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What if Planck's Universe isn't flat?

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Inflationary theory predicts that the observable Universe should be very close to flat, with a spatial-curvature parameter $|\Omega_K| \lesssim 10^{-4}$. The WMAP satellite currently constrains $|\Omega_K| \lesssim 0.01$, and the Planck satellite will be sensitive to values near 10^{-3}. Suppose that Planck were to find $\Omega_K \neq 0$ at this level. Would this necessarily be a serious problem for inflation? We argue that an apparent departure from flatness could be due either to a local (wavelength comparable to the observable horizon) inhomogeneity, or a truly superhorizon departure from flatness. If there is a local inhomogeneity, then secondary cosmic microwave background (CMB) anisotropies distort the CMB frequency spectrum at a level potentially detectable by a next-generation experiment. We discuss how these spectral distortions would complement constraints on the Grishchuk-Zel'dovich effect from the low-ℓ CMB power spectrum in discovering the source of the departure from flatness.

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Inflation predicts that the observable Universe should be very nearly flat, with a spatial-curvature parameter $|\Omega_K| < 10^{-4}$ in most models [1]. WMAP data currently constrain $|\Omega_K| \lesssim 10^{-2}$ (95% C.L.) [2], and Planck should be sensitive to Ω_K at around the 10^{-3} level [3], improving to $\sim 10^{-4}$ when combined with 21-cm intensity maps [4] (which represents the limit of detectability [5]).

Suppose that Planck were to find a nonzero value for Ω_K. What might this mean for inflation? Such an observation would nominally be evidence for a genuine departure from flatness on superhorizon scales, with wide-ranging implications for a broad class of inflationary models; for example, a measurement of $\Omega_K < -10^{-4}$ is sufficient to rule out the majority of eternal inflation scenarios with high confidence [6].

Before jumping to such conclusions, though, one might wonder whether the deviation could be explained simply by a local inhomogeneity that biases our determinations of cosmological parameters. This would allow us to preserve flatness (and thus some relatively natural sort of inflation) by explaining the discrepancy as the result of systematic distortions of, e.g., the distance-redshift relation due to lensing by the inhomogeneity [7]. Although a local density fluctuation of a large enough amplitude ($\Phi \gtrsim 10^{-3}$) would be inconsistent with the simplest inflationary models, it might conceivably arise if there is some strongly scale-dependent non-Gaussianity, or perhaps if some sort of semiclassical fluctuation arises at the beginning or end of inflation [8].

For a sufficiently large and smooth local inhomogeneity, it would be difficult to definitively distinguish these two situations using standard cosmological tests. Purely geometric observables such as distance measures would be inhibited by degeneracies with evolving-dark-energy models [9], and the deviation from flatness would be too small to significantly affect the growth of structure.

In this paper, we show that a class of observables based on spectral distortions of the cosmic microwave background (CMB) offer the prospect to disentangle the two scenarios. These observables exploit the strong relationship between spatial homogeneity and the isotropy of spacetime; by using them to measure the dipole anisotropy of the CMB about distant points, it is possible to place stringent constraints on the possible size of a local inhomogeneity [10]. Furthermore, these observables unambiguously distinguish between subhorizon and superhorizon effects, owing to a cancellation of the dipole induced by superhorizon perturbations [11,12].

We begin by calculating the bias in Ω_K due to a local inhomogeneity. We take the form of this local inhomogeneity throughout to be a spherically symmetric potential perturbation $\Phi(r, t) = D(t)\Phi_0 \exp[-(r/r_0)^2]$, where the linear-theory growth factor is normalized to $D = 1$ today. The presence of a large, local inhomogeneity modifies the apparent distance to last scattering through a combination of lensing, integrated Sachs-Wolfe effect, gravitational redshift, and Doppler shift. Reference [13] derived a full expression for the (subhorizon) luminosity-distance perturbation δd_L up to linear order in perturbations. [The observed distance $d_L(z) = \bar{d}_L(1 + \delta d_L)$, where the overbar denotes a background quantity.] When considering the CMB, it is useful to rewrite this as a perturbation $\delta d_A = \delta d_L - 2\delta z/(1 + z)$ to the angular-diameter distance.

An observer sitting at the center of a spherically symmetric inhomogeneity will measure a distance to last scattering which deviates from the background quantity by a uniform amount over the whole sky. This introduces a shift in angular scale of the entire CMB power spectrum.

The value of Ω_K inferred from observations depends primarily on the angular scale of the first few CMB acoustic peaks [14], and will therefore be biased away from its background value. Figure 1 shows the distance perturbation as a function of the depth and width of a local inhomogeneity, compared with the change in (background) distance between a flat model, and one with $|\Omega_K| = 10^{-3}$. (For numerical work we take $[h, \Omega_m, \Omega_\Lambda, \sigma_8] = [0.71, 0.266, 0.734, 0.8]$ and redshifts of reionization and last scattering to be $z_{\rm re} = 10$ and $z_* = 1090.79$, respectively.) Based on the distance to last scattering alone, an inhomogeneity with $\Phi_0 \sim 10^{-3}$ would induce an apparent shift in Ω_K of order 10^{-3} for a wide range of widths. (The null in δd_A near $r_0 = 2.3$ Gpc results from cancellation of different contributions to the distance perturbation. The location of this null depends on the shape of the inhomogeneity.)

The inhomogeneity will also cause the observed redshift z_* of the surface of last scattering, to differ from its background value, $\bar{z}_* = z_* - \delta z_*$, where [13]

$$\delta z = (1 + z_s)\left[\Phi_s - \Phi_o + (\mathbf{v}_o - \mathbf{v}_s)\mathbf{n} + 2\int_{\eta_s}^{\eta_o} d\eta \mathbf{n}\nabla\Phi\right], \tag{1}$$

and $\mathbf{n}$ is a unit vector along the line of sight. For the central observer, the effect of the redshift perturbation is to change the inferred conformal time (and thus expansion rate) of last scattering, which will bias the estimation of parameters such as Ω_m. For an observer who is *off center* in the inhomogeneity, however, an additional anisotropy will also be induced in the CMB. This is because the redshift perturbation, Eq. (1), depends on direction; a line of sight looking towards the center of the inhomogeneity will experience a different change in redshift to one looking away from it, and thus there will be a direction-dependent change in temperature.

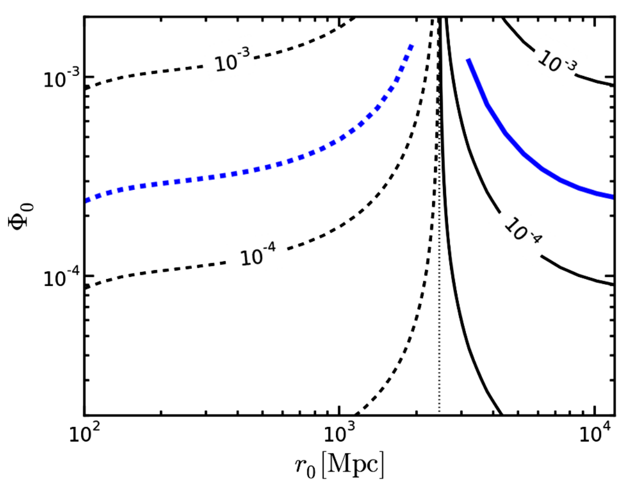

FIG. 1 (color online). The change δd_A in the distance to last scattering as a function of the width and depth of the inhomogeneity. Solid (dashed) lines denote a positive (negative) δd_A. The thick blue lines plot the difference in distance (in background) between models with $|\Omega_K| = 10^{-3}$ and $\Omega_K = 0$ (with identical h and Ω_m), equivalent to $\delta d_A = 2.7 \times 10^{-4}$.

In general, anisotropies will be induced over a range of angular scales, but at least for observers close to the center of a large (wide) inhomogeneity, the dipole β will dominate. While there is also a dipole contribution due to the peculiar velocity of the observer, velocity perturbations due to the matter distribution on smaller scales are expected to be Gaussian random distributed with mean zero, whereas the dipole due to a large inhomogeneity will generally present a *systematic* trend in redshift and angle on the sky. This allows us to distinguish between the two contributions. For a spherical inhomogeneity, axial symmetry dictates that the dipole will be aligned in the radial direction, and that all spherical-harmonic modes of the induced anisotropy with $m \neq 0$ on the sky of the observer will be zero, so that $\beta \propto \int \delta z_*(\theta) \cos\theta \sin\theta d\theta$.

We now discuss spectral distortions due to a local inhomogeneity. There is a close relationship between homogeneity and the isotropy of spacetime. A number of observational tests that are sensitive to CMB anisotropies about distant points can be used to exploit this link and detect local inhomogeneities of the kind that would cause a systematic bias in measurements of Ω_K.

The strength of the connection between homogeneity and isotropy is most clearly demonstrated by the Ehlers-Geren-Sachs (EGS) theorem [15]. According to EGS, if all comoving observers in a patch of spacetime see an isotropic CMB radiation field, then that patch is uniquely Friedmann-Lemaître-Robertson-Walker (i.e., it is necessarily homogeneous and isotropic). Generalizations of this result show that it is perturbatively stable, in the sense that small departures from perfect isotropy imply only small departures from homogeneity (up to some assumptions; see Ref. [16] for a critique). A corollary to the EGS theorem is that observers in an inhomogeneous region of spacetime will in general see an anisotropic CMB sky. We can therefore use measurements of the anisotropy of the CMB about a collection of spacetime points to constrain the degree of inhomogeneity inside our Hubble volume [17].

Compton scattering of CMB radiation by ionized gas provides a way to detect anisotropy about remote points. The scattered radiation spectrum consists of a weighted superposition of spectra from all directions on the scatterer's sky, $I'_\nu \sim \int \tau(1 + \cos^2\theta) I_\nu(\theta, \phi) d\Omega$. If the scatterer's sky is a perfectly isotropic blackbody of uniform temperature, the scattered spectrum is simply a blackbody of the same temperature, plus spectral distortions due to the random thermal motions of the electrons in the scattering medium [the thermal Sunyaev-Zel'dovich (TSZ) effect [18]]. If its sky is anisotropic, however, the resulting spectrum is a combination of blackbodies of different temperatures. This induces additional blackbody spectral distortions and shifts the temperature of the "base" blackbody spectrum as seen by an observer [10,19]. If the dipole anisotropy dominates, we call these the Compton-y distortion and the kinematic Sunyaev-Zel'dovich (KSZ) effect [20], respectively.

By measuring the Compton-y distortion and KSZ effects for many scattering regions on our own sky, we can build up a picture of the degree of anisotropy, and thus inhomogeneity, within our past light cone. We will now outline three observational tests based on these effects and estimate their sensitivities to a local inhomogeneity.

We begin with the KSZ effect from galaxy clusters. Galaxy clusters contain a significant amount of ionized gas. Since they are effectively individual collapsed objects, they can be used to sample the dipole anisotropy induced by a local inhomogeneity at discrete points in space. This is useful to reconstruct the systematic trend in dipole anisotropy as a function of redshift that a local inhomogeneity produces. Each cluster has a characteristic integrated optical depth of $\tau \sim 10^{-3}$–10^{-2}. The KSZ signal due to a single galaxy cluster at redshift z is $\Delta T/T = -\beta(z)\tau$, and can be extracted from CMB sky maps given a sufficiently accurate component separation method and low-noise data.

The KSZ effect from individual clusters is difficult to measure owing to the smallness of the signal, confusion with primary CMB anisotropies, and other dominant systematic errors. Currently, only upper limits are available, but this is likely to change as data from Planck and small-scale CMB experiments such as Atacama Cosmology Telescope and SPT become available. Current data have nevertheless been used to constrain inhomogeneous relativistic cosmological models for dark energy [21].

We now consider the KSZ angular power spectrum from gas in the intergalactic medium. This angular power spectrum is easier to measure than the KSZ effect from individual clusters because it is an integrated quantity and has additional contributions from the diffuse intergalactic medium that is not associated with clusters (sometimes called the Ostriker-Vishniac effect [22]). The KSZ power spectrum from a large inhomogeneity is [23]

$$C_\ell \approx 8\pi^3 \int_0^{r_{\rm re}} dr r^{-3}[\beta(z)(d\tau/dr)]^2 P(k(r), z(r)). \quad (2)$$

The Limber approximation has been used, giving $k(r) = (2\ell+1)/2r(z)$. We model the distribution of scatterers in the late Universe with $d\tau/dz \propto \sigma_T f_b \rho(z)/H(z)$ [23], and take reionization to be an abrupt transition at $z_{\rm re}$. At high ℓ, the KSZ signal is strongly dependent on the nonlinear matter power spectrum $P(k,z)$, which we model using HaloFit/CLASS [24]. Results for our toy model are shown in Fig. 2.

At a characteristic angular scale of $\ell \sim 3000$ (where the primary CMB signal becomes subdominant), the signal is dominated by contributions from small-scale matter inhomogeneities at lower redshifts, where the induced anisotropy is mostly dipolar. These scales are accessible to CMB experiments such as Atacama Cosmology Telescope and SPT, which have recently put stringent upper limits on the combined TSZ + KSZ power [25]. Accessing the bare KSZ signal is complicated by difficulties in modeling the distribution of extragalactic point sources [26], and contains a theoretical uncertainty due to the unknown "patchiness" of reionization, which also contributes a KSZ effect [27].

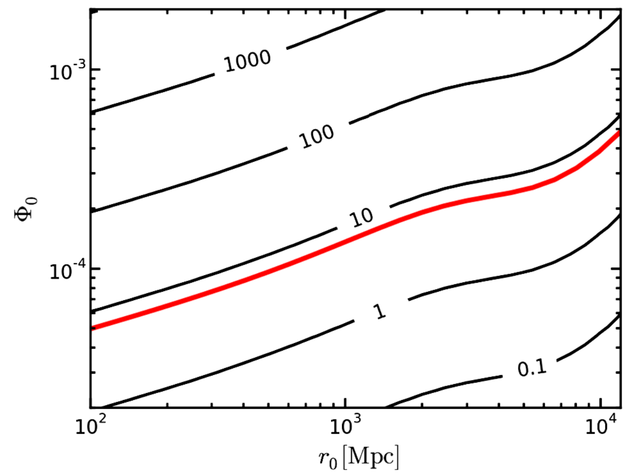

FIG. 2 (color online). The KSZ power $D_\ell = \ell(\ell+1)C_\ell T_0^2/2\pi$ at $\ell = 3000$, in μK^2. The thick red line is the SPT upper limit of $D_{3000} < 6.7\ \mu$K^2 (95% C.L.).

We now turn to the Compton-y distortion induced by the inhomogeneity. Spectral distortions arising from the Compton scattering of an anisotropic CMB can be parametrized as a Compton-y blackbody distortion. When the dipole dominates, the observed Compton-y distortion is a monopole [23]:

$$y = (7/10) \int_0^{r_{\rm re}} dr (d\tau/dr) \beta^2(r). \quad (3)$$

Results for our model are shown in Fig. 3.

Measurement of the Compton-y distortion requires an instrument for which an absolute calibration of the spectral response can be obtained. This excludes most recent CMB experiments, and so the best current constraints come from COBE/FIRAS [28]. The planned PIXIE mission [29] could improve the determination of y by some 4 orders of magnitude.

Our toy-model calculations give some sense of the effectiveness of the different spectral-distortion tests in constraining the size of a local inhomogeneity. A depth of $\Phi_0 \sim 3 \times 10^{-4}$ is sufficient to induce a bias in the inferred spatial curvature of $\Delta\Omega_K \approx 10^{-3}$ for a wide range of r_0 (Fig. 1). Existing upper limits on the KSZ power at $\ell = 3000$ from SPT are sufficient to rule out an inhomogeneity of this depth with a width less than around 8 Gpc, although larger r_0 are still allowed (Fig. 2). The Compton-y distortion, on the other hand, provides much weaker constraints even with the great increase in precision that

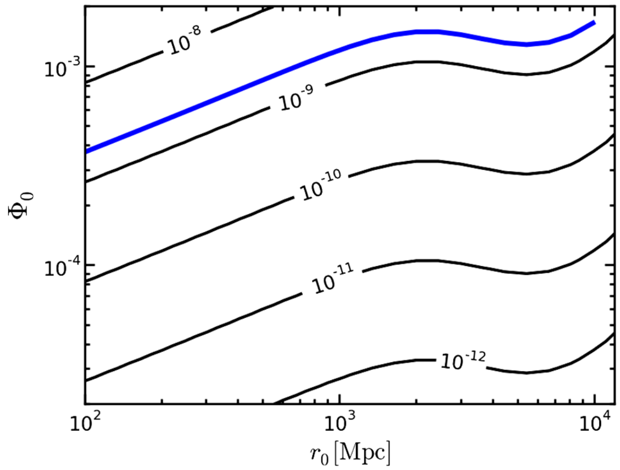

FIG. 3 (color online). The Compton-y distortion induced by the inhomogeneity. Also plotted is the projected upper limit from PIXIE (thick blue line).

would be possible with PIXIE (Fig. 3). Part of the reason for the relative effectiveness of the KSZ power spectrum is its density weighting, which enhances the signal at the high ℓ probed by precision CMB experiments.

The above calculations are only intended to be illustrative, and more detailed modeling would be required to produce firmer constraints. For example, the KSZ angular power spectrum is sensitive to the nonlinear contributions to $P(k)$ [23], and the form of $\beta(z)$ to the shape of the potential, $\Phi(r)$, so uncertainties in these functions should be treated carefully. For wider inhomogeneities, there is also a (relatively minor) dependence on the details of reionization. Finally, the assumption that the inhomogeneity is perfectly spherically symmetric, and that we are exactly at its center, should also be relaxed. A realistic inhomogeneity cannot be too asymmetric, or place us too far from the center, however, without violating limits on isotropy, moderating a CMB dipole that is observed locally [30], or inducing a CMB statistical anisotropy [8].

Why should we expect to find ourselves near to the center of a large inhomogeneity in the first place? Although such a situation may seem unlikely [30], there are inflationary mechanisms known in the literature which preferentially place observers near the center of large underdensities [31]. Furthermore, Ellis [32] has argued that it would be inconsistent to rule out such inhomogeneities on strictly *a priori* probabilistic grounds, since we currently accept features in our cosmological models that are substantially less probable anyway. As such, observations should be the final arbiter in deciding whether a large inhomogeneity exists or not.

Would its presence have not already been discovered through other observational probes? Inhomogeneities of the kind considered here modify the low-ℓ CMB, causing alignment of low-ℓ multipoles [33], and changes in the integrated Sachs-Wolfe effect signal, temperature-polarization cross-spectrum, and associated modifications to the reionization history [34]. Unfortunately, the induced effects tend either to be smaller than cosmic variance at the relevant scales or strongly dependent on the details of the model, rendering these tests inconclusive.

Superhorizon perturbations also produce fluctuations in the low-ℓ CMB through the Grishchuk-Zel'dovich effect [11]. In a number of cosmological models (including ΛCDM), it has been shown that there is a cancellation between the anisotropy and peculiar velocity induced by such perturbations, resulting in no net dipole to first order [12]. Constraints from the low-ℓ CMB are therefore complementary to spectral-distortion tests of the sort outlined above: A deviation from spatial flatness caused by a local inhomogeneity results in a net dipole about many locations within our horizon, which can be measured using, e.g., the KSZ effect, whereas a superhorizon deviation from flatness will produce no such signal, instead causing an enhancement of the quadrupole and higher moments of our local CMB.

In conclusion, an observation of $|\Omega_K| \gtrsim 10^{-4}$ would have considerable implications for inflation but would not, on its own, be sufficient to rule out eternal inflation. It would also have to be shown that the inferred deviation from flatness was not caused by the effects of a local inhomogeneity instead. Observations of CMB spectral distortions such as the KSZ effect and Compton-y distortion, taken with constraints on the size of the Grishchuk-Zel'dovich effect from the low-ℓ CMB power spectrum, present a viable method to constrain the source of a seeming departure from flatness.

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