PHYSICAL REVIEW D **87,** 083511 (2013)

Lee-Wick radiation induced bouncing universe models

Kaushik Bhattacharya,[1,*] Yi-Fu Cai,[2,†] and Suratna Das[3,‡]

[1]*Department of Physics, Indian Institute of Technology, Kanpur, Kanpur 208016, India*
[2]*Department of Physics, McGill University, Montréal, Quebec H3A 2T8, Canada*
[3]*Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai 400005, India*
(Received 9 January 2013; published 9 April 2013)

The present article discusses the effect of a Lee-Wick partner infested radiation phase of the early universe. As Lee-Wick partners can contribute negative energy density it is always possible that at some early phase of the universe when the Lee-Wick partners were thermalized the total energy density of the universe became very small making the effective Hubble radius very big. This possibility gives rise to the probability of a bouncing universe. As will be shown in the article a simple Lee-Wick radiation is not enough to produce a bounce. There can be two possibilities which can produce a bounce in the Lee-Wick radiation phase. One requires a cold dark matter candidate to trigger the bounce and the other possibility requires the bouncing temperature to be fine-tuned such as all the Lee-Wick partners of the standard fields are not thermalized at the bounce temperature. Both the possibilities give rise to a blue-tilted power spectrum of metric perturbations. Moreover the bouncing universe model can predict the lower limit of the masses of the Lee-Wick partners of chiral fermions and massless gauge bosons. The mass limit intrinsically depends upon the bounce temperature.

DOI: 10.1103/PhysRevD.87.083511 PACS numbers: 98.80.−k, 11.10.−z

I. INTRODUCTION

Recently the idea of constructing a unitary, gauge-invariant, Lorentz-invariant and divergence free theory of quantum electrodynamics by invoking unusual partners of the standard particles in the Lagrangian, originally proposed by Lee and Wick [1,2], has been generalized to construct a Lee-Wick theory of the standard model of particle physics [3]. This model, while it still suffers from unsettled issues such as quantum instability of the vacuum due to gravitational interaction [4], attracts a lot of attention in the literature for several phenomenological benefits. Namely, the mass of the Higgs field in such a quantum field theory is perturbatively stable against the quadratically divergent radiative corrections and thus the well-known "hierarchy puzzle" of the standard model of particle physics could be avoided. There are many other interesting applications of the Lee-Wick idea of which some are discussed here. In Ref. [5] a minimal extension of the Lee-Wick standard model (LWSM) is considered; in Ref. [6] a LWSM with more than one Lee-Wick (LW) partner for each standard model particle is studied. Reference [7] deals with gauge-coupling unification in the Lee-Wick framework and in Refs. [8,9] analysis of two-Higgs doublet models where one of the doublet contains Lee-Wick fields is presented. In Ref. [10] the process $gg \to h_0 \to \gamma\gamma$ is studied in the framework of LWSM where the authors predict small changes in the rate of these processes, due to the presences of Lee-Wick fields, from those rates calculated from other models such as universal extra dimensions. In Ref. [11] Higgs pair production processes $gg \to h_0 h_0$ and $gg \to h_0 \tilde{p}_0$ are studied in LWSM framework.

It was recently realized in Ref. [12] that a model constructed out of a Lee-Wick type scalar field theory is capable of yielding a bounce during the evolution of the universe within the framework of a homogeneous and isotropic cosmological background. In this model the energy density and pressure of all other kinds of fields are supposed to be negligible. In this scenario the Lee-Wick partners, which arise from higher derivative operators, evolve as a tracking solution to the normal matter (the standard scalar field) and can break certain energy conditions when the energy scale of the universe becomes high enough. Therefore, a nonsingular bounce takes place which avoids the big bang singularity widely existing in the standard Friedmann-Robertson-Walker (FRW) cosmologies. Interests in such Lee-Wick nonsingular bouncing models have increased recently as these models [13,14] can be considered as an alternative to inflation and can be used to explain the origin of large scale structure in the Universe. In this paradigm the primordial vacuum fluctuations leave the Hubble radius during a matter dominated contracting phase and then form a nearly scale-invariant power spectrum after the bounce.

However, it was found that nonsingular Lee-Wick bouncing cosmologies induced by pressureless matter are in general unstable while accompanied with radiation and a bouncing solution might be achieved only when an extremely fine tuning of the initial phases of the field configuration is assumed [15,16]. This problem can be roughly understood as follows. The normal matter which scales as

*kaushikb@iitk.ac.in
†yifucai@physics.mcgill.ca
‡suratna@tifr.res.in

a^{-3} as a function of the scale factor a could be easily suppressed by radiation matter which scales as a^{-4} in the contracting phase. If we simply add a radiation component to nonrelativistic matter within a framework of a Lee-Wick bounce model without fine-tuning of the initial values of radiation then the radiation would become dominant, during the contraction phase of the universe, and produce a big crunch singularity before the bounce takes place. In Ref. [15], the effects of Lee-Wick partners of radiation fields were taken into account as well but the energy scale of the universe is assumed to be below the scale of thermalization of Lee-Wick radiation. Then, it was found the effective energy density of Lee-Wick radiation increases slower than that of regular radiation and thus cannot help to realize the bounce at the low temperature regime.

But unlike these models of bouncing cosmologies, where the energy density and pressure of the universe arises from a scalar field (whose energy scales as a^{-3}) or a radiation field (whose energy scales as a^{-4}), one can conceive a phase of the early universe where we have the normal standard model particles and their Lee-Wick parters (as virtual resonances) in thermal equilibrium as shown in Refs. [17,18]. This phase is a nonstandard radiation dominated phase where one can have a state parameter $w \sim 1$ and consequently the energy density scales $\sim a^{-6}$. It is explicitly described in Ref. [18] that in such kind of a Lee-Wick partner infested radiation dominated universe the time-temperature relation in standard cosmology changes and the universe may go through out-of-equilibrium processes when some of the Lee-Wick partners become nonrelativistic. It was noticed that these kinds of cosmologies can produce negative energy densities and pressure when one includes chiral fermions and massless gauge bosons [19,20]. This particular point turned out to be a serious drawback for these models. It was shown that only under some specific conditions this kind of an early radiation domination can exist. On the other hand bouncing cosmologies naturally require a time (bounce time) when the Hubble parameter turns out to be zero (indicating a zero energy density). It turns out that one can construct an interesting model of a bouncing cosmology from the nonstandard Lee-Wick radiation phase.

In this paper we study the possibility of realizing a Lee-Wick bounce at high temperature by taking into account the radiation fields. The Lee-Wick resonances are produced dramatically when the temperature of the radiation fluid is much higher than the mass of heavy Lee-Wick fields. As the contribution of Lee-Wick radiation to the total energy density is always negative it enlarges the viable phase space for the universe to experience the nonsingular bounce. Unlike the method developed in Ref. [15] in which the radiation fields are described by the field configuration of gauge fields, we directly study thermodynamics of normal particles and their Lee-Wick partners following Refs. [17,18]. However, for a radiation dominated universe purely dominated by standard fields and their Lee-Wick partners, one still cannot obtain a nonsingular bounce. This is because, at the bouncing moment when the energy density of Lee-Wick resonances are able to cancel that of normal radiation, the pressure of the universe keeps positive and then prevents the background from evolving into an expanding phase. Thus we introduce a cold dark matter (CDM) component in our model which can easily solve this problem. As a consequence, our cosmological model, which realizes a nonsingular bouncing solution in the thermal Lee-Wick theory, consists of cold dark matter fluid, radiation, and Lee-Wick resonances.

If one does not include the CDM component then one cannot obtain the required condition of the bounce as discussed in the last paragraph. This fact is true if all the standard particles and their Lee-Wick partners are in thermal equilibrium near the bounce time. But if it happens that the bounce temperature is such that some of the Lee-Wick partners' masses are more than the bounce temperature then the plasma around the bounce time will have fewer Lee-Wick partners. Interestingly, this kind of a model where the bounce temperature is less than some of the masses of the Lee-Wick partners can give rise to a nonsingular bounce. This model does not require any CDM candidate to make the pressure negative at the bounce time. In the present article we will also discuss this alternative bouncing model. Both of the bouncing models yield a blue-tilted power spectrum of metric perturbations in the expanding phase after bounce.

The outline of this paper is as follows: In Sec. II we briefly introduce the thermodynamics of Lee-Wick particles. In Sec. III, we study the condition for a Lee-Wick thermal bounce and then provide a concrete cosmological model which involves both kinds of bounce scenarios as described above. Section IV is devoted to the discussion of cosmological perturbations in both of these models. The final section presents some conclusions and discussion. For completeness we have attached an Appendix at the end of the article which discusses the mode matching conditions near the bounce point.

II. THERMODYNAMICS OF LEE-WICK PARTICLES REVISITED

In this section we will give a brief account of the thermodynamics of Lee-Wick particles. We will show that in a realistic model of Lee-Wick theory, where all the degrees of freedom of the standard particles along with their Lee-Wick partners are taken into account properly, the energy density of the Lee-Wick resonance dominated universe would become very small when the heaviest of the Lee-Wick partners thermalize due to the very high temperature of the universe. This property is in favor of a bouncing scenario which requires the Hubble parameter (which is directly proportional to the square root of the total energy density of the universe) to vanish at the time of bounce.

It is noted in Refs. [17,18] that the Lee-Wick particles behave very differently from their standard partners in the relativistic regime. For the standard bosons and fermions the energy density, pressure and the entropy density read as

$$\rho_b^{(\mathrm{sm})} = \frac{g\pi^2 T^4}{30}, \qquad p_b^{(\mathrm{sm})} = \frac{g\pi^2 T^4}{90}, \qquad s_b^{(\mathrm{sm})} = \frac{2g\pi^2 T^3}{45}, \tag{1}$$

and

$$\rho_f^{(\mathrm{sm})} = \frac{7g\pi^2 T^4}{240}, \qquad p_f^{(\mathrm{sm})} = \frac{7g\pi^2 T^4}{720}, \qquad s_f^{(\mathrm{sm})} = \frac{7g\pi^2 T^3}{180}, \tag{2}$$

respectively. Here the subscripts b and f stand for bosons and fermions and g stands for any number of internal degrees of freedom of the relativistic species. On the other hand, it is shown in Refs. [17,18] that the Lee-Wick particles at high energies contribute negatively to the energy density, pressure and entropy density as

$$\rho_b^{(\mathrm{LW})} = -g\left(\frac{\pi^2 T^4}{30} - \frac{M^2 T^2}{24}\right), \tag{3}$$

$$p_b^{(\mathrm{LW})} = -g\left(\frac{\pi^2 T^4}{90} - \frac{M^2 T^2}{24}\right), \tag{4}$$

$$s_b^{(\mathrm{LW})} = -g\left(\frac{2\pi^2 T^3}{45} - \frac{M^2 T}{12}\right) \tag{5}$$

for the bosonic Lee-Wick partners and

$$\rho_f^{(\mathrm{LW})} = -g\left(\frac{7\pi^2 T^4}{240} - \frac{M^2 T^2}{48}\right), \tag{6}$$

$$p_f^{(\mathrm{LW})} = -g\left(\frac{7\pi^2 T^4}{720} - \frac{M^2 T^2}{48}\right), \tag{7}$$

$$s_f^{(\mathrm{LW})} = -g\left(\frac{7\pi^2 T^3}{180} - \frac{M^2 T}{24}\right) \tag{8}$$

for the fermionic Lee-Wick partners. In the above equations M is the mass of a generic Lee-Wick partner and as the system is relativistic $T \gg M$. The Lee-Wick particles were initially introduced as resonance particles to overcome the divergences appearing in a theory and consequently when $T \gg M$ the standard model particles are naturally ultrarelativistic. If one considers a toy model where each standard particle is accompanied by one Lee-Wick partner and the number of degrees of freedom of the Lee-Wick partner is the same as that of its standard partner then in such a scenario the total energy density, pressure and entropy density turn out to be positive as [17,18]

$$\rho_b = \rho_b^{(\mathrm{sm})} + \rho_b^{(\mathrm{LW})} = \frac{gM^2T^2}{24}, \tag{9}$$

$$p_b = p_b^{(\mathrm{sm})} + p_b^{(\mathrm{LW})} = \frac{gM^2T^2}{24}, \tag{10}$$

$$s_b = s_b^{(\mathrm{sm})} + s_b^{(\mathrm{LW})} = \frac{gM^2T}{12} \tag{11}$$

for the bosonic sector and

$$\rho_f = \rho_f^{(\mathrm{sm})} + \rho_f^{(\mathrm{LW})} = \frac{gM^2T^2}{48}, \tag{12}$$

$$p_f = p_f^{(\mathrm{sm})} + p_f^{(\mathrm{LW})} = \frac{gM^2T^2}{48}, \tag{13}$$

$$s_f = s_f^{(\mathrm{sm})} + s_f^{(\mathrm{LW})} = \frac{gM^2T}{24} \tag{14}$$

for the fermionic sector. But this simple scenario, where each standard particle is accompanied by one Lee-Wick particle with equal number of degrees of freedom, does not serve the purpose while dealing with realistic scenarios. It is discussed in Ref. [19] that in the fermionic sector each chiral fermion requires two Lee-Wick partners to eliminate the higher derivative terms in an initial higher derivative Lagrangian to construct a Lee-Wick theory. Such discrepancies also arise in the bosonic sector when one considers massive Lee-Wick partners (with three degrees of freedom) of massless gauge bosons (with two longitudinal degrees of freedom) [20]. Both these cases lead to unacceptable scenarios with negative energy density. It is shown in Ref. [20] that considering both the fermionic and bosonic sector together one still can come up with a scenario where the total energy density, pressure and entropy density can become positive.

In a realistic scenario, where each fermion is accompanied by two Lee-Wick partners and the extra degrees of freedom of the massive Lee-Wick partners of each massless gauge boson are taken into account, the total energy density of the realistic Lee-Wick plasma can be written as [20]

$$\rho = \frac{\tilde{M}^2}{24}\tilde{g}_{*N}T^2 - \frac{7\pi^2}{240}\tilde{g}_F T^4 - \frac{\pi^2}{30}nT^4. \tag{15}$$

Here the new number of degrees of freedom $\tilde{g}_{*N}$ is given as

$$\tilde{g}_{*N} = \sum_{i=\mathrm{bosons}} g_{iN}\left(\frac{M_i}{\tilde{M}}\right)^2\left(\frac{T_i}{T}\right)^2 + \sum_{i=\mathrm{fermions}} g_{iF}\left(\frac{M_i}{\tilde{M}}\right)^2\left(\frac{T_i}{T}\right)^2, \tag{16}$$

where g_{iN} for bosonic particles stands for the number of internal degrees of freedom g_i for the partners of massive standard bosons (which may be 2 or 1), while for standard massless vector boson partners it equals $g_i + 1$ where primarily $g_i = 2$. Also the unpaired fermionic contribution comes with $\tilde{g}_F$ where

$$\tilde{g}_F = \sum_{i=\text{fermions}} g_{iF}\left(\frac{T_i}{T}\right)^4, \tag{17}$$

where $\tilde{g}_F$ solely arises from the unpaired fermionic Lee-Wick partners of the standard model particles. The quantity n is defined as

$$n = \sum_{i=\text{massive vect. bosons}} \left(\frac{T_i}{T}\right)^4. \tag{18}$$

Here T_i is the temperature at which the ith massive Lee-Wick vector boson partner of a standard massless gauge boson is equilibrated and n denotes the number of massive vector boson partners of massless standard gauge bosons if all the species are in thermal equilibrium at the same temperature T. The sum appearing in Eq. (18) does not include all the massive Lee-Wick vector boson partners, but includes only those which are partners of massless standard gauge bosons. The total pressure of this Lee-Wick infested plasma is given by

$$p = \frac{\tilde{M}^2}{24}\tilde{g}_{*N}T^2 - \frac{7\pi^2}{720}\tilde{g}_F T^4 - \frac{\pi^2}{90}nT^4. \tag{19}$$

It must be noted that if the energy density of the Lee-Wick partner infested universe is positive then the pressure of the same universe must be positive. It is also to be noted that the negative contributions to the total energy density and pressure are coming from the Lee-Wick partners of the chiral fermions and massless gauge bosons.

III. MODEL(S) OF LEE-WICK THERMAL BOUNCE

Our main aim is to construct a nonsingular bouncing cosmology within the arena of the thermal Lee-Wick scenario in which the universe initially starts its evolution in a contracting phase and evolves into a standard thermal expanding phase smoothly and continuously through a nonsingular bounce. We first start with a general discussion on the conditions required for a nonsingular bounce and would consider a simple model where the universe consists of bosonic and fermionic radiation fields and their thermalized Lee-Wick resonances. We will show that this simple toy model, which consists of only radiation fields accompanied by their thermal Lee-Wick partners, is incapable of achieving the conditions required for a nonsingular bounce and then we will try to illustrate two possible modifications to this simple model where a bounce can be achieved.

We start with a spatially flat FRW metric

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2, \tag{20}$$

where $a(t)$ stands for the scale factor. The dynamics of the FRW universe is described by the Hubble parameter $H \equiv \dot{a}/a$ and its time derivative $\dot{H}$ which obey the well-known Friedman equations:

$$H^2 = \frac{8\pi G}{3}\rho, \tag{21}$$

$$\dot{H} = -4\pi G(\rho + p). \tag{22}$$

At the moment of the nonsingular bounce, i.e., at $t = t_B$, the Hubble parameter vanishes $H(t_B) = 0$ and in order to ensure that the universe enters an expanding phase one also requires $\dot{H}(t_B) > 0$ at the same moment.

Let us consider that in the contracting phase the universe was initially radiation dominated with the standard particles (with $w = 1/3$). As the scale factor contracts the temperature of the Universe increases and the Lee-Wick particles (which are heavier than their standard model partners) will gradually start to contribute thermally as $t \to 0^-$ and just before the bounce the energy density of the universe would be given by Eq. (15). To satisfy the first condition of bounce i.e., $H(t_B) = 0$ one requires $\rho(t_B) = 0$ which can be seen from Eq. (21). Thus, assuming that just before the bounce all the particle species and their Lee-Wick partners are thermalized, it can be seen from Eq. (15) that at the time of bounce the energy density vanishes, i.e., $\rho(t_B) = 0$, to yield

$$\frac{\tilde{M}^2}{24}\tilde{g}_{*N}T_B^2 = \frac{7\pi^2}{240}\tilde{g}_F T_B^4 + \frac{\pi^2}{30}nT_B^4, \tag{23}$$

where T_B is the temperature of the universe at bouncing time t_B. This shows that at bounce the pressure given in Eq. (19) would be positive as

$$p(t_B) = \frac{7\pi^2}{360}\tilde{g}_F T_B^4 + \frac{\pi^2}{45}nT_B^4 > 0. \tag{24}$$

Thus at the bouncing point one has from Eq. (22) that

$$\dot{H}(t_B) = -4\pi G(\rho(t_B) + p(t_B)) < 0, \tag{25}$$

which indicates that after the bounce the universe fails to enter into an expanding phase. This means that in this simple cosmological model which is comprised of pure radiation and their thermalized Lee-Wick resonances, one cannot obtain a bouncing solution for a universe. Now we will discuss two distinct cases which slightly deviate from this simple scenario discussed above and where a bouncing universe scenario can be realized.

A. Lee-Wick infested radiation plasma and a CDM component

First we consider a CDM component along with the radiation plasma which is composed of thermal standard particles and their thermalized Lee-Wick partners. The CDM candidate is nonrelativistic throughout the whole evolution and hence we can neglect its pressure in the following analysis. The CDM component too can have Lee-Wick partners according to the Lee-Wick scenario, but these partners would be heavier than the CDM particle under consideration. Hence the Lee-Wick partners of the CDM component could never thermalize in the evolution (as the CDM remains nonrelativistic throughout) and would remain off shell. Thus these Lee-Wick partners of the CDM component would not contribute thermally to the energy density of the radiation plasma. Thus the energy density and pressure of the CDM component would be

$$\rho_{\rm D} = n_{\rm D} m, \qquad p_{\rm D} \sim 0, \tag{26}$$

respectively, where $n_{\rm D}$ and m are the number density and mass of the CDM particles, respectively, and $m \gg T$ during all the phases of evolution of the universe.

Let us briefly illustrate the scenario under consideration. We naively imagine an inverse picture of evolution of the expanding phase of the universe in the contracting phase. Hence in the contracting phase the universe initially evolves in a CDM dominated phase with $w = 0$. As the universe contracts the temperature of the universe increases and the universe enters a standard radiation dominated phase with $w = 1/3$. As the temperature of the universe keeps on increasing with the contraction the Lee-Wick resonances start to contribute thermally and negatively to the energy density of the radiation bath. In that case, though the temperature of the universe increases with contraction the energy density starts to decrease with more and more Lee-Wick particles (mainly the partners of chiral fermions and massless gauge bosons) contributing thermally. The CDM component, during this period, evolves nonrelativistically with the background evolution decoupled from the thermal bath. Once the energy density of the Lee-Wick infested radiation bath becomes slightly smaller than the energy density of the CDM component, the later dominates the energy density of the universe and helps a bounce to occur, as will be shown next. This is not a conventional matter domination phase as during this period the state parameter of the cosmic fluid would not be zero. This whole scenario is shown graphically in Fig. 1.

According to the above discussion, the total energy density of the universe in this scenario, which consists of the energy density of the Lee-Wick infested radiation plasma $\rho_R(t_B)$ and that of the CDM particles $\rho_D(t_B)$, at the bouncing time t_B turns out to be

$$\begin{aligned}\rho(t_B) &\equiv \rho_R(t_B) + \rho_D(t_B)\\ &= \frac{\tilde{M}^2}{24}\tilde{g}_{*N}T_B^2 - \frac{7\pi^2}{240}\tilde{g}_F T_B^4 - \frac{\pi^2}{30}nT_B^4 + \rho_{\rm D}(t_B),\end{aligned} \tag{27}$$

whereas the expression for the pressure of this Lee-Wick infested radiation plasma aided with the CDM particles remains the same as given in Eq. (19). If one wants to make $p(t_B) < 0$, required for a bounce at a temperature $T_{\rm B}$, then the first constraint on the model is given by

$$\frac{\tilde{M}^2}{24}\tilde{g}_{*N}T_{\rm B}^2 < \frac{7\pi^2}{720}\tilde{g}_F T_{\rm B}^4 + \frac{\pi^2}{90}nT_{\rm B}^4. \tag{28}$$

If this constraint on the system is fulfilled, then one can indeed have a bouncing universe where the energy density of the CDM candidate has to satisfy the second constraint:

$$\rho_{\rm D}(t_B) > \frac{7\pi^2}{360}\tilde{g}_F T_{\rm B}^4 + \frac{\pi^2}{45}nT_{\rm B}^4, \tag{29}$$

which is obtained by requiring that $\rho(t_B) = 0$ and using the first constraint given in Eq. (28). Along with these two

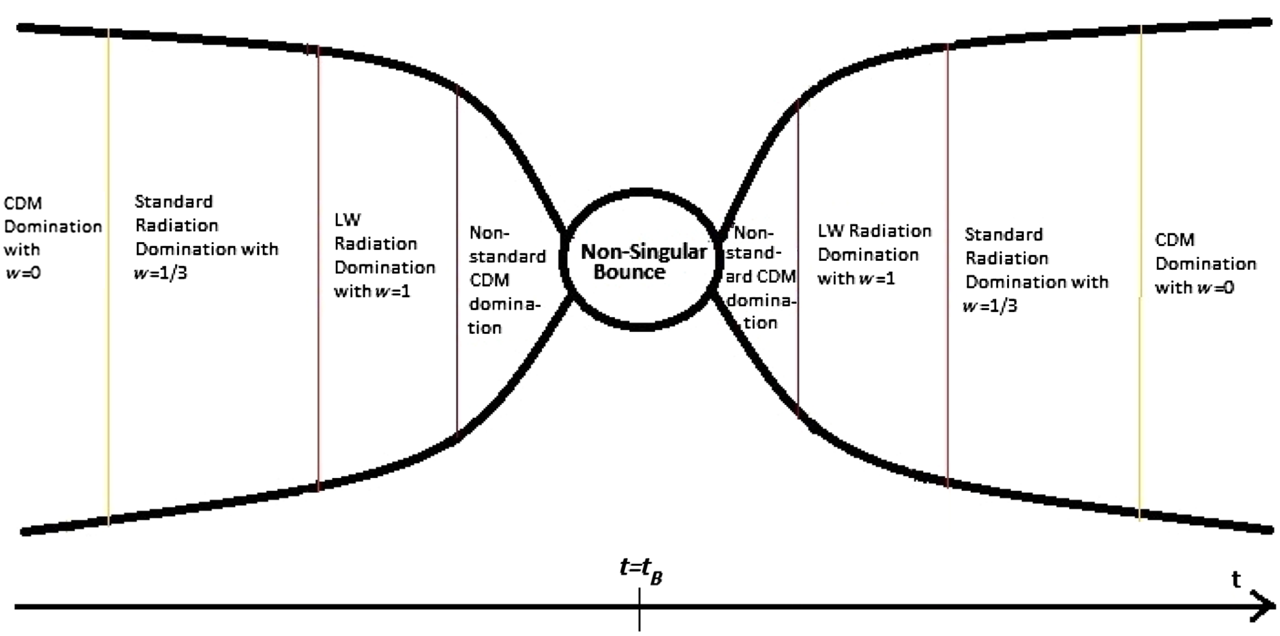

FIG. 1 (color online). Figure showing the bounce point and the energy balance of the universe. The CDM domination near the bounce point is due to the addition of the nonstandard CDM-like component in the theory.

constraints we also must have the other constraint about the relativistic nature of the heaviest LW particle at T_B which reads as

$$T_{\rm B} > \tilde{M}, \tag{30}$$

This condition along with Eq. (28) yields the constraint on $\tilde{g}_{*N}$ as

$$\tilde{g}_{*N} \geq \frac{7\pi^2}{30}\tilde{g}_F + \frac{4\pi^2}{15}n. \tag{31}$$

If we fix the free parameters of this model as $T_B \sim 10^{16}$ GeV and roughly use the standard model values for calculating the number of internal degrees of freedom as $\tilde{g}_F \sim 40$ and $n \sim 10$, then these three parameters yield $\rho_D(t_B) > 4 \times 10^{64}$ GeV^4 and $\tilde{g}_{*N} \geq 100$. Also to get $\rho(t_B) = 0$ one requires from Eq. (27) $\rho_R(t_B) = -\rho_D(t_B)$ which yields $\tilde{M} \approx 9 \times 10^{15}$ GeV which is the mass of the heaviest Lee-Wick particle in the model under consideration.

B. Bouncing Universe within thermal LW framework without a CDM component

We can have another scenario where only radiation can trigger the bounce. But for this to happen some conditions have to be met. The conditions are related to the mass of some Lee-Wick partners of the normal particles and the bouncing temperature T_B. In this case we consider that the bounce temperature T_B is smaller than the masses of some Lee-Wick particles. If this happens then some standard particles' Lee-Wick partners will not be able to thermalize as their masses are higher than T_B, but their standard partners being lighter would be thermalized at T_B. Due to this effect these (unpaired) standard particles will contribute only with positive radiation energy density. Such standard particles whose Lee-Wick partners could not thermalize at T_B can both be bosonic and fermionic. In this scenario the total energy density of the Universe can be written as

$$\rho(t_B) = \frac{\tilde{M}^2}{24}\tilde{g}_{*N}T_B^2 - \frac{7\pi^2}{240}\tilde{g}_F T_B^4 - \frac{\pi^2}{30}nT_B^4 + \frac{\tilde{c}}{4}T_B^4, \tag{32}$$

and the total pressure would be

$$p(t_B) = \frac{\tilde{M}^2}{24}\tilde{g}_{*N}T_B^2 - \frac{7\pi^2}{720}\tilde{g}_F T_B^4 - \frac{\pi^2}{90}nT_B^4 + \frac{\tilde{d}}{4}T_B^4. \tag{33}$$

Here $\tilde{c}$ and $\tilde{d}$ encapsulate all the numbers of degrees of freedom and the corresponding factors of those standard particles whose Lee-Wick partners could not thermalize at the bouncing point and $\tilde{M}$ is the mass of the heaviest Lee-Wick particle which could thermalize at or before t_B. Thus in this case too we have $\tilde{M} < T_B$. To satisfy the bouncing condition $p(t_B) < 0$ one requires [using Eq. (33)]

$$\frac{\tilde{M}^2}{24}\tilde{g}_{*N}T_B^2 < \frac{7\pi^2}{720}\tilde{g}_F T_B^4 + \frac{\pi^2}{90}nT_B^4 - \frac{\tilde{d}}{4}T_B^4, \tag{34}$$

and to achieve the other bouncing condition i.e., $\rho(t_B) = 0$ one gets

$$\tilde{c} - \tilde{d} > \frac{7\pi^2}{90}\tilde{g}_F T_B^4 + \frac{4\pi^2}{45}nT_B^4, \tag{35}$$

where we have used Eqs. (32) and (34). This scheme of a bouncing universe is also a possibility as for any thermalized bosonic or fermionic standard model particle $p = \frac{1}{3}\rho$ and thus $\tilde{d} = \frac{1}{3}\tilde{c}$. Hence the above condition can be satisfied by choosing an appropriate set of parameters.

IV. COSMOLOGICAL PERTURBATIONS IN BOUNCING UNIVERSE

In the previous section two distinct thermal Lee-Wick scenarios are considered where a nonsingular bouncing universe scenario can be achieved. We devote this section to study the dynamics of linear cosmological perturbations generated in these models of thermal Lee-Wick bounce and to investigate the nature of the power spectrum generated in such models. We focus on adiabatic fluctuations and consider matter components without anisotropic stress (we refer to Ref. [21] for a comprehensive review on the theory of cosmological perturbations). In general, there are mainly two methods of analyzing cosmological perturbations in bouncing scenarios. One is to introduce a canonical variable of perturbation mode v (known as the Mukhanov-Sasaki variable) of which the quadratic action is of canonical form in the frame of a conformal time coordinate. This variable is associated with the curvature fluctuation in comoving gauge ζ through $v = z\zeta$ where z is a background dependent coefficient and is roughly proportional to the scale factor if the background evolution is stable. A detailed calculation using a generalized canonical variable v in nonsingular bounce cosmologies was carried out in Ref. [22] and it was shown that such a method is more suitable for those early universe models in which the primordial perturbations originate from quantum fluctuations. Another method is to study the gravitational potential Φ directly. The advantage of this method over the other is that it is much easier to impose initial conditions while analyzing gravitational potential Φ as cosmological perturbations [23]. We will follow the second method for our analysis of cosmological perturbations.

Before going into the details of the analysis of cosmological perturbations and the corresponding power spectra let us discuss briefly how the modes of these primordial perturbations evolve in the contracting and the expanding phases. For the discussion let us only consider the case with the CDM component and Fig. 2 explains the essence of this particular bouncing model. Similar analysis also holds for the other case which does not consider any CDM component. In Fig. 2 it is shown how the Hubble radius $|\mathcal{H}|^{-1}$ behaves in the space-time diagram. The lowermost heavy curve shows that the universe is undergoing a contracting phase where the energy density of the universe is changing

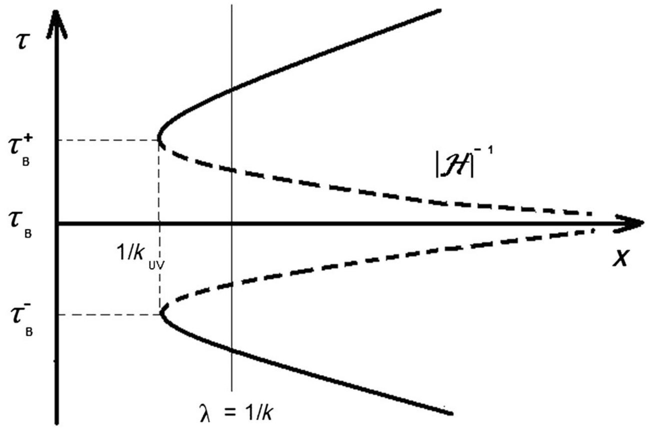

FIG. 2. Figure showing the contracting and expanding phases of the universe in the bouncing model. The horizontal axis is a comoving spatial coordinate and the vertical axis is conformal time. The main plot is of the Hubble radius $|\mathcal{H}|^{-1}$ and the wavelength λ of fluctuations with comoving wave number k. $k_{\rm UV}$ is roughly the wave number at the onset of the bouncing phase whose value is dictated by the microphysics of the bounce. The heavy dashed curves on the main figure correspond to the bounce phase.

from the standard radiation dominated phase ($\rho \propto T^4$) to the mildly Lee-Wick radiation phase ($\rho \propto T^2$). In this phase, the dark matter energy density is not important as it is mainly a radiation dominated phase. At the beginning of this Lee-Wick infested radiation phase, as all the Lee-Wick partners are not thermalized, the effective energy density will look like

$$\rho \sim aT^4 + bT^2 - cT^4, \tag{36}$$

where a, b and c are some constants whose exact values are not of much importance in this discussion. The first term aT^4 is the dominant term in the standard radiation domination as shown in Fig. 1 and arises because the radiation domination is mainly dictated by the standard fields. The bT^2 term designates that some of the Lee-Wick partners have entered the scene and they have paired up with their normal partners to give the T^2 nature of energy density. The last term cT^4 is the contribution from the unpaired Lee-Wick partners of chiral fermions and massless gauge bosons. The schematic form of energy density, in Eq. (36), is similar to the form of energy density in Eq. (15) except that here one also takes into account the standard thermal radiation part (aT^4). In presenting the energy density of Lee-Wick thermal radiation in Eq. (15) one assumed that all of the standard field's Lee-Wick partners are in thermal equilibrium. In the initial stage of the contracting phase of the universe when the temperature of the system is not very high it is natural that most of the Lee-Wick partners would not have thermalized and so Eq. (36) holds true. In this phase $b \gg c$ and $a \gg c$, which amounts to an assumption that most of the Lee-Wick partners of the chiral fermions and the massless gauge bosons (which give rise to the negative radiation energy density) are predominantly heavier than the other Lee-Wick partners. If this was not the case then the Lee-Wick partners of the chiral fermions and massless gauge bosons should have dominated the energy density at the onset of the Lee-Wick infested radiation era and the total energy density of the universe should have decreased dramatically.

At the start of the contracting phase, when $a > b$, the state parameter $w \sim 1/3$ designates a standard radiation phase. After some time $b > a$ (and $b \gg c$) and the state parameter becomes $w \sim 1$. This phase is shown as the Lee-Wick radiation domination phase in Fig. 1. Because of the b and c terms in the energy density the effective energy density of this phase rises but its rise is slowed down (as a result the growth of the Hubble parameter is inhibited) and consequently the Hubble radius goes on decreasing but with a slower rate. During this phase the modes with wavelength $\lambda = 1/k$ leave the Hubble radius as shown in Fig. 2.

As soon as the conformal time is about τ_B^- the c term in the effective energy density starts to dominate and now $c > a$, b and the radiation part of the effective energy density starts to decrease rapidly. In this phase the radiation energy density starts to decrease as more and more Lee-Wick partners of the chiral fermions and massless gauge bosons thermalize, the Hubble parameter effectively decreases rapidly, and the CDM energy density starts to dominate. This phase of the universe is represented by the nonstandard CDM domination phase in Fig. 1. The CDM contribution to the energy density ultimately balances the negative energy density due to Lee-Wick radiation and leads to a bounce.

If we assume symmetry in time then a symmetric analysis can be presented for the expanding phase of the universe. The heavy dashed curves from τ_B^- to τ_B^+ correspond to the bouncing phase of the universe where the microphysics is mainly governed by the Lee-Wick partners of chiral fermions and massless gauge bosons and the dark matter sector. The solid heavy curves stand for the contracting and expanding phases of the universe. The modes with wavelength λ which went out during the contracting phase start to reenter the Hubble radius after τ_B^+ in the expanding phase. In Fig. 2, τ_B is the time of bounce whereas $\tau_B^{\pm}$ stands for the time where there should have been big-bang- or big-crunch-like singularities in the absence of the bounce. In Fig. 2, $k_{\rm UV}$ is the maximum possible wave number, which becomes super-Hubble, in such a bouncing scenario as it corresponds to the minimum radius of the possible Hubble radius. In the absence of the bounce $k_{\rm UV}$ could have been much bigger, effectively of the Planckian order.

A. Cosmological perturbations in contracting and expanding phases and their matching conditions

Here we will discuss how cosmological perturbations generate in a contracting phase and evolve into the expanding phase in a generic bouncing universe scenario. The linearly perturbed FRW metric in longitudinal gauge can be written as [21]

$$ds^2 = a(\tau)^2[(1+2\Phi)d\tau^2 - (1-2\Phi)d\vec{x}^2], \quad (37)$$

where Φ is the gravitational potential which characterizes metric fluctuations and τ is the conformal time. At linear order the scalar metric fluctuations evolve independently. Thus we are able to study the evolution of Φ by following one of its Fourier modes with a fixed comoving wave number k and the perturbation equation of Φ_k is given by

$$\Phi_k'' + 2\eta\mathcal{H}\Phi_k' + (c_s^2k^2 - 2\epsilon\mathcal{H}^2 + 2\eta\mathcal{H}^2)\Phi_k = 0, \quad (38)$$

where $\mathcal{H} \equiv a'/a$ and the prime denotes the derivative with respective to τ. The sound speed parameter c_s in the above equation is usually determined by the thermodynamical property of the background system. Moreover, we have defined two background dependent parameters $\epsilon \equiv -\dot{H}/H^2$ and $\eta \equiv -\ddot{H}/2H\dot{H}$. For a constant background equation of state (say w) these two parameters are equal and are totally determined by the background equation of state as

$$\epsilon = \eta = \frac{3}{2}(1+w). \quad (39)$$

The situation which we are interested in here is a contracting phase of the universe dominated by radiation fields along with their thermalized Lee-Wick partners evolving in a stable background which evolves with a constant state parameter w. In that case the scale factor and the Hubble parameter of the Universe can be written as

$$a \sim (\tau - \tau_B^i)^{\frac{2}{1+3w}}, \qquad \mathcal{H} \simeq \frac{2}{(1+3w)(\tau - \tau_B^i)}, \quad (40)$$

respectively, where τ_B^i is some moment at which the big bang or the big crunch singularity would occur in absence of the nonsingular bouncing phase. In our particular model $\tau_B^i = \tau_B^+, \tau_B^-$ as shown in Fig. 2. As a consequence, the perturbation equation given in Eq. (38) can be simplified using Eq. (39) as

$$\Phi_k'' + \frac{1+\nu}{\tau - \tau_B^i}\Phi_k' + c_s^2k^2\Phi_k \simeq 0, \quad (41)$$

with

$$\nu = \frac{5+3w}{1+3w}. \quad (42)$$

Generically, the solutions to Eq. (41) are composed of two linearly independent Bessel functions. On super-Hubble scales they correspond to a constant mode (called the D mode) and a time-evolving mode (called the S mode). Specifically, the general solution to the perturbation equation on super-Hubble scales can be expressed as

$$\Phi_k^{\pm} = D_{\pm} + S_{\pm}\left(\frac{\tau_B - \tau_B^{\pm}}{\tau - \tau_B^{\pm}}\right)^{\nu}, \quad (43)$$

where D and S are the mode coefficients and the subscripts $\pm$ represent the expanding and the contracting phases of the universe, respectively. In the above expression τ_B is the conformal time at the bouncing point. It is also assumed in the above solutions that the background dynamics of the universe both in the contracting and in the expanding phases are governed by the same steady equation of state w.

Now one needs to know how to transfer the primordial fluctuations generated in the contracting phase of the universe through the bounce. This issue was initially studied by replacing the bounce with a matching surface across which the perturbation modes are connected by using the Hwang-Vishniac [24] (or Deruelle-Mukhanov [25]) matching conditions. Later, it was found that for a nonsingular bounce one can evolve the fluctuations through bounce both numerically and analytically [12,26,27]. Thus, relations between the mode coefficients in the expanding phase and those in the contracting phase can be calculated explicitly [12] (also see Ref. [23] for a general discussion[1]). A general transfer relation between these coefficients in the contracting and expanding phase can be written as

$$D_+ = \mathcal{O}(1)D_- + \mathcal{O}(1)\left(\frac{k}{k_{\rm UV}}\right)^2 S_-, \quad (44)$$

where $k_{\rm UV}$ is a normalization scale which is set by the microphysics of the bounce. It corresponds to the inverse length scales near around $\tau_B^{\pm}$ in our model as shown in Fig. 2. A brief derivation of the above relation is presented in the Appendix.

B. Thermal fluctuations in the case which includes CDM

In the model which we consider here the universe experiences a Lee-Wick infested radiation dominated period in the contracting phase just before the CDM component takes over to yield a bounce. In the following we will study the generation of the primordial power spectrum of curvature perturbation arisen from thermal fluctuations of Lee-Wick infested radiation plasma. Due to the existence of Lee-Wick partners, the heat capacity of the radiation plasma is different from the conventional one and thus affects the scale dependence of the power spectrum.

[1]It is interesting to observe that for a nonsingular bounce model based on a closed space geometry, the primordial perturbation would be dramatically affected by the spatial curvature during the bounce even at large scale limit [28]. However, since in the model under consideration the bouncing solution is achieved by introducing matter components which violate the null energy condition in the frame of flat spatial coordinates, the curvature term will not be involved in the following transfer relation, as has been verified explicitly in the cold Lee-Wick bounce model [12].

Then we will study the stability of this cosmological system and estimate the mass bounds of the Lee-Wick partners.

1. Generation of primordial power spectrum of thermal fluctuations in IR regime

To determine the IR regime of the primordial power spectrum of the thermal fluctuations generated in the contracting phase of the universe, one should consider the modes which leave the horizon at earliest times of the contracting phase. From Fig. 1 we see that at the onset of the radiation dominated era in the contracting phase the radiation fluid is dominated mainly by standard particles (as Lee-Wick partners being heavy thermalize at a later stage) yielding $w_r \approx 1/3$. The radiation energy density during that time thus can be written as

$$\rho_r \approx g_{\rm sm} T^4/4, \tag{45}$$

where $g_{\rm sm}$ takes into account the number of degrees of freedom of the standard particles thermalized initially. We also consider that the background during these initial phases of radiation domination still evolves under the influence of the CDM yielding $w \sim 0$ and the index $\nu = 5$ introduced in Eq. (42).

The correlation function of the energy density of such a cosmic fluid in thermal equilibrium can be written as

$$\langle \delta\rho^2 \rangle|_R = \frac{k^3}{2\pi^2}\langle \delta\rho_k^2 \rangle = C_V \frac{T^2}{V^2}, \tag{46}$$

where V is a fixed volume determined by the correlation length R of the thermal system which in a cosmological setup is roughly of the same order of the Hubble radius $R \sim 1/H$ and a Fourier transform of any generic quantity ϕ with the corresponding Fourier mode ϕ_k is defined as

$$\phi(t, \mathbf{x}) = \sqrt{V}\int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \phi_k(t). \tag{47}$$

The parameter C_V is the heat capacity of the radiation plasma and is defined in terms of the expectation value of the internal energy within the thermally correlated volume. These fluctuations in the energy density of the radiation fluid are coupled to the metric perturbations Φ_- during the contracting phase through the time-time component of the perturbed Einstein equation which is given as [21]

$$-3\mathcal{H}(\mathcal{H}\Phi_- + \Phi_-') + \nabla^2\Phi_- = a^2\delta\rho/2M_p^2, \tag{48}$$

where M_P is the reduced Planck mass. When these perturbations leave the Hubble radius during the contracting phase (i.e., when the corresponding wave number of the mode is of the order of the comoving Hubble radius $k \sim aH$), the three terms contribute equally to the amplitude of Φ_- yielding

$$|\Phi_k^-| \sim \frac{a(\tau_k)^2\delta\rho_k}{2k^2M_p^2} \simeq \frac{\pi a^2(\tau_k) C_V^{1/2}(\tau_k) T(\tau_k)}{2^{1/2}k^{7/2}M_p^2 V(\tau_k)}, \tag{49}$$

up to a constant of order $\mathcal{O}(1)$. We have used Eq. (48) to derive the second expression and τ_k is the conformal time at the time of the Hubble crossing of the mode k. All the quantities in the above equation are to be derived at the time of the Hubble crossing of the mode k.

Now, the key issue is to find out the explicit form of the heat capacity of the thermal system under consideration. By definition, the heat capacity of a thermal fluid is determined by

$$C_V \equiv V\frac{\partial\rho}{\partial T}. \tag{50}$$

As has been mentioned before, the volume V is determined by the thermal correlation length which is of the order of the Hubble radius. Consequently, using Eq. (45) the heat capacity at the Hubble-crossing moment of the IR modes is given by

$$C_V(\tau_k) \approx \frac{4\pi}{3} g_{\rm sm}(\tau_k)\frac{T(\tau_k)^3}{H(\tau_k)^3}. \tag{51}$$

We note here that the perturbations in the radiation energy density scale with the scale factor w_r as

$$\delta\rho \sim a^{-3(1+w_r)}, \tag{52}$$

where the entropy density of a fluid is defined as

$$s = \frac{\delta\rho + \delta p}{T} = (1 + w_r)\frac{\delta\rho}{T}. \tag{53}$$

For an adiabatic expansion of the Universe the comoving entropy remains conserved which means $a^3 s$ is a conserved quantity. Thus the temperature of the radiation plasma evolves as

$$T \sim a^{-3w_r}, \tag{54}$$

which follows from the previous two equations. Moreover, the scale factor of the universe evolves as $a \sim \tau^{\frac{1}{2}(\nu-1)}$ in the contracting background which can be seen from Eq. (40) and at the time of the Hubble crossing of the mode k one has $\tau_k = -\frac{1}{k}$. Then we have $a = a_*(k/k_*)^{-2}$. Thus, following the above expressions, the temperature of the universe at the time when the mode k leaves the Hubble radius can be determined as

$$T(\tau_k) \simeq T_*\left(\frac{k}{k_*}\right)^{\frac{3w_r}{2}(\nu-1)}, \tag{55}$$

where k_* and T_* are associated with the initial moment of thermal equilibrium as introduced in the previous subsection. Similarly the Hubble parameter $H(\tau_k) \equiv \frac{\mathcal{H}(\tau_k)}{a(\tau_k)}$ during that time can also be derived using Eq. (40) which reads as

$$H(\tau_k) \simeq H_* \left(\frac{k}{k_*}\right)^{\frac{1}{2}(\nu+1)}, \tag{56}$$

where H_* again is associated with k_*. Thus the above two equations yield

$$T(\tau_k) \simeq T_* \left(\frac{k}{k_*}\right)^2, \qquad H(\tau_k) \simeq H_* \left(\frac{k}{k_*}\right)^3, \tag{57}$$

if the perturbation modes exit the Hubble radius before the universe is fully thermalized. Here, k_* is a normalization scale which is associated with the initial moment of thermal equilibrium, T_* is the corresponding temperature at that moment, and H_* is the Hubble parameter corresponding to k_* as well. Thus the heat capacity is given by

$$C_V(\tau_k) \approx \frac{4\pi}{3} g_{\rm sm}(\tau_k) \frac{T_*^3}{H_*^3} \left(\frac{k}{k_*}\right)^{-3}, \tag{58}$$

at the moment of Hubble crossing during initial radiation contraction. Then we get the amplitude of gravitational potential as

$$\Phi_H \equiv \Phi_k^-(\tau_H) \simeq \frac{(3\pi)^{1/2}}{2^{3/2} M_p^2} \frac{\sqrt{g_{\rm sm}} T_*^{5/2}}{a_*^{7/2} H_*^4} k^2, \tag{59}$$

where Eq. (49) was applied and the scale factor a_* is associated with the moment of thermal equilibrium. Following Eq. (43), we see that the amplitudes of the constant D_- mode and the growing S_- mode during the initial radiation contraction are related to the metric fluctuation as

$$D_-(k) \simeq \Phi_H, \qquad S_-(k) \simeq \left(\frac{\mathcal{H}_{\rm UV}}{k}\right)^5 \Phi_H. \tag{60}$$

Here $\mathcal{H}_{\rm UV}$ is the maximal scale of the Hubble parameter, in the beginning of the bouncing phase, which is roughly of the same order of $k_{\rm UV}$ introduced in Eq. (44). Thus from the above equation and Eq. (44) we easily find out that the amplitude of the D_+ mode in the expanding phase is mainly contributed by the S_- mode in the IR regime. Correspondingly, we derive the primordial power spectrum of metric perturbation in the IR regime as

$$P_\Phi = \frac{k^3}{2\pi^2} |D_+|^2 \simeq \frac{3 g_{\rm sm} \mathcal{H}_{\rm UV}^6 T_*^5}{16\pi M_p^4 a_*^6 H_*^7} \left(\frac{k}{k_*}\right), \tag{61}$$

and the corresponding spectral index is given by

$$n_\Phi \equiv 1 + \frac{d \ln P_\Phi}{d \ln k} = 2, \tag{62}$$

which implies that the power spectrum of metric perturbations seeded by thermal fluctuations during the onset of the radiation dominated contracting phase is a blue spectrum.

2. Generation of primordial power spectrum of thermal fluctuations in UV regime

As we have pointed out before, the thermodynamics of the radiation fluid in the contracting phase would be greatly affected by the Lee-Wick resonances at very high temperatures ($T > \tilde{M}$) and the corresponding equation of state of the radiation fluid evolves from the conventional value of $w_r = 1/3$ to a nonconventional value of $w_r \approx 1$ when more and more Lee-Wick resonances thermalize with the increasing temperature of the universe. Therefore, the generation of the primordial power spectrum of thermal fluctuations is dramatically changed in the UV regime as the background and the radiation fluid at these later stages of the evolution will evolve with state parameters as $w \sim w_r \sim 1$.

During the Hubble crossing of the mode k the cosmic fluid would be dominated by Lee-Wick resonance infested radiation plasma and the energy density of that fluid can be expressed as given in Eq. (15) (where we can neglect the contribution of the CDM component required for the nonsingular bounce). Also, to let the Hubble radius of the universe contract which allows the mode to become super-Hubble, the first term in Eq. (15) would be the dominant term in the total energy density (as discussed before). If during that time the mass of the heaviest thermalized Lee-Wick particle is $\tilde{M}$ then using Eq. (15) in Eq. (50) one gets an expression for the heat capacity of the radiation fluid at the time of the Hubble crossing of the mode k as

$$C_V(\tau_k) \approx V(\tau_k)\alpha(\tau_k)\tilde{M}^2 T(\tau_k), \tag{63}$$

where we have introduced

$$\alpha(\tau_k) = \frac{\tilde{g}_{*N}(\tau_k)}{12}, \tag{64}$$

which is associated with the number of internal degrees of freedom of the thermal system at the time τ_k. As the energy density and pressure of the radiation fluid are dominated by the α term [as given in Eq. (15)] during the Hubble crossing of the modes the equation of state of that plasma would be $w_r(\tau_H) \approx 1$. In that case, following Eq. (63), the heat capacity of the universe would be

$$C_V(\tau_k) \approx \frac{4\pi}{3} \alpha(\tau_k) \tilde{M}^2 \frac{T_*}{H_*^3} \left(\frac{k}{k_*}\right)^{-3}, \tag{65}$$

where we have used Eqs. (55) and (56). Thus, if we also consider in this scenario that the background cosmology of the universe is also evolving due to the dominance of the Lee-Wick infested radiation field and consider $w = w_r \approx 1$ during the Hubble exit of the mode k, then we get the amplitude of the metric fluctuation corresponding to mode k using Eq. (49) as

$$\Phi_H \equiv \Phi_k^-(\tau_H) \simeq \frac{(3\pi)^{1/2}}{2^{3/2} M_p^2} \frac{\sqrt{\alpha} \tilde{M} T_*^{3/2}}{a_*^{3/2} H_*^2}, \tag{66}$$

where a_* is the scale factor corresponding to k_* through $k_* = a_* H_*$. Following Eq. (66), we see that the amplitudes of the constant D_- mode and the growing S_- mode during the contracting phase are related to the metric fluctuation as

$$D_-(k) \simeq \Phi_H, \qquad S_-(k) \simeq \left(\frac{\mathcal{H}_{\rm UV}}{k}\right)^2 \Phi_H, \qquad (67)$$

where $\nu = 2$ as $w = 1$ in this particular case. Here $\mathcal{H}_{\rm UV}$ is the maximal scale of the Hubble parameter, in the beginning of the bouncing phase, which is roughly of the same order of $k_{\rm UV}$ introduced in Eq. (44). Thus from the above equation and Eq. (44) we see that both the modes D_- and S_- contribute equally to the amplitude of the D_+ mode which is the dominant mode of the expanding phase (as the other S_+ mode is the decaying one and its amplitude falls off with time).[2] Hence, we obtain the primordial power spectrum of the expanding phase of the universe as

$$P_\Phi = \frac{k^3}{2\pi^2}|D_+|^2 \simeq \frac{\tilde{g}_{*N}\tilde{M}^2 T_*^3}{64\pi M_p^4 H_*}\left(\frac{k}{k_*}\right)^3, \qquad (68)$$

and the corresponding spectral index is given by

$$n_\Phi \equiv 1 + \frac{d\ln P_\Phi}{d\ln k} = 4, \qquad (69)$$

which implies that the power spectrum of metric perturbations seeded by thermal fluctuations during the Lee-Wick resonance dominated contracting phase is highly blue in the UV regime.

3. Stability analysis and a rough estimate of the mass bounds of the Lee-Wick partners of chiral fermions and massless gauge bosons

As the primordial power spectrum of metric perturbation generated in the Lee-Wick radiation phase is deeply blue, its amplitude would become secondary in the infrared limit which corresponds to a large length scale. However, one needs to be aware of the potential concern that the power spectrum would become too large in the ultraviolet regime where k takes a large value. Fortunately, for all nonsingular bounce models, there is a natural ultraviolet cutoff on k modes due to the existence of the bouncing scale. This is because, if a perturbation mode has not yet evolved into the super-Hubble scale before the bouncing phase, then it will never take place unless there is a period of inflation after the bounce. As a consequence, one can easily read that the maximal value of the power spectrum takes place right at the beginning of the bouncing phase.

The universe is in thermal equilibrium, with standard particles, in the phase of traditional radiation domination and thus one can assume the benchmark value of the Hubble parameter to be $H_* \simeq \frac{g^{1/2}\pi T_*^2}{3\sqrt{10}M_p}$ where g is the number of internal degrees of freedom of traditional radiation. Using this equation and the relation between H and k from Eq. (56), one can calculate the maximal value of the primordial power spectrum as

$$P_\Phi^{\rm max} \simeq \frac{27\tilde{g}_{*N}\tilde{M}^2 H_{\rm UV}^2}{64\pi g^{3/2} M_p T_*^3}. \qquad (70)$$

In the above equation $H_{\rm UV}$ is roughly similar to $k_{\rm UV}$ whose value is set by the microphysics of the bounce. In our notation it is trivial to show that

$$\langle\Phi(\mathbf{x})^2\rangle = V\int_0^{k_{\rm UV}} \frac{dk}{k} P_\Phi(k), \qquad (71)$$

where $P_\Phi(k) = k^3|\Phi_k|^2/2\pi^2$ is the power spectrum of metric perturbation Φ. Note that this UV cutoff can be selected as the Planck scale $k_{\rm UV} \sim aM_p$ in inflationary cosmology, and due to the quasiexponential expansion of the background universe, it leads to the well-known trans-Planckian problem for inflationary perturbation [30,31] (see also Ref. [32]). This issue does not happen in this model of bouncing cosmology as there is another natural cutoff, $k_{\rm UV}$ as shown in Fig. 2, which is much lower than the Planck scale, i.e., the bounce scale (the maximal absolute value that the Hubble parameter can reach throughout the whole evolution). If $P_\Phi(k) \propto k^n$ (where $n = 3$ in our bounce model) then we obtain $\langle\Phi(\mathbf{x})^2\rangle \sim \frac{k_{\rm UV}^n}{n} \sim \frac{P_\Phi^{\rm max}}{n}$. As $\Phi(\mathbf{x})$ is supposed to be the metric perturbation we expect $\frac{1}{V}\langle\Phi(\mathbf{x})^2\rangle$ to be smaller than unity. Consequently the above equation implies the maximal value of the power spectrum to be less than unity as well.

If we take $g \simeq 107$ which is the number of internal degrees of freedom of the standard model of particle physics, then we can roughly obtain an upper bound on a combination of the Lee-Wick mass and the bounce scale as

$$\tilde{M}|H_{\rm UV}| \lesssim 100\left(\frac{M_p T_*^3}{\tilde{g}_{*N}}\right)^{1/2}, \qquad (72)$$

by requiring $P_\Phi^{\rm max} \lesssim 1$. We further take $\tilde{g}_{*N} \sim 100$ and assume $T_* \sim 100$ GeV, and then get $\tilde{M}|H_{\rm UV}| \lesssim 10^{14}$ GeV^2. However, since the bounce scale must be higher than the mass scale of Lee-Wick partners $|H_{\rm UV}| > \tilde{M}$, it roughly implies that $\tilde{M} \lesssim 10^7$ GeV.

This theoretical constraint restricts the masses of the Lee-Wick partners in the expanding or in the contracting phase of the universe to be less than 10^7 GeV as only the terms proportional to α are taken into account in the heat capacity. The constraint does not work in the bouncing phase of the universe (in the time between τ_B^- and τ_B^+) where much heavier Lee-Wick partners can get thermalized. In the present models where a dark component is used we have calculated the maximum mass of the Lee-Wick partner to be of the order of 10^{15} GeV, but this does not

[2] We would like to refer to Refs. [12,22,26,27,29] for extensive analysis on microscopic description of the nonsingular bouncing phase in a wide class of linear bounce models.

contradict the analysis given above as the heavily massive Lee-Wick particle is only thermalized at the bounce point, near τ_B. Predominantly in the bouncing phase the Lee-Wick partners of the chiral fermions and massless gauge bosons will contribute and consequently we can infer that most of the Lee-Wick partners of the chiral fermions and massless gauge bosons will have their mass in the range 10^7–10^{15} GeV if the temperature near the bounce is 10^{16} GeV.

4. Numerical analysis of the background and the perturbation in the bouncing phase

Previously the problem of mode matching near the bounce was done analytically for modes in the IR or UV regime. In the analytic way one matches the modes which were super-Hubble before t_B^- and after t_B^+. The super-Hubble modes briefly reenter the causal horizon during the bouncing phase (in the time interval between t_B^+ and t_B^-). To see what happens inside the bouncing phase we use numerical techniques.

To ensure that the model under consideration indeed yields a bouncing phase of the Universe we analyze the behavior of the background plasma on which the perturbations live. It is seen that with some benchmark values of the theory the model gives rise to predictable bouncing behavior. In the numerical code we set the scale factor $a(t) = 1$ at the bouncing time $t = t_B \equiv 0$. We also set the temperature of bounce, T_B, and the mass of the heaviest Lee-Wick partner's mass during the bouncing phase, $\tilde{M}$, as

$$\tilde{M} = 10^{-5} M_p, \qquad T_B = 10^{-4} M_p, \tag{73}$$

where M_p is the Planck mass which is set to 1, $M_p = 1$. In this numerical analysis the time unit is set to be the Planck time. The other parameters appearing in Eq. (27) are set as

$$\tilde{g}_{*N} = 120, \qquad \tilde{g}_F = 40, \qquad n = 10. \tag{74}$$

It is to be noted that as we treat the CDM component to be nonrelativistic throughout the evolution of the universe, the energy density of this CDM component evolves as

$$\rho_D(t) = \frac{\rho_B}{a(t)^3}, \tag{75}$$

where the constant ρ_B is found from the condition that at the bouncing time $t = t_B$ one must have $\rho_R(t_B) + \rho_D(t_B) = 0$.

With these typical parameter values it is seen that the background properties of our model encoded in $a(t)$, $H(t)$, $T(t)$, $\rho(t)$ and $p(t)$ show a continuous nonsingular bounce. From Fig. 3 it is clear that the scale factor $a(t)$ has a minimum at $t = 0$. Before $t = 0$ it is seen that $a(t)$ is decreasing, specifying a contracting universe which smoothly transforms into an expanding universe after the bounce. In Fig. 4 the variation of the Hubble parameter $H(t)$ across the bounce is depicted. The variation of temperature of the background near the bouncing point is shown in Fig. 5. If one compares the variation of energy density with time, as shown in Fig. 6, with the variation in temperature near the bouncing point one notices that the temperature of the system is the maximum at the bounce point when the total energy density is the minimum. This point shows the nature of Lee-Wick particle dominated cosmology where the Lee-Wick partners contribute to the energy density of the cosmic plasma with a negative contribution. In Fig. 7 the pressure of the background plasma, $p(t)$, is plotted against time. The figure clearly shows that pressure is negative in the bouncing region, which is a prerequisite of the bouncing model.

The metric perturbation amplitude $|\Phi_k|$ for three values of the wave number are plotted in Fig. 8. Numerical analysis shows that the perturbations remain almost constant across the bouncing point. The perturbations smoothly evolve through the bouncing point and that allows one to match the mode functions in the expanding

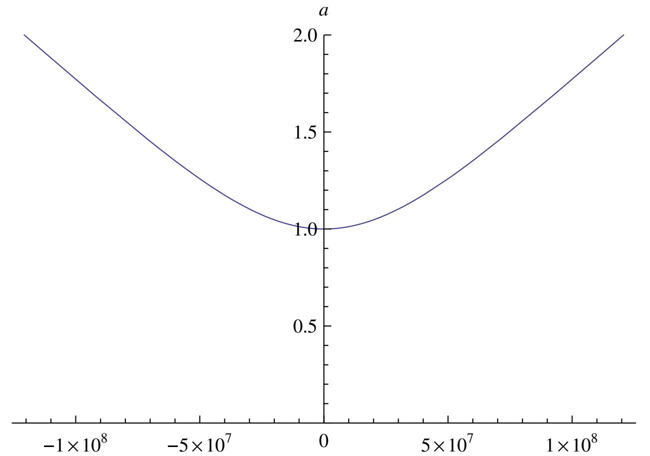

FIG. 3 (color online). Figure showing the behavior of the scale factor $a(t)$ in the bouncing phase.

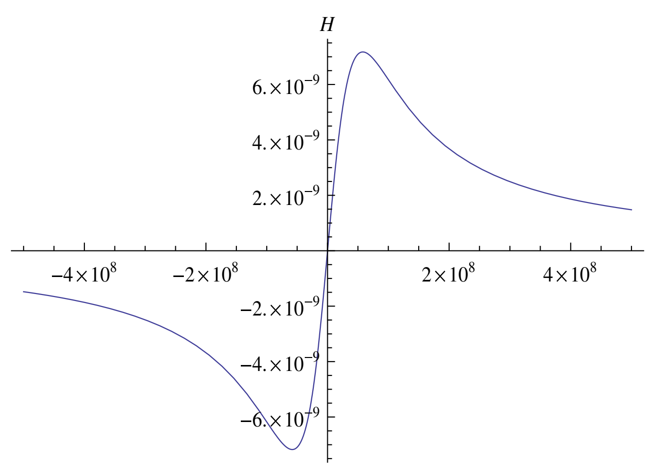

FIG. 4 (color online). Figure showing the behavior of the Hubble parameter $H(t)$ in the bouncing phase.

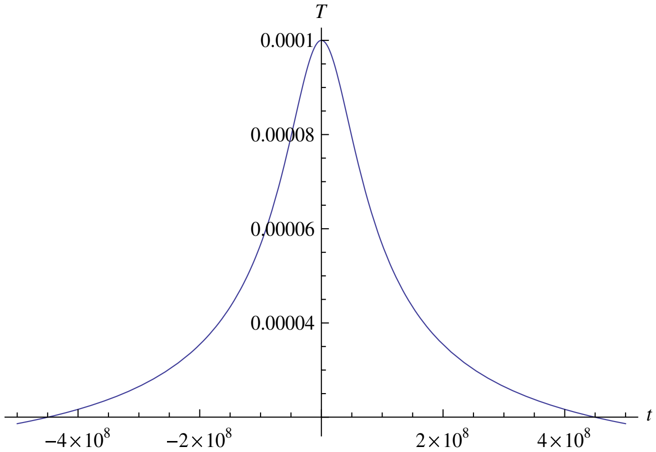

FIG. 5 (color online). Figure showing the behavior of the background temperature $T(t)$ in the bouncing phase.

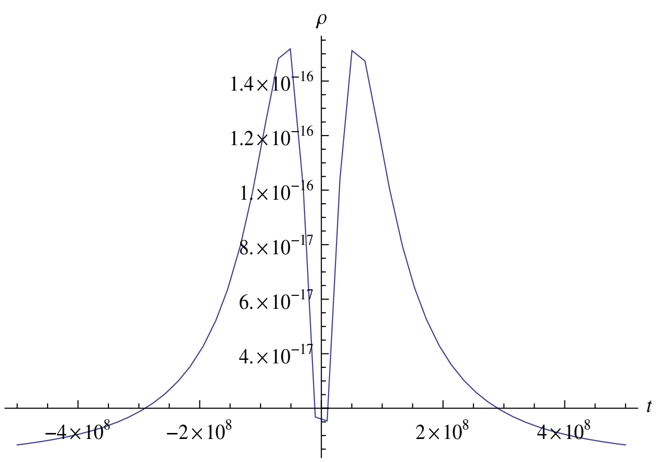

FIG. 6 (color online). Figure showing the behavior of the total energy density $\rho(t)$ in the bouncing phase.

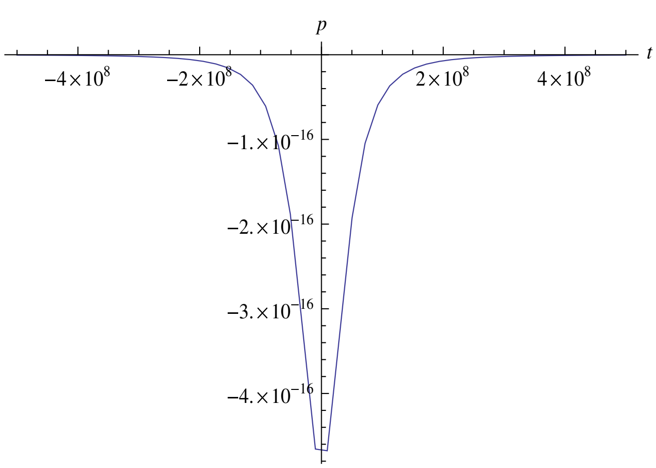

FIG. 7 (color online). Figure showing the behavior of pressure $p(t)$ in the bouncing phase.

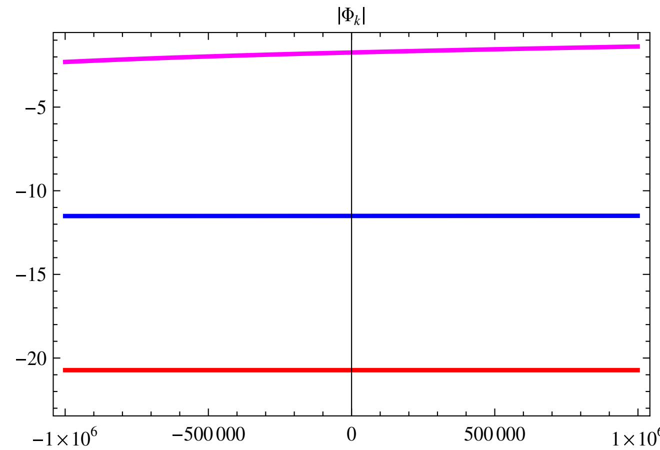

FIG. 8 (color online). Figure showing the behavior of the perturbations across the bouncing point. In the above figure we have numerically evolved the perturbations along the bounce point for the following values of the wave number: from top the wave numbers are $k = 8 \times 10^{-7}$, 8×10^{-9}, 8×10^{-11}, respectively. The y axis is plotted in the logarithmic scale.

phase after the bounce. While comparing these different Fourier modes, one finds that the perturbation mode with the highest value of k has the largest amplitude. Thus, it shows explicitly a blue spectrum is achieved in our model based on the thermal initial condition. Therefore, the numerical result is in agreement with the analytic analysis performed previously.

C. Thermal fluctuations in the case which does not include CDM

Here we will discuss the scenario where a bounce can be achieved in the contracting phase without invoking a CDM component in the cosmic plasma. The main feature of this model is that the bounce would occur before all the heaviest Lee-Wick resonances could thermalize. So, some of the standard model particles will contribute to the radiation plasma without their Lee-Wick resonances (like the standard radiation phase) near the bounce. In this case the dominant part of energy density before τ_B^- or after τ_B^+ is assumed to be coming from the standard radiation part, namely from $\tilde{c}T^4/4$, in Eq. (32). The energy density starts to decrease (or increase) rapidly in the bouncing phase. Consequently in this scenario the dominant nature of the radiation plasma would be more like a conventional radiation fluid with $w_r \approx 1/3$. Hence, for the modes which would leave the Hubble radius in the contracting phase, much before the bounce takes place, one can consider $w = w_r \approx 1/3$.

The analysis of the IR regime is the same as that in the regular radiation era. We need to analyze the UV regime separately as now the background and radiation fluid evolve as $w \sim w_r \sim 1/3$. Following the arguments given in the previous subsections, one can see that the heat capacity of the plasma during the Hubble exit of mode k in this case would be

$$C_V(\tau_k) \approx \frac{4\pi}{3}\tilde{c}(\tau_k)\frac{T_*^3}{H_*^3}\left(\frac{k}{k_*}\right)^{-3}, \tag{76}$$

and the amplitude of the metric fluctuation with wave number k would become

$$\Phi_H \equiv \Phi_k^-(\tau_H) \simeq \frac{(3\pi)^{1/2}}{2^{3/2}M_p^2}\frac{\sqrt{\tilde{c}}T_*^{5/2}}{a_*^{3/2}H_*^2}, \tag{77}$$

where we have applied $w = w_r \simeq 1/3$ and consequently $\nu = 3$. In this case, by matching the formula (77) and the general solution (43) in the contracting phase, the D_- and S_- modes can be expressed as

$$D_-(k) \simeq \Phi_H \propto k^0, \qquad S_-(k) \simeq \left(\frac{\mathcal{H}_{\rm UV}}{k}\right)^3\Phi_H \propto \frac{1}{k^3}, \tag{78}$$

which show that the contribution of the D_- mode to the D_+ mode in the expanding phase is scale free whereas the contribution of the S_- mode for the same is proportional to $1/k$ [following Eq. (44)]. Thus, if the S_- mode (being the growing mode in the contracting phase) contributes dominantly to the D_+ mode in the expanding phase, then we get the power spectrum as follows:

$$P_\Phi = \frac{k^3}{2\pi^2}|D_+|^2 = \frac{3\tilde{c}a_B^2H_{\rm UV}^2T_*}{4\pi M_p^2a_*^2H_*}\left(\frac{k}{k_*}\right), \tag{79}$$

which is also a blue-tilted spectrum as before with a spectral index

$$n_\Phi \equiv 1 + \frac{d\ln P_\Phi}{d\ln k} = 2. \tag{80}$$

Note that, if the D_+ mode in the expanding phase is mainly inherited from the D_- mode in the contracting phase for some reason, the corresponding power spectrum is proportional to k^3.

Thus from the above analysis we see that here the spectrum remains equally blue tilted in both IR and UV regimes. In the other case, where a CDM component is added, the spectrum is more blue titled in the UV regime than that of IR. Therefore, it is not difficult to distinguish these two models observationally.

V. CONCLUSION

In this article an attempt has been made to apply the esoteric topic of Lee-Wick thermodynamics in the physics of the early universe undergoing a nonsingular bounce. Bouncing cosmologies try to evade the big bang singularity which plagues most of the recent cosmological models. Moreover the bouncing universe models presented in the article do not require inflation to solve the standard cosmological problems as an inflationlike scenario is built into the bouncing models.

If one includes Lee-Wick partners of standard fields in the theory then it naturally leads to a radiation dominated phase where the partners of some of the standard fields contribute with a negative sign in the effective energy density of the universe. This negative radiation can lead to a bouncing phase of the universe when the total energy density reaches zero. There can be two options to produce the bounce. The first option includes a dark matter component which helps to produce a negative pressure near the bounce point.[3] A brief numerical analysis of the background evolution and the evolution of the perturbations across the bouncing point for this option is presented in the article. The other option does not require any dark matter component but it requires the temperature of the universe near the bounce point to be less than the masses of some of the Lee-Wick partners of the standard fields.

It has been shown that for the bounce mechanism to succeed one has to have the Lee-Wick partners of most of the chiral fermions and massless gauge bosons to be heavier than a mass scale. This scale depends on the bounce energy scale. This fact gives us a hint about the mass of the elusive Lee-Wick partners.

In the bouncing universe scenarios presented in this article, the relevant perturbation modes leave the Hubble radius while the universe is contracting as the Hubble radius starts to shrink. These modes lie outside the Hubble radius for a brief time. After the bounce is over and the Hubble radius starts to increase these modes start to reenter. In this way the bouncing universe scenarios can mimic the effects of inflationary universe models. The bouncing universe models naturally set a wave number cutoff; in our case it is called $k_{\rm UV}$. In the Lee-Wick thermal radiation induced bouncing models discussed here we have calculated the power spectrum of the metric perturbation Φ. In both the cases one gets a blue-tilted power spectrum. Therefore, the primordial power spectrum induced by thermal fluctuations in the contracting phase can hardly explain the CMB data since cosmological observations have proven a nearly scale-invariant power spectrum. However, this problem can be generally circumvented by introducing another light scalar field during the matter contracting phase, which is known as the bounce curvaton mechanism [33,34].

The theories of Lee-Wick partners are still in their infancy as they were neglected for a long time because there were other interesting scenarios in particle physics which could address the most pressing problems, as the hierarchy problem and other related problems, there. At the present moment Lee-Wick standard model and Lee-Wick thermodynamics related works are also trying to address the problems plaguing particle physics and cosmology. In this regard the present article discusses an interesting

[3]It helps to evade the null energy condition near the bounce point.

property of the probable early universe utilizing the properties of the Lee-Wick theories.

ACKNOWLEDGMENTS

We thank Robert Brandenberger and Guy Moore for helpful comments on the manuscript. S. D. would like to thank Archisman Ghosh for useful discussions. The work of Y.-F. C. is supported in part by an NSERC Discovery Grant.

APPENDIX: TRANSFER RELATION BETWEEN THE MODE SOLUTIONS IN THE CONTRACTING AND EXPANDING PHASES

Here we will give a brief description of how to match the mode functions of the two phases, contracting and expanding, at the nonsingular bouncing point by making use of the analysis of Ref. [25]. In Ref. [25], the matching conditions of modes have been extensively derived in a situation where the stress-energy tensor undergoes a finite discontinuity at a phase transition. If this sudden change in the stress-energy tensor is due to a sudden change in the equation of state w, then the energy density remains constant on the hypersurface Σ of the phase transition. In such a situation the two matching conditions in the conformal Newtonian gauge (longitudinal gauge) read

$$\Phi_k^\pm = 0, \qquad \hat{v}_k^\pm = \left[v_k - \frac{k^2}{3}(\mathcal{H}' - \mathcal{H}^2)^{-1}\Phi_k \right]_\pm = 0, \tag{A1}$$

where v is the Mukhanov-Sasaki variable which can be written in terms of Φ as

$$v = \Phi + \frac{2}{3}\frac{H^{-1}\dot{\Phi} + \Phi}{(1+w)} = \Phi + \frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'}(\Phi' + \mathcal{H}\Phi). \tag{A2}$$

Using $\tau^{-\nu} = \frac{\mathcal{H}}{a^2}$ the first matching condition yields

$$D_+ = D_- + \frac{S_-\mathcal{H}_- - S_+\mathcal{H}_+}{a^2}, \tag{A3}$$

where we have used the solutions for $\Phi_k^\pm$ given in Eq. (43). On the other hand the second matching condition gives

$$\begin{aligned} &\frac{1}{\mathcal{H}'_- - \mathcal{H}_-^2}\left[\left(\mathcal{H}_-^2 + \frac{k^2}{3}\right)D_- \right. \\ &\qquad \left. + \frac{S_-\mathcal{H}_-}{a^2}\left(\mathcal{H}'_- - \mathcal{H}_-^2 + \frac{k^2}{3}\right)\right] \\ &= \frac{1}{\mathcal{H}'_+ - \mathcal{H}_+^2}\left[\left(\mathcal{H}_+^2 + \frac{k^2}{3}\right)D_+ \right. \\ &\qquad \left. + \frac{S_+\mathcal{H}_+}{a^2}\left(\mathcal{H}'_+ - \mathcal{H}_+^2 + \frac{k^2}{3}\right)\right]. \end{aligned} \tag{A4}$$

Using the first relation given in Eq. (A3) in the above equation one gets

$$\begin{aligned} &\frac{2\mathcal{H}'_+ - \mathcal{H}_+^2}{\mathcal{H}'_+ - \mathcal{H}_+^2}D_+ \\ &= \left[\frac{2\mathcal{H}'_- - \mathcal{H}_-^2}{\mathcal{H}'_- - \mathcal{H}_-^2} + \frac{k^2}{3}\left(\frac{1}{\mathcal{H}'_- - \mathcal{H}_-^2} - \frac{1}{\mathcal{H}'_+ - \mathcal{H}_+^2}\right)\right]D_- \\ &\quad - \frac{S_-\mathcal{H}_-}{a^2}\left(\frac{k^2}{3}\right)\left(\frac{1}{\mathcal{H}'_- - \mathcal{H}_-^2} + \frac{1}{\mathcal{H}'_+ - \mathcal{H}_+^2}\right). \end{aligned} \tag{A5}$$

If the universe goes through a symmetric evolution around the bounce then the above equation leads to

$$D_+ = AD_- + Bk^2 S_-, \tag{A6}$$

where A and B are constants of $\mathcal{O}(1)$.

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