

[70, ℓ^+] baryons in large N_c QCD revisited: The effect on Regge trajectories

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A simple procedure within the $1/N_c$ expansion method where all the N_c quarks are treated on the same footing has been found successful in describing mixed symmetric negative parity baryon states belonging to the $[70, \ell^-]$ multiplets of the $N = 1$ and 3 bands. Presently it is applied to mixed symmetric positive parity $[70, 0^+]$ and $[70, 2^+]$ multiplets of the $N = 2$ band. We search for the most dominant terms in the mass formula. The results are compared to those obtained in the procedure where the system is separated into a core and an excited quark. We find that both the spin and flavor operators of the entire system of N_c quarks play dominant roles in describing the data, like for negative parity states. As a by-product we present the contribution of the leading spin-flavor singlet term as a function of the band number, which hints at distinct Regge trajectories for the symmetric and mixed symmetric states.

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I. INTRODUCTION

The $1/N_c$ expansion method, where N_c is the number of colors [1,2], is based on the discovery that, for N_f flavors, the ground state baryons display an exact contracted $SU(2N_f)$ spin-flavor symmetry in the large N_c limit of QCD [3]. Presently it is considered to be a model independent, powerful and systematic tool for baryon spectroscopy. It has been applied with great success to the ground state baryons ($N = 0$ band), described by the symmetric representation **56** of $SU(6)$, where $N_f = 3$ [3–9]. At $N_c \rightarrow \infty$ the ground state baryons are degenerate. At large, but finite N_c , the mass splitting starts at order $1/N_c$.

The extension of the $1/N_c$ expansion method to excited states is based on the observation these states can approximately be classified as $SU(2N_f)$ multiplets, and that the resonances can be grouped into excitation bands, $N = 1, 2, \dots$, as in quark models, each band containing a number of $SU(6) \times O(3)$ multiplets. The symmetric multiplets of these bands were analyzed by analogy to the ground state. In this case the splitting starts at order $1/N_c$ as well.

The study of mixed symmetric multiplets was less straightforward, being technically more complicated. Two procedures have been proposed and applied to the excited states belonging to the $[70, 1^-]$ multiplet ($N = 1$ band). The first one is based on the separation of the system into a ground state core + an excited quark [10–16]. It is an extension of the ground state treatment to excited states inspired by the Hartree picture. Later on it was supported by the authors of Ref. [17]. In the second method, proposed by us [18], the system of N_c quarks is treated as a whole. All identical quarks are considered on the same footing and

therefore the Pauli principle is satisfied. It has successfully been applied to the negative parity multiplet $[70, 1^-]$ of the $N = 1$ band [19] and recently to the multiplets $[70, \ell^-]$ (with $\ell = 1, 2, 3$) of the $N = 3$ band [20]. The advantage is that our mass formula has fewer terms than the ground state core + excited quark method, so that it is physically more transparent.

It is worth mentioning that in both procedures the mass splitting of mixed symmetric states starts at order N_c^0 and they are both compatible with the meson-nucleon scattering picture [21–24]. For the ground state core + excited quark approach this has been shown in Refs. [16,25] by using a mass formula with three leading operators (one of order N_c , two of order N_c^0) generating three sets of degenerate states called *three towers of states* [16] for $\ell = 1$. In Ref. [26] we gave an explicit proof of the degeneracy of mass eigenvalues for $\ell = 3$. In a similar way, but with three different leading operators in our approach, we have also proven the above compatibility for mixed symmetric $\ell = 1$ [27] states in $SU(4)$. Note that in both procedures only operators containing components of the angular momentum start at order N_c^0 .

Here we wish to test our method [18] on mixed symmetric positive parity multiplets, studied so far within the ground state core + excited quark approach [28,29]. Our study is especially motivated by the fact that a recent multichannel partial wave analysis has revealed the existence of new positive parity resonances which appeared in the 2012 version of the Review of Particle Properties (PDG) [30].

The paper is organized as follows. In the next section we shortly review previous studies on positive parity resonances within the symmetric core + excited quark method and present the orbital-flavor-spin wave function in the new approach [18]. In Sec. III we introduce the mass

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operator and derive the analytic expressions of the matrix elements of the required operators as a function of N_c , using the method of Ref. [18]. In Sec. IV the experimental situation is shortly reviewed. In Sec. V our results for mixed symmetric positive parity states are presented. Section VI is devoted to Regge trajectories and the last section contains some conclusions.

II. THE LOWEST EXCITED POSITIVE PARITY STATES

The lowest excited positive parity resonances belong to the $N = 2$ band, which contains the multiplets $[56', 0^+]$, $[56, 2^+]$, $[70, 0^+]$, $[70, 2^+]$ and $[20, 1^+]$. Radially excited states $[56', 0^+]$ have been studied in Ref. [31] with a simplified Gürsey-Radicati type mass formula. The masses of the baryons supposed to belong to the multiplet $[56, 2^+]$ have been calculated within the $1/N_c$ expansion method in Ref. [32]. The approach of Ref. [32] has been extended to higher excitations belonging to the $[56, 4^+]$ multiplet ($N = 4$ band) [33]. We recall that in the symmetric representation it is not necessary to distinguish between excited and core quarks, thus the wave function has a simple structure [32].

As already mentioned, the mixed symmetric multiplets $[70, 0^+]$ and $[70, 2^+]$ have been studied by applying the symmetric core + an excited quark approach [28,29]. The structure of the intrinsic orbital wave function was rather complicated, containing a term with an excited symmetric core in contrast to the wave function of the $N = 1$ band, where the symmetric core was in the ground state [13]. Such a wave function has been constructed in Ref. [28] by using generalized Jacobi coordinates [34] and fractional parentage techniques [35]. It had been first applied to SU(4) ($N_f = 2$) [28] and next extended to SU(6) ($N_f = 3$) baryons [29].

Presently we treat the system of N_c quarks as a whole, no quark separation. Both the orbitally excited and the spin-flavor (SF) parts of the total wave function are described by the partition $[f] = [N_c - 1, 1]$. By inner product rules of the permutation group one can form a totally symmetric orbital-spin-flavor wave function described by the partition $[N_c]$. Following Ref. [19] the most general form of such a wave function in $SU(6) \times O(3)$, having a total angular momentum J and projection J_3 is given by

$$|\ell S; JJ_3; (\lambda\mu)YII_3\rangle = \sum_{m_\ell, S_3} \begin{pmatrix} \ell & S & J \\ m_\ell & S_3 & J_3 \end{pmatrix} \times |\ell m_\ell\rangle |[f](\lambda\mu)YII_3; SS_3\rangle, \quad (1)$$

where the orbital part of the wave function of the entire system, denoted by $|\ell m_\ell\rangle$, has a permutation symmetry $[f]$, for simplicity not specified, the same as its flavor-spin part $|[f](\lambda\mu)YII_3; SS_3\rangle$. The wave function (1), together with an $SU_c(3)$ color singlet $[1^{N_c}]$ forms a totally antisymmetric N_c quark state.

In order to calculate the expectation value of the mass operator defined in the following section one needs to know the matrix elements of the SU(6) generators S^i , T^a and G^{ia} between the states $|[f](\lambda\mu)YII_3; SS_3\rangle$ of a given SU(3) symmetry $(\lambda\mu)$ and a spin S , associated to the entire system of N_c quarks. In Ref. [36] these matrix elements were presented under the form of a generalized Wigner-Eckart theorem, containing isoscalar factors of SU(3) and SU(6). Tables for the most needed isoscalar factors of SU(6) were produced in the same paper for the 2_8 , 4_8 , ${}^2_{10}$ and 2_1 SU(3) \times SU(2) multiplets. Extended tables were obtained in Ref. [19].

As already mentioned, applications to the $[70, 1^-]$ multiplet of the $N = 1$ band and to the $[70, \ell^-]$ ($\ell = 1, 2, 3$) of the $N = 3$ band were made in Refs. [19,20], respectively. In the following we shall follow a similar approach.

III. THE MASS OPERATOR

The general form of the mass operator, where the SU(3) symmetry is broken, has first been proposed in Ref. [8] as

$$M = \sum_i c_i O_i + \sum_i d_i B_i. \quad (2)$$

This is inspired by the perturbative expansion in powers of $1/N_c$ proposed by 't Hooft [1] where the operators O_i represent $1/N_c$ corrections to the leading spin-flavor singlet operator O_1 proportional to N_c . The contributions of O_i with $i > 1$ estimate the amount of SF symmetry breaking. Accordingly, the operators O_i are defined as the scalar products

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{\text{SF}}^{(k)}, \quad (3)$$

where $O_\ell^{(k)}$ is a k -rank tensor in SO(3) and $O_{\text{SF}}^{(k)}$ a k -rank tensor in SU(2) spin, but invariant in $SU(N_f)$. Thus O_i are rotational invariant. For the ground state one has $k = 0$. The excited states also require $k = 1$ and $k = 2$ terms. The rank $k = 1$ tensor has as components the generators L^i of SO(3). The components of the $k = 2$ tensor operator of SO(3) are

$$L^{(2)ij} = \frac{1}{2} \{L^i, L^j\} - \frac{1}{3} \delta_{i,-j} \vec{L} \cdot \vec{L}, \quad (4)$$

which, like L^i , act on the orbital wave function $|\ell m_\ell\rangle$ of the whole system of N_c quarks [see Ref. [28] for the normalization of $L^{(2)ij}$]. According to the large N_c counting rules [2] an n -body operator carries a coefficient $1/N_c$ reflecting the minimum of $n - 1$ gluon exchanges between two quarks in QCD.

The operators B_i break SU(3) explicitly and are defined to have zero expectation values for nonstrange baryons. Only first order SU(3) breaking terms have been considered so far.

TABLE I. List of dominant operators and their coefficients in the mass formula (2) obtained in three distinct numerical fits.

Operator	Fit 1	Fit 2	Fit 3
$O_1 = N_c \mathbb{1}$	616 ± 11	616 ± 11	616 ± 11
$O_2 = \ell^i s^i$	150 ± 239	52 ± 44	243 ± 237
$O_3 = \frac{1}{N_c} S^i S^i$	149 ± 30	152 ± 29	136 ± 29
$O_4 = \frac{1}{N_c} [T^a T^a - \frac{1}{12} N_c (N_c + 6)]$	66 ± 55	57 ± 51	86 ± 55
$O_5 = \frac{3}{N_c} L^i T^a G^i$	-22 ± 5		-25 ± 52
$O_6 = \frac{15}{N_c} L^{(2)ij} G^{ia} G^{ja}$	14 ± 5	14 ± 5	
$B_1 = -S$	23 ± 38	24 ± 38	-22 ± 35
χ_{dof}^2	0.61	0.52	2.27

Using the experimental data described below we have performed several numerical fits to obtain the unknown coefficients c_i and d_i , which encode the QCD dynamics. As the data are still scarce we had to restrict the number of terms in the mass formula, therefore we had to choose the most relevant operators. They were suggested by our previous experience with negative parity states and are exhibited in Table I.

The first is the trivial spin-flavor singlet operator O_1 of order $\mathcal{O}(N_c)$. The first nontrivial operator is the spin-orbit operator O_2 , which we identify with the single-particle operator

$$\ell \cdot s = \sum_{i=1}^{N_c} \ell(i) \cdot s(i), \quad (5)$$

the matrix elements of which are of order N_c^0 and are given in Ref. [29] The analytic expression of the matrix elements of O_2 can be found in Appendix A of Ref. [20].

The spin operator O_3 and the flavor operator O_4 are two-body and linearly independent. The expectation value of O_3 is $\frac{1}{N_c} S(S+1)$ where S is the spin of the entire system of N_c quarks. The expression of the operator O_4 given in Table I is consistent with the usual $1/N_c(T^a T^a)$ definition in SU(4). In extending it to SU(6) we had to subtract the quantity $(N_c + 6)/12$ as explained in Ref. [36]. Then, as one can see from Tables II, III, and IV, the expectation

TABLE II. Matrix elements of O_i for octet resonances.

	O_1	O_2	O_3	O_4	O_5	O_6
$48[70, 2^+]_{\frac{7}{2}}^+$	N_c	$\frac{2}{3}$	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$\frac{3(N_c+3)}{2N_c}$	$-\frac{15(N_c-1)}{4N_c}$
$28[70, 2^+]_{\frac{5}{2}}^+$	N_c	$\frac{2}{9N_c}(2N_c-3)$	$\frac{3}{4N_c}$	$\frac{3}{4N_c}$	$\frac{3}{N_c}$	0
$48[70, 2^+]_{\frac{5}{2}}^+$	N_c	$-\frac{1}{9}$	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{N_c+3}{4N_c}$	$\frac{75(N_c-1)}{8N_c}$
$48[70, 0^+]_{\frac{3}{2}}^+$	N_c	0	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	0	0
$28[70, 2^+]_{\frac{3}{2}}^+$	N_c	$-\frac{1}{3N_c}(2N_c-3)$	$\frac{3}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{9}{2N_c}$	0
$48[70, 2^+]_{\frac{3}{2}}^+$	N_c	$-\frac{2}{3}$	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{3(N_c+3)}{2N_c}$	0
$28[70, 0^+]_{\frac{1}{2}}^+$	N_c	0	$\frac{3}{4N_c}$	$\frac{3}{4N_c}$	0	0
$48[70, 2^+]_{\frac{1}{2}}^+$	N_c	-1	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{9(N_c+3)}{4N_c}$	$-\frac{105(N_c-1)}{8N_c}$

TABLE III. Matrix elements of O_i for decuplet resonances.

	O_1	O_2	O_3	O_4	O_5	O_6
$2^{10}[70, 2^+]_{\frac{5}{2}}^+$	N_c	$-\frac{2}{9}$	$\frac{3}{4N_c}$	$\frac{15}{4N_c}$	$\frac{3(N_c+1)}{2N_c}$	0
$2^{10}[70, 2^+]_{\frac{3}{2}}^+$	N_c	$\frac{1}{3}$	$\frac{3}{4N_c}$	$\frac{15}{4N_c}$	$-\frac{9(N_c+1)}{4N_c}$	0
$2^{10}[70, 0^+]_{\frac{1}{2}}^+$	N_c	0	$\frac{3}{4N_c}$	$\frac{15}{4N_c}$	0	0

values of O_4 are positive for octets and decuplets and of order N_c^{-1} , as in SU(4), and negative and of order N_c^0 for flavor singlets.

By construction, the operators O_5 and O_6 have non-vanishing contributions for orbitally excited states only. They are also two-body, which means that they carry a factor $1/N_c$ in the definition. The operator O_6 contains the irreducible spherical tensor (4) and the SU(6) generator G^{ja} both acting on the whole system. The latter is a coherent operator which introduces an extra power N_c so that the order of the matrix elements of O_6 is $\mathcal{O}(1)$, as it can be seen from Table II. For decuplets and singlets its matrix elements vanish, see Tables III and IV, respectively.

The matrix elements of O_5 and O_6 were obtained from the formulas (B2) and (B4) of Ref. [19] where the multiplet $[70, 1^-]$ has been discussed. The contribution of O_5 cancels out for flavor singlets when $N_c = 3$, like for $\ell = 1$ [19] and $\ell = 3$ [20]. This property follows from the analytic expression of the isoscalar factors given in Ref. [19].

Therefore in the mass formula there is one operator, namely O_1 , of order $\mathcal{O}(N_c)$ and two operators, O_2 and O_6 of order $\mathcal{O}(N_c^0)$. They have been used in Refs. [26,27] where the compatibility of the present approach with the meson-nucleon scattering picture has been proven, as mentioned in the Introduction.

We remind that the advantage of the present procedure over the standard one, where the system is separated into a ground state core + an excited quark [13], is that the number of relevant operators needed in the fit is usually smaller than the number of data and it allows a better understanding of their role in the mass formula, in particular the role of the flavor operator O_4 which has been omitted in the symmetric core + excited quark procedure in the analysis of mixed symmetric negative parity states [13,15]. We should also mention that in our approach the permutation symmetry remains exact in all applications.

A comment is in order for the flavor breaking operators B_i . In the procedure where the system is separated into a core and an excited quark one deals with two operators

TABLE IV. Matrix elements of O_i for singlet resonances.

	O_1	O_2	O_3	O_4	O_5	O_6
$2^1[70, 2^+]_{\frac{5}{2}}^+$	N_c	$\frac{2}{3}$	$\frac{3}{4N_c}$	$-\frac{2N_c+3}{4N_c}$	$-\frac{N_c-3}{2N_c}$	0
$2^1[70, 2^+]_{\frac{3}{2}}^+$	N_c	-1	$\frac{3}{4N_c}$	$-\frac{2N_c+3}{4N_c}$	$\frac{3(N_c-3)}{4N_c}$	0
$2^1[70, 0^+]_{\frac{1}{2}}^+$	N_c	0	$\frac{3}{4N_c}$	$-\frac{2N_c+3}{4N_c}$	0	0

$B_1 = t^8 - \frac{1}{2\sqrt{3}}$ and $B_2 = T_c^8 - \frac{N_c-1}{2\sqrt{3}}$ acting on the excited quark and the core respectively (lower case indicates operators acting on the excited quark and subscript c indicates those acting on the core). These two operators have distinct matrix elements in each sector 28_J , 48_J , 210_J and 21_J [15,29]. In the present method there is a single operator $T^8 = t^8 + T_c^8$ which generates the flavor breaking operator

$$B_1 = -\frac{2}{\sqrt{3}}\left(T^8 - \frac{1}{2\sqrt{3}}\mathcal{O}_1\right) \quad (6)$$

the matrix element of which is $\langle B_1 \rangle = -\mathcal{S}$, the same for all sectors, as indicated in Table I, where \mathcal{S} is the strangeness. Such a result is consistent with Table V of Ref. [29] from which we get

$$\langle T^8 \rangle = \frac{N_c + 3\mathcal{S}}{2\sqrt{3}}, \quad (7)$$

for all sectors, as in Ref. [8]. This considerably simplifies the situation and implies that the flavor symmetry breaking picture is different in the present approach as compared to the symmetric core + excited quark approach, inasmuch as in the first the breaking is independent of the sector and in the second it is not. This may provide an explanation of the unexpectedly large $\Lambda\Sigma$ splitting obtained in the sector 48 , with the symmetric core + excited quark approach, see Ref. [29], while presently, where $\langle B_1 \rangle = -\mathcal{S}$, there is no splitting at all.

IV. THE EXPERIMENTAL SITUATION

In our previous work [29] we have made use of the Baryon Particle Listings of the Particle Data Group before 2012 and made averages over the baryon masses of the Karlsruhe-Helsinki group [37] and the Carnegie Mellon-Berkeley group [38] or considered some values obtained by Manley and Saleski [39]. Here we rely on the 2012 version of the Review of Particle Properties (PDG) [30] which incorporates the new multichannel partial wave analysis of the Bonn-Gatchina group [40]. The changes in PDG for positive parity resonances (for a summary of the Bonn-Gatchina group see Ref. [41]) are important for our work.

First, the resonance $P_{13}(1900)$ has been upgraded from two to three stars with a Breit-Wigner mass of 1905 ± 30 MeV. Second, the resonance $N(2000)5/2^+$ has been split into two two-star resonances $N(1860)5/2^+$ and $N(2000)5/2^+$ with masses indicated in Table V. The suggestion was that $N(1860)5/2^+$ belongs to a quartet [42]. There is a new one-star resonance $N(2040)3/2^+$ observed in the decay $J/\psi \rightarrow p\bar{p}\pi^0$. There is also a new two-star resonance $N(1880)1/2^+$ observed by the Bonn-Gatchina group with a mass of 1870 ± 35 MeV [40], which confirms a previous observation by Manley and Saleski [39] where a mass of 1885 ± 30 MeV has been found.

V. RESULTS AND DISCUSSION

We have performed several numerical fits for finding the unknown coefficients c_i and d_i of the mass formula (2) using the 2012 Review of Particle Properties (PDG) [30] which incorporates the new multichannel partial wave analysis of the Bonn-Gatchina group [40], implying the changes described in Sec. IV. In Table I we present three of the most favorable fits.

Actually we have started by including all experimentally known resonances located in the appropriate mass region, except for those which were supposed to belong to the $[56, 2^+]$ multiplet [32]. Finally we found out that only a selective choice of resonances give a reasonable fit when described by the formalism presented above.

The final result includes 11 resonances, having a status of three, two or occasionally one star. There are no four-star resonances as candidates for the $[70, \ell^+]$ multiplet. The selection we have made is described below. As experimental masses we took either the Bonn-Gatchina group results, or we averaged over all values indicated in the Particle Listings of PDG [30]. For example, for $\Delta(2000)5/2^{+**}$ and $\Sigma(1880)1/2^{+**}$ we averaged over three and eleven experimental values respectively.

As a matter of fact we have included the new $N(1860)5/2^{+**}$ and $N(2040)3/2^{+**}$ resonances and obtained a better numerical fit when interpreting the $N(1860)5/2^{+**}$ resonance as a member of a spin doublet (see Table V) instead of a quartet, as proposed in Ref. [42]. The reason is that the spin operator O_3 contributes with a quantity proportional to $S(S+1)$ and c_3 is positive, see Table I, so that a doublet member should be below a spin quartet member with $J^P = 5/2^+$. The latter is thus expected to have a mass larger than 1860 MeV. However we agree with Ref. [42] that the resonance $N(1880)1/2^{+**}$ belongs to a spin quartet, see Table V.

On the other hand, in order to obtain natural sizes for the coefficient c_i [16], from the final fit we have removed several resonances which were included in our previous work based on the excited quark + symmetric core procedure [29], but which are not compatible with the present approach. These are the $N(1710)1/2^{+***}$ and the $\Sigma(1770)1/2^{+**}$ resonances. The theoretical argument is that their masses are too low. On the experimental side one can justify the removal of the $N(1710)1/2^{+***}$ resonance as due to the latest GWU analysis of Arndt *et al.* [43] where it has not been seen. This is anyhow a controversial resonance.

We had also ignored the $\Delta(1750)1/2^{+**}$ resonance, considered previously [29], inasmuch as, neither Arndt *et al.* [43] nor Anisovich *et al.* [40] find evidence for it.

From Table I one can see that χ^2_{dof} of Fits 1, 2 and 3 are 0.61, 0.52, and 2.27, respectively. In Fit 1 the mass formula contains the operators up to order $1/N_c$ included, which, according to our previous experience with mixed symmetric negative parity states, see, for example, Ref. [20], are

TABLE V. The partial contribution and the total mass (MeV) predicted by the $1/N_c$ expansion using Fit 2 of Table I. The last two columns give the empirically known masses and the 2012 status in the Review of Particles Properties [30].

	Part. contrib. (MeV)						Total (MeV)	Exp. (MeV)	Name, status
	$c_1 O_1$	$c_2 O_2$	$c_3 O_3$	$c_4 O_4$	$c_6 O_6$	$d_1 B_1$			
$^4 N[70, 2^+]_{\frac{7}{2}}$	1848	35	190	14	-36	0	2051 ± 44	2016 ± 104	$N(1990)7/2^{+***}$
$^4 \Lambda[70, 2^+]_{\frac{7}{2}}$						24	2075 ± 63	2094 ± 78	$\Lambda(2020)7/2^{+**}$
$^4 \Sigma[70, 2^+]_{\frac{7}{2}}$						24	2075 ± 63		
$^4 \Xi[70, 2^+]_{\frac{7}{2}}$						48	2099 ± 93		
$^2 N[70, 2^+]_{\frac{5}{2}}$	1848	12	38	14	0	0	1912 ± 31	1860 ± 70	$N(1860)5/2^{+***}$
$^2 \Lambda[70, 2^+]_{\frac{5}{2}}$						24	1936 ± 54		
$^2 \Sigma[70, 2^+]_{\frac{5}{2}}$						24	1936 ± 54		
$^2 \Xi[70, 2^+]_{\frac{5}{2}}$						48	1959 ± 88		
$^4 N[70, 2^+]_{\frac{5}{2}}$	1848	-6	190	14	89	0	2136 ± 39	2090 ± 120	$N(2000)5/2^{+***}$
$^4 \Lambda[70, 2^+]_{\frac{5}{2}}$						24	2159 ± 60	2112 ± 40	$\Lambda(2110)5/2^{+***}$
$^4 \Sigma[70, 2^+]_{\frac{5}{2}}$						24	2159 ± 60		
$^4 \Xi[70, 2^+]_{\frac{5}{2}}$						48	2183 ± 92		
$^4 N[70, 0^+]_{\frac{3}{2}}$	1848	0	190	14	0	0	2052 ± 18	2052 ± 20	$N(2040)3/2^{+**}$
$^4 \Lambda[70, 0^+]_{\frac{3}{2}}$						24	2076 ± 49		
$^4 \Sigma[70, 0^+]_{\frac{3}{2}}$						24	2076 ± 49		
$^4 \Xi[70, 0^+]_{\frac{3}{2}}$						48	2100 ± 86		
$^2 N[70, 2^+]_{\frac{3}{2}}$	1848	-17	38	14	0	0	1883 ± 26	1905 ± 30	$N(1900)3/2^{+****}$
$^2 \Lambda[70, 2^+]_{\frac{3}{2}}$						24	1907 ± 52		
$^2 \Sigma[70, 2^+]_{\frac{3}{2}}$						24	1907 ± 52		
$^2 \Xi[70, 2^+]_{\frac{3}{2}}$						48	1931 ± 87		
$^4 N[70, 2^+]_{\frac{3}{2}}$	1848	-35	190	14	0	0	2018 ± 30		
$^4 \Lambda[70, 2^+]_{\frac{3}{2}}$						24	2041 ± 55		
$^4 \Sigma[70, 2^+]_{\frac{3}{2}}$						24	2041 ± 55		
$^4 \Xi[70, 2^+]_{\frac{3}{2}}$						48	2065 ± 90		
$^2 N[70, 0^+]_{\frac{1}{2}}$	1848	0	38	14	0	0	1900 ± 27		
$^2 \Lambda[70, 0^+]_{\frac{1}{2}}$						24	1924 ± 52		
$^2 \Sigma[70, 0^+]_{\frac{1}{2}}$						24	1924 ± 52		
$^2 \Xi[70, 0^+]_{\frac{1}{2}}$						48	1948 ± 87		
$^4 N[70, 2^+]_{\frac{1}{2}}$	1848	-52	190	14	-125	0	1875 ± 34	1870 ± 35	$N(1880)1/2^{+***}$
$^4 \Lambda[70, 2^+]_{\frac{1}{2}}$						24	1899 ± 58		
$^4 \Sigma[70, 2^+]_{\frac{1}{2}}$						24	1899 ± 58		
$^4 \Xi[70, 2^+]_{\frac{1}{2}}$						48	1923 ± 92		
$^2 \Delta[70, 2^+]_{\frac{5}{2}}$	1848	-12	38	72	0	0	1946 ± 58	1892 ± 143	$\Delta(2000)5/2^{+***}$
$^2 \Sigma'[70, 2^+]_{\frac{5}{2}}$						24	1970 ± 67		
$^2 \Xi'[70, 2^+]_{\frac{5}{2}}$						48	1994 ± 92		
$^2 \Omega[70, 2^+]_{\frac{5}{2}}$						71	2018 ± 124		
$^2 \Delta[70, 2^+]_{\frac{3}{2}}$	1848	17	38	72	0	0	1975 ± 64		
$^2 \Sigma'[70, 2^+]_{\frac{3}{2}}$						24	1999 ± 71		
$^2 \Xi'[70, 2^+]_{\frac{3}{2}}$						48	2023 ± 95		
$^2 \Omega[70, 2^+]_{\frac{3}{2}}$						71	2046 ± 126		
$^2 \Delta[70, 0^+]_{\frac{1}{2}}$	1848	0	38	72	0	0	1958 ± 59		
$^2 \Sigma'[70, 0^+]_{\frac{1}{2}}$						24	1982 ± 68	1896 ± 95	$\Sigma(1880)1/2^{+***}$
$^2 \Xi'[70, 0^+]_{\frac{1}{2}}$						48	2005 ± 93		
$^2 \Omega[70, 0^+]_{\frac{1}{2}}$						71	2029 ± 124		
$^2 \Lambda'[70, 2^+]_{\frac{5}{2}}$	1848	35	38	-43	0	24	1901 ± 84		
$^2 \Lambda'[70, 2^+]_{\frac{3}{2}}$	1848	-52	38	-43	0	24	1815 ± 87		
$^2 \Lambda'[70, 0^+]_{\frac{1}{2}}$	1848	0	38	-43	0	24	1867 ± 77	1791 ± 64	$\Lambda(1810)1/2^{+***}$

thought to be the most dominant. Note that despite a good χ_{dof}^2 , the coefficient c_2 of the spin-orbit operator is not well determined. Its central value is consistent with predictions from the $1/N_c$ expansion for the $N = 1$ band [16] but we expect a smaller c_2 in the $N = 2$ band, inasmuch as the contribution from the spin-orbit operator decreases with the excitation energy [28].

In Fit 2 we have removed the operator O_5 and obtained a reasonable value for c_2 . In Fit 3 we have removed the operator O_6 , which like O_2 is of order N_c^0 , crucial for the compatibility of the quark-shell picture used here and the more fundamental meson-nucleon scattering picture, as discussed in Ref. [27]. In Fit 3 the coefficient d_1 of the SU(3) breaking term becomes negative which is not good for the mass sequence within a multiplet. Thus the presence of O_6 is necessary. It implies once more that the model used here is compatible with the contracted SU($2N_f$) symmetry which is exact when $N_c \rightarrow \infty$.

Thus Fit 2 is the best fit. The baryon masses calculated from this fit with the formula (2) are exhibited in Table V, together with the partial contributions of various operators. One can clearly see that the flavor operator O_4 , neglected in the symmetric core + excited quark studies of the $N = 1$ band is crucial for the fit. To the masses of the decuplet members it contributes nearly two times more than the spin operator O_3 . As its matrix elements are negative for flavor singlets, see Table IV, it also allows a good description of the $\Lambda(1810)1/2^+$ resonance. The important role of O_4 is in agreement with the conclusion of previous studies on negative parity states [19,20].

VI. REGGE TRAJECTORIES

In Ref. [28] we searched for a systematic global behavior of some c_i coefficients as a function of the excitation energy, i.e., as a function of the band number N . Accordingly, we have plotted some of the known c_i at that time for $N \leq 4$. The points corresponding to mixed symmetric states were obtained from the symmetric core + excited quark approach. There were no studies of the $N = 3$ band available yet. We found that c_1 increases linearly as a function of N , while c_2 and the spin term coefficient decrease as a function of N , as expected from quark models.

These findings inspired further studies to establish a connection between the $1/N_c$ expansion method and a simple semi-relativistic quark model with a Y-junction confinement potential plus a hyperfine interaction generated by one gluon exchange, both for nonstrange and strange baryons [44,45]. The band number N emerged naturally from both approaches. We found that the large N_c results for c_1^2 are practically indistinguishable from the quark model results and they followed a linear Regge trajectory as a function of N . The linear Regge trajectories are a manifestation of the nonperturbative aspect of QCD dynamics, which at long distance becomes dominated by

confinement [46]. Indeed, let us denote by M_{qqq} the contribution of the kinetic plus the confinement energy in the quark model. Then from the identification of this contribution with the leading spin-flavor singlet operator of the large N_c mass formula one has

$$c_1^2 = M_{qqq}^2/9, \quad (8)$$

where we have set $N_c = 3$, so that the values of c_1^2 were compared to the quark model results, see Fig. 1 of Ref. [47] where a review can also be found.

Presently, we have a consistent description of mixed symmetric positive and negative parity states corresponding to $N = 1, 2$ and 3 bands. It is interesting to revisit the Regge trajectory problem. In Fig. 1 we plot c_1^2 as a function of the band number N for $N \leq 4$. The value of c_1 at $N = 3$ is presently known [20], while in Ref. [28] the corresponding point was missing. One can see that two distinct trajectories emerge from this new picture, one for symmetric [56]-plets, the other for mixed symmetric [70]-plets. This behavior reminds that obtained in Ref. [48] where two distinct trajectories have been found for the evolution of $(N_c c_1)^2$ as a function of the angular momentum $\ell \leq 6$ (Chew-Frautschi plots). Note that in Ref. [48] the mixed symmetric states were described within the ground state core + excited quark approach. The mass operator was reduced to the contribution of the $\mathcal{O}(N_c)$ spin-flavor singlet, the $\mathcal{O}(1/N_c)$ hyperfine spin-spin interaction, acting between core quarks only, and SU(3) breaking terms. As a consequence there is no contribution from the spin dependent terms in flavor singlets because their core has $S_c = 0$. There are no $\mathcal{O}(N_c^0)$ contributions. For a consistent treatment, in Ref. [48] the hyperfine interaction was restricted to core quarks in symmetric states as well. It was not

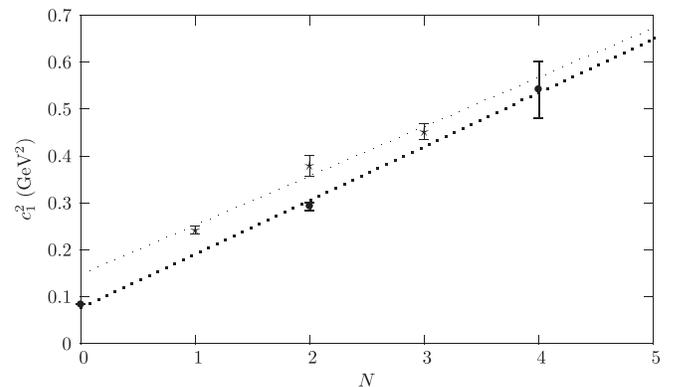


FIG. 1. The coefficient $c_1^2(\text{GeV}^2)$ as a function of the band number N . The numerical values of c_1 were taken from Ref. [44] for $N = 0$, from Ref. [19] Fit 3 for $N = 1$, from Ref. [32] for $N = 2$ [56, 2^+], from the present work Fit 2 for $N = 2$ [70, ℓ^+] ($\ell = 0, 2$), from Ref. [20] Fit 3 for $N = 3$ [70, ℓ^-] ($\ell = 1, 2, 3$), from Ref. [33] for $N = 4$ [56, 4^+]. The heavy dots refer to [56]-plets and the stars to [70]-plets. The best fit of these data was obtained with two distinct linear trajectories.

necessary to specify whether or not the core is excited, due to the simplicity of the mass operator.

In our case, the symmetric and mixed symmetric states are treated on an equal basis: there is no distinction between the core and an excited quark (the core may be excited as well), the Pauli principle is always fulfilled and all quark-quark interaction terms are included. The existence of two distinct Regge trajectories, one for symmetric, another for mixed symmetric states, may be due to the existence of terms of order N_c^0 in the mass formula for mixed symmetric states, which often bring a negative contribution, see e.g., the operator O_6 , while for symmetric states the expansion starts at order $1/N_c$. This may require the coefficient c_1 to be larger for mixed symmetric states, for compensating the negative contribution of operators of order N_c^0 .

VII. CONCLUSIONS

We have revisited the mass spectrum of resonances that are supposed to belong to the [70, ℓ^+] multiplets of the $N = 2$ band with $\ell = 0$ or 2 in the light of a recent multi-channel partial wave analysis which enriched the Review of Particle Properties in 2012. We found that the new resonances can well be described as belonging to the above multiplets. However, we found it more appropriate to describe the resonance $N(1860)5/2^+$ as a member of a doublet rather than that of a spin quartet, at variance with the suggestion of Ref. [42]. The three-star resonance $N(1710)1/2^{+***}$ does not fit into our treatment of the

[70, ℓ^+] multiplets. It would be useful to better understand its nature.

We point out that the $1/N_c$ expansion method allows us to search for a classification of excited baryons into $SU(6) \times O(3)$ multiplets, as presently shown in Table V. This is a natural and useful extension of the classification of ground state baryons. It allows us to make predictions for the mass range of unknown baryons as members of octets or decuplets, which may guide experimentalists in the search for highly excited or strange baryons for which data are scarce.

Like for the $N = 1$ and 3 bands, we found that both the quark spin and flavor operators, acting on the entire system, play dominant roles in describing the data. In the symmetric core + excited quark approach applied to the $N = 1$ band [13] the latter was split into the core part T_c^2 (where its contribution is identical to that of the spin part S_c^2) and a term, $\frac{1}{N_c} t^a T_c^a$ acting between the excited quark and the core, which was ignored. In our previous work [29] we found that $\frac{1}{N_c} t^a T_c^a$ contributes with some important amount to the mass.

Two distinct Regge trajectories have been found for symmetric and mixed symmetric states. An important remark is that the bases of operators used for different mixed symmetric multiplets was the same, irrespective of the resonance parity. The extension of our studies to resonances belonging to $N > 4$ would help in better understanding the existence of two Regge trajectories, although we are aware that the number of experimental data is very limited at higher energies.

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