# Neutron-star mergers in scalar-tensor theories of gravity

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Scalar-tensor theories of gravity are natural phenomenological alternatives to General Relativity, where the gravitational interaction is mediated by a scalar degree of freedom, besides the usual tensor gravitons. In regions of the parameter space of these theories where constraints from both solar system experiments and binary-pulsar observations are satisfied, we show that binaries of neutron stars present marked differences from General Relativity in both the late-inspiral and merger phases. In particular, phenomena related to the spontaneous scalarization of isolated neutron stars take place in the late stages of the evolution of binary systems, with important effects in the ensuing dynamics. We comment on the relevance of our results for the upcoming Advanced LIGO/Virgo detectors.

General Relativity (GR) has passed stringent tests in the solar system [1] and in binary pulsars [2]. However, these tests involve weak gravitational fields and/or velocities $v \ll c$, so the theory remains essentially untested in the strong-field, highly dynamical $v \sim c$ regime, where high-energy corrections may appear. Strong-field regimes are provided by systems containing black holes and/or neutron stars (NSs), which are the target of existing (Advanced LIGO/Virgo) and future gravitational-wave (GW) detectors. Thus, the final stages of the evolution of compact binaries provide excellent opportunities to explore gravity at extreme conditions [3].

A natural alternative to GR is given by scalar-tensor (ST) theories [4–6], where the gravitational interaction is mediated by the usual tensor gravitons and by a (non-minimally coupled) scalar field. Not only are several phenomenological gravity theories exactly equivalent to ST theories (e.g., $f(R)$ gravity [4,7]), but a gravitational scalar (besides other degrees of freedom) is also generally expected based on string theory. ST theories date back to Jordan [8], Fierz [9], Brans and Dicke [10], and bounds have been placed on them with solar system experiments [1], isolated NSs [11–15] and binary pulsars [16–19]. Stricter constraints may be obtained by detecting GWs from a gravitational collapse [20], from vibrating NSs [21], or with future-generation GW detectors [22–25]. However, the viable parameter space of ST theories is still sizable, so they are a rather natural choice to investigate strong-field deviations from GR.

We consider NS binaries and focus on strong-field/highly dynamical effects during the late inspiral/plunge until the merger (black holes in these theories are not expected to show significant deviations from GR [26–29]). We show that for a class of viable ST theories, NS binaries can present strong-field effects that are qualitatively different from GR

and related to the "spontaneous scalarization" of isolated NSs in ST theories, first discovered in Refs. [11,12] (see also Refs. [14,30]). Although the merger of NS binaries is only marginally detectable with Advanced LIGO/VIRGO in GR, for the class of viable ST theories that we consider (i) large deviations from GR appear which are not captured by weak-field analyses; (ii) these effects cannot be reproduced within GR, even with an exotic equation of state; (iii) distinct features will be detectable with Advanced LIGO/VIRGO, even in the late-inspiral/plunge and merger, unlike in GR; and (iv) these features may even have astrophysical implications in possible models for energetic electromagnetic events.

## I. METHODOLOGY

We consider a generic ST theory with action

$$S = \int d^4 x \frac{\sqrt{-g}}{2\kappa} \left[ \phi R - \frac{\omega(\phi)}{\phi} \partial_\mu \phi \partial^\mu \phi \right] + S_M[g_{\mu\nu}, \psi], \tag{1}$$

where $\kappa = 8\pi G$ (adopting $c = 1$ throughout this Letter), $R$ and $g$ are the Ricci scalar and determinant of the metric, $\phi$ is the gravitational scalar, and $\psi$ collectively describes the matter degrees of freedom. Although Eq. (1) is not the most general action giving second-order field equations, as Galileon-type terms may be present [31,32], it includes a large family of theories and thus provides a suitable framework for studying nonlinear interactions. For instance, Jordan-Fierz-Brans-Dicke theory corresponds to $\omega = $ const, while $\omega(\phi) = -3/2 - \kappa/(4\beta \log \phi)$ correspond to the theories of Refs. [11,12], which give large deviations from GR for isolated NSs ("spontaneous scalarization") and sufficiently negative $\beta$. Also, as already mentioned, $f(R)$ gravity (both in the metric and Palatini formalism)

can be remapped into the form (1) (although one has to allow the presence of a potential for the scalar $\phi$) [4,7].

One can reexpress the ("Jordan-frame") action (1) into the so-called "Einstein-frame" action through a conformal transformation $g_{\mu\nu}^E = \phi g_{\mu\nu}$, which yields

$$S = \int d^4x \sqrt{-g^E} \left( \frac{R^E}{2\kappa} - \frac{1}{2} g_E^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right) + S_M \left[ \frac{g_{\mu\nu}^E}{\phi(\varphi)}, \psi \right], \qquad (2)$$

where $\varphi$ is defined by $(\mathrm{d}\log\phi/\mathrm{d}\varphi)^2 = 2\kappa/[3 + 2\omega(\phi)]$. Imposing $\varphi = 0$ for $\phi = 1$, this gives

$$\phi = \exp\left( \sqrt{\frac{2\kappa}{3 + 2\omega}} \varphi \right), \qquad \phi = \exp\left( -\beta\varphi^2 \right), \quad (3)$$

respectively for Jordan-Fierz-Brans-Dicke theory and for the theories of Refs. [11,12]. (Our $\varphi$ is related to the scalar $\varphi_{\mathrm{DEF}}$ used there via $\varphi = \varphi_{\mathrm{DEF}}/\sqrt{4\pi G}$.)

In the Einstein frame the field equations are

$$G_{\mu\nu}^E = \kappa(T_{\mu\nu}^\varphi + T_{\mu\nu}^E), \qquad (4)$$

$$\Box^E \varphi = \frac{1}{2} \frac{d\log\phi}{d\varphi} T_E, \qquad (5)$$

$$\nabla_\mu^E T_E^{\mu\nu} = -\frac{1}{2} T_E \frac{d\log\phi}{d\varphi} g_E^{\mu\nu} \partial_\mu \varphi, \qquad (6)$$

where

$$T_E^{\mu\nu} = \frac{2}{\sqrt{-g^E}} \frac{\delta S_M}{\delta g_{\mu\nu}^E} \quad \text{and} \qquad (7)$$

$$T_{\mu\nu}^\varphi = \partial_\mu \varphi \partial_\nu \varphi - \frac{g_{\mu\nu}^E}{2} g_E^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \qquad (8)$$

are the matter and scalar-field stress-energy tensors in the Einstein frame, and $T_E \equiv T_E^{\mu\nu} g_{\mu\nu}^E$. Indices are raised/lowered with the Einstein-frame metric $g_E$, and the relation between the stress-energy tensors in the two frames is $T_E^{\mu\nu} = T^{\mu\nu} \phi^{-3}$, $T_{\mu\nu}^E = T_{\mu\nu} \phi^{-1}$. Also, $u^\mu = \sqrt{\phi} u_E^\mu$ (from $g_{\mu\nu}^E u_E^\mu u_E^\nu = -1$); $\rho = \phi^2 \rho_E$ (from $\rho_E = u_E^\mu u_E^\nu T_{\mu\nu}^E$) and $p = \phi^2 p_E$ (from $T = \phi^2 T_E$). Last, to preserve the same equation of state in both frames, the rest-mass densities must be related by $\rho_0 = \phi^2 \rho_0^E$. Baryon number conservation in the Jordan frame ($\nabla_\mu j^\mu = 0$ with $j^\mu = \rho_0 u^\mu$) then gives

$$\nabla_\mu^E j_E^\mu = -\frac{1}{2} \frac{\mathrm{d}\log\phi}{\mathrm{d}\varphi} j_E^\mu \partial_\mu \varphi, \qquad (9)$$

with $j_E^\mu = \rho_0^E u_E^\mu$. Therefore, solving the system (4)–(6) and (9) and transforming back to the Jordan frame provides a solution to the original problem. We adopt this approach here.

## II. PHYSICAL SETUP

We model the NSs with a perfect fluid coupled to the full field equations (4)–(6) and (9) to accurately represent the strong gravitational effects during the binary's evolution. Our techniques for solving these equations have been described and tested previously [33–38]. The initial data are evolved in a cubical computational domain $x^i \in [-350, 350]$ km, and we employ an adaptive mesh refinement that tracks the compact objects with cubes slightly larger than their radii and resolution $\Delta x = 0.5$ km. We consider an unequal-mass binary system, initially on a quasicircular orbit with separation of 60 km and angular velocity $\Omega = 1295$ rad/s, constructed with LORENE [39]. The stars are described by a polytropic equation of state $(p/c^2 = K\rho_0^\Gamma)$ with $\Gamma = 2$ and $K = 123 G^3 M_\odot^2/c^6$ (which yields a maximum ADM mass of about $1.8 M_\odot$ both in GR and in the ST theories we consider). We adopt a mass ratio of $q \equiv 0.937$, possible for progenitors of gamma-ray bursts [40], and choose individual baryon masses $\{1.78, 1.90\} M_\odot$, corresponding to gravitational masses $\{1.58, 1.67\} M_\odot$.

For the gravity theory, we consider $\omega(\phi) = -3/2 - \kappa/(4\beta\log\phi)$ [corresponding to $\phi = \exp(-\beta\varphi^2)$]. As mentioned, these theories are equivalent to those of Refs. [11,12]. Besides the constant $\beta$, the gravity theory is also characterized by the asymptotic value $\varphi_0$ of the scalar [11]. Binary pulsar measurements require $\beta/(4\pi G) \gtrsim -4.5$ [17], while the Cassini experiment constrains $\varphi_0 < \varphi_{\mathrm{Cassini}} \equiv 2(G\pi)^{1/2}/[|\beta|(3 + 2\omega_0)^{1/2}] \approx 1.26 \times 10^{-2} G^{1/2}/|\beta|$ (with $\omega_0 = 4 \times 10^4$ [1,17]). Moreover, from $\beta/(4\pi G) \sim -4$ to $\beta/(4\pi G) = -4.5$, the allowed value for $\varphi_0$ decreases from $\varphi_{\mathrm{Cassini}}$ to 0, again due to constraints from binary pulsars [17]. For the system described above, we tried various values of $\varphi_0 \lesssim 10^{-5} G^{-1/2}$, and the results do not change significantly when $\beta$ is fixed. (As will become clear from Fig. 3 and associated discussion, larger values of $\varphi_0$, even when allowed by existing constraints, induce even larger deviations from GR.)

## III. GW EXTRACTION AND BACKREACTION

The response of a GW detector is encoded in the curvature scalars in the physical (Jordan) frame [41]. These are obtained from the Einstein frame components as $\psi_4 = -R_{\ell\bar{m}l\bar{m}} = \phi\psi_4^E$, , $\psi_3 = -R_{\ell k\ell\bar{m}}/2 = \phi\psi_3^E + \cdots$, $\psi_2 = -R_{\ell k\ell k}/6 = \phi\psi_2^E + \cdots$ and $\phi_{22} = -R_{\ell m\ell\bar{m}} = \phi(\phi_{22}^E - l^\nu l^\mu \nabla_\nu \nabla_\mu \log\phi/2 + \cdots)$ (with ... denoting subleading terms in the distance to the detector and $l$, $m$ being components of a null tetrad adapted to outgoing wavefronts). Far from the source, one expects $\varphi = \varphi_0 + \varphi_1/r + \cdots$, with $\varphi_0 = $ const and $\varphi_1$ a function of $x^\mu$, so because of the peeling property in the Einstein frame, $\psi_2$ and $\psi_3$ decay faster than $1/r$ and do not produce observable effects on a GW detector at infinity. However, using $\log\phi = -\beta\varphi^2 = -\beta(\varphi_0^2 + 2\varphi_0\varphi_1/r) + \cdots$, one obtains

$\phi_{22} \sim \beta\varphi_0 \partial_t^2 \varphi_1/r$. Thus, the radiative degrees of freedom (decaying as $1/r$ and observable by GW detectors) are $\psi_4$ (i.e., tensor gravitons) and $\phi_{22} \sim \beta\varphi_0 \partial_t^2 \varphi_1/r$ (i.e., a purely transverse, radiative scalar mode [41]). Nonetheless, for $\varphi_0 \to 0$ the $1/r$ radiative component of $\phi_{22}$ vanishes. Thus, since $\varphi_0$ is constrained to small values, for viable ST theories the scalar mode couples weakly to GW detectors [17], which makes its direct detection problematic.

In fact, it is easy to get convinced, already at the level of the action (2), that the scalar mode is not observable directly in the limit $\varphi_0 \to 0$. The detection of GWs is based on free-falling test masses, so to analyze the detector's response one needs to look at the Jordan frame metric $g_{\mu\nu}^E/\phi(\varphi)$, to which the matter fields $\psi$ couple [cf. Eq. (2)]. Far from the source, in suitable coordinates one has $g_{\mu\nu}^E \approx \eta_{\mu\nu} + h_{\mu\nu}$ and $\varphi \approx \varphi_0 + \delta\varphi$, where $h_{\mu\nu}$ and $\delta\varphi$ are small perturbations. If $\varphi_0 = 0$, we have $\phi = \exp(-\beta\varphi^2) \approx 1 - \beta\delta\varphi^2$, and therefore $g_{\mu\nu}^E/\phi(\varphi) \approx \eta_{\mu\nu} + h_{\mu\nu}$ at linear order. This means that the motion of the detector's test masses is only sensitive to the tensor waves $h_{\mu\nu}$ in the limit $\varphi_0 \to 0$.

Still, although weakly coupled to a GW detector at infinity, the scalar mode carries energy away from the source [cf. Eqs. (5) and (6)], and exerts a significant back-reaction on it, because the scalar fluxes appear at 1.5 PN order, while the quadrupolar tensor fluxes of GR appear at 2.5 PN. More precisely, for a quasicircular binary with masses $m_1$ and $m_2$, and scalar charges $\alpha_1$ and $\alpha_2$ [with $\alpha_i \approx \sqrt{4\pi/G}\varphi_1^i/m_i$, where $\varphi_1$ is defined, as above, by $\varphi = \varphi_0 + \varphi_1/r + \cdots$], the dipolar scalar emission is [13,16,18]

$$\dot{E}_{\text{dipole}} = \frac{G}{3c^3}\left(\frac{G_{\text{eff}}m_1m_2}{r^2}\right)^2(\alpha_1 - \alpha_2)^2. \qquad (10)$$

Here, $G_{\text{eff}} = G(1 + \alpha_1\alpha_2)$ is the effective gravitational constant appearing in the Newtonian interaction between the stars, i.e., the gravitational force gets modified by the exchange of scalar gravitons and becomes [13]

$$F = \frac{G_{\text{eff}}m_1m_2}{r^2}. \qquad (11)$$

The quadrupole tensor emission is instead [13,18]

$$\dot{E}_{\text{quadrupole}} = \frac{32G}{5c^3}\left(\frac{G_{\text{eff}}m_1m_2}{r^2}\right)^2\left(\frac{v}{c}\right)^2, \qquad (12)$$

where $v = [G_{\text{eff}}(m_1 + m_2)/r]^{1/2}$ is the relative velocity of the two stars. Therefore, the dipolar scalar fluxes are produced abundantly during the inspiral if the charges $\alpha_1$ and $\alpha_2$ are different, and they dominate over the tensor quadrupole fluxes, which are suppressed by $(v/c)^2$ relative to them.

## IV. RESULTS AND COMPARISON TO GR

Our simulations confirm the qualitative features described above but also highlight a more intricate phenomenology. Specifically, in ST theories with $\beta/(4\pi G) \lesssim -4.2$, NS binaries merge at significantly lower frequency than in GR; e.g., in Fig. 1, the plunge starts already when the stars' centers are $\sim 52$ km apart, corresponding to an angular velocity $\Omega \sim 1850$ rad/s (i.e., a GW frequency $f \sim 586$ Hz, within Advanced LIGO/Virgo's sensitivity bands) and results in the formation of a rotating bar (whose long-lived GW signal is seen in the lower panel). Remarkably, plunges starting so early cannot be obtained in GR, because even with exotic equations of state, NS radii are constrained to $R_{\text{NS}} \lesssim 14$ km [42], so the interaction between the two stars does not trigger a plunge until a separation $\sim 2R_{\text{NS}} \lesssim 28$ km. Clearly, because a NS binary spends a large part of its inspiral within LIGO/Virgo's sensitivity bands, these early plunges will not produce a signal-to-noise ratio very different from GR and will not jeopardize the source's detection. Given the magnitude of the differences highlighted in Fig. 1 and the fact that they appear well within advanced detectors' frequency windows, however, it appears likely that a suitable post-detection analysis (i.e., at the parameter-estimation stage) will be able to highlight them. (A more detailed analysis of this point goes beyond the scope of this paper and will be presented elsewhere.)

The cause of these earlier mergers is not simply the backreaction of the scalar fluxes (10) (absent in GR). In fact, even though our initial data essentially maximize the
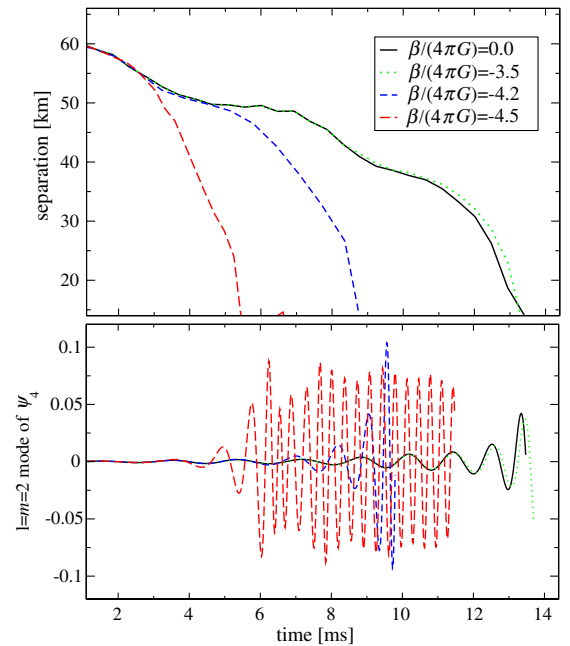


FIG. 1 (color online). The separation and the dominant mode of the $\psi_4$ scalar (encoding the effect of GWs) for a binary with gravitational masses $\{1.58, 1.67\}M_\odot$ and for different values of $\beta$.
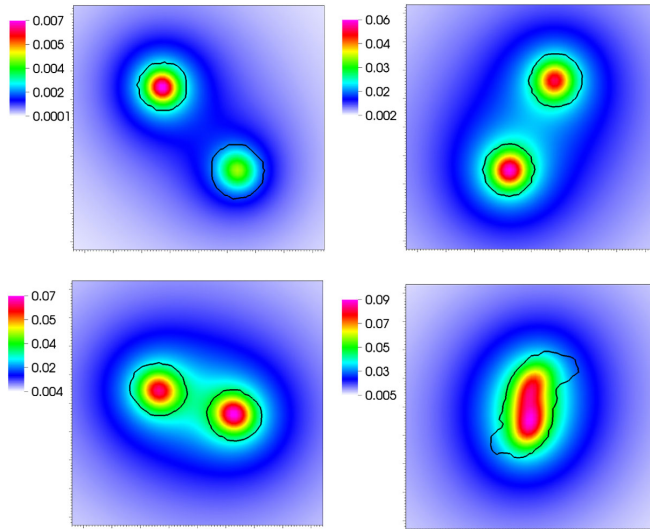
FIG. 2 (color online). The scalar field $\varphi G^{1/2}$ (color code) and the NS surfaces (solid black line) at $t = \{1.8, 3.1, 4.0, 5.3\}$ ms for $\beta/(4\pi G) = -4.5$, and the binary of Fig. 1.

dipolar emission (10) by giving the first star a charge close to the maximum value allowed by the ST theory ($\alpha_1 \sim \alpha_{max}$), and an almost zero scalar charge to the second star ($\alpha_2 \approx 0$), the scalar field grows rapidly inside the second star, which quickly develops a charge $\alpha_2 \approx \alpha_1$ when the binary becomes sufficiently close (cf. Fig. 2). This shuts off the dipolar flux (10) but enhances the Newtonian pull (11). Therefore, the earlier mergers are caused by the combination of dissipative [Eq. (10)] and conservative [Eq. (11)] effects. As a qualitative test, we integrated the PN equations of motion of GR with $G$ replaced by $G_{eff} = G(1 + \alpha_1\alpha_2)$ [so as to mimic Eq. (11), with $\alpha_1, \alpha_2 \sim 0.2$–0.4 set to values compatible with our simulations] and confirmed that the enhanced gravitational pull induces quicker mergers.

The growth of the scalar field and charge of nonscalarized stars getting close to scalarized ones can be understood in simple terms. The scalar field extends beyond the radius of the baryonic matter [11,12]. Indeed, defining an effective radius $L$ for the scalar as that enclosing a fixed fraction, e.g., 90%, of its mass, one gets $L/R_{NS} \sim 4$–5 for isolated stars (cf. also Fig. 2). When the nonscalarized star enters this scalar "halo" of the scalarized star, it grows a significant charge. This can be seen by studying isolated NSs [11,12] and imposing a nonzero asymptotic value $\varphi_0$ for the scalar field, in order to mimic the effect of the "external" scalar field produced by the other (scalarized) star. The effect of $\varphi_0$ is shown in Fig. 3, where we used a static, spherically symmetric code to calculate the scalar charge of NSs as a function of the baryonic mass, for a ST theory with $\beta/(4\pi G) = -4.5$. As can be seen, even modest values of $\varphi_0$ induce significant scalar charges. This phenomenon, known as "induced scalarization" [11–13], has also been observed for boson stars in ST theory [43] and is similar, energetically, to the magnetization of a
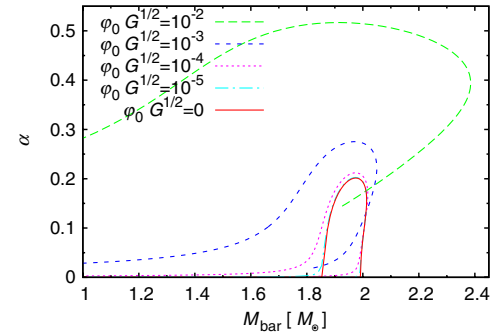


FIG. 3 (color online). Effect of an external scalar field $\varphi_0$, for $\beta/(4\pi G) = -4.5$.

ferromagnetic material in a sufficiently strong magnetic field [11,12,44]. Here, the external scalar field makes the configuration with nonzero charge energetically preferred over the initial noncharged one.

Quite remarkably, the growth of the scalar field inside stars that are sufficiently close seems quite robust, (though its magnitude naturally depends on the values of $\beta$ and $\varphi_0$). In fact, it happens also in systems where induced scalarization is likely unable to trigger the scalar's initial growth, e.g., in (at least) some binaries whose stars are initially nonscalarized and far from the "critical mass" $M_{bar} \approx 1.85 M_\odot$, marking the onset of spontaneous scalarization for small $\varphi_0$ in Fig. 3. For instance, in Fig. 4 we show the waveforms for an equal-mass binary whose stars have baryon mass $1.625 M_\odot$, gravitational mass $1.47 M_\odot$ and radius $R_{NS} = 13$ km, for GR and a ST theory with $\beta/(4\pi G) = -4.5$ and $\varphi_0\sqrt{G} = 10^{-5}$. Clear deviations from GR arise at $t \sim 10$ ms, corresponding to a separation $R \sim 40$ km and $f \sim 645$ Hz. These deviations will occur at later (earlier) times for smaller (larger) $\varphi_0$. We will study these smaller-mass systems more in future work, but this result is not entirely surprising. The spontaneous scalarization of isolated stars occurs when a nonzero value $\varphi_c$ of the scalar at the center becomes energetically favored over $\varphi_c = 0$. References [11,12,44] noted indeed that the star's
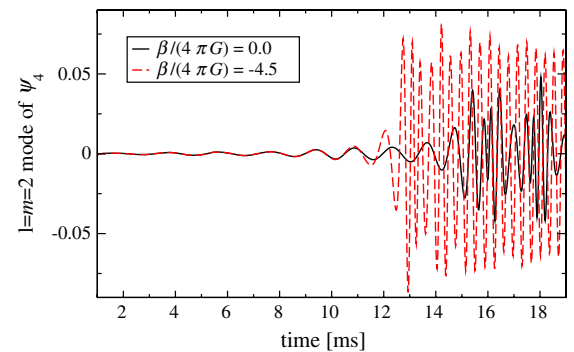


FIG. 4 (color online). The dominant mode of the $\psi_4$ scalar, with $\beta/(4\pi G) = -4.5$, $\varphi_0 = 10^{-5} G^{-1/2}$, for an equal-mass binary with individual baryon masses of $1.625 M_\odot$.

energy is $E \sim \int[(\nabla\varphi)^2/2 + \rho \exp(\beta\varphi^2)]d^3x \sim \varphi_c^2 L + M\exp(\beta\varphi_c^2)$, where the length $L \sim |\varphi/\nabla\varphi|$ regulates the scalar's gradients. [As mentioned, $L \sim 4$–$5R_{\mathrm{NS}}$, because $\varphi$ decays slowly ($\sim 1/r$) outside the star]. One can easily check that if $M/L$ is large enough (i.e., if the star is compact enough) and $\beta < 0$, $E$ may have a minimum at $\varphi_c \neq 0$, and the star will spontaneously scalarize. In a tight binary, however, the scalar field will change on the scale of the separation $R$ (cf. Fig. 2), so $E \sim \varphi_c^2 R + M \exp(\beta\varphi_c^2)$, suggesting that at sufficiently small separations, the energy's minimum will lie at $\varphi_c \neq 0$, i.e., $\varphi$ may grow inside stars that would not scalarize spontaneously in isolation.

Finally, our findings might have implications for short gamma-ray bursts, of which NS binaries are likely progenitors. While restricted to ST theories with $\beta/(4\pi G) \lesssim -4.2$ (and possibly to binary masses $M_{\mathrm{ADM}} \gtrsim 3M_\odot$), our results show that in principle, modifications to the gravity theory may cause the GW signal and the orbital evolution to differ from GR. This may be important for coincident searches of GW and electromagnetic signals from NS binaries, and for energetic events possibly associated with NS mergers and their after-merger remnant (because the extra scalar channel carrying energy away from the binary can affect its energy budget).

[1] C. M. Will, Living Rev. Relativity **4**, 4 (2001).
[2] J. H. Taylor and J. M. Weisberg, Astrophys. J. **253**, 908 (1982); T. Damour and J. H. Taylor, Phys. Rev. D **45**, 1840 (1992).
[3] J. Abadie *et al.* (LIGO Scientific and Virgo Collaborations), Classical Quantum Gravity **27**, 173001 (2010).
[4] R. Wagoner, Phys. Rev. D **1**, 3209 (1970).
[5] P. G. Bergmann, Int. J. Theor. Phys. **1**, 25 (1968).
[6] K. Nordtvedt, Astrophys. J. **161**, 1059 (1970).
[7] T. P. Sotiriou, Classical Quantum Gravity **23**, 5117 (2006).
[8] P. Jordan, Z. Phys. **157**, 112 (1959).
[9] C. M. Will, Living Rev. Relativity **9**, 3 (2006).
[10] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).
[11] T. Damour and G. Esposito-Farese, Phys. Rev. Lett. **70**, 2220 (1993).
[12] T. Damour and G. Esposito-Farese, Phys. Rev. D **54**, 1474 (1996).
[13] T. Damour and G. Esposito-Farese, Classical Quantum Gravity **9**, 2093 (1992).
[14] T. Harada, Phys. Rev. D **57**, 4802 (1998).
[15] H. Sotani, Phys. Rev. D **86**, 124036 (2012).
[16] D. M. Eardley, Astrophys. J. Lett. **196**, L59 (1975).
[17] T. Damour and G. Esposito-Farese, Phys. Rev. D **58**, 042001 (1998); P. C. C. Freire, N. Wex, G. Esposito-Farese, J. P. W. Verbiest, M. Bailes, B. A. Jacoby, M. Kramer, I. H. Stairs, J. Antoniadis, and G. H. Janssen, Mon. Not. R. Astron. Soc. **423**, 3328 (2012).
[18] C. M. Will and H. W. Zaglauer, Astrophys. J. **346**, 366 (1989).
[19] J. Alsing, E. Berti, C. M. Will, and H. Zaglauer, Phys. Rev. D **85**, 064041 (2012).
[20] J. Novak, Phys. Rev. D **57**, 4789 (1998).
[21] H. Sotani and K. D. Kokkotas, Phys. Rev. D **70**, 084026 (2004).
[22] C. M. Will and N. Yunes, Classical Quantum Gravity **21**, 4367 (2004); E. Berti, A. Buonanno, and C. M. Will, Phys. Rev. D **71**, 084025 (2005).
[23] V. Cardoso, S. Chakrabarti, P. Pani, E. Berti, and L. Gualtieri, Phys. Rev. Lett. **107**, 241101 (2011); N. Yunes, P. Pani, and V. Cardoso, Phys. Rev. D **85**, 102003 (2012).
[24] E. Berti, L. Gualtieri, M. Horbatsch, and J. Alsing, Phys. Rev. D **85**, 122005 (2012).
[25] K. G. Arun, Classical Quantum Gravity **29**, 075011 (2012).
[26] S. W. Hawking, Commun. Math. Phys. **25**, 167 (1972).
[27] K. S. Thorne and J. J. Dykla, Astrophys. J. Lett. **166**, L35 (1971).
[28] J. Healy, T. Bode, R. Haas, E. Pazos, P. Laguna, D. M. Shoemaker, and N. Yunes, arXiv:1112.3928.
[29] S. Mirshekari and C. M. Will, arXiv:1301.4680.
[30] P. Pani, V. Cardoso, E. Berti, J. Read, and M. Salgado, Phys. Rev. D **83**, 081501 (2011).
[31] G. W. Horndeski, Int. J. Theor. Phys. **10**, 363 (1974).
[32] C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, Phys. Rev. D **84**, 064039 (2011).
[33] M. Anderson, E. Hirschmann, S. L. Liebling, and D. Neilsen, Classical Quantum Gravity **23**, 6503 (2006).

[34] C. Palenzuela, I. Olabarrieta, L. Lehner, and S. L. Liebling, Phys. Rev. D **75**, 064005 (2007).

[35] S. L. Liebling, Phys. Rev. D **66**, 041703 (2002).

[36] M. Anderson, E. W. Hirschmann, L. Lehner, S. L. Liebling, P. M. Motl, D. Neilsen, C. Palenzuela, and J. E. Tohline, Phys. Rev. D **77**, 024006 (2008).

[37] M. Anderson, E. W. Hirschmann, L. Lehner, S. L. Liebling, P. M. Motl, D. Neilsen, C. Palenzuela, and J. E. Tohline, Phys. Rev. Lett. **100**, 191101 (2008).

[38] http://www.had.liu.edu/.

[39] http://www.lorene.obspm.fr.

[40] K. Belczynski, R. W. O'Shaughnessy, V. Kalogera, F. Rasio, R. Taam, and T. Bulik, Astrophys. J. Lett. **680**, L129 (2008).

[41] D. M. Eardley, D. L. Lee, and A. P. Lightman, Phys. Rev. D **8**, 3308 (1973).

[42] A. W. Steiner, J. M. Lattimer, and E. F. Brown, Astrophys. J. **722**, 33 (2010).

[43] M. Ruiz, J. C. Degollado, M. Alcubierre, D. Nunez, and M. Salgado, Phys. Rev. D **86**, 104044 (2012).

[44] G. Esposito-Farese, AIP Conf. Proc. **736**, 35 (2004).