Predictive description of Planck-scale-induced spacetime fuzziness

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Several approaches to the quantum-gravity problem predict that spacetime should be "fuzzy," but so far these approaches have been unable to provide a crisp physical characterization of this notion. An intuitive picture of spacetime fuzziness has been proposed on the basis of semiheuristic arguments and, in particular, involves an irreducible Planck-scale contribution to the uncertainty of the energy of a particle. These arguments also inspired a rather active phenomenological program that looks for the blurring of images of distant astrophysical sources that would result from such energy uncertainties. Here we report the first ever physical characterization of spacetime fuzziness derived constructively within a quantum picture of spacetime, the one provided by spacetime noncommutativity. Our results confirm earlier heuristic arguments suggesting that spacetime fuzziness, while irrelevantly small on terrestrial scales, could be observably large for propagation of particles over cosmological distances. However, we find no Planck-scale-induced lower bound on the uncertainty of the energy of particles; we observe that this changes how we should picture a quantum spacetime, and it also imposes a reanalysis of the associated phenomenology.

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There has been growing interest [1–5] in the possibility of testing the hypothesis of spacetime fuzziness at the Planck scale ($E_P \sim 10^{28}$ eV), exploiting an associated effect of blurring the images of distant astrophysical sources, such as quasars. The arguments providing encouragement for these phenomenological studies are merely heuristic, but this could be a rare opportunity [6–8] for experimentally testing an aspect of the interplay between gravitational and quantum-mechanical phenomena. The scenario considered in Refs. [1–5] (building on earlier analogous pictures, such as those in Refs. [9–12]) is centered on the possibility that the quantum-gravity contribution to the fuzziness of a particle's worldline might grow with propagation distance, in such a way that, in spite of its ultra-microscopic characteristic scale (which we assume [6–8] to be of the order of the Planck length $\ell \simeq 10^{-35}$ m), it could turn into a macroscopic effect for propagation over suitably large (cosmological) distances.

The most crucial aspect of the phenomenological proposals is the assumption [1–5,11,12] that in a quantum spacetime there should be an irreducible Planck-scale contribution to the uncertainty of energies, governed by a law of the form

$$\delta E_{[\ell]} \sim \ell^{\alpha} E^{1+\alpha}, \tag{1}$$

where we assume that α [1–5,11,12] takes values between 1/2 and 1, since it is the single parameter that should discriminate, in this respect, among different proposals for a quantum spacetime. [We shall consistently place an index "$[\ell]$," as done in (1), when reporting an estimate of the Planck-scale contribution to a given quantity.]

As first observed in Ref. [11], Eq. (1) would in turn produce uncertainties in the phase velocity $v_p = E/p$ and in the group velocity $v_g = dE/dp$, both of order $\delta E_{[\ell]}/E$ and uncorrelated. This would imply that as a wave propagates it will experience [11] essentially random mismatches, of order $\delta E_{[\ell]}/E$, between its phase velocity and its group velocity. In turn, one can then notice [11] that during the propagation time $t_{\rm prop} = D/v_g$ the phase should normally advance from its initial value by an amount $2\pi v_p t_{\rm prop}/\lambda$, i.e., $2\pi(v_p/v_g)D/\lambda$ (denoting with λ the wavelength). The net result would be [1–5,11,12] an uncertainty for the phase of a wave governed by a law of the type

$$\delta\phi_{[\ell]} = 2\pi \frac{D^{\beta}}{\lambda^{\beta}} \delta\left[\frac{v_p}{v_g}\right]_{[\ell]} \sim \frac{D^{\beta}}{\lambda^{\beta}} \frac{2\pi}{E} \delta E_{[\ell]}, \tag{2}$$

where β is an additional phenomenological parameter concerning whether or not the random mismatches between phase and group velocity should add coherently: if they add coherently [11] then $\beta = 1$, while, according to the most popular picture where they do not add coherently [1], one should have $\beta = 1 - \alpha$.

The debate revolving around the blurring of images of distant astros that would result from this phase uncertainty has revolved exclusively around the β parameter and, therefore, the hypotheses needed for the coherent (versus incoherent) addition of mismatches between phase and group velocity. Instead, the aspect which is of primary interest for us here, concerning the Planck-scale-induced energy uncertainty $\delta E_{[\ell]}$ in (2), and its description according to (1), has so far remained unchallenged, and it provides the core feature of this whole research line. This assumption is motivated, in the relevant heuristic arguments, by essentially noticing that energy is operatively

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a notion derived from spacetime observations, and with spacetime being "fuzzy" at the Planck scale, one should, according to Refs. [1–4,11], inevitably get "fuzzy energies."

Besides being, as we stressed, the key ingredient of an active phenomenological program, this link from spacetime fuzziness to irreducible contributions to energy uncertainty could be a very significant characterization of the quantum-gravity realm. But its only bases are indeed heuristic arguments. Even in the most studied formalizations of quantum properties of spacetime at the Planck scale, such as loop quantum gravity [13] and spacetime noncommutativity [14,15], spacetime fuzziness has so far been characterized only at a rather formal level, unsuitable for phenomenology and inconclusive for what concerns the description of energy uncertainties.

Here we report significant progress in formalizing and analyzing worldline fuzziness within the quantum-spacetime framework of spacetime noncommutativity. And, in particular, we establish some severe limitations to the applicability of the arguments summarized above, suggesting that in a quantum spacetime there should be an irreducible Planck-scale contribution to energy uncertainties. We do this by considering the two most studied examples of spacetime noncommutativity, the case of Moyal noncommutativity [16] and the case of κ-Minkowski noncommutativity [15,17]. For reasons that shall be clear in light of the outcome of our analyses, we first specialize to the case of a 2D (1 + 1-dimensional) κ-Minkowski spacetime, so that the noncommutativity of coordinates is fully specified by

$$[x_1, x_0] = i\ell x_1. \tag{3}$$

We are evidently working in units such that the speed-of-light scale c and the Planck scale $\hbar$ are set to unity, and most of our results are derived at leading order in ℓ, which suffices for the purposes of the relevant phenomenology.

The starting point of this analysis is provided by our previous study in Ref. [18], where we addressed one of the challenges which had obstructed the characterization of worldline fuzziness in noncommutative spacetimes. In κ-Minkowski spacetime the time coordinate is a noncommutative observable, whereas in the standard formulation of quantum mechanics the time coordinate is merely an evolution parameter. This difficulty can be circumvented [18] by resorting to results [19–21] that establish a covariant formulation of quantum mechanics, where both the spatial coordinates and the time coordinates play the same type of role. Spatial and time coordinates are well-defined operators on a "kinematical Hilbert space," which is just an ordinary Hilbert space of normalizable wave functions [21]. Observable features of the quantum theory are coded on the "physical Hilbert space," obtained from the kinematical Hilbert space by enforcing the on-shellness constraint (this constraint codifies dynamics in the same sense as the covariant formulation of classical mechanics; see, e.g., Chapter 4 of Ref. [22]).

Within this formulation of quantum mechanics, time and spatial coordinates of course commute among themselves but do not commute with their conjugate momenta; in particular, in the 2D case one has [21]

$$\begin{aligned}[\pi_0, q_0] &= i,\\ [\pi_1, q_1] &= -i,\\ [\pi_1, q_0] &= [\pi_0, q_1] = 0.\end{aligned} \tag{4}$$

We observed in Ref. [18] that the κ-Minkowski defining commutator (3) is satisfied by posing a relationship between the κ-Minkowski coordinates and the phase-space operators of the covariant formulation of quantum mechanics [the ones of Eq. (4), viewed here merely as formal auxiliary operators [18]) of the following form:

$$x_1 = e^{\ell\pi_0} q_1, \qquad x_0 = q_0. \tag{5}$$

We also showed in Ref. [18] that the translational symmetries of κ-Minkowski spacetime, which in terms of the x_1, x_0 coordinates require a rather sophisticated formalization (see, e.g., Refs. [15,17,23]), can be implemented in terms of the auxiliary variables q_0, q_1, π_0, π_1 as standard translation transformations:

$$\begin{aligned}\mathcal{T}_{a^\mu} \triangleright f(x_0, x_1) \equiv\ & f(q_0, q_1 e^{\ell\pi_0})\\ & - i a^\mu [\pi_\mu, f(q_0, q_1 e^{\ell\pi_0})].\end{aligned} \tag{6}$$

We also established [18] that the "on-shellness operator" (the operator which, for massless particles, should vanish on physical states) can be written in terms of the auxiliary variables π_1, π_0 as follows:

$$\mathcal{H} = \left(\frac{2}{\ell}\right)^2 \sinh^2\left(\frac{\ell\pi_0}{2}\right) - e^{-\ell\pi_0}\pi_1^2. \tag{7}$$

One more result of Ref. [18] that is relevant for the observations we are reporting here concerns the measure for integration over momenta, which is needed for evaluating scalar products when working in the "momentum representation." We found that the implementation of relativistic symmetries in terms of the κ-Minkowski x_1, x_0 coordinates induces an ℓ-deformed π_0, π_1-integration measure: $d\pi_0 d\pi_1 \rightarrow \exp(-\ell\pi_0) d\pi_0 d\pi_1$.

In Ref. [18] we only reached the point of analyzing the kinematical Hilbert space. The form of the operator $\mathcal{H}$ was established, but we did not explore the implications of enforcing the Hamiltonian constraint in obtaining the physical Hilbert space. Since we are interested here in the fuzziness of the worldline of a physical particle, we must progress to that next level. More precisely, we characterize physical observables of free relativistic quantum particles in κ-Minkowski spacetime following the covariant prescription adopted in Ref. [21]: we obtain the needed feature of invariance of physical observables under the action of $\mathcal{H}$ by introducing a new scalar product [21] that projects all the orbit of the gauge transformation generated by $\mathcal{H}$ on the same state. This allows us to formally refer to states in the kinematical Hilbert space (but only as representatives of an orbit) and, for free massless particles, leads to the study of scalar products of the type $\langle\psi|\phi\rangle_{\mathcal{H}} = \langle\psi|\delta(\mathcal{H})\Theta(\pi_0)|\phi\rangle$, where $\Theta(\pi_0)$ specifies a restriction [21] to positive-energy solutions of the on-shellness constraint. Accordingly, in the "momentum-space representation" one has

$$\langle\psi|\phi\rangle_{\mathcal{H}} = \int e^{-\ell\pi_0} d\pi_1 d\pi_0 \delta(\mathcal{H})\Theta(\pi_0)\psi^*(\pi)\phi(\pi).$$

We focus here on the case of a localized massless particle, describable in terms of a Gaussian state[1]

$$\Psi_{\bar{q}_0,\bar{q}_1}(\pi_\mu;\bar{\pi}_\mu,\sigma_\mu) = Ne^{-\frac{(\pi_0-\bar{\pi}_0)^2}{4\sigma_0^2}-\frac{(\pi_1-\bar{\pi}_1)^2}{4\sigma_1^2}} e^{i\pi_0\bar{q}_0 - i\pi_1\bar{q}_1},$$

where N is a normalization constant,

$$N^{-2} = \int e^{-\ell\pi_0} d\pi_1 d\pi_0 \delta(\mathcal{H})\Theta(\pi_0)|\Psi_{\bar{q}_0,\bar{q}_1}(\pi_\mu;\bar{\pi}_\mu,\sigma_\mu)|^2,$$

and $\Psi_{\bar{q}_0,\bar{q}_1}$ is evidently written in the momentum-space representation, with parameters $\bar{\pi}_0$, $\bar{\pi}_1$, σ_0, σ_1, $\bar{q}_0$, $\bar{q}_1$ (with $\bar{q}_0$, $\bar{q}_1$ highlighted, in the notation $\Psi_{\bar{q}_0,\bar{q}_1}$, since the issue of localization of the particle is predominantly connected with those two parameters).

Our $\Psi_{\bar{q}_0,\bar{q}_1}$ gives a state on our physical Hilbert space of relativistic free-particle quantum mechanics, so it identifies a fuzzy worldline [21], as shall be evident also in what follows. The expectation in $\Psi_{\bar{q}_0,\bar{q}_1}$ of the measurable quantity described by a self-adjoint operator $\mathcal{O}$ is computed in terms of $\langle\Psi_{\bar{q}_0,\bar{q}_1}|\mathcal{O}|\Psi_{\bar{q}_0,\bar{q}_1}\rangle_{\mathcal{H}}$.

The next hurdle we must face concerns the identification of a well-defined observable suitable for the characterization of the fuzziness of the worldline. The apparently obvious choices, x_1 and x_0, are actually not suitable for this task, since they are not self-adjoint operators on our physical Hilbert space (in particular, they do not commute with $\mathcal{H}$). We propose to remedy this by focusing on the following "intercept operator" $\mathcal{A}$:

$$\mathcal{A} = e^{\ell\pi_0}\left(q_1 - \mathcal{V}q_0 - \frac{1}{2}[q_0, \mathcal{V}]\right), \tag{8}$$

where $\mathcal{V}$ is shorthand for $\mathcal{V} \equiv (\partial\mathcal{H}/\partial\pi^0)^{-1}\partial\mathcal{H}/\partial\pi^1$.

One may notice that $\mathcal{A}$ is describable as an ℓ-deformed Newton-Wigner operator [24]. And it is well known that within special-relativistic quantum mechanics there is no better estimator of localization than the Newton-Wigner operator (it can only be questioned for localization comparable to the Compton wavelength of the particle [24], but this merely conceptual limit of ideal localization is evidently irrelevant for the level of localization achieved by particle production at, e.g., quasars). For our purposes it is important to notice that $\mathcal{A}$ is a good observable on our physical Hilbert space (self-adjoint, commuting with $\mathcal{H}$), and, evidently, in the classical limit $\mathcal{A}$ reduces to the intercept of the particle worldline with the x_1 axis.

Let us focus, for conceptual clarity, on the analysis of the properties of $\mathcal{A}$ for the case of $\Psi_{0,0}$, i.e., for $\bar{q}_0 = 0$, $\bar{q}_1 = 0$. One then easily finds that

$$\langle\Psi_{0,0}|\mathcal{A}|\Psi_{0,0}\rangle_{\mathcal{H}} = 0,$$

so this is a case where the particle intercepts the observer Alice in her origin. The fact that this intercept is fuzzy reflects the fuzziness of the worldline described by $\Psi_{0,0}$, and, in particular, the leading ℓ-dependent contribution to this fuzziness is characterized by

$$\delta\mathcal{A}^2_{[\ell]} = (\langle\Psi_{0,0}|\mathcal{A}^2|\Psi_{0,0}\rangle_{\mathcal{H}})_{[\ell]} \approx \ell\langle\pi_0\rangle\sigma^{-2}/2, \tag{9}$$

where, for simplicity, we assumed (as we shall do throughout) that σ_1 is small enough, in comparison to σ_0, $\bar{\pi}_1$, to allow a saddle point approximation in the π_1 integration; then σ (without indices) is the effective Gaussian width after the saddle point approximation in π_1: $\sigma^{-2} \equiv \sigma_1^{-2} + \langle\mathcal{V}\rangle^2\sigma_0^{-2}$.

In our proposed interpretation of the formalism, Eq. (9) gives the fuzziness of the worldline "at Alice" (at the point of crossing the origin of Alice's reference frame). It is interesting to also consider the perspective of observers reached by the particle at cosmological distances from Alice. These observers are those who are connected to Alice by a pure translation so that, for them, the state of the particle is Ψ_{a_0,a_1}, and they are such that $\langle\mathcal{A}\rangle = 0$, i.e., $\langle\Psi_{a_0,a_1}|\mathcal{A}|\Psi_{a_0,a_1}\rangle_{\mathcal{H}} = 0$. Finding these observers amounts to finding the translation parameters a^0, a^1 such that $\langle\Psi_{0,0}|\mathcal{T}^{-1}\mathcal{A}\mathcal{T}|\Psi_{0,0}\rangle_{\mathcal{H}} = 0$, where $\mathcal{T}$ is the one of Eq. (6). Of course, this leads to a one-parameter family of solutions (the family of observers "on the worldline"), which unsurprisingly takes the form $a^1 = \langle\mathcal{V}\rangle a^0$.

Crucial for us is that these observers with a vanishing expectation value for the intercept have values of the uncertainty in the intercept $\delta\mathcal{A}$ given by

$$\begin{aligned}\delta\mathcal{A}^2_{[\ell]} &= (\langle\Psi_{a^0,\langle\mathcal{V}\rangle a^0}|\mathcal{A}^2|\Psi_{a^0,\langle\mathcal{V}\rangle a^0}\rangle_{\mathcal{H}})_{[\ell]} \\ &\approx \left(\frac{\ell\langle\pi_0\rangle}{2\sigma^2} + \ell^2 a_0^2\sigma^2\right).\end{aligned} \tag{10}$$

So, we do have here a quantum-spacetime picture that fits within the intuition inspiring spacetime-fuzziness phenomenology: one can in fact interpret our observer Alice, the observer on the worldline for whom the fuzziness of the intercept takes the minimum value, as the observer at the source (where the particle is produced), and then the

[1]Of course, in the massless-particle limit of interest here, one must proceed cautiously: $\Psi_{\bar{q}_0,\bar{q}_1}(\pi_\mu;\bar{\pi}_\mu,\sigma_\mu)$ must be replaced by $\Psi^{\xi}_{\bar{q}_0,\bar{q}_1}(\pi_\mu;\bar{\pi}_\mu,\sigma_\mu) = \exp(-\xi/\pi_0^2)\Psi_{\bar{q}_0,\bar{q}_1}(\pi_\mu;\bar{\pi}_\mu,\sigma_\mu)$, with ξ a small infrared regulator which never actually matters in the results we exhibit here.

intercept of the particle worldline with the origin of the reference frames of observers distant from Alice (where the particle could be detected) has bigger uncertainty.

However, our formalization provides a quantification of the relevant effects that differs from what had been suggested heuristically. A crucial aspect of these differences is uncovered by evaluating the energy uncertainty δE. In Ref. [18] we established that π_0 does have a standard role of energy for particles in our κ-Minkowski spacetime, and therefore δE is given by $\langle \Psi_{a^0,\langle\mathcal{V}\rangle a^0}|\pi_0^2|\Psi_{a^0,\langle\mathcal{V}\rangle a^0}\rangle_{\mathcal{H}} - \langle \Psi_{a^0,\langle\mathcal{V}\rangle a^0}|\pi_0|\Psi_{a^0,\langle\mathcal{V}\rangle a^0}\rangle_{\mathcal{H}}^2$, for which we find

$$\delta E^2 \simeq \sigma^2 - 2\ell E\sigma^2. \tag{11}$$

Remarkably, this shows that, contrary to what has been assumed on the basis of heuristic arguments, in our quantum spacetime there is no irreducible Planck-scale contribution to energy uncertainties (since σ can take unboundedly small values in our Gaussian states). This is perhaps our most significant result, which we feel has very strong implications for both the phenomenology and the theoretical understanding of spacetime fuzziness. Phenomenologically, the fact that we found no irreducible Planck-scale contribution to energy uncertainties renders the experimental bounds on spacetime fuzziness derived in Refs. [1–5,11,12] completely inapplicable to κ-Minkowski spacetime.

And, it is also relatively easy to apply the strategy of analysis adopted above for κ-Minkowski spacetime to the other much-studied noncommutative spacetime, the one with Moyal noncommutativity. Denoting with X_μ the coordinates of the Moyal spacetime, one has [16] that

$$[X_\mu, X_\nu] = i\ell^2\theta_{\mu\nu},$$

where the dimensionless noncommutativity parameters $\theta_{\mu\nu}$ are coordinate independent. Applying our approach centered on the manifestly covariant formulation of quantum mechanics to the Moyal case is indeed easier than for the κ-Minkowski case, because of the simplicity of the relationship between the Moyal coordinates and the phase-space operators of the covariant formulation of quantum mechanics [the ones of Eq. (4), here viewed again as formal auxiliary operators]: by posing

$$X_\mu = q_\mu + \ell^2\frac{\theta_{\mu\nu}}{2}\pi^\nu, \tag{12}$$

the Moyal commutation relations are automatically satisfied. Comparison of (12) for the Moyal case to the complexity of its nonlinear κ-Minkowski counterpart (5) already suggests how much simpler it is to adopt our strategy of analysis when considering Moyal noncommutativity. In particular, the seed for our κ-Minkowski findings is in the associated description of translations, which according to (6) are such that

$$x'_0 = x_0 + a_0 = q_0 + a_0,$$
$$x'_j = (q_j + a_j)e^{\ell\pi_0} = x_j + a_j e^{\ell\pi_0}.$$

One can easily trace back to the nontrivial $e^{\ell\pi_0}$ operatorial factor, for translations of spatial coordinates, the source of the mechanism that gives increasing fuzziness as the particle propagates. For the Moyal case from (12) it follows that under translations one simply has

$$X'_\mu = q_\mu + a_\mu + \ell^2\frac{\theta_{\mu\nu}}{2}\pi^\nu = X_\mu + a_\mu,$$

and in light of this result it is easy to follow the steps of our κ-Minkowski analysis by adapting them to the Moyal case, ultimately finding that there is no Planck-scale-induced increase of the fuzziness as the particle propagates.

This also explains why we chose to keep our primary focus on κ-Minkowski spacetime: the Moyal case is simpler but does not even provide the starting ingredient of the heuristic pictures of spacetime fuzziness we intended to investigate. In κ-Minkowski spacetime we did at least find that, as argued by the heuristic arguments, the fuzziness of a particle's worldline increases as the particle propagates from emission to detection. But even κ-Minkowski spacetime provides no support for the other key aspect of the intuition about (and the phenomenology on) spacetime quantization provided by the heuristic arguments, which concerns an irreducible Planck-scale contribution to energy uncertainties.

In closing, it is perhaps useful to summarize what we feel are the key findings reported in this paper. Here we performed the first ever constructive/deductive (no heuristics) derivation of the properties of fuzzy worldlines in a class of quantum spacetimes. We established firmly that the phenomenological parametrization based exclusively on the parameters α and β (in the notation adopted here in the opening remarks), which had been suggested by several heuristic arguments, is at least not sufficiently general. Specifically, we established that this phenomenological parametrization is not applicable to the two most-studied noncommutative spacetimes, the Moyal type and the κ-Minkowski type. In the κ-Minkowski case we did find that, as stated by the heuristic arguments, the fuzziness of a particle's worldline increases as the particle propagates from emission to detection: combining (10) and (11) one gets for κ-Minkowski spacetime

$$\delta\mathcal{A}_{[\ell]} \simeq \ell D\delta E, \tag{13}$$

where we only included the contribution growing with the propagation distance and we used $D = a^0$ since the translation parameter a^0 connecting the observer at emission and the observer at detection is just the propagation distance (any difference from this would only manifest itself at subleading orders in ℓ). The propagation-distance-dependent amplification of Planck-scale effects shown in

(11) can provide a natural target for quantum-gravity phenomenology, and now these plans can be pursued at a level that goes beyond heuristics. But the phenomenology will need to adapt to the fact that within our κ-Minkowski picture the δE in (13) receives no irreducible Planck-scale contributions.

The absence of irreducible Planck-scale contributions to δE also characterizes our Moyal-case results [and in the Moyal case even the propagation-distance amplification of type (13) is absent]. This is rather significant since the presence of irreducible Planck-scale contributions to δE, of the type noted here in Eq. (1), was a key aspect of the conceptualization of spacetime fuzziness provided by previous heuristic arguments and a key ingredient of the associated phenomenology developed in Refs. [1–4,11,12] and references therein. The ''experimental bounds on spacetime fuzziness'' derived in Refs. [1–4,11,12] and references therein are therefore evidently inapplicable to Moyal and κ-Minkowski noncommutativity. And it would be reductive to view our study as a counterexample to a general feature: this is the first time that the expectations of the relevant heuristic arguments have been tested in actual formalizations of the notion of a quantum spacetime, and they were found to fail. There may well be some quantum spacetimes where the relevant heuristic predictions do happen to apply, but there is at present (with two actual spacetime pictures analyzed, and the heuristic predictions found to be inapplicable there) no reason to assume that those predictions will be generic. It is still plausible that those bounds do apply, for example, to the picture of quantum spacetime emerging from loop-quantum-gravity research [13], and we feel that establishing rigorously such a link (or the lack thereof) should be one of the next main targets for this research program.

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