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# Cosmological phase transition, baryon asymmetry, and dark matter $Q$-balls

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We consider a mechanism of dark matter production in the course of the first-order phase transition. We assume that there is an asymmetry between X and $\bar{\mathrm{X}}$ particles of the dark sector. In particular, it may be related to the baryon asymmetry. We also assume that the phase transition is so strongly first order that X particles do not permeate into the new phase. In this case, as the bubbles of the old phase collapse, X particles are packed into $Q$-balls with a huge mass defect. These $Q$-balls compose the present dark matter. We find that the required present dark matter density is obtained for the energy scale of the theory in the ballpark of 1–10 TeV. As an example, we consider a theory with an effective potential of one-loop motivated form.

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## I. INTRODUCTION

The idea that the baryon asymmetry and dark matter in the Universe may have common origin, is of considerable interest [1–21]. In particular, dark matter particles $(\mathrm{X}, \bar{\mathrm{X}})$ may have their own conserved charge and be created, together with baryons, in asymmetric decays of heavier particles [4–9,16,18,21]. In this scenario, the initial asymmetry in the dark matter particles is roughly of the order of the baryon asymmetry,

$$n_{\mathrm{X}} - n_{\bar{\mathrm{X}}} \sim n_B. \tag{1}$$

Assuming that the X-$\bar{\mathrm{X}}$ annihilation cross section is not particularly small, one obtains the estimate for the present dark matter mass density:

$$\rho_{\mathrm{X}} \sim m_{\mathrm{X}} n_B. \tag{2}$$

Thus, the correct value of $\rho_{\mathrm{X}}$ appears to require the mass of X particles in the range of a few GeV.

New physics, manifestation of which would be the existence of X particles, may well be characterized by a much higher energy scale, and the X-particle mass may grossly exceed a few GeV. One may wonder whether the coproduction of the baryon asymmetry and dark matter can still work in this case. The estimate (2) shows that heavy X particles are overproduced, so one needs a mechanism that makes the actual mass density of dark matter much lower than that given by Eq. (2).

In this paper, we consider a possible scenario of this sort. The idea is that X particles may be packed into $Q$-balls. A $Q$-ball made of X particles of total number $Q$ typically has a mass $m_Q$, which is much smaller than $Q \cdot m_{\mathrm{X}}$ [22–26]. Hence, the mass density of the dark matter $Q$-balls is naturally well below the estimate (2). A mechanism that packs free particles into $Q$-balls applies to the Friedberg-Lee-Sirlin $Q$-balls [22–25] (as opposed to the Coleman $Q$-balls [26] explored in supersymmetric theories [27,28]) and is as follows [29].

Let us assume that X particles obtain their mass due to the interaction with an additional scalar field $\phi$, so that $m_{\mathrm{X}} = h\phi$, where $h$ is the coupling constant. Let us also assume that there is the first-order cosmological phase transition at some temperature $T_c$ from the phase $\phi = 0$ to the phase $\phi = \phi_c \neq 0$ and, furthermore, that the X-particle mass is large in the new phase, $h\phi_c \gg T_c$. Then X particles get trapped in the remnants of the old phase, and these remnants eventually shrink to very small size and become $Q$-balls (see Fig. 1).

We find that this mechanism indeed works in a certain range of couplings characterizing the model. Interestingly, the correct value of the present $Q$-ball mass density is obtained for X-particle mass in the ballpark of 1–10 TeV. This makes the scenario potentially testable in collider experiments. The $Q$-ball mass and charge are in the range $m_Q \sim 10^{-6}$–$10^{-3}$ g and $Q \sim 10^{19}$–$10^{22}$, respectively. In this respect, our $Q$-ball dark matter is not very different from that discussed in the context of supersymmetric theories [27,28]. Phenomenology of our dark matter $Q$-balls is also similar to that of supersymmetric $Q$-balls [27,28], except that the latter may be stabilized by baryon number and hence eat up baryons.

The paper is organized as follows. We begin with a brief description of $Q$-balls (Sec. II). In Sec. III, we discuss the creation of $Q$-balls in the course of the phase transition and relate the dark matter $Q$-ball parameters and their present mass density to the properties of the phase transition. We also consider the conditions of validity of our scenario. In Sec. IV, we give a concrete example based on one-loop motivated form of the finite temperature effective potential and present the ranges of parameters for which our mechanism is viable. We conclude in Sec. V.

 

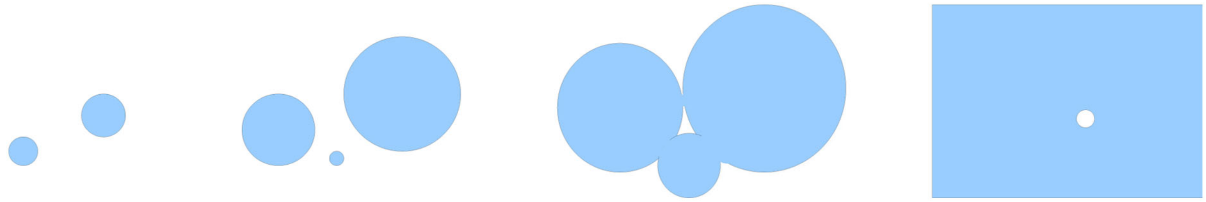

FIG. 1 (color online). $Q$-ball formation during the first-order phase transition from the phase $\phi = 0$ (white) to the phase $\phi \neq 0$ (blue).

## II. $Q$-BALLS

### A. $Q$-ball configuration

$Q$-balls are compact objects that exist in some models possessing a global symmetry and associated conserved charge. One of the simplest models admitting Q-balls is of the Friedberg-Lee-Sirlin type [22–25]. Its Lagrangian is[1]

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - U(\phi) + (\partial_\mu \chi)^*(\partial_\mu \chi) - h^2\phi^2\chi^*\chi, \tag{3a}$$

$$U(\phi) = \lambda(\phi^2 - v^2)^2, \tag{3b}$$

where $\phi$ is a real scalar field and $\chi$ is a complex scalar field.[2] The field $\chi$ is meant to describe X particles whose mass in vacuo equals

$$m_X = hv.$$

These particles carry global charge, associated with the $U(1)$-symmetry $\chi \to e^{i\alpha}\chi$. The lowest energy state of large enough charge is a spherical $Q$-ball with $\phi = 0$ inside and $\phi = v$ outside. At large $Q$, its size $R$ and energy $E$ are determined by the balance of the energy of $Q$ massless $\chi$ quanta confined in the potential well of radius $R$ and the potential energy of the field $\phi$ in the interior, i.e., $R$ and $E$ are found by minimizing

$$E(R) = \frac{\pi Q}{R} + \frac{4\pi}{3}R^3 U_0, \tag{4}$$

where $U_0 = U(0) - U(v) = \lambda v^4$. Hence, the $Q$-ball parameters are

$$R_Q = \left(\frac{Q}{4U_0}\right)^{1/4}, \qquad m_Q = \frac{4\sqrt{2}\pi}{3}Q^{3/4}U_0^{1/4}. \tag{5}$$

Note that the surface energy is proportional to $R_Q^2 \propto Q^{1/2}$ and therefore negligible at large $Q$. The $Q$-ball is stable, provided its energy is smaller than the rest energy of $Q$ massive $\chi$ quanta in the vacuum $\phi = v$,

$$m_Q < m_X Q. \tag{6}$$

A $Q$-ball is a classical object, since its radius is much larger than its Compton wavelength.

### B. Radius of cosmological $Q$-balls

Assuming that $Q$-balls made of X particles compose dark matter and that the X asymmetry is related to the baryon asymmetry via Eq. (2), we can obtain an estimate for the present radius of a typical $Q$-ball already at this point. Indeed, we find from Eq. (5) that

$$R_Q = 4\pi\frac{Q}{3m_Q}.$$

Now, since X particles are packed into $Q$-balls, we have

$$\frac{(n_X - n_{\bar{X}})}{s} = \frac{n_Q Q}{s},$$

where $n_Q$ is the number density of $Q$-balls and $s$ is the entropy density. Therefore,

$$\frac{n_Q Q}{s} = \frac{n_q - n_{\bar{q}}}{s} = 3\Delta_B, \tag{7}$$

where $\Delta_B = n_B/s = 0.9 \times 10^{-10}$, and we assume for definiteness that the X-particle asymmetry is equal to the quark asymmetry. Making use of the relation $\rho_{DM} = m_Q n_Q$, we obtain

$$\left.\frac{Q}{m_Q}\right|_{t_0} = \frac{3\Delta_B s_0}{\rho_{DM}}, \tag{8}$$

where the subscript 0 refers to the present epoch. Hence, we obtain from Eq. (5) that the present radius is

$$R_Q = 4\pi\frac{\Delta_B s_0}{\rho_{DM}} \simeq 6 \times 10^{-14}\ \text{cm}. \tag{9}$$

Thus, even though the $Q$-ball mass and charge depend on the parameters of the model, its typical radius does not.

## III. $Q$-BALL PRODUCTION AT THE COSMOLOGICAL PHASE TRANSITION

In this section, we give a general description of the $Q$-ball formation without specifying the form of the finite

[1] As it stands, this model has the discrete symmetry $\phi \to -\phi$, and, hence, is not viable because of the domain wall problem. To get around this problem, one can either assume that the discrete symmetry is explicitly broken or consider the field $\phi$ carrying some gauge or global charge. This qualification is irrelevant for what follows.

[2] Note that our notations are opposite to notations in Ref. [22]: we denote the complex field by $\chi$, while it is denoted by $\phi$ in Ref. [22]; the real field is denoted by $\phi$ here and by $\chi$ in Ref. [22].

temperature effective potential. We assume only that it has a minimum $\phi = 0$ at high temperature and that the phase transition from $\phi = 0$ to $\phi \neq 0$ is of the first order.

### A. Bubble nucleation rate

Let $T_c$ be the critical temperature at which the effective potential has two degenerate minima at $\phi = 0$ and $\phi = \phi_c$. Below this temperature, the new phase $\phi = \phi_c$ has lower free energy density,

$$V_{\rm eff}(\phi_c) - V_{\rm eff}(0) = -\rho < 0. \tag{10}$$

Thermal fluctuations lead to the creation of the bubbles of the new phase. In the thin wall approximation, the free energy of a bubble of radius $R$ is

$$F(R) = -\frac{4}{3}\pi R^3 \rho + 4\pi R^2 \sigma, \tag{11}$$

where $\sigma$ is surface free energy density. The extremum of Eq. (11) gives the free energy of the critical bubble,

$$F_c = \frac{16\pi}{3}\frac{\sigma^3}{\rho^2}. \tag{12}$$

It, in turn, determines the bubble nucleation rate,

$$\Gamma = \kappa T_c^4 e^{-\frac{F_c(T)}{T_c}},$$

where $\kappa$ is a factor roughly of order 1. Let us introduce

$$\tau = \frac{T_c - T}{T_c} \tag{13}$$

and assume that this parameter is small. To the leading order in $\tau$, we have

$$\rho = \rho_1 \tau, \tag{14}$$

$$\sigma = \sigma_0, \tag{15}$$

where $\rho_1$ and $\sigma_0$ are constants independent of $\tau$. Thus,

$$\Gamma = \kappa T_c^4 e^{-\frac{A}{\tau^2}}, \tag{16}$$

where

$$A = \frac{16\pi}{3}\frac{\sigma_0^3}{\rho_1^2}. \tag{17}$$

As the temperature decreases, the bubble nucleation rate rapidly grows.

### B. Temperature of the phase transition

To estimate the number density and charge of $Q$-balls produced during the phase transition, we need an estimate of the transition temperature. Since the duration of the phase transition is often much shorter than the Hubble time (this corresponds to $\tau \ll 1$), we neglect the cosmological expansion and write for the fraction of volume occupied by the old phase at time $t$ [30]

$$x(t) = \exp[-\Delta(t)],$$

where

$$\Delta(t) = \int_{t_c}^{t} V(t, t')\Gamma(t')dt',$$

$V(t, t') = \frac{4\pi}{3}[u(t - t')]^3$ is the volume, at time $t$, of a bubble of the new phase born at time $t'$, and $u$ is the velocity of the bubble wall. We make use of the standard relation between the Hubble parameter and temperature, $H = T^2/M_{\rm Pl}^*$, where $M_{\rm Pl}^* = \frac{M_{\rm Pl}}{1.66\sqrt{g_*}}$ and $g_*$ hereafter denotes the effective number of the degrees of freedom at the phase transition temperature. We obtain for $\tau \ll 1$

$$\Delta = \kappa u^3 \left(\frac{M_{\rm Pl}^*}{T_c}\right)^4 \int_0^{\tau} (\tau - \tau')^3 e^{-\frac{A}{\tau'^2}} d\tau'. \tag{18}$$

This integral is saturated near the upper limit of integration, and we get

$$\Delta \sim \kappa u^3 \left(\frac{M_{\rm Pl}^*}{T_c}\right)^4 \frac{\tau^{12}}{A^4} e^{-\frac{A}{\tau^2}}.$$

The phase transition occurs when $\Delta$ is roughly of order 1, which happens in a narrow interval of temperatures around

$$\tau_* = A^{1/2} L^{-1/2}, \tag{19}$$

where

$$L = \ln\left[\kappa u^3 A^2 \left(\frac{M_{\rm Pl}^*}{T_c}\right)^4\right], \tag{20}$$

with logarithmic accuracy. Our estimate is valid provided that $\tau_* \ll 1$, i.e.,

$$A \ll L. \tag{21}$$

In what follows, we assume that this is indeed the case; see also Sec. IV B.

Note that at the time of the phase transition, the bubble nucleation rate (16) is still small,

$$\Gamma \sim \frac{T_c^4 A^4}{u^3 \tau_*^{12}} \left(\frac{T_c}{M_{\rm Pl}^*}\right)^4. \tag{22}$$

This is, of course, a consequence of the slow cosmological expansion.

### C. $Q$-balls in the end of the phase transition

We are now ready to estimate the volume from which X particles are collected into a single $Q$-ball. This volume will determine the number density of $Q$-balls immediately after the phase transition and the typical $Q$-ball charge. One way to obtain the estimate is to notice that within a factor of order 1, this volume is the same as the volume of a remnant of the old phase in the midst of the phase transition; see Fig. 1. We estimate the size $R_*$ of a remnant by requiring that it shrinks to small size before a bubble of the new phase is created inside it. The lifetime of a remnant of size $R$ is $R/u$, so the latter requirement gives

$$R_*^3\Gamma(T)\frac{R_*}{u}\sim 1.$$

Making use of Eq. (22), we obtain

$$R_*\sim\frac{u\tau_*^3M_{\rm Pl}^*}{T_c^2A}. \tag{23}$$

Another way to estimate the volume that will shrink to one $Q$-ball is to consider bubbles of the new phase instead and estimate the typical size of a bubble in the midst of the transition. At time $t$, the average bubble volume is

$$\frac{4\pi}{3}R^3(t)=N^{-1}(t)\int_{t_c}^{t}V(t,t')\Gamma(t')x(t')dt',$$

where $N(t)=\int_{t_c}^{t}\Gamma(t')x(t')dt'$ estimates the number density of the bubbles. These integrals are evaluated in the same way as Eq. (18), and we get

$$\frac{4\pi}{3}R_*^3(t)=2\pi\left(\frac{u\tau_*^3M_{\rm Pl}^*}{T_c^2A}\right)^3.$$

This gives the same estimate as Eq. (23), which demonstrates the consistency of the approach.

From now on, we use the following expression for the volume from which X particles are collected into a single $Q$-ball:

$$V_*=\frac{4\pi}{3}R_*^3=\xi\left(\frac{uA^{1/2}M_{\rm Pl}^*}{T_c^2L^{3/2}}\right)^3,$$

where we inserted $\tau_*$ given by Eq. (19), and $\xi$ is a parameter of order 1 that parmetrizes the uncertainty of our estimate.

The number density of $Q$-balls immediately after the phase transition equals $n_Q(T_c)=V_*^{-1}$, and its ratio to the entropy density is

$$\frac{n_Q(T_c)}{s(T_c)}=\frac{45}{2\pi^2g_*}\frac{1}{T_c^3V_*}. \tag{24}$$

Note that for given values of couplings, this ratio is suppressed by $(T_c/M_{\rm Pl})^3$. Since one $Q$-ball contains all excess of X particles in volume $V_*$, its charge is [again assuming X asymmetry equal to quark asymmetry, cf. Eq. (7)]

$$Q=(n_{\rm X}-n_{\bar{\rm X}})V_*=3\Delta_Bs(T_c)V_*. \tag{25}$$

This charge is large, since it is proportional to $(M_{\rm Pl}/T_c)^3$. The X particles are packed into $Q$-balls rather efficiently.

### D. $Q$-balls at present

Once the phase transition completes and $Q$-balls get formed, the ratio of their number density to entropy density stays constant and is given by Eq. (24). With the $Q$-ball charge (25), its mass is found from Eq. (5),

$$m_Q=\frac{4\pi\sqrt{2}}{3}U_0^{1/4}[3\Delta_Bs(T_c)]^{3/4}V_*^{3/4}$$
$$=7.3\cdot\xi^{3/4}\cdot\Delta_B^{3/4}g_*^{3/4}M_{\rm Pl}^{*9/4}\frac{u^{9/4}U_0^{1/4}A^{9/8}}{T_c^{9/4}L^{27/8}}. \tag{26}$$

Hence, $Q$-balls are dark matter candidates, provided that

$$\frac{m_Qn_Q}{s}=f\Delta_B^{3/4}\frac{T_c^{3/4}U_0^{1/4}}{M_{\rm Pl}^{*3/4}}=\frac{\rho_{\rm DM}}{s_0}, \tag{27}$$

where $\rho_{\rm DM}\simeq 1\times 10^{-6}\ {\rm GeV}\cdot{\rm cm}^{-3}$ and $s_0\simeq 3000\ {\rm cm}^{-3}$ are the present mass density of dark matter and entropy density, respectively, and

$$f=19.0\frac{\xi^{-1/4}L^{9/8}}{A^{3/8}g_*^{1/4}u^{3/4}} \tag{28}$$

is a combination of dimensionless parameters. As we see, the dependence on parameter $\xi$ is weak, so we set $\xi=1$ from now on. At this point, we can make a rough estimate for the relevant energy scale. Assuming $U_0\sim T_c^4$, $A\sim 1$, $u\sim 0.1$, $g_*\sim 100$, and $L\sim 100$, we get from Eq. (27) that $T_c$ should be in a ballpark of

$$T_c\sim 1\div 10\ {\rm TeV}. \tag{29}$$

As we pointed out in the introduction, this relatively low energy scale makes the scenario interesting from the viewpoint of collider experiments.

### E. Validity of calculation

A particle physics model in which our mechanism can work must satisfy several requirements. One is the condition (21) or

$$A\ll 4\ln\left(\frac{M_{\rm Pl}^*}{T_c}\right). \tag{30}$$

Another is that X particles do not penetrate into the new phase in the course of the phase transition. This is the case if their mass in the new phase is sufficiently larger than the temperature. Quantitatively, we require that the mass density of remaining free X particles is negligible compared to the mass density of dark matter $Q$-balls,

$$\frac{(n_{\rm X}+n_{\bar{\rm X}})m_{\rm X}|_{(T=0)}}{s}\ll\frac{\rho_{\rm DM}}{s_0}. \tag{31}$$

Let us see what this condition means in terms of parameters.

The dynamics of penetration of X particles into the new phase depends on the bubble wall velocity and the strength of X-particle interaction with cosmic plasma. There are two extreme cases: very slow motion of the bubble wall and very fast motion. Let us consider them in turn.

*Slow wall* Let the wall velocity be so small that there is complete thermal equilibrium for X particles across the wall. Then the chemical potentials in the old and new phases are equal, and since X particles in the old phase are massless, the chemical potential is negligibly small: indeed, in the old phase, $\mu T^2 \sim (n_X - n_{\bar{X}}) \sim \Delta_B T^3$; hence, $\mu/T \sim \Delta_B$. The number density of X particles in the new phase is given by the equilibrium formula for nonrelativistic species, and we have

$$\frac{n_X + n_{\bar{X}}}{s} = s^{-1} \cdot 2\left(\frac{m_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-m_X/T} \sim \frac{1}{g_*}\left(\frac{m_X}{2\pi T}\right)^{3/2} \mathrm{e}^{-m_X/T}, \quad (32)$$

where all quantities, including $m_X$, are evaluated at $T_c$.

*Fast wall* In the opposite case of the fast wall, all X particles that penetrate the new phase stay there. Flux of X particles and $\bar{X}$ particles, for which the momentum normal to the wall exceeds $m_X$, is

$$j_{X\bar{X}} = \frac{2}{(2\pi)^3} \int d^2 p_L \int_{m_X}^{\infty} dp_T \mathrm{e}^{-\sqrt{p_T^2 + p_L^2}/T}.$$

This integral is saturated at $p_T$ near $m_X$ and $p_L \ll p_T$. Hence, we write

$$\sqrt{p_T^2 + p_L^2} = p_T + \frac{p_L^2}{2p_T} = p_T + \frac{p_L^2}{2m_X}$$

and obtain

$$j_{X\bar{X}} = \frac{1}{2\pi^2} m_X T^2 \mathrm{e}^{-m_X/T}.$$

As the wall moves, its radius increases by $dR = udt$, and the number of penetrated particles is

$$j_{X\bar{X}} S dt = j_{X\bar{X}} u^{-1} 4\pi R^2 dR.$$

So, the number density in the new phase is, in the end,

$$n_X + n_{\bar{X}} = u^{-1} j_{X\bar{X}}. \quad (33)$$

This gives the estimate similar to that in the slow wall case, except that instead of particle velocity $\sqrt{T/m_X}$, it involves the wall velocity.

If $X\bar{X}$ annihilation is switched off in the new phase, then the ratio $(n_X + n_{\bar{X}})/s$ stays constant after the phase transition. For any of the above cases, and, hence, for all intermediate ones, up to logarithmic corrections, we get from Eq. (31)

$$\frac{m_X|_{T_c}}{T_c} > \ln\left(\frac{\rho_{DM}}{s_0 T_c}\right) \quad (34)$$

or

$$\frac{h\phi_c}{T_c} > \ln\left(\frac{3 \times 10^{-10}\ \mathrm{GeV}}{T_c}\right). \quad (35)$$

This condition does not apply if $X\bar{X}$ annihilation is efficient in the new phase. If Eq. (35) does not hold, the behavior of X particles in the new phase is similar to that of weakly interacting massive particles [note that Eqs. (32) and (33) show that the X-particle abundance in the new phase just after the phase transition either coincides with or exceeds the equilibrium abundance]. In that case, the condition (31) implies that the $X\bar{X}$-annihilation cross section exceeds the standard weakly-interacting-massive-particle-annihilation cross section. Since the energy scale inherent in the model is high [see Eq. (29)], the latter scenario is not particularly plausible. In what follows, we assume that the inequality (35) holds.

## IV. EXAMPLE: ONE-LOOP MOTIVATED EFFECTIVE POTENTIAL

### A. Critical temperature and $Q$-ball parameters

As an example, let us consider effective potential of a particular one-loop motivated form:

$$U(\phi, T) = \alpha(T^2 - T_{c2}^2)\phi^2 - \gamma T\phi^3 + \lambda\phi^4, \quad (36)$$

where $T_{c2}$, $\alpha$, and $\gamma$ are parameters depending on particle physics at temperature $T$, and we neglect temperature corrections to the quartic self-coupling. We treat $T_{c2}$, $\alpha$, $\gamma$, and $\lambda$ as parameters of the model but keep in mind the relation $\alpha T_{c2}^2 = 2\lambda v^2$, which follows from Eq. (3b).

We assume in what follows that $9\gamma^2 < 32\alpha\lambda$. Then at high temperatures, the effective potential has only one local minimum at $\phi = 0$. As the Universe cools down, the minimum at $\phi \neq 0$ develops. It becomes deeper than the minimum at $\phi = 0$ at $T < T_c$, where

$$T_c^2 = \frac{4\alpha\lambda}{4\alpha\lambda - \gamma^2} T_{c2}^2. \quad (37)$$

In what follows, we need the expression for the height of the scalar potential and mass of the $\phi$ particle, both at zero temperature, in terms of the parameters entering Eqs. (36) and (37):

$$U_0 = \frac{(4\alpha\lambda - \gamma^2)^2}{64\lambda^3} T_c^4, \qquad m_\phi = \left(\frac{4\alpha\lambda - \gamma^2}{\lambda}\right)^{1/2} T_c. \quad (38)$$

In this way, we trade the parameter $v$ in the original Lagrangian (3a) for the parameters relevant for the phase transition.

Let us now rewrite the results of Sec. III in terms of the parameters used in Eq. (36). At small $\tau \equiv \frac{T_c - T}{T_c}$, one has

$$\rho = -\frac{\gamma^2(4\alpha\lambda - \gamma^2)}{8\lambda^3} T_c^4 \tau, \quad (39)$$

$$\sigma = \int_0^{\phi_c} \sqrt{2U(\phi, T_c)} d\phi = \frac{\gamma^3}{24\sqrt{2}\lambda^{\frac{5}{2}}} T_c^3. \quad (40)$$

Thus,

$$A = \frac{\pi}{81\sqrt{2}} \frac{\gamma^5}{\lambda^{3/2}(4\alpha\lambda - \gamma^2)^2} \quad (41)$$

and

$$V_* = \left(\frac{uA^{1/2}M_{\rm Pl}^*}{T_c^2 L^{3/2}}\right)^3 = \left(\frac{\sqrt{\pi}u}{9\cdot 2^{1/4}}\frac{M_{\rm Pl}^*}{T_c^2}\frac{\gamma^{5/2}}{\lambda^{3/4}(4\alpha\lambda-\gamma^2)}\times L^{-3/2}\right)^3, \quad (42)$$

where $L$ is defined in Eq. (20). We find from Eq. (27) the present $Q$-ball dark matter mass density

$$\rho_{\rm DM} = K_\rho \Delta_B^{3/4} s_0 M_{\rm Pl}^{*-3/4} g_*^{-1/4} u^{-3/4} \times \gamma^{-15/8}\lambda^{-3/16}(4\alpha\lambda-\gamma^2)^{5/4}T_c^{7/4}L^{9/8}, \quad (43)$$

where

$$K_\rho = 2^{15/16}3^{7/4}5^{1/4}\pi^{1/8} \approx 22.5.$$

Equating $\rho_{\rm DM}$ to the actual dark matter density, we obtain the critical temperature in terms of other parameters:

$$T_c = K_T \rho_{\rm DM}^{4/7}\Delta_B^{-3/7}s_0^{-4/7}M_{\rm Pl}^{*3/7}g_*^{1/7}u^{3/7}\gamma^{15/14} \times \lambda^{3/28}(4\alpha\lambda-\gamma^2)^{-5/7}\times L^{-9/14}, \quad (44)$$

where

$$K_T = 2^{-15/28}3^{-1}5^{-1/7}\pi^{-1/14} \approx 0.17.$$

In further computations, we use $T_c = 10$ TeV in the argument of logarithm (20), see Eq. (29). Finally, Eqs. (25) and (26) give for the $Q$-ball parameters

$$Q = K_Q \rho_{\rm DM}^{-12/7}\Delta_B^{16/7}s_0^{12/7}M_{\rm Pl}^{*12/7}g_*^{4/7} \times \gamma^{30/7}u^{12/7}\alpha^{29/7}\lambda^{9/7}(4\alpha\lambda-\gamma^2)^{-5}L^{-18/7}, \quad (45)$$

$$m_Q = K_{m_Q}\rho_{\rm DM}^{-5/7}\Delta_B^{9/7}s_0^{5/7}M_{\rm Pl}^{*12/7}g_*^{4/7}u^{12/7} \times \gamma^{30/7}\lambda^{-18/7}(4\alpha\lambda-\gamma^2)^{-6/7}L^{-18/7}, \quad (46)$$

where

$$K_Q = 2^{69/7}\pi^{26/7}3^{-4}5^{-4/7} \approx 0.063,$$
$$K_{m_Q} = 2^{13/7}3^{-5}5^{-4/7}\pi^{26/7} \approx 0.42.$$

The present number density of $Q$-balls is, of course, equal to $\rho_{\rm DM}/m_Q$.

### B. Parameter space

In Sec. III E, we pointed out two conditions that the model should obey. In terms of the parameters of the effective potential, the condition (30) takes the following form [see Eq. (41)]:

$$\frac{\pi}{81\sqrt{2}}\frac{\gamma^5}{\lambda^{3/2}(4\alpha\lambda-\gamma^2)^2} \ll 4\ln\left(\frac{M_{\rm Pl}^*}{T_c}\right) \simeq 120. \quad (47)$$

The concrete form of the condition (35) is obtained by noticing that

$$\phi_c \equiv \langle\phi\rangle|_{T=T_c} = \frac{\gamma}{2\lambda}T_c. \quad (48)$$

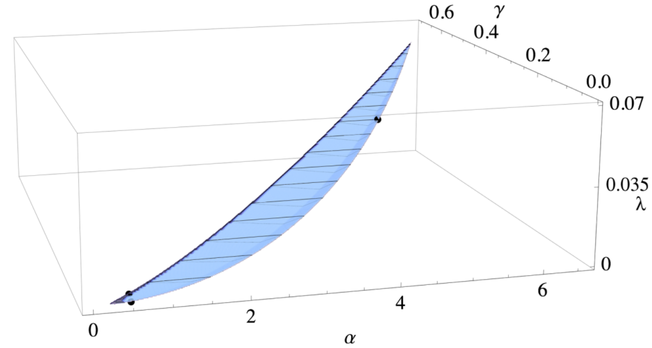


FIG. 2 (color online). The region in the parameter space consistent with the constraints (47), (49), and (50) for $h = 5$. Dots are the points for which the mass, critical temperature, and $Q$-ball parameters are listed in Tables I and II.

We find

$$h\frac{\gamma}{2\lambda} > 25. \quad (49)$$

We also assume that the quartic self-coupling of the field $\phi$ is not particularly small and impose a mild constraint motivated by naturalness argument,

$$\lambda > \frac{\alpha^2}{64\pi^2}. \quad (50)$$

The conditions (47), (49), and (50) are actually quite restrictive. In particular, they require that the $\phi$-$\chi$ coupling is rather strong. For $h\sim 5$, the available region in the parameter space is fairly large, as shown in Fig. 2. This region becomes considerably smaller already for $h\sim 3$; see Fig. 3.

We scanned the available regions and found the following range of masses $m_\phi$ and critical temperatures, at which our mechanism does work:

$$m_\phi \in 3\times 10^2 \div 1\times 10^4\ \text{GeV},$$
$$T_c \in 1\times 10^3 \div 3\times 10^4\ \text{GeV}.$$

For the $Q$-ball parameters we obtained the range

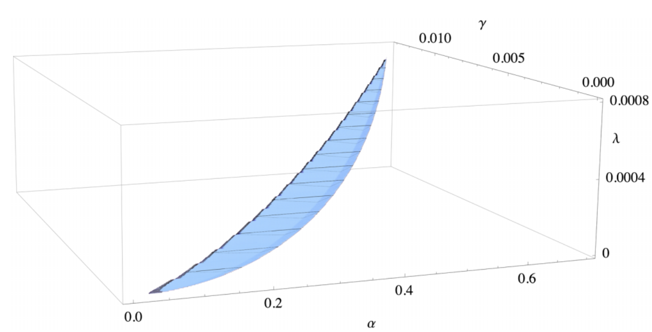


FIG. 3 (color online). Same as in Fig. 2 but for $h = 3$.

TABLE I. $Q$-ball parameters for particular values of couplings (three left columns) and bubble wall velocity $u = 0.03$.

| | | | $m_\phi$, GeV | $T_c$, GeV | $Q$ | $m_Q$, g | $n_Q$, cm$^{-3}$ |
|---|---|---|---|---|---|---|---|
| $\alpha = 5$ | $\gamma = 0.4$ | $\lambda = 0.04$ | $1.9 \times 10^4$ | $4.8 \times 10^3$ | $8.7 \times 10^{19}$ | $4.2 \times 10^{-5}$ | $4.5 \times 10^{-26}$ |
| $\alpha = 0.5$ | $\gamma = 0.02$ | $\lambda = 0.002$ | $8.1 \times 10^3$ | $6.1 \times 10^3$ | $7.2 \times 10^{19}$ | $2.3 \times 10^{-5}$ | $8.1 \times 10^{-26}$ |
| $\alpha = 0.5$ | $\gamma = 0.004$ | $\lambda = 0.0004$ | $3.7 \times 10^3$ | $2.6 \times 10^3$ | $1.9 \times 10^{19}$ | $5.6 \times 10^{-6}$ | $3.4 \times 10^{-25}$ |

TABLE II. Same but for $u = 0.3$.

| | | | $m_\phi$, GeV | $T_c$, GeV | $Q$ | $m_Q$, g | $n_Q$, cm$^{-3}$ |
|---|---|---|---|---|---|---|---|
| $\alpha = 5$ | $\gamma = 0.4$ | $\lambda = 0.04$ | $4.9 \times 10^4$ | $1.2 \times 10^4$ | $3.9 \times 10^{21}$ | $1.9 \times 10^{-3}$ | $1.0 \times 10^{-27}$ |
| $\alpha = 0.5$ | $\gamma = 0.02$ | $\lambda = 0.002$ | $2.1 \times 10^4$ | $1.6 \times 10^4$ | $3.2 \times 10^{21}$ | $1.0 \times 10^{-3}$ | $1.9 \times 10^{-27}$ |
| $\alpha = 0.5$ | $\gamma = 0.004$ | $\lambda = 0.0004$ | $9.4 \times 10^3$ | $6.7 \times 10^3$ | $8.2 \times 10^{20}$ | $2.5 \times 10^{-4}$ | $7.8 \times 10^{-27}$ |

$$m_Q \in 3 \times 10^{-7} \div 3 \times 10^{-3} \text{ gram},$$
$$Q \in 10^{19} \div 10^{22},$$
$$n_Q \in 1 \times 10^{-27} \div 3 \times 10^{-24} \text{ cm}^{-3}.$$

Three particular examples, corresponding to the points in Fig. 2, are listed in Tables I and II for two values of the bubble wall velocity.

## V. CONCLUSION

We considered a mechanism for producing $Q$-balls in the course of the first-order phase transition and described it quantitatively. As we have seen, this mechanism efficiently packs massive stable particles, which otherwise would be overproduced, and, thus, drastically reduces the mass density.

Using a well-studied model of Friedberg-Lee-Sirlin $Q$-balls, we obtained formulas for the dark matter properties, such as charge, mass, and concentration of dark matter $Q$-balls, as well as masses of scalar fields, depending on the parameters of the transition. As an example, we considered this mechanism in a theory with the effective potential of one-loop motivated form.

We have seen that the main parameter is the temperature of the phase transition, whose adjustment yields the right value of the present dark matter density. A remarkable property of the mechanism is that for a wide range of parameters, independently of particle physics and an effective potential model, the estimate for the energy scale is $T_c \sim$ 1–10 TeV.

The main requirement for effective packing of particles into $Q$-balls is that the phase transition is strongly first order. For one-loop motivated effective potential, this implies strong $\phi$-$\chi$ coupling and gives the main constraint on available parameter space. However, for other models, this is not necessarily the case, since there are other ways to make the first-order phase transition strong enough (see, e.g., Refs. [31,32] and references therein).

Our study was motivated by the possibility that X-particle asymmetry is related to the baryon asymmetry. However, this is optional. The mechanism described works in the same way, with $3\Delta_B$ replaced by $\Delta_X$, if $n_X$ is considered as yet another free parameter.

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