investigation is to work out the quantum stress tensor when one employs, as a background spacetime, the classical relativistic examples of gravitational collapse and to study its effects. In the latter, in particular, the metric representing pressureless fluid collapse in spherical symmetry, the Lemaitre-Tolman-Bondi (LTB) metric, is a common choice. We follow this line of investigation in this work.

The classical LTB metric exhibits both naked and covered singularities which can be easily related to data on the initial Cauchy surface. The transition from covered to naked singularities appears to occur on account of inhomogeneities in initial data. Furthermore, at the semiclassical level it appears that this increasing inhomogeneity causes a drastic change in the behavior of the quantum stress tensor. In fact, it is found that the quantum stress tensor diverges in the naked singularity case where the Cauchy horizon is expected to form, whereas there is no divergence anywhere in the covered singularity case. This divergence is physically interpreted as an energy burst on the Cauchy horizon [5].

It is still not clear if such a behavior is generic, as the results obtained are in fact exact in $1 + 1$-dimensional cases and for a few cases of background metrics. Also, the calculations involve a background metric which remains unaffected by the quantum stress tensor. In other words, the backreaction is not implicitly incorporated in the results. However, physical arguments are offered for the divergence which rely on high curvature regions of spacetime causing particle production. Such regions would not be exposed in the covered case, unlike the naked case, and it is *likely* that the semiclassical divergence is actually a general feature.

Semiclassical gravity is, however, fraught with its share of interpretational issues, apart from operational difficulties like backreaction. Since the idea by Rosenfeld of using the quantum stress tensor on the right of the Einstein field equations, several issues have been raised [6]. At least some of them involve serious objections to employing an averaged quantity for the stress tensor. Indeed, one can imagine that, in situations where fluctuations are important, the use of the semiclassical quantum-averaged stress tensor would not be justified for any physical interpretation like the one above. So for a complete consideration it seems natural to address the fluctuations as well, whenever they can be argued to be significant.

Stochastic gravity seeks to remedy this situation, addressing the effect of including fluctuations of the energy-momentum tensor and possibly its backreaction on the metric as well. This is accomplished using the Einstein-Langevin equation [7,8]. The central object in the calculations of stochastic gravity is the noise kernel, which gives an idea of the stochastic source, in addition to the quantum stress tensor source. The noise kernel is the vacuum expectation value of the symmetrized stress-energy bitensor

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for a quantum field in curved spacetime [9,10]. It characterizes fluctuations of the stress tensor which play, as mentioned above, a significant role in the analysis of quantum effects in curved spacetime. These fluctuations lead to fluctuations in the metric, which are themselves understood as *induced* or *passive* fluctuations [7]. Induced fluctuations have played an important role in many studies involving backreaction. Backreaction problems in gravity and cosmology, for example, have been addressed [11] using Einstein-Langevin equations.

Thus far, the effect of quantum fluctuations has not been studied in the context of gravitational collapse, the physical scenario for studying cosmic censorship. This is, first of all, because of the operational difficulty in calculating the central quantity, the noise kernel, which gives us an idea of the stochastic source term. Second, the quantum stress tensor does not add significantly to the classical source for most of the spacetime regions. This would support any interpretation ignoring backreaction effects of the quantum stress, especially regarding the effects on quantum fields like pair creation or Hawking radiation (at ''late'' times) [12]. However, it is physically important to assure that the quantum-averaged source is indeed a good approximation to the quantum field contribution to the source. Any effect of the fluctuations, which appear as the stochastic source described above, would have to be negligible. However, it has been a challenge in stochastic gravity to come up with a method to calculate the noise kernel for specific background metrics. There are various approaches that have been used for such calculations, in the case of Minkowski, de Sitter, anti–de Sitter, and Schwarzschild spacetimes in the coincidence limit [13–18]. Recently, Eftekharzadeh *et al.* [19] have devised a method to compute the noise kernel for conformally invariant quantum fields in Schwarzschild spacetime under the Gaussian approximation. This is based on the method developed by Page [20] and provides a possibly general way to estimate the noise kernel approximately.

We seek to apply stochastic gravity techniques in the study of cosmic censorship to the gravitational collapse scenario, particularly to the LTB metric. This metric has been hitherto studied only at the level of quantum fields on spacetime. It has been demonstrated that the stress energy tensor diverges at the Cauchy horizon, despite the Cauchy horizon being perfectly regular [5]. Such results do not take into account backreaction of the quantum fields on spacetime. The motivation for calculating the noise kernel in a LTB background is to test the validity of the above-mentioned result in view of backreaction. Our aim in carrying out this work has been to investigate this in the context of gravitational collapse and suggest interpretations of the particular calculations for the noise kernel. In this paper, we employ a similar method to that of Ref. [19] and compute the noise kernel on the self-similar LTB metric background for conformally invariant fields under a Gaussian approximation. We obtain the noise kernel, in this case, which diverges at the Cauchy horizon, seemingly making backreaction important for consideration and hence questioning any attempt to study the spacetime in a semiclassical or perturbative manner.

The paper is organized as follows. In Sec. II we introduce the building blocks of stochastic gravity, the noise kernel, and the required Wightman function. Section III describes the conformal rescaling of the Tolman-Bondi metric, and its conformally equivalent ultrastatic form is obtained. We obtain the expansions of the Synge function and the Van Vleck-Morette determinant in the coordinates of this ultrastatic metric. In Sec. IV we relate the noise kernel of this metric to that of the self-similar Tolman-Bondi metric. We argue here for a plausible divergence. In the end we summarize the results and discuss the interpretation of the divergence.

II. NOISE KERNEL IN ULTRASTATIC SPACETIMES

The noise kernel is introduced briefly and related to the contribution of stress fluctuations to the source in Einstein equations. The nature of the assumed quantum state is described for this situation. Its relation to the noise kernel is mentioned, and its expression is provided in the case of an ultrastatic spacetime.

A. Einstein-Langevin equation and the noise kernel

The noise kernel for a quantized matter field is given by

$$N_{abc'd'}(x, x') = \frac{1}{2}\langle\{\hat{t}_{ab}(x), \hat{t}_{c'd'}(x')\}\rangle, \tag{1}$$

where

$$\hat{t}_{ab}(x) \equiv \hat{T}_{ab}(x) - \langle\hat{T}_{ab}(x)\rangle$$

and $\langle\cdots\rangle$ is the quantum expectation value taken with respect to a normalized state. $\hat{T}_{ab}$ is the stress tensor operator of the field. The classical stress tensor of a conformally invariant scalar field ϕ is given by

$$T_{ab} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla^c\phi\nabla_c\phi + \frac{1}{6}(g_{ab}\Box - \nabla_a\nabla_b + G_{ab})\phi^2. \tag{2}$$

Then, the scalar field ϕ is quantised raising it to the level of and operator, whereas $g_{\mu\nu}$ is treated classically.

The noise kernel embodies the contributions of the higher correlation functions in the quantum field, on account of which it may be used to interpret issues related to the quantum nature of spacetime. Two point functions of the energy-momentum tensor involve fourth order correlations of the quantum field, for example, which would affect the coherence of the geometry [21], if we were to employ the Einstein-Langevin equations (often referred to as semiclassical Einstein-Langevin equations)

$$G_{\mu\nu} = \langle \hat{T}_{\mu\nu} \rangle + \hat{\xi}_{\mu\nu}, \tag{3}$$

where $\hat{\xi}_{\mu\nu}$ is a random variable (tensor) which is closely related to the noise kernel. The relations

$$\langle \hat{\xi}_{\mu\nu}(x) \rangle_s = 0 \quad \langle \hat{\xi}_{\alpha\beta}(x) \hat{\xi}_{\mu'\nu'}(x') \rangle_s = N_{\alpha\beta\mu'\nu'}(x,x') \tag{4}$$

completely characterize the variable $\hat{\xi}_{\alpha\beta}(x)$. The statistical expectation $\langle \ \rangle_s$ is taken over various stochastic realizations of the Gaussian source $\hat{\xi}_{\mu\nu}(x)$. Each of the realizations leads to a metric solution

$$h_{\alpha\beta}(x) = h^{(0)}_{\alpha\beta}(x) + 8\pi \int d^4y' \sqrt{-g(y')} G^{(\mathrm{ret})}_{\alpha\beta\gamma'\delta'}(x,y') \xi^{\gamma'\delta'}(y'), \tag{5}$$

where $h^{(0)}_{\alpha\beta}$ is the solution to the semiclassical equation

$$G_{\mu\nu} = \langle \hat{T}_{\mu\nu} \rangle$$

and $G^{(\mathrm{ret})}_{\alpha\beta\gamma'\delta'}$ serves as the retarded propagator of the semiclassical Einstein-Langevin equations with vanishing initial conditions. The stochastic realization contains information about coherence of geometry, with $\hat{\xi}^{\mu\nu}(x)$ being treated as a random variable.

B. The quantum state for the noise kernel: Wightman function

Before physically interpreting any particular noise calculation, the nature of the quantum state of the field needs to be taken into account. The state we consider in our calculations turns out to be of Hadamard type, in addition to our assumptions of it being quasi-free and thermal. We discuss these three qualifiers of the state below.

First, the Hadamard nature of the state guarantees that the stress tensor is well defined within the maximal Cauchy development and obeys the Wald axioms [2]. We will analytically continue the state across the Cauchy horizon. This consideration can be straightforwardly generalized to Schwarzschild spacetime, as considered in Ref. [19]. Operationally, we are not interested in the stress tensor and would not require any coincidence limits, even for the noise kernel, for addressing issues of fluctuations. However, the fluctuations obtained would be specific to the quantum state, and a physically reasonable choice of state is warranted. Hence, the Hadamard nature of the state is accorded physical significance in the discussion of our results.

In our calculations below, we have emphasized the singularity structure of the symmetrized form of the two point function we use. Represented by a Hadamard expansion, we conclude that the quantum state being used is of Hadamard type. The expressions we use are for a general quasistatic spacetime and based on the approximation carried out by Page, and so the conclusion would be more general. However, a careful interpretation of the divergent expression in $1/\sigma$, with σ^2 being the Synge function, is required to support the claim that the state is of Hadamard type. Such an interpretation, in the sense of distributions, is provided by Kay and Wald [22]. A rigorous definition is provided by Radzikowski [23] based on microlocal analysis, but we do not attempt to use that in this paper. Second, we operationally assume that the Wightman two point function defined as

$$\mathcal{G}(x,x') = \langle \phi(x)\phi(x') \rangle$$

determines the quantum state of the field. Indeed, that is true if one is working with quasi-free (or Gaussian) states. Quasi-free states have been extensively used [2] in curved spacetime. They conveniently lead to Fock space representations of one particle Hilbert spaces via the Gelfand-Naimark-Segal construction on the Weyl algebra of observables. One advantage of this is that it can admit a concept of particles in a limited manner, despite having curved spacetime at hand. For example, large fluctuations implied by the noise kernel could be interpreted as highly fluctuating particle creation.

Third, it is assumed that the state is thermal or of the Kubo-Martin-Schwinger type with temperature κ. It turns out that the Wightman function can be approximated for thermal states in a particular manner, called Gaussian approximation. We employ the related expression in the last subsection for ultrastatic spacetimes.

Our results would thus be applicable to a quasi-free thermal Hadamard state of a conformally invariant field. The state-specific information is not contained in the singularity structure. So changing to a different quasi-free Hadamard state would amount to adding an analytic function to the symmetrized two point function. We find that the noise kernel expression would amount to adding an analytic function to the symmetrized two point function, and that the noise kernel expression would not change at the κ^0 level. The divergence on the Cauchy horizon, being at all orders in κ, is expected to persist for any quasi-free Hadamard state.

C. Noise kernel in terms of the Wightman function

The properties and the expressions of a noise kernel, given a quasi-free (or Gaussian) state of a quantum matter field, are obtained in Ref. [24]. Furthermore, for conformally invariant fields the noise kernel is given in terms of the Wightman function as [19]

$$N_{abc'd'} = \mathrm{Re}\{\bar{K}_{abc'd'} + g_{ab}\bar{K}_{c'd'} + g_{c'd'}\bar{K}'_{ab} + g_{ab}g_{c'd'}\bar{K}\}, \tag{6}$$

where

$$9\bar{K}_{abc'd'} = 4(\mathcal{G}_{;c'b}\mathcal{G}_{;d'a} + \mathcal{G}_{;c'a}\mathcal{G}_{;d'b}) + \mathcal{G}_{;c'd'}\mathcal{G}_{;ab} + \mathcal{G}\mathcal{G}_{;abc'd'} - 2(\mathcal{G}_{;b}\mathcal{G}_{;c'ad'} + \mathcal{G}_{;a}\mathcal{G}_{;c'bd'} + \mathcal{G}_{;d'}\mathcal{G}_{;abc'} + \mathcal{G}_{;c'}\mathcal{G}_{;abd'}) + 2(\mathcal{G}_{;a}\mathcal{G}_{;b}R_{c'd'} + \mathcal{G}_{;c'}\mathcal{G}_{;d'}R_{ab}) - (\mathcal{G}_{;ab}R_{c'd'} + \mathcal{G}_{;c'd'}R_{ab})\mathcal{G} + \frac{1}{2}R_{c'd'}R_{ab}\mathcal{G}^2, \tag{7a}$$

$$36\bar{K}'_{ab} = 8(-\mathcal{G}_{;p'b}\mathcal{G}_{;}{}^{p'}{}_{a} + \mathcal{G}_{;b}\mathcal{G}_{;p'a}{}^{p'} + \mathcal{G}_{;a}\mathcal{G}_{;p'b}{}^{p'}) + 4(\mathcal{G}_{;}{}^{p'}\mathcal{G}_{;abp'} - \mathcal{G}_{;p'}{}^{p'}\mathcal{G}_{;ab} - \mathcal{G}\mathcal{G}_{;abp'}{}^{p'}) - 2R'(2\mathcal{G}_{;a}\mathcal{G}_{;b} - \mathcal{G}\mathcal{G}_{;ab}) - 2(\mathcal{G}_{;p'}\mathcal{G}_{;}{}^{p'} - 2\mathcal{G}\mathcal{G}_{;p'}{}^{p'})R_{ab} - R'R_{ab}\mathcal{G}^2, \tag{7b}$$

$$36\bar{K} = 2\mathcal{G}_{;p'q}\mathcal{G}_{;}{}^{p'q} + 4(\mathcal{G}_{;p'}{}^{p'}\mathcal{G}_{;q}{}^{q} + \mathcal{G}\mathcal{G}_{;p}{}^{p}{}_{q'}{}^{q'}) - 4(\mathcal{G}_{;p}\mathcal{G}_{;q'}{}^{pq'} + \mathcal{G}_{;}{}^{p'}\mathcal{G}_{;q}{}^{q}{}_{p'}) + R\mathcal{G}_{;p'}\mathcal{G}_{;}{}^{p'} + R'\mathcal{G}_{;p}\mathcal{G}^{;p} - 2(R\mathcal{G}_{;p'}{}^{p'} + R'\mathcal{G}_{;p}{}^{p})\mathcal{G} + \frac{1}{2}RR'\mathcal{G}^2 \tag{7c}$$

are given in terms of the Wightman function. Primes on indices represent the point x', while the entities at x are the unprimed ones. R_{ab} is the Ricci tensor and R is the Riemann curvature.

D. Wightman function in ultrastatic spacetime

Next, we give a short review of the method used in Ref. [19] for an ultrastatic metric. The metric in a static spacetime takes the form

$$ds^2 = g_{\tau\tau}(\vec{x})d\tau^2 + g_{ij}(\vec{x})dx^i dx^j. \tag{8}$$

This can be transformed into an ultrastatic form, called the optical metric, by a conformal transformation. This optical metric takes the following form:

$$ds^2 = dt^2 + g_{ij}(\vec{x})dx^i dx^j, \tag{9}$$

where the metric functions g_{ij} are independent of time t. The Synge function (half of the square of the proper distance of the shortest geodesic between two points) can thus, in ultrastatic spacetime, take the form

$$\sigma(x, x') = \frac{1}{2}((t - t')^2 - \mathbf{r})^2,$$

where $\mathbf{r}^2$ is twice the spatial part of the Synge function, and it depends only on spatial coordinates as in Ref. [19].

The Wightman function in an optical background metric for a thermal (Kubo-Martin-Schwinger) state can be calculated under the Gaussian approximation [19]. This approximation was first carried out by Page using the Schwinger-De Witt expansion in the context of calculating quantum stress tensors. In this scheme

$$\mathcal{G}(\delta t, \vec{x}, \vec{x}') = \frac{\kappa \sinh \kappa \mathbf{r}}{8\pi^2 \mathbf{r}[\cosh(\kappa \mathbf{r}) - \cosh(\kappa \delta t)]} U(\delta t, \vec{x}, \vec{x}'), \tag{10}$$

where

$$\mathbf{T} = \frac{\kappa}{2\pi}$$

is the temperature of the thermal state considered[1] and

$$\delta t = t - t'$$

while

$$U(x, x') \equiv \Delta^{1/2}(x, x'), \tag{11}$$

$$\Delta(x, x') \equiv \frac{1}{\sqrt{-g(x)}\sqrt{-g(x')}} \det(\sigma;_{ab'}), \tag{12}$$

where $\Delta(x, x')$ is known as the Van Vleck-Morette determinant.

III. CONFORMALLY OPTICAL FORM OF SELF-SIMILAR TOLMAN-BONDI METRIC

In order to evaluate the noise kernel for Schwarzschild spacetime, Eftekharzadeh *et al.* [19] have used an approach of conformally rescaling the metric to the form of an ultrastatic (optical) metric. First, the noise kernel is evaluated in this ultrastatic spacetime. Then, the noise kernel of Schwarzschild spacetime is obtained by just rescaling back the result in the optical case. We will be applying a similar technique to the self-similar Tolman-Bondi (TB) metric, which is given by the line element

$$ds^2 = dt^2 - R'dr^2 - R^2 d\theta^2 - R^2 \sin^2\theta d\phi^2, \tag{13}$$

where $R(t, r)$ is the area radius, and

$$R^{3/2}(t, r) = r^{3/2}\left(1 - \frac{3}{2}\frac{t}{r}\sqrt{\lambda}\right) \tag{14}$$

is characterized by a dimensionless parameter λ. The collapse rate can also be given in terms of this parameter,

$$\dot{R} = -r\sqrt{\frac{\lambda}{R}}. \tag{15}$$

The initial data for this metric are regular, and a curvature singularity eventually forms at $r = 0$. The singularity is naked if $\frac{\lambda^{3/2}}{12} \leq \frac{26}{3} - 5\sqrt{3}$ and is covered otherwise.

For conformal transformation it is useful to work in a new coordinate system (t, z, θ, ϕ) where $z = t/r$. Then

[1] In Schwarzschild spacetime, the κ has been chosen to be the surface gravity on the event horizon null surface. However, one can work with a general thermal state without ascribing this significance to it.

$$R^{3/2}(z) = r^{3/2}\left(1 - \frac{3}{2}z\sqrt{\lambda}\right), \quad (16)$$

$$R'(z) = \frac{1 - \frac{z}{2}\sqrt{\lambda}}{(1 - \frac{3z\sqrt{\lambda}}{2})^{1/3}}. \quad (17)$$

The self-similar TB metric can now be put in the conformally ultrastatic form as

$$ds^2 = \Omega^2[dT^2 - f_1(z)^2dz^2 - z^2 f_2(z)^2 d\Omega^2], \quad (18)$$

where

$$\Omega^2 = \left(1 - \frac{R'^2}{z^2}\right)t^2, \quad (19)$$

$$f_1(z)^2 = \left(\frac{R'}{z^2 - R'^2}\right)^2, \quad (20)$$

$$f_2(z)^2 = \frac{(1 - \frac{3}{2}z\sqrt{\lambda})^{4/3}}{z^2(z^2 - R'^2)}, \quad (21)$$

and

$$dT = \frac{dt}{t} + \frac{R'^2}{z(z^2 - R'^2)}dz.$$

Since the metric (18) is conformally related to an ultrastatic metric, the Synge function can readily be obtained in the following form as is done in Ref. [19] for the Schwarzschild metric in optical form. We apply the same procedure for the TB metric in the optical form. The Synge function expansion used here becomes

$$\sigma = \sum_{ijk} s_{ijk}(z)\delta T^{2i}\eta^j\delta z^k,$$

where

$$\delta T = (T - T'), \qquad \delta z = (z - z'),$$
$$\eta + 1 = \cos(\theta)\cos(\theta') + \sin(\theta)\sin(\theta')\cos(\phi - \phi').$$

The expression for the Synge function is obtained as

$$\begin{aligned}\sigma(x,x') = \frac{1}{2}[&\delta T^2 - \delta z^2 f_1(z)^2 + 2\eta z^2 f_2(z)^2 \\ &- 2\delta z\eta(zf_2(z)^2 + z^2 f_2(z)f_2'(z)) \\ &+ \delta z^3 f_1(z)f_1'(z)] + O[(x-x)^4]. \end{aligned} \quad (22)$$

The function $U(x,x')$ present in the Wightman function (10) can be expanded in powers of $(x - x')$ using the Synge function (11) and (12) as follows:

$$\begin{aligned}U(x,x') = 1 &+ \delta z^2\left[\frac{f_1'(z)}{6zf_1(z)} - \frac{f_2'(z)}{3zf_2(z)} + \frac{f_1'(z)f_2'(z)}{6f_1(z)f_2(z)}\right] \\ &- \delta z^2\left[\frac{f_2''(z)}{6f_2(z)}\right] - \eta\left[\frac{1}{6} - \frac{f_2(z)^2}{6f_1(z)^2} + \frac{zf_2(z)^2f_1'(z)}{6f_1(z)^3}\right. \\ &\left.- \frac{2zf_2(z)f_2'(z)}{3f_1(z)^2}\right] - \eta\left[\frac{z^2f_2(z)f_1'(z)f_2'(z)}{6f_1(z)^3}\right. \\ &\left.- \frac{z^2f_2'(z)^2}{6f_1(z)^2} - \frac{z^2f_2(z)f_2''(z)}{6f_1(z)^2}\right] + O[(x-x')^3]. \end{aligned} \quad (23)$$

With U and the Synge function obtained for the optical self-similar TB spacetime, we can now evaluate the Wightman function which will be used to obtain the noise kernel.

IV. NOISE KERNEL EXPRESSION FOR (SELF-SIMILAR) TOLMAN-BONDI SPACETIME

The Wightman function for a thermal state with temperature κ in the Gaussian approximation on ultrastatic spacetime is given by [19]

$$\begin{aligned}\mathcal{G}(x,x') = \frac{1}{8\pi}&\left[\frac{1}{\sigma} + \frac{\kappa^2}{6} - \frac{\kappa^4}{180}(2\delta T^2 + \sigma) + O[(x-x')^4]\right] \\ &\times U(x,x'). \end{aligned} \quad (24)$$

At this point we note that the U above is analytic in its domain and the prefactor contains the singularity structure. The same singularity structure would be present in the symmetrized two point function. We have a Hadamard singularity structure above, in fact, without any $\log\sigma$ term.

The noise kernel, when points are separated, can now be computed using the above Green's function, in the usual way as given in Ref. [19]. Here we will present one component of the noise kernel for demonstrating some important results for our metric. The expression calculated is for the noise kernel component given by $N_{TTT'T'}$ for points separated in T, where $\delta z = \eta = 0$ and $\delta T \neq 0$. The noise kernel $N_{TTT'T'}$ can be expanded in terms of various powers of κ. These coefficients in such an expansion are presented in Table I.

The above expansion for the noise kernel has been obtained after conformal transformation of the optical form back to the original form (18). We have displayed the expression of $N_{TTT'T'}$ for points separated in the T coordinate only. This turns out to be interesting, as it yields a divergence at the Cauchy horizon with leading orders in separation. This takes place despite the Cauchy horizon itself being regular in that there is no curvature singularity along it.

TABLE I. Noise kernel component $N_{TTT'T'}$ displayed as coefficients of powers of κ the inverse temperature.

Coefficient of κ^0	$\frac{z^4 f_1^2}{R'^2 t^2 (t-\delta t)^2}\Big\{\frac{13}{12\delta T^8 \pi^4} + [-4f_1^4 + 3z^2 f_2^2 f_1'^2 + 2z f_1 f_2\{-2z f_1' f_2' + f_2(-2f_1' + z f_1'')\} + 4f_1^2\{2f_2^2 + 2z^2 f_2'^2 + z f_2(6f_2' + z f_2'')\}]/[72\delta T^6 \pi^4 z^2 f_1^4 f_2^2] + [27 f_1^7 + 50 z f_1^4 f_2 f_1'(f_2 + z f_2') + 360 z^3 f_2^3 f_1'^3 (f_2 + z f_2') - 10 f_1^5[3f_2^2 + 3z^2 f_2'^2 + z f_2(16 f_2' + 5z f_2'')] - 5z^2 f_1 f_2^2 f_1'(z^2 f_1' f_2'^2 + f_2^2 f_1' + 48 z f_1'') + 2z f_2[24 z f_2' f_1'' + f_1'(73 f_2' + 36 z f_2'')]] + 2z f_1^2 f_2\{-49 z^3 f_1' f_2'^3 + z^2 f_2 f_2'[12 z f_2' f_1'' - f_1'(209 f_2' + 31 z f_2'')] + f_2^3[-49 f_1' + 12z(f_1'' + z f_1^{(3)})] + z f_2^2[12z\{4z f_1'' f_2'' + f_2'(10 f_1'' + z f_1^{(3)})\} + f_1'[-209 f_2' + z(185 f_2'' + 72 z f_2^{(3)})]]\} + f_1^3\{3f_2^4 + 3z^4 f_2'^4 + 2z^3 f_2 f_2'^2(104 f_2' + 49 z f_2'') + z^2 f_2^2[678 f_2'^2 + 67 z^2 f_2''^2 - 8z f_2'(-49 f_2'' + 3z f_2^{(3)})] + 2z f_2^3[104 f_2' + z(13 f_2'' - 12z(5f_2^{(3)} + z f_2^{(4)}))]\}/[25920\delta T^4 \pi^4 z^4 f_1^7 f_2^4]\Big\}$
Coefficient of κ^2	$\frac{z^4 f_1^2}{R'^2 t^2 (t-\delta t)^2}\Big\{\frac{5}{72\delta T^6 \pi^4} + [2f_1^4 + 9z^2 f_2^2 f_1'^2 + 2z f_1 f_2\{8z f_1' f_2' + f_2(8f_1' + 3z f_1'')\} + 2f_1^2\{5f_2^2 + 5z^2 f_2'^2 - 2z f_2(3f_2' + 4z f_2'')\}]/[2592\delta T^4 \pi^4 z^2 f_1^4 f_2^2]\Big\}$
Coefficient of κ^4	$\frac{z^4 f_1^2}{R' t^2 (t-\delta t)^2}\Big[-\frac{53}{1080\delta T^4 \pi^4}\Big] + O[(x-x')^3]$

A. Structure for naked singularity and comparison with covered case

In the case of a naked singularity solution, the Cauchy horizon is given by the smaller root z_- of $(z^2 - R'(z)^2)$ for the self-similar TB metric [5]. This leads to divergence of the factors $f_1(z)$ and $f_2(z)$ if evaluated on the Cauchy horizon. We could choose T for the point on the Cauchy horizon and T' for the one elsewhere. The behavior thus obtained would be clearly of the point separated kernel.

The terms for various orders of κ displayed above diverge on account of the appearance of $f_1(z)$ in the factor [the appearance of $f_1(z) \sim \frac{1}{z-z_-}$ and $f_2(z) \sim \frac{1}{\sqrt{z-z_-}}$ occurring in terms in the brackets essentially does not contribute any singular structure] leading to our main result, the divergence of the noise kernel on the Cauchy horizon. It should be noted that the above divergence has been obtained using metric (18). We have been constrained by our approach to use such a metric in coordinates ill behaved at the Cauchy horizon.[2] Suitable coordinate transformations can be used to remedy this. From the metric one can observe that the coordinate transformation factors acting on $N_{TTT'T'}$ above would need to diverge at the Cauchy horizon as $\frac{1}{z-z_-}$, similar to Ref. [5]. The divergence of the expression of the noise kernel above is rather enhanced if we demand that the metric is regular at the Cauchy horizon.

[2]The metric in the earlier (t, r, θ, ϕ) coordinate system is perfectly regular on the Cauchy horizon, and we have conformally rescaled back our results. Our results would thus hold for any other metric related to it by regular coordinate system transformations.

In the case of a covered singularity $(z^2 - R'(z)^2)$ has no real roots. So the factors of $f_1(z)$ and $f_2(z)$ do not yield divergences. Nor do the rest of the factors. The metric used is also regular everywhere.

B. Singularity structure

The issue of regularity also occurs in the noise expressions of Ref. [19], as the Schwarzschild metric used is expressed in the usual coordinates rather than, say, Kruskal coordinates at the event horizon. For interpreting the expressions on the event horizon, further coordinate transformations would be necessary on the noise kernel.

The above analysis is restricted to the noise kernel with only $\delta T \neq 0$. We obtain a similar divergence on the Cauchy horizon (in the naked singularity case) for $\delta z \neq 0$ separation as well. The corresponding noise expressions, however, are not short enough for an explicit display in this communication.

Our results above for the noise kernel would be applicable to a particular quasi-free thermal Hadamard state of a conformally invariant field on self-similar LTB spacetime. It would be of interest to examine if this could be applicable to any quasi-free thermal Hadamard state.

The state-specific information is not contained in the singularity structure. So changing to a different quasi-free Hadamard state would amount to adding an analytic function to the symmetrized two point function. If the above expressions of the noise kernel are examined, we find that they would not change at the κ^0 level. The divergence on the Cauchy horizon (in the naked singularity case), being at all orders in κ, is expected to persist for any quasi-free Hadamard state.

V. RESULTS AND DISCUSSION

We have thus obtained the noise kernel for a conformally coupled scalar field in the self-similar Tolman-Bondi metric in an approximate form for a quasi-free thermal Hadamard state. We seek to interpret this result below.

We know that the noise kernel affects the induced fluctuations of the metric. In Eq. (5), however, the G^{ret} function required would need a background metric for calculating it. Unfortunately, the largely fluctuating stochastic source as implied by the noise kernel for the naked singularity metric does not allow us to treat the backreaction as negligible. So, our results cannot easily be extended for studying the induced fluctuations on the Cauchy horizon.

At the same time, we note that noise kernel is not very large before it gets near the Cauchy horizon (in the naked singularity case), and so the background appears to be a good approximation despite backreaction.

Even with the above difficulty regarding the metric fluctuations, the source fluctuations can still be directly used for the purpose of interpretation. The semiclassical interpretation of the stress tensor divergence on the Cauchy horizon [5] suggested that an energy burst on the Cauchy horizon is plausible. Such an interpretation would need to be reconsidered in view of our analysis. The highly fluctuating source near the Cauchy horizon would render it difficult to interpret any of its stochastic realizations as physically significant, in particular, the semiclassical one of an energy burst on the Cauchy horizon.

The covered singularity case is in contrast with the above results. This is similar to the difference seen in the semiclassical behavior of the metric. In the latter analysis, the quantum stress tensor diverges on the Cauchy horizon in the naked singularity case, as opposed to regular behavior in the covered case. The contrast is borne out by analysis of fluctuations as well.

The divergence of the quantum stress tensor in the semiclassical analysis has been attributed to the fact that high curvature regions are exposed in the naked singularity case, leading to high energy effects. It would seem that high curvatures lead to diverging noise as well. Thus, the contrast in the behavior of local quantum fields seems to be tied to the exposure of high curvature regions, or the lack of it. We suggest that this plausible connection could be investigated further.

Finally, the stochastic source fluctuating highly near the Cauchy horizon could be interpreted for its effect as an *environment* (quantum fields) on the *system* (background metric) [7]. The spacetime could be highly sensitive to such *environmental* decoherence effects very near the Cauchy horizon. In particular, as we have a quasi-free state, one could interpret this, for example, as spacetime reaction effects of particle creation. We suggest that this issue could be pursued separately.

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