

Next-to-leading order corrections to the gluon-induced forward jet vertex from the high energy effective action

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(Received 2 January 2013; published 22 April 2013)

We determine both real and virtual next-to-leading order corrections to the gluon-induced forward jet vertex from the high energy effective action proposed by Lipatov. For these calculations we employ the same regularization and subtraction formalism developed in our previous work on the quark-initiated vertex. We find agreement with previous results in the literature.

DOI: [10.1103/PhysRevD.87.076009](https://doi.org/10.1103/PhysRevD.87.076009)

PACS numbers: 11.10.-z, 11.55.Jy, 12.38.Cy, 25.75.-q

I. THEORETICAL FRAMEWORK

In this work we present the calculation of the vertex describing the production of a jet in a forward direction very close in the detector to one of the hadrons in hadron-hadron interactions at very high energies. This is done in the kinematic approximation, where the jet is well separated in rapidity from other jets also produced in the scattering process. As a calculational technique we make use of Lipatov's effective action [1], designed to ease the derivation of scattering amplitudes in the high energy limit of QCD. Here we focus on the gluon-initiated jet vertex, which is a more complicated counterpart of the quark-initiated vertex, derived with similar techniques in Ref. [2]. A convenient regularization and subtraction procedure, taken from Ref. [2], is shown to give the correct results (in agreement with previous calculations in the literature [3–8]) at next-to-leading (NLO) accuracy. The convolution of this jet vertex with the NLO Balitsky-Fadin-Kuraev-Lipatov (BFKL) gluon Green function plays a very important role in the description of jet production at the LHC physics program. Many interesting studies have been performed in this direction in recent years; see Refs. [9–20].

A complete description of the high energy effective action used in this work can be found in Refs. [1,5]. For a more recent discussion directly related to our calculation, we refer the reader to Ref. [21]. The calculation of the quark contributions to the gluon Regge trajectory at two loops using Lipatov's effective action has been performed by us in Refs. [22,23]. Here we will just briefly explain the general structure of this action and then describe in some detail our calculation of the gluon-initiated jet vertex.

The high energy effective action is based on the interplay between QCD particles and Reggeized degrees of freedom, which are introduced as independent fields interacting with the standard ones via new vertices. These effective interactions, dominant in the high energy limit of QCD, appear inside an extra term added to the QCD action. Reggeized quarks and gluons “propagate” in the t

channel, with modified propagators connecting two regions of different rapidities, and play the role of suppressing any real emission in that interval. At the endpoints of these intervals there can be particle production—single production in multi-Regge kinematics (MRK) and double production in quasi-multi-Regge kinematics. Within these clusters of particle production there are no kinematic restrictions, and interactions are the usual QCD ones. A representation of this effective clustering is shown in Fig. 1(a). The ordering in rapidity of the produced clusters is of the form $y_0 \gg y_1 \gg \dots \gg y_{n+1}$, where $y_k = \frac{1}{2} \ln \frac{k^+}{k^-}$ and with all particles in each cluster being emitted with a similar rapidity. Defining the light-cone vectors $n^{+,-} = 2p_{a,b}/\sqrt{s}$ we work with Sudakov expansions for four-dimensional vectors of the form $k = \frac{1}{2}(k^+ n^- + k^- n^+) + \mathbf{k}$, with \mathbf{k} being transverse. The strong ordering of the clusters simplifies the polarization tensor of the t -channel Reggeized particles, which can be written as $g_{\mu\nu} \rightarrow \frac{1}{2}(n^+)_{\mu}(n^-)_{\nu} + \mathcal{O}(\frac{1}{s})$, with \sqrt{s} being the center-of-mass energy of the scattering process, carrying mainly transverse momenta, $q_i^2 = -q_i^{\perp 2}$.

The effective interaction between Reggeized and usual particles, like the one shown in Fig. 1, consists of two pieces: the projection of the QCD vertex onto high energy kinematics and additional induced contributions. This structure can be obtained from the following form of the effective action:

$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind}};$$

$$S_{\text{ind}} = \int d^4x \text{Tr}[(W_+[v(x)] - \mathcal{A}_+(x))\partial_{\perp}^2 \mathcal{A}_-(x)]$$

$$+ \{+ \leftrightarrow -\}, \quad (1)$$

where \mathcal{A}_{\pm} are the gauge-invariant Reggeized gluon fields which satisfy the following kinematic constraint:

$$\partial_+ \mathcal{A}_-(x) = 0 = \partial_- \mathcal{A}_+(x). \quad (2)$$

Their couplings to the QCD gluon field are given in terms of two nonlocal functionals $W_{\pm}[v] = v_{\pm} \frac{1}{D_{\pm}} \partial_{\pm} = v_{\pm} - g v_{\pm} \frac{1}{\partial_{\pm}} v_{\pm} + \dots$ with $D_{\pm} = \partial_{\pm} + g v_{\pm}$.

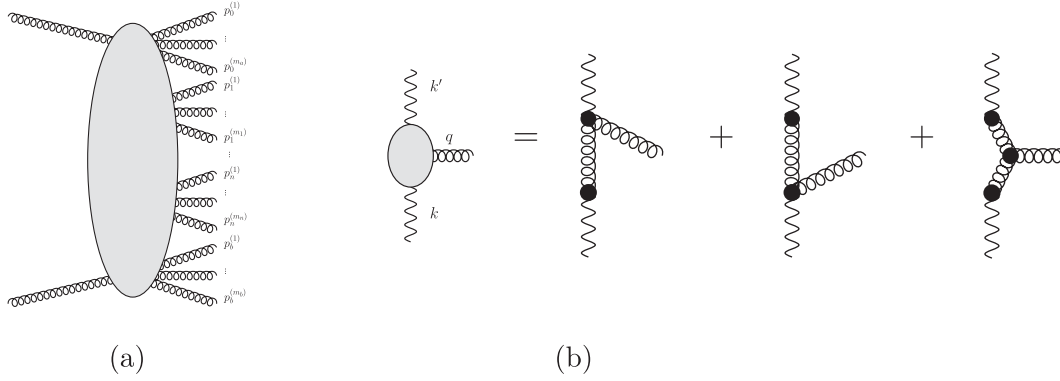


FIG. 1. (a) Quasi-multi-Regge kinematics. (b) Reggeized gluon-Reggeized gluon-gluon effective vertex. The first two diagrams are the induced contributions.

$$\begin{aligned}
 & \begin{array}{c} k, c, \nu \\ \text{wavy line} \\ q, a, \pm \end{array} = -i\mathbf{q}^2 \delta^{ac} (n^\pm)^\nu, \quad k^\pm = 0. \\
 & \begin{array}{c} + \\ \text{wavy line} \\ a \\ q \\ \text{wavy line} \\ - \\ b \end{array} = \delta^{ab} \frac{i/2}{\mathbf{q}^2} \\
 & \begin{array}{c} k_1, c_1, \nu_1 \quad k_2, c_2, \nu_2 \\ \text{wavy lines} \\ q, a, \pm \end{array} = g f^{c_1 c_2 a} \frac{\mathbf{q}^2}{k_1^\pm} (n^\pm)^{\nu_1} (n^\pm)^{\nu_2}, \quad k_1^\pm + k_2^\pm = 0 \\
 & \begin{array}{c} k_2, c_2, \nu_2 \\ \text{wavy lines} \\ k_1, c_1, \nu_1 \quad k_3, c_3, \nu_3 \\ \text{wavy lines} \\ q, a, \pm \end{array} = ig^2 \mathbf{q}^2 \left(\frac{f a_3 a_2 e f a_1 e a}{k_3^\pm k_1^\pm} + \frac{f a_3 a_1 e f a_2 e a}{k_3^\pm k_2^\pm} \right) (n^\pm)^{\nu_1} (n^\pm)^{\nu_2} (n^\pm)^{\nu_3}, \quad k_1^\pm + k_2^\pm + k_3^\pm = 0
 \end{aligned}
 \tag{a} \tag{b} \tag{c} \tag{d}$$

FIG. 2. Feynman rules for the lowest-order effective vertices. Wavy lines denote Reggeized fields and curly lines gluons. (a) is the direct transition vertex and (b) the Reggeized gluon propagator. We also show the unregulated order g (c) and order g^2 (d) induced vertices.

The lowest order Feynman rules for the induced vertices are shown in Fig. 2. The contributions in the vertices of the form $1/k^\pm$ generate a new type of divergence which will be related to high energy logarithms. These divergences call for a regularization, or equivalently, a suitable definition of the nonlocal operator ∂_\pm^{-1} in the Wilson lines. A convenient regularization scheme was defined in Refs. [2,22] where n^+ and n^- are replaced by tilted light-cone vectors of the form $a = n^- + e^{-\rho} n^+$ and $b = n^+ + e^{-\rho} n^-$. These tilted light-cone vectors form a hyperbolic angle ρ in Minkowski space which can be interpreted as $\ln s$, implying that at high energies we are interested in the $\rho \rightarrow \infty$ limit. In the following we treat ρ as an external parameter which, at the end, we consider in the $\rho \rightarrow \infty$ limit, similar to the treatment of $\epsilon \rightarrow 0$ in $d = 4 + 2\epsilon$ dimensional regularization.

After this brief introduction we turn in the following to give details of our calculation of the gluon-initiated

forward jet vertex. In Sec. II we provide a description of the virtual corrections to the gluon-gluon-Reggeized gluon vertex, while in Sec. III we explain the key details for the calculation of the real corrections. Finally, Sec. IV contains our conclusions and an outlook for future calculations. The Appendix collects further technical details.

II. VIRTUAL CORRECTIONS TO THE GLUON-GLUON-REGGEIZED GLUON VERTEX

Let us consider the process $gg \rightarrow gg$, where the external gluons are on-shell: $p_a^2 = (p_a - k)^2 = p_b^2 = (p_b + k)^2 = 0$. In the high energy limit the corresponding scattering amplitude factorizes into a Reggeized gluon exchange in the t channel and its couplings to the external particles, the so-called impact factors. At tree level this is shown in Fig. 3.

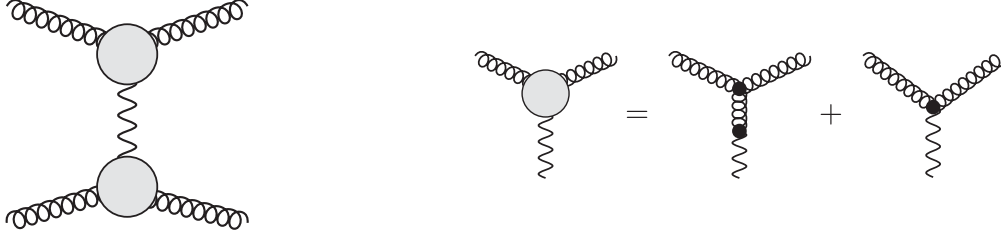


FIG. 3. Tree-level contribution to the gluon-gluon scattering amplitude in terms of effective vertices.

Light-cone momenta are defined making use of the momenta of the incoming particles p_a and p_b through $p_a = p_a^+ n^- / 2$ and $p_b = p_b^- n^+ / 2$ with $2p_a \cdot p_b = s = p_a^+ p_b^-$. From Eq. (2) one obtains for the upper vertex the constraint $k^+ = 0$, while $k^- = 0$ for the lower one. Both constraints can be understood as the leading term in the expansion in the small k^+ / p_a^+ and k^- / p_b^- ratios, respectively. The polarization vectors must be physical, satisfying for the upper vertex $\varepsilon \cdot p_a = 0$ and $\varepsilon^* \cdot (p_a - k) = 0$. The last relation implies that $\varepsilon^* \cdot p_a = \varepsilon^* \cdot k$. Gauge invariance of the effective action enables us to choose different gauges for the upper and lower gluon-gluon-Reggeized gluon couplings. We therefore impose the condition $\varepsilon(p_a) \cdot n^+ = \varepsilon^*(p_1) \cdot n^+ = 0$ for the upper vertex and the condition $\varepsilon(p_b) \cdot n^- = \varepsilon^*(p_2) \cdot n^- = 0$ for the lower vertex, which implies the following polarization sum,

$$\sum_{\text{polarizations}} \varepsilon_\mu^\lambda(p, n) (\varepsilon_\nu^{\lambda'})^*(p, n) = -g_{\mu\nu} + \frac{p_\mu n_\nu + p_\nu n_\mu}{p \cdot n}, \quad (3)$$

with p being the gluon momentum and $n = n^\pm$.

To define the impact factors we start from the general definition for the differential cross section for m -particle production in terms of the corresponding matrix elements and the phase space integral, i.e.,

$$d\sigma = \frac{1}{2s} |\mathcal{M}_{i \rightarrow f}|^2 d\Pi_m (2\pi)^d \delta^d\left(p_a + p_b - \sum_{j=1}^m p_j\right);$$

$$d\Pi_m = \prod_{j=1}^m \frac{d^d p_j}{(2\pi)^{d-1}} \delta_+(p_j^2). \quad (4)$$

In the special case where all final state particles are produced either in the fragmentation region of particle a or particle b , we rewrite, with $m = m_a + m_b$, the overall delta function for global momentum conservation as follows:

$$(2\pi)^d \delta^d\left(p_a + p_b - \sum_{j=1}^m p_j\right)$$

$$= \int \frac{d^d k}{(2\pi)^d} (2\pi)^{2d} \delta^d\left(p_a + k - \sum_{j=1}^{m_a} p_j\right)$$

$$\times \delta^d\left(p_b - k - \sum_{l=1}^{m_b} p_l\right). \quad (5)$$

The generalization to the additional production of n -particle clusters at central rapidities (see also Fig. 1(a), with $m = m_a + m_b + \sum_i^n m_i$, reads

$$(2\pi)^d \delta^d\left(p_1 + p_2 - \sum_{j=1}^m p_j\right)$$

$$= \prod_{i=0}^n \int \frac{d^d k_i}{(2\pi)^d} (2\pi)^{(2+n)d} \delta^d\left(p_a + k_0 - \sum_{l_0=1}^{m_a} p_{l_0}\right)$$

$$\times \delta^d\left(p_b - k_n - \sum_{l_n=1}^{m_b} p_{l_n}\right)$$

$$\times (2\pi)^{nd} \delta^d\left(k_0 - k_1 - \sum_{l_1=1}^{m_1} p_{l_1}\right)$$

$$\times \cdots \delta^d\left(k_{n-1} - k_n - \sum_{l_n=1}^{m_n} p_{l_n}\right). \quad (6)$$

Restricting from now on to the $gg \rightarrow gg$ amplitude at tree-level, we use in our next step the fact that the effective action naturally factorizes the amplitude $i\mathcal{M}_{gr^* \rightarrow g_1}$ into two products of $i\mathcal{M}_{gr^* \rightarrow g_1}$ times the square root of the Reggeized gluon propagator $i/2q^2$. Squaring, averaging over color and polarization of the initial gluon and summing over color and polarization of the final state and Reggeized gluon (at the level of the $gr^* \rightarrow g$ amplitudes), the $2 \rightarrow 2$ tree-level amplitude takes the following factorized form:

$$\overline{|\mathcal{M}_{g_a g_b \rightarrow g_1 g_2}^{(0)}|^2} = \frac{\overline{|\mathcal{M}_{g_a r^* \rightarrow g_1}^{(0)}|^2}}{2k^2 \sqrt{N_c^2 - 1}} \times \frac{\overline{|\mathcal{M}_{g_b r^* \rightarrow g_2}^{(0)}|^2}}{2k^2 \sqrt{N_c^2 - 1}}. \quad (7)$$

Defining now the impact factors $= h_{a,b}^{(0)}(\mathbf{k})$ through the relation

$$d\sigma_{ab} = h_a^{(0)}(\mathbf{k}) h_b^{(0)}(\mathbf{k}) d^{2+2\epsilon} \mathbf{k}, \quad (8)$$

we are led to the following general expression in terms of the effective action matrix elements:

$$h_{a,\text{gluon}}^{(0)}(\mathbf{k})$$

$$= \frac{(2\pi)^{d/2}}{2p_a^+} \int dk^- \frac{\overline{|\mathcal{M}_{g_a r^* \rightarrow g_1}^{(0)}|^2}}{2k^2 \sqrt{N_c^2 - 1}} d\Pi_1 \delta^{(d)}(p_a + k - p_1), \quad (9)$$

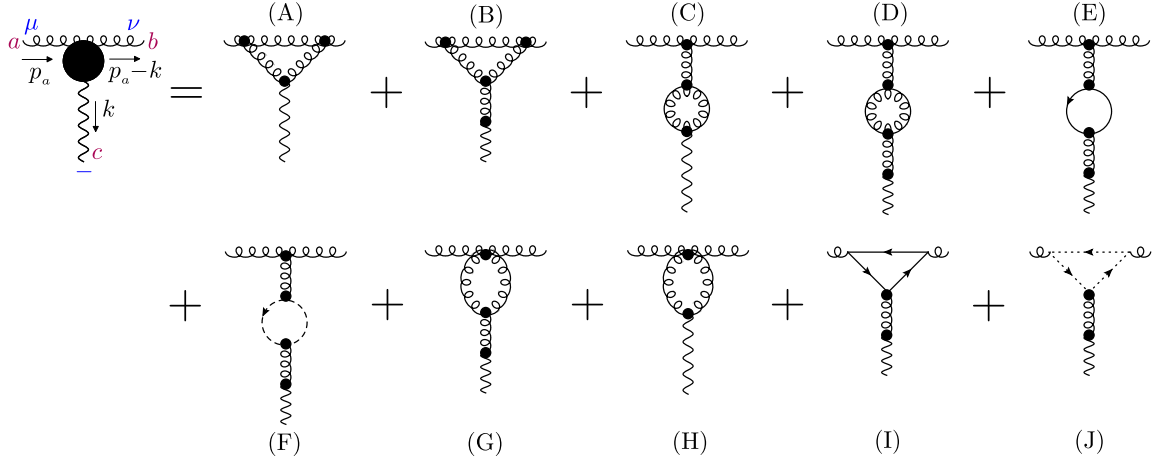


FIG. 4 (color online). One-loop virtual corrections to the gluon-initiated jet vertex.

with a natural generalization to m_b -particle production in and the fragmentation region of particle b .

The $i\mathcal{M}_{gr^* \rightarrow g_1}$ amplitude itself receives at tree-level two contributions (see Fig. 3)—one from the gluon-gluon-Reggeized gluon vertex,

$$gf_{abc} \frac{k^2}{p_a^+} (n^+)^{\mu_1} (n^+)^{\mu_2} \varepsilon_{\mu_1} \varepsilon_{\mu_2}^*,$$

and the other from the projection of the 3-gluon vertex,

$$gf_{abc} [2g^{\mu_1\mu_2} p_a^+ - (n^+)^{\mu_1} (p_a - k)^{\mu_2} - (n^+)^{\mu_2} (2k + p_a)^{\mu_1}] \varepsilon_{\mu_1} \varepsilon_{\mu_2}^*.$$

It is possible to verify that this amplitude is gauge invariant and satisfies the necessary Ward/Slavnov-Taylor identities, see, e.g., Refs. [1,21]. At the amplitude level we arrive at

$$\mathcal{M}_{gr^* \rightarrow g_1} = 2gf_{abc} \varepsilon \cdot \varepsilon^* \quad (10)$$

$$\begin{aligned} h_{a,\text{gluon}}^{(0)}(\mathbf{k}) &= \frac{N_c}{\sqrt{N_c^2 - 1}} \frac{g^2}{k^2} \frac{1}{(2\pi)^{1+\epsilon}} \\ &= \frac{2^{1+\epsilon} \alpha_s C_A}{\mu^{2\epsilon} \Gamma(1-\epsilon) \sqrt{N_c^2 - 1}} \frac{1}{k^2}; \\ \alpha_s &\equiv \frac{g^2 \mu^{2\epsilon} \Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}}. \end{aligned} \quad (11)$$

The one-loop corrections to this gluon-gluon-Reggeized gluon vertex are shown in Fig. 4. All diagrams are evaluated in the limit $\rho \rightarrow \infty$, while we only keep track of divergent ($\mathcal{O}(\rho)$) and finite ($\mathcal{O}(\rho^0)$) terms; ϵ is on the other hand kept finite. Details about the calculation of individual diagrams can be found in the Appendix A1. The final result for the one-loop $gr^* \rightarrow g$ amplitude reads

$$\begin{aligned} i\mathcal{M}_{gr^* \rightarrow g_1}^{(1)} &= -\frac{g^3 \mu^{2\epsilon} p_a^+}{(4\pi)^{2+\epsilon}} f_{abc} \left(\frac{k^2}{\mu^2} \right)^\epsilon \left\{ N_c \varepsilon \cdot \varepsilon^* \frac{\Gamma(1-\epsilon) \Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left[2 \ln \left(\frac{p_a^+}{|k|} \right) + \rho - i\pi + \psi(1) - 2\psi(\epsilon) + \psi(1-\epsilon) \right] \right. \\ &+ 8[N_c(1+\epsilon) - n_f] \left[-\frac{1}{2} \varepsilon \cdot \varepsilon^* \frac{\Gamma^2(1+\epsilon) \Gamma(-\epsilon)}{\Gamma(4+2\epsilon)} + \frac{\varepsilon \cdot q \varepsilon^* \cdot q}{k^2} \frac{\Gamma(\epsilon) \Gamma(1-\epsilon)}{\Gamma(4+2\epsilon)} (2\Gamma(1+\epsilon) + \Gamma(2+\epsilon)) \right] \\ &+ 8[N_c(1+\epsilon) - n_f] \frac{\varepsilon \cdot q \varepsilon^* \cdot q}{k^2} \frac{\Gamma(-\epsilon) \Gamma^2(1+\epsilon)}{(2+2\epsilon) \Gamma(2+2\epsilon)} + \varepsilon \cdot \varepsilon^* (4N_c - n_f) \frac{\Gamma(-\epsilon) \Gamma^2(1+\epsilon)}{\epsilon \Gamma(2+2\epsilon)} \\ &- \varepsilon \cdot \varepsilon^* (4N_c - n_f) \frac{1+2\epsilon}{\epsilon} \frac{\Gamma(-\epsilon) \Gamma^2(1+\epsilon)}{\Gamma(2+2\epsilon)} + 2N_c \varepsilon \cdot \varepsilon^* \frac{\Gamma(-\epsilon) \Gamma^2(1+\epsilon)}{\Gamma(2+2\epsilon)} + 2\varepsilon \cdot \varepsilon^* [(1+\epsilon)N_c - n_f] \\ &\times \left. \frac{\Gamma(-\epsilon) \Gamma^2(1+\epsilon)}{(3+2\epsilon) \Gamma(2+2\epsilon)} - 2\varepsilon \cdot \varepsilon^* [4N_c - n_f] \frac{\Gamma(-\epsilon) \Gamma^2(1+\epsilon)}{\Gamma(2+2\epsilon)} \right\}. \end{aligned} \quad (12)$$

Our result contains two types of tensor structures involving the initial and final polarization tensors, namely, $\varepsilon \cdot \varepsilon^*$ and $\varepsilon \cdot q \varepsilon^* \cdot q / k^2$. These are related to the helicity conserving and helicity violating terms [8]

$$\varepsilon \cdot \varepsilon^* = \delta_{\lambda_a, \lambda_1}; \quad \varepsilon \cdot \varepsilon^* + \frac{2}{k^2} \varepsilon \cdot q \varepsilon^* \cdot q = -\delta_{\lambda_a, -\lambda_1}. \quad (13)$$

The renormalization factors Z^\pm cancel for the complete scattering amplitude and can be parametrized as follows

$$Z^\pm\left(\frac{M^\pm}{\sqrt{k^2}}, \rho; \epsilon, \frac{k^2}{\mu^2}\right) = \exp\left[\left(\frac{\rho}{2} - \ln \frac{M^\pm}{\sqrt{k^2}}\right)\omega\left(\epsilon, \frac{k^2}{\mu^2}\right) + f^\pm\left(\epsilon, \frac{k^2}{\mu^2}\right)\right], \quad (23)$$

where the gluon Regge trajectory has the following perturbative expansion,

$$\omega\left(\epsilon, \frac{k^2}{\mu^2}\right) = \omega^{(1)}\left(\epsilon, \frac{k^2}{\mu^2}\right) + \omega^{(2)}\left(\epsilon, \frac{k^2}{\mu^2}\right) + \dots, \quad (24)$$

with the one-loop expression given in Eq. (17). The function $f(\epsilon, k^2)$ parametrizes finite contributions and is, in principle, arbitrary. While symmetry of the scattering amplitude requires $f^+ = f^- = f$, Regge theory suggests fixing it in such a way that at one loop, the non- ρ -enhanced contributions of the one-loop Reggeized gluon self energy

are entirely transferred to the gluon-Reggeized gluon couplings. This leads to

$$f^{(1)}\left(\epsilon, \frac{k^2}{\mu^2}\right) = -\frac{\alpha_s N_c \Gamma^2(1 + \epsilon)}{4\pi\Gamma(1 + 2\epsilon)} \left(\frac{k^2}{\mu^2}\right)^\epsilon \left[\frac{5 + 3\epsilon - \frac{n_f}{N_c}(2 + 2\epsilon)}{2(1 + 2\epsilon)(3 + 2\epsilon)}\right]. \quad (25)$$

Let us remark that in principle other alternative and even asymmetric $f^+ \neq f^-$ choices are possible, as long as they are in agreement with UV-renormalizability of QCD and collinear factorization. Using now $M^+ = p_a^+$, $M^- = p_b^-$ we can see that this choice for f keeps the full s -dependence of the amplitude inside the Reggeized gluon exchange. The renormalized gluon-gluon-Reggeized gluon couplings allows then to extract the NLO corrections to the gluon impact factor. Extracting the Born contribution and decomposing into helicity conserving and non-conserving parts

$$\mathcal{C}_{g^* \rightarrow g}^{\mathcal{R}}\left(1; \epsilon, \frac{k^2}{\mu^2}\right) = 2gf_{abc} \cdot [\Gamma_a^{(+)} \delta_{\lambda_a, \lambda_1} + \Gamma_a^{(-)} \delta_{\lambda_a, -\lambda_1}], \quad (26)$$

where the helicity tensors are for finite ϵ defined through Eq. (13), we have

$$\begin{aligned} \Gamma_a^{(+)} &= -\frac{1}{2}\omega^{(1)}\left(\epsilon, \frac{k^2}{\mu^2}\right) \left[-\psi(1) + 2\psi(2\epsilon) - \psi(1 - \epsilon) + \frac{1}{4(1 + 2\epsilon)(3 + 2\epsilon)} + \frac{7}{4(1 + 2\epsilon)} - \frac{n_f}{N_c} \frac{1 + \epsilon}{(1 + 2\epsilon)(3 + 2\epsilon)}\right] \\ &= \frac{\alpha_s N_c}{4\pi} \left(\frac{k^2}{\mu^2}\right)^\epsilon \left[-\frac{1}{\epsilon^2} + \frac{\beta_0}{2\epsilon} - \frac{(67 - \pi^2)N_c - 10n_f}{18}\right] + \mathcal{O}(\epsilon); \\ \Gamma_a^{(-)} &= -\frac{1}{2}\omega^{(1)}\left(\epsilon, \frac{k^2}{\mu^2}\right) \left[\frac{\epsilon}{(1 + \epsilon)(1 + 2\epsilon)(3 + 2\epsilon)} \left(1 + \epsilon - \frac{n_f}{N_c}\right)\right] = \frac{\alpha_s}{12\pi} (N_c - n_f) + \mathcal{O}(\epsilon). \end{aligned}$$

with $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$. The result is in precise agreement with the literature² [3–6,8].

III. REAL CORRECTIONS AND ONE-LOOP JET VERTEX

In the high energy limit, the real corrections to the Born-level process are naturally cast into three contributions to the $gg \rightarrow ggg$ amplitude where the additional gluon is either produced at central rapidities or close to the fragmentation region of one of the initial gluons (quasielastic gluon production); see Fig. 5. A second class of corrections due to the possible fragmentation of one of the initial gluons into a $q\bar{q}$ pair which only contributes to the quasi-elastic region, see Fig. 6. In the same way as the effective action generates high energy divergences near the light cone when computing virtual corrections, a cutoff in rapidity must be enforced in the longitudinal integrations.

The central production amplitude yields the unintegrated real part of the forward leading order BFKL kernel and is obtained from the sum of the following three effective diagrams:

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 + \text{Diagram}_3. \quad (27)$$

The squared amplitude for (27), averaged over color of the incoming Reggeized fields and summed over final state color and helicities reads [2]

$$\overline{|\mathcal{M}|^2}_{r^* \rightarrow g} = \frac{16g^2 N_c}{N_c^2 - 1} \frac{k'^2 k^2}{q^2} \quad (28)$$

with $k' = k - q$. The central production vertex $V^{(0)}$ can be then defined through the differential cross section

²As noted in Ref. [6], the original result [3] contains several misprints. The correct expressions can be found, e.g., in Refs. [5,6,8].

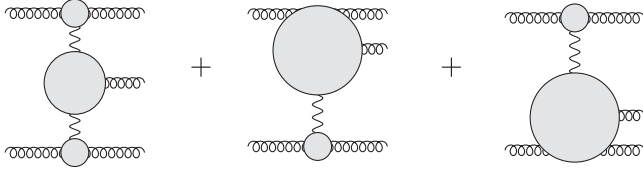


FIG. 5. Central (left) and quasielastic (middle and right) gluon production.

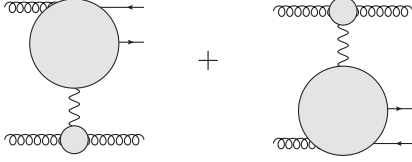


FIG. 6. Quark production is restricted to the quasielastic region.

where the gluon with transverse momentum \mathbf{q} is produced at central rapidities and the gluons with transverse momenta \mathbf{k}' and \mathbf{k} in the forward and backward direction, respectively,

$$d\hat{\sigma}_{ab}(\mathbf{k}', \mathbf{q}, \mathbf{k}) = h_a^{(0)}(\mathbf{k}') h_b^{(0)}(\mathbf{k}) \mathcal{V}(\mathbf{k}, \mathbf{k}'; \mathbf{q}) d^{2+2\epsilon} \mathbf{k}' d^{2+2\epsilon} \mathbf{k} dy. \quad (29)$$

We obtain, using the decomposition in Eq. (6) and with $M^2 = k^+ k^-$:

$$\begin{aligned} V(\mathbf{k}, \mathbf{k}'; \mathbf{q}) &= \frac{N_c^2 - 1}{8} \int dM^2 \frac{|\overline{\mathcal{M}}_{g_r^* r^* \rightarrow g}^2|}{k^2 k'^2} d\Pi_1 \delta^{(d)}(k' - k - \mathbf{q}) \\ &= \frac{\alpha_s N_c}{\pi_\epsilon \pi \mathbf{q}^2}; \quad \pi_\epsilon \equiv \pi^{1+\epsilon} \Gamma(1-\epsilon) \mu^{2\epsilon}. \end{aligned} \quad (30)$$

While the rapidities of the gluons in the forward and backward directions are fixed through energy-momentum conservation, $y_a = \ln p_a^+ / |\mathbf{k}'|$ and $y_b = -\ln p_b^- / |\mathbf{k}|$, the integral over the rapidity of the central gluon leads for inclusive observables, where the integral over gluon rapidity extends up to $y \rightarrow \pm\infty$, to a divergence. We therefore introduce upper and lower bounds $\eta_{a,b}$ with $\eta_a > y > -\eta_b$ to define such integrals, with the regulators evaluated in the limit $\eta_{a,b} \rightarrow \infty$. This leads then to the definition of the regularized production vertex $\mathcal{V}(\mathbf{k}, \mathbf{k}'; \mathbf{q}; \eta_a, \eta_b) \equiv V(\mathbf{k}, \mathbf{k}') \Theta(\eta_a - y) \Theta(y - \eta_b)$. The complete exclusive differential cross section is then given by

$$\begin{aligned} d\hat{\sigma}_{ab}^{(c)} &= \frac{1}{3!} \left[h_a^{(0)}(\mathbf{k}') h_b^{(0)}(\mathbf{k}) \mathcal{V}(\mathbf{k}, \mathbf{k}'; \mathbf{q}) \right. \\ &\quad \left. \times d^{2+2\epsilon} \mathbf{k}' d^{2+2\epsilon} \mathbf{k} dy + \text{perm} \right] \end{aligned} \quad (31)$$

Here ‘perm’ denotes all possible permutations of the four momenta of the three final state gluons, while the factor $1/3!$ arises due to the indistinguishability of three gluons in the final state.

The remaining part of our calculation addresses the quasielastic contribution $g(p_a) r^*(k) \rightarrow g(p) g(q)$ (see the

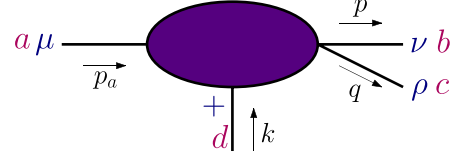


FIG. 7 (color online). Notation for external momenta and color indices in the quasielastic contribution.

notation in Fig. 7). The sum of all the effective diagrams is given in Fig. 8, where we include both the gg and the $q\bar{q}$ final state. In the computation of these diagrams we will employ the following choice for the polarization vectors: $\epsilon_a \cdot p_a = \epsilon_b \cdot p = \epsilon_c \cdot q = \epsilon_a \cdot n^+ = \epsilon_b \cdot n^+ = \epsilon_c \cdot n^+ = 0$; together with the Sudakov decomposition

$$\begin{aligned} p_a &= p_a^+ \frac{n^-}{2}, \quad p = (1-z)p_a^+ \frac{n^-}{2} + p^- \frac{n^+}{2} + \mathbf{k}', \\ k &= k^- \frac{n^+}{2} + \mathbf{k}, \quad q = zp_a^+ \frac{n^-}{2} + (k^- - p^-) \frac{n^+}{2} + \mathbf{q}, \end{aligned} \quad (32)$$

where we used $q^+ = zp_a^+$ and $p_a^+ = (1-z)p_a^+$. Squaring and averaging over initial colors and polarizations and summing over the final state we obtain with $\Delta = \mathbf{q} - z\mathbf{k}$

$$\begin{aligned} |\overline{\mathcal{M}}^2|_{gg \rightarrow ggg} &= 8z(1-z)(p_a^+)^2 g^4 C_a (1+\epsilon) \mathcal{P}_{gg}(z) \frac{k^2}{k'^2} \\ &\quad \times \left[\frac{z^2 k'^2 + (1-z)^2 q^2 - z(1-z) \mathbf{q} \cdot \mathbf{k}'}{q^2 \Delta^2} \right], \end{aligned} \quad (33)$$

where $\mathcal{P}_{gg}(z) = C_a \frac{1+z^4+(1-z)^4}{z(1-z)}$ is the gluon-gluon Altarelli-Parisi splitting function. In the same way, the joined contribution of the diagrams (j), (k) and (l) for the quark-antiquark final state gives

$$\begin{aligned} |\overline{\mathcal{M}}^2|_{gg \rightarrow q\bar{q}} &= 8z(1-z)(p_a^+)^2 g^4 n_f N_c (1+\epsilon) \mathcal{P}_{qg}(z, \epsilon) \\ &\quad \times \frac{k^2}{q^2 k'^2} \left[\frac{C_f}{C_a} + z(1-z) \frac{\mathbf{q} \cdot \Delta}{\Delta^2} \right], \end{aligned} \quad (34)$$

with $\mathcal{P}_{qg}(z, \epsilon) = \frac{1}{2} \left[1 - \frac{2z(1-z)}{1+\epsilon} \right]$ the quark-gluon splitting function. Generalizing our definition in Eq. (9) to two final state particles we obtain

$$\begin{aligned} h_{a,gg}^{(1)}(\mathbf{k}) &= \frac{(2\pi)^{d/2}}{2p_a^+} \\ &\quad \times \int d\mathbf{k}^- \frac{|\overline{\mathcal{M}}_{g_a r^* \rightarrow g g}^{(0)}|^2}{2k^2 \sqrt{N_c^2 - 1}} d\Pi_2 \delta^{(d)}(p_a + k - p - q) \\ &= \int dz d^{2+2\epsilon} \mathbf{k}' F_{ggg}(\mathbf{k}, \mathbf{k}', z) h_{a,\text{gluon}}^{(0)}(\mathbf{k}'), \end{aligned} \quad (35)$$

and

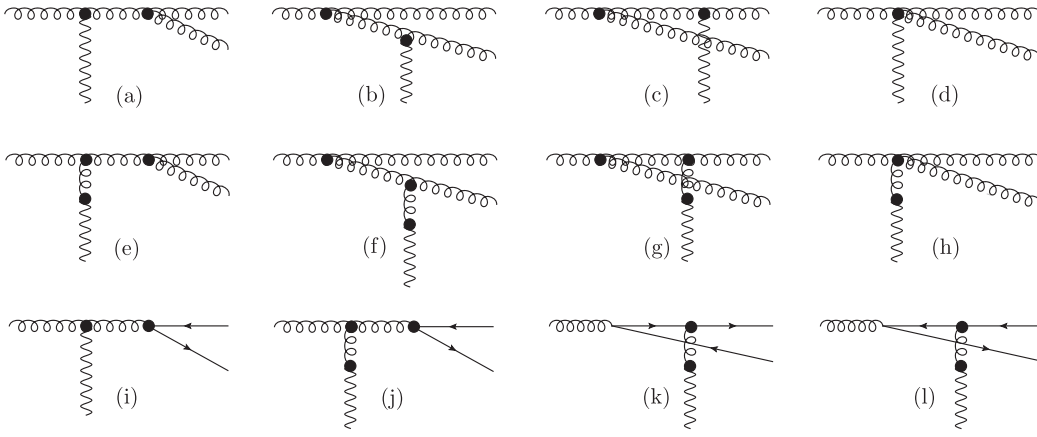


FIG. 8. Diagrams involved in the computation of the real corrections. In the case of final $q\bar{q}$ state, the external quark has momentum p and the external antiquark momentum q .

$$\begin{aligned}
 h_{a,q\bar{q}}^{(1)}(\mathbf{k}) &= \frac{(2\pi)^{d/2}}{2p_a^+} \\
 &\times \int dk^- \frac{|\overline{\mathcal{M}}_{g_a r^* \rightarrow gg}^{(0)}|^2}{2k^2 \sqrt{N_c^2 - 1}} d\Pi_2 \delta^{(d)}(p_a + k - p - q) \\
 &= \int dz d^2+2\epsilon \mathbf{k}' F_{gq\bar{q}}(\mathbf{k}, \mathbf{k}', z) h_{a,\text{gluon}}^{(0)}(\mathbf{k}'), \quad (36)
 \end{aligned}$$

with

$$\begin{aligned}
 F_{ggg}(\mathbf{k}, \mathbf{k}', z) &= \frac{1}{2} \frac{\alpha_s}{2\pi\pi_\epsilon} \mathcal{P}_{gg}(z) \\
 &\times \left[\frac{z^2 \mathbf{k}'^2 + (1-z)^2 \mathbf{q}^2 - z(1-z) \mathbf{q} \cdot \mathbf{k}'}{q^2 \Delta^2} \right], \quad (37)
 \end{aligned}$$

and

$$F_{gq\bar{q}}(\mathbf{k}, \mathbf{k}', z) = \frac{\alpha_s}{2\pi\pi_\epsilon} n_f \mathcal{P}_{qg}(z, \epsilon) \frac{1}{q^2} \left[\frac{C_f}{C_a} + z(1-z) \frac{\mathbf{q} \cdot \Delta}{\Delta^2} \right], \quad (38)$$

where the overall factor 1/2 for the gg final state stems from the indistinguishability of identical bosons in the final state.

If we parametrize the momentum fraction z in terms of the rapidity difference $\Delta y \equiv y_p - y_q$ of the final state gluons, i.e.,

$$z = \frac{e^{\Delta y}}{(\mathbf{k}'^2/q^2)^{1/2} + e^{\Delta y}}, \quad (39)$$

we can see that F_{ggg} reduces in the limits $\Delta y \rightarrow \pm\infty$ (including the corresponding Jacobian factor) to half of the central production vertex of Eq. (30). To regularize the resulting divergence of the rapidity integral we require $\ln(p_a^+/k') + \eta_b > \Delta y > -\ln(p_a^+/q) - \eta_b$, where η_b is again taken in the limit $\eta_b \rightarrow \infty$, and define $\mathcal{F}_{ggg}(\mathbf{k}, \mathbf{k}', z, \eta_b) = F_{ggg}(\mathbf{k}, \mathbf{k}', z) \Theta(\Delta y - \ln(p_a^+/k') - \eta_b) \Theta(\Delta y + \ln(p_a^+/q) + \eta_b)$. As in the case of virtual corrections

it is now necessary to subtract the contribution from gluon production at central rapidities to construct the complete differential cross section, schematically,

$$\begin{aligned}
 &\text{Diagram 1} = \text{Diagram 2} - \text{Diagram 3} \quad (40)
 \end{aligned}$$

This leads to the definition of the coefficient,

$$\begin{aligned}
 \mathcal{G}_{ggg}^{(a)}(\mathbf{k}, \mathbf{k}', z, \eta_b) &= \mathcal{F}_{ggg}(\mathbf{k}, \mathbf{k}', z, \eta_b) \\
 &- \frac{1}{2} \left[\frac{1}{1-z} \mathcal{V}(\mathbf{k}, \mathbf{k}'; \mathbf{q}; \eta_a, \eta_b) \right. \\
 &\left. + \frac{1}{z} \mathcal{V}(\mathbf{k}, \mathbf{q}; \mathbf{k}'; \eta_a, \eta_b) \right], \quad (41)
 \end{aligned}$$

where the factor 1/2 in front of the subtraction term takes into account Bose symmetry of the produced two gluon system. The cross section for quasielastic production of two gluons then reads

$$\begin{aligned}
 d\hat{\sigma}_{ab,gg}^{(qea)} &= \frac{1}{3!} [h_{a,\text{gluon}}^{(0)}(\mathbf{k}') \mathcal{G}_{ggg}^{(a)}(\mathbf{k}, \mathbf{k}', z, \eta_b) \\
 &\times h_{b,\text{gluon}}^{(0)} dz d^2+2\epsilon \mathbf{k} d^2+2\epsilon \mathbf{k}' + \text{perm}], \quad (42)
 \end{aligned}$$

where ‘‘perm’’ denotes all possible permutations of the four momenta of the three final state gluons. The cross section for quasielastic production of a $q\bar{q}$ pair is given by

$$\begin{aligned}
 d\hat{\sigma}_{ab,q\bar{q}}^{(qea)} &= h_{a,\text{gluon}}^{(0)}(\mathbf{k}') F_{gq\bar{q}}^{(a)}(\mathbf{k}, \mathbf{k}', z, \eta_b) \\
 &\times h_{b,\text{gluon}}^{(0)} dz d^2+2\epsilon \mathbf{k} d^2+2\epsilon \mathbf{k}'. \quad (43)
 \end{aligned}$$

The high energy limit of the differential cross section for the production of three final state partons is finally obtained as the sum of central and quasielastic contributions,

$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{(c)} + d\hat{\sigma}_{ab,gg}^{(qea)} + d\hat{\sigma}_{ab,q\bar{q}}^{(qea)} + d\hat{\sigma}_{ab,gg}^{(qeb)} + d\hat{\sigma}_{ab,q\bar{q}}^{(qeb)}. \quad (44)$$

Integrated over the gluon rapidity y , it yields a finite result with a well-defined limit $\eta_{a,b} \rightarrow \infty$. This completes our calculation of the gluon-initiated forward jet vertex at NLO using the high energy effective action constructed by Lipatov.

IV. CONCLUSIONS AND OUTLOOK

We have performed the calculation of the forward jet vertex at next-to-leading order in the BFKL formalism, offering an explicit derivation of the gluon-initiated contribution. This adds to that previously calculated for the quark-initiated vertex and completes the derivation of the full vertex. We have found agreement with previous results in the literature. Our method of calculation is based on the high energy effective action for QCD proposed by Lipatov, which is proven to be very useful to streamline our calculations. Our subtraction and regularization procedure in order to avoid overcounting of kinematic regions has been proven to work well in both gluon and quark cases. Compared to more standard calculations, in this approach the number of Feynman diagrams finally contributing to the physical observables is reduced, and gauge invariance is readily ensured. We are now confident this method will help in the calculation of further amplitudes at loop level. We understand that it is important to test these high energy resummations in exclusive observables at the Large Hadron Collider. Typical processes where our approach can be applied include the inclusive production of a pair of forward (Mueller-Navelet) jets. Similar techniques are now being used to calculate the production of a forward jet coupled to a color singlet, associated to diffractive events (Mueller-Tang jets) [24,25] at next-to-leading order.

ACKNOWLEDGMENTS

We acknowledge partial support from the European Commission under Contract No. LHCPHenoNet (PITN-GA-2010-264564), the Comunidad de Madrid through

Proyecto HEPHACOS ESP-1473, MICINN (FPA2010-17747), and Spanish MINECO's "Centro de Excelencia Severo Ochoa" Programme under Grant No. SEV-2012-0249. M.H. acknowledges support from the German Academic Exchange Service (DAAD), the U.S. Department of Energy under Contract No. DE-AC02-98CH10886 and a BNL "Laboratory Directed Research and Development" Grant (No. LDRD 12-034). G.C. thanks the partial support of the Research Executive Agency (REA) of the European Union under the Grant Agreement No. PIEF-GA-2011-298582 and MICINN (No. FPA2011-23778, No. FPA2007-60323, and No. CSD2007-00042 CPAN).

APPENDIX

In the following we present further details on the NLO calculation of both virtual and real corrections

1. Virtual corrections

For the notation, we refer to Fig. 4. Tadpole contributions vanish in dimensional regularization. The contribution for each of these diagrams can be written in terms of master integrals labelled with the following notation: M, S, P, and Q denote, respectively, the existence of a propagator of the form k^+ ($\rightarrow b \cdot k$), k^2 , $(k - p_a)^2$ or $(k - q)^2$. The number at the end (0, 1, 2, or 3) indicates how many tensor indices are present in the numerator (e.g., 2 stands for a factor $k^\mu k^\nu$). $\xi = a^2 = b^2 = 4e^{-\rho}$ are chosen to indicate the squares of the new light-cone vectors. For diagrams (E), (F), (I) and (J) in Fig. 4, the contribution with reversed arrows is included. Diagrams (G) and (H) turn out to vanish completely. The symmetry factors for the diagrams, which are included, are equal to one apart from diagrams (C) and (D), for which it is two. In more detail, following are all the contributing expressions:

$$\begin{aligned}
i\mathcal{M}_{(A)} &= -\frac{ig^3}{2} \mathbf{k}^2 f_{abc} N_c \int \frac{d^d l}{(2\pi)^d} \frac{(l^+ - 2p_a^+)^2 \varepsilon \cdot \varepsilon^* + 4\xi \varepsilon \cdot l \varepsilon^* \cdot (l - k)}{l^+ l^2 (l - p_a)^2 (l - k)^2} \\
&= -\frac{ig^3}{2} \mathbf{k}^2 f_{abc} N_c \{16e^{-\rho} \varepsilon_\mu \varepsilon_\nu^* [\text{MSPQ2}] - 16e^{-\rho} \varepsilon^* \cdot k \varepsilon_\mu [\text{MSPQ1}] \\
&\quad - 4p_a^+ \varepsilon \cdot \varepsilon^* [\text{SPQ0}] + 4(p_a^+)^2 \varepsilon \cdot \varepsilon^* [\text{MSPQ0}] + (n^+)_\mu \varepsilon \cdot \varepsilon^* [\text{SPQ1}]\}; \\
i\mathcal{M}_{(B)} &= -\frac{ig^3}{2} f_{abc} N_c \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2 (l - p_a)^2 (k - l)^2} [4p_a^+ \{\varepsilon \cdot \varepsilon^* ((l - p_a)^2 - k^2) + 4(\varepsilon \cdot l \varepsilon^* \cdot k - \varepsilon \cdot k \varepsilon^* \cdot l)\} \\
&\quad + l^+ \{7\mathbf{k}^2 \varepsilon \cdot \varepsilon^* + (18 + 16\varepsilon) \varepsilon \cdot l \varepsilon^* \cdot (l - k) + 16\varepsilon \cdot k \varepsilon^* \cdot k\}] \\
&= -\frac{ig^3}{2} f_{abc} N_c \{(18 + 16\varepsilon)(n^+)_\mu \varepsilon_\nu \varepsilon_\rho^* [\text{SPQ3}] - (18 + 16\varepsilon)(n^+)_\mu \varepsilon_\nu \varepsilon^* \cdot k [\text{SPQ2}] \\
&\quad + [(n^+)_\mu (16\varepsilon \cdot k \varepsilon^* \cdot k + 7\mathbf{k}^2 \varepsilon \cdot \varepsilon^*) - 16p_a^+ (\varepsilon \cdot k \varepsilon_\mu^* - \varepsilon^* \cdot k \varepsilon_\mu)] [\text{SPQ1}] \\
&\quad - 4p_a^+ \mathbf{k}^2 \varepsilon \cdot \varepsilon^* [\text{SPQ0}] + 4p_a^+ \varepsilon \cdot \varepsilon^* [\text{SQ0}]\};
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_{(C)} &= -\frac{ig^3}{2}f_{abc}N_c \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^+ l^2(l-k)^2} [4\xi(\varepsilon \cdot l\varepsilon^* \cdot k - \varepsilon \cdot k\varepsilon^* \cdot l) + \varepsilon \cdot \varepsilon^*(4l^+ p_a^+ + \xi(2(l-p_a)^2 - \mathbf{k}^2))] \\
&= -\frac{ig^3}{2}f_{abc}N_c \{-16e^{-\rho}(\varepsilon \cdot \varepsilon^* p_{a\mu} + \varepsilon \cdot k\varepsilon_\mu^* - \varepsilon^* \cdot k\varepsilon_\mu)[\text{MSQ1}] - 4e^{-\rho}\mathbf{k}^2\varepsilon \cdot \varepsilon^*[\text{MSQ0}] + 4p_a^+ \varepsilon \cdot \varepsilon^*[\text{SQ0}]\};
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_{(D)} &= \frac{ig^3}{2k^2}f_{abc}N_c \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l-k)^2} [-8p_a^+ \mathbf{k}^2\varepsilon \cdot \varepsilon^* + (5+4\varepsilon)l^+ \{\varepsilon \cdot \varepsilon^*(\mathbf{k}^2 - 2(l-p_a)^2) \\
&\quad + 4\varepsilon \cdot k\varepsilon^* \cdot l - 4\varepsilon \cdot l\varepsilon^* \cdot k\}] = \frac{ig^3}{2k^2}f_{abc}N_c \{(20+16\varepsilon)(n^+)_{\mu}(\varepsilon \cdot k\varepsilon_\nu^* - \varepsilon^* \cdot k\varepsilon_\nu + \varepsilon \cdot \varepsilon^* p_{a\nu})[\text{SQ2}] \\
&\quad + (5+4\varepsilon)\mathbf{k}^2\varepsilon \cdot \varepsilon^*(n^+)_{\mu}[\text{SQ1}] - 8p_a^+ \mathbf{k}^2\varepsilon \cdot \varepsilon^*[\text{SQ0}]\};
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_{(E)} &= \frac{2ig^3}{k^2}f_{abc}n_f \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l-k)^2} [p_a^+ \mathbf{k}^2\varepsilon \cdot \varepsilon^* + l^+ [\varepsilon \cdot \varepsilon^*(2(l-p_a)^2 - \mathbf{k}^2) + 4(\varepsilon \cdot l\varepsilon^* \cdot k - \varepsilon \cdot k\varepsilon^* \cdot l)]] \\
&= -\frac{2ig^3}{k^2}f_{abc}n_f \{4(n^+)_{\mu}(\varepsilon \cdot k\varepsilon_\nu^* - \varepsilon^* \cdot k\varepsilon_\nu + \varepsilon \cdot \varepsilon^* p_{a\nu})[\text{SQ2}] + \varepsilon \cdot \varepsilon^* \mathbf{k}^2(n^+)_{\mu}[\text{SQ1}] - p_a^+ \mathbf{k}^2\varepsilon \cdot \varepsilon^*[\text{SQ0}]\};
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_{(F)} &= \frac{ig^3}{2k^2}f_{abc}N_c \int \frac{d^d l}{(2\pi)^d} \frac{l^+}{l^2(l-k)^2} [\varepsilon \cdot \varepsilon^*(2(l-p_a)^2 - \mathbf{k}^2) + 4(\varepsilon \cdot l\varepsilon^* \cdot k - \varepsilon \cdot k\varepsilon^* \cdot l)] \\
&= -\frac{ig^3}{2k^2}f_{abc}N_c \{4(n^+)_{\mu}(\varepsilon \cdot \varepsilon^* p_{a\nu} + \varepsilon \cdot k\varepsilon_\nu^* - \varepsilon^* \cdot k\varepsilon_\nu)[\text{SQ2}] + \mathbf{k}^2\varepsilon \cdot \varepsilon^*(n^+)_{\mu}[\text{SQ1}]\};
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_{(I)} &= ig^3 f_{abc}n_f \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l-p_a)^2(l-k)^2} [p_a^+ (-\mathbf{k}^2\varepsilon \cdot \varepsilon^* + 2(\varepsilon \cdot l\varepsilon^* \cdot k - \varepsilon \cdot k\varepsilon^* \cdot l)) \\
&\quad + l^+ (\varepsilon \cdot \varepsilon^*(\mathbf{k}^2 - 2(l-p_a)^2) + 8\varepsilon \cdot l\varepsilon^* \cdot (l-k) + 2\varepsilon \cdot k\varepsilon^* \cdot k)] \\
&= ig^3 f_{abc}n_f \{8(n^+)_{\mu}\varepsilon_\nu \varepsilon_\rho^*[\text{SPQ3}] - 8\varepsilon^* \cdot k(n^+)_{\mu}\varepsilon_\nu[\text{SPQ2}] \\
&\quad + [(\mathbf{k}^2\varepsilon \cdot \varepsilon^* + 2\varepsilon \cdot k\varepsilon^* \cdot k)(n^+)_{\mu} + 2p_a^+ \varepsilon^* \cdot k\varepsilon_\mu - 2p_a^+ \varepsilon \cdot k\varepsilon_\mu^*][\text{SPQ1}] - \mathbf{k}^2 p_a^+ \varepsilon \cdot \varepsilon^*[\text{SPQ0}]\};
\end{aligned}$$

$$i\mathcal{M}_{(J)} = ig^3 f_{abc}N_c \int \frac{d^d l}{(2\pi)^d} \frac{l^+}{l^2(l-p_a)^2(l-k)^2} \varepsilon \cdot l\varepsilon^* \cdot (l-k) = ig^3 f_{abc}N_c (n^+)_{\mu} \{\varepsilon_\nu \varepsilon_\rho^*[\text{SPQ3}] - \varepsilon^* \cdot k\varepsilon_\nu[\text{SPQ2}]\}.$$

Those integrals which are not suppressed in the $\rho \rightarrow \infty$ limit are

$$\begin{aligned}
[\text{SQ0}] &= \frac{i}{(4\pi)^{2+\varepsilon}} (\mathbf{k}^2)^\varepsilon \frac{\Gamma(-\varepsilon)\Gamma^2(1+\varepsilon)}{\Gamma(2+2\varepsilon)}; \quad [\text{SQ2}] = (g^{\mu\nu}\mathbf{k}^2 + k^\mu k^\nu(4+2\varepsilon)) \frac{1}{4(3+2\varepsilon)} [\text{SQ0}]; \\
[\text{SPQ0}] &= \frac{1+2\varepsilon}{\varepsilon k^2} [\text{SQ0}]; \quad [\text{SPQ1}] = \left(k^\mu + \frac{1}{\varepsilon} p_a^\mu\right) [\text{SQ0}]; \\
[\text{SPQ2}] &= \left\{ \frac{1}{2+2\varepsilon} \left[\frac{1}{2} g^{\mu\nu} + \left(\frac{k^\mu p_a^\nu + p_a^\mu k^\nu}{\mathbf{k}^2} \right) + \frac{2}{\varepsilon} \frac{p_a^\mu p_a^\nu}{\mathbf{k}^2} \right] + \frac{1}{\mathbf{k}^2} k^\mu k^\nu \right\} [\text{SQ0}]; \\
[\text{SPQ3}] &= \frac{1}{\varepsilon(1+\varepsilon)(3+2\varepsilon)} \left\{ \frac{1}{\mathbf{k}^2} \left[p_a^\mu p_a^\nu p_a^\rho + \varepsilon k^\mu p^\nu p^\rho + \frac{1}{2} \varepsilon(1+\varepsilon) k^\mu k^\nu p^\rho - \frac{1}{6} (1-\varepsilon)(2+\varepsilon)^2 k^\mu k^\nu k^\rho \right] \right. \\
&\quad \left. + \frac{\varepsilon}{4} [p^\mu g^{\nu\rho} + (1+\varepsilon)k^\mu g^{\nu\rho}] + \text{cyclic permutations of } \mu, \nu \text{ and } \rho \right\} [\text{SQ0}]; \\
[\text{MSQ1}] &= \frac{b^\mu}{\xi} [\text{SQ0}] + \frac{1}{2} q^\mu \left\{ \frac{-i}{(4\pi)^{2+\varepsilon}} \frac{\Gamma^2(\frac{1}{2}+\varepsilon)\Gamma(\frac{1}{2}-\varepsilon)\Gamma(\frac{1}{2})}{\Gamma(1+2\varepsilon)(\mathbf{k}^2)^{\frac{1}{2}-\varepsilon}\xi^{\frac{1}{2}}} \right\}; \\
[\text{MSPQ0}] &= -\frac{i}{(4\pi)^{2+\varepsilon}} \frac{(\mathbf{k}^2)^{\varepsilon-1}}{p_a^+} \frac{\Gamma(1-\varepsilon)\Gamma^2(\varepsilon)}{\Gamma(2\varepsilon)} \left(\ln \left[\frac{p_a^+}{|\mathbf{k}|} \right] + \frac{\rho}{2} + \frac{\psi(1) - 2\psi(\varepsilon) + \psi(1-\varepsilon)}{2} \right).
\end{aligned}$$

2. Details on the real corrections

We refer to Figs. 7 and 8 for our notation. We have

$$p^- = \frac{(\mathbf{k} - \mathbf{q})^2}{(1-z)p_a^+}, \quad k^- = \frac{(\mathbf{q} - z\mathbf{k})^2 + z(1-z)\mathbf{k}^2}{z(1-z)p_a^+}. \quad (\text{A1})$$

Within the given choice of polarization vectors, diagrams (a), (b), (c), (d) and (i) are immediately zero, while diagram (h) turns out to vanish as well. The amplitudes for the nonvanishing diagrams can be written in the following form:

$$\begin{aligned} i\mathcal{M}_{(e)} &= \varepsilon_{a\mu}\varepsilon_{b\nu}^*\varepsilon_{c\rho}^* 2ig^2 \frac{f_{ade}f_{bce}}{s} p_a^+ \{g^{\nu\rho}[k^\mu(1-2z) - p^\mu + q^\mu] + g^{\mu\nu}(2p^\rho + q^\rho) - g^{\mu\rho}(2q^\nu + p^\nu)\}; \\ i\mathcal{M}_{(f)} &= \varepsilon_{a\mu}\varepsilon_{b\nu}^*\varepsilon_{c\rho}^* (-ig^2) \frac{f_{abe}f_{cde}}{t} p_a^+ \{-4z(g^{\nu\rho}p^\mu + g^{\mu\rho}p_a^\nu) + g^{\mu\nu}[k^\rho(2-z) + p^\rho(2+z) + p_a^\rho(-2+3z)]\}; \\ i\mathcal{M}_{(g)} &= \varepsilon_{a\mu}\varepsilon_{b\nu}^*\varepsilon_{c\rho}^* (-ig^2) \frac{f_{ace}f_{bde}}{u} p_a^+ \{-4(1-z)[g^{\mu\nu}p_a^\rho + g^{\nu\rho}q^\mu] + g^{\mu\rho}[k^\nu(1+z) + p_a^\nu(1-3z) + q^\nu(3-z)]\}; \\ i\mathcal{M}_{(j)} &= -\frac{2g^2 f_{acd}t^c}{s} \varepsilon_{a\mu}[k^\mu(n^+)^\sigma - p_a^+ g^{\mu\sigma}] \bar{u}(p) \gamma_\sigma v(q); \\ i\mathcal{M}_{(k)} &= -\frac{ig^2 t^d t^a}{u} \varepsilon_{a\mu} \bar{u}(p) \not{t}^+ (\not{p}_a - \not{q}) \gamma^\mu v(q); \\ i\mathcal{M}_{(l)} &= \frac{ig^2 t^a t^d}{t} \varepsilon_{a\mu} \bar{u}(p) \gamma^\mu (\not{p}_a - \not{p}) \not{t}^+ v(q). \end{aligned} \quad (\text{A2})$$

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