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Strictly anomaly mediated supersymmetry breaking

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We consider an extension of the minimal supersymmetric Standard Model with anomaly mediation as the only source of supersymmetry breaking, and the tachyonic slepton problem solved by a gauged U(1) symmetry. The extra gauge symmetry is broken at high energies in a manner preserving supersymmetry, while also introducing both the seesaw mechanism for neutrino masses, and the Higgs μ-term. We call the model *strictly* anomaly mediated supersymmetry breaking. We present typical spectra for the model and compare them with those from so-called *minimal* anomaly mediated supersymmetry breaking. We find a Standard Model-like Higgs of mass 125 GeV with a gravitino mass of 140 TeV and $\tan\beta = 16$. However, the muon anomalous magnetic moment is 3σ away from the experimental value. The model naturally produces a period of hybrid inflation, which can exit to a false vacuum characterized by large Higgs vacuum expectation values, reaching the true ground state after a period of thermal inflation. The scalar spectral index is reduced to approximately 0.975, and the correct abundance of neutralino dark matter can be produced by decays of thermally produced gravitinos, provided the gravitino mass (and hence the Higgs mass) is high. Naturally light cosmic strings are produced, satisfying bounds from the cosmic microwave background. The complementary pulsar timing and cosmic ray bounds require that strings decay primarily via loops into gravitational waves. Unless the loops are extremely small, the next generation pulsar timing array will rule out or detect the string-derived gravitational radiation background in this model.

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I. INTRODUCTION

The Standard Model (SM) Higgs-like particle of mass 125 GeV recently discovered at the LHC [1,2] strongly constrains future model building, while recent negative results from both the Tevatron and LHC in searches for sparticles place increasing pressure on models with low energy supersymmetry. Here we explore a specific supersymmetric model in which the low energy spectrum is that of the minimal supersymmetric Standard Model (MSSM), but the gauge symmetry is augmented by an extra gauged U(1) symmetry, $U(1)'$, spontaneously broken at high energies in a manner which affects both physics at the supersymmetry-breaking scale and physics at high scales characterizing inflation and cosmic strings.

The broad features of the model are independent of the source of supersymmetry breaking, but if we assume that this source is in fact anomaly mediation (AMSB) [3–5], then there arises an interesting interplay between the low energy physics (and in particular the Higgs μ-term) and the high energy physics involving strings and inflation. Moreover the breaking of $U(1)'$ solves the tachyonic slepton problem characteristic of AMSB [5,6].

We first presented this specific model in Ref. [7], in a form where we also introduced a Fayet-Iliopoulos (FI) term for the $U(1)'$. Here we concentrate on the minimal formulation when there is no such term [8]. The model implements a form of AMSB which we refer to as *strictly* anomaly mediated supersymmetry breaking (sAMSB), by which we mean that there are no other sources of supersymmetry breaking beside the F-term of the conformal compensator field. As a consequence the soft parameters have an elegant renormalization group (RG) invariant form. It therefore differs from so-called minimal AMSB (mAMSB), which posits an extra source of supersymmetry breaking, instead of extra fields, in order to solve the tachyonic slepton problem. Our model is not quite a complete sAMSB implementation, in that it requires an extension to determine the soft parameter associated with the Higgs μ-term.

We begin by describing the symmetries and field content of the model and explaining in detail how the spontaneous breakdown of the $U(1)'$ symmetry at a large scale M not only solves the AMSB tachyonic slepton problem, but also generates a Higgs μ-term and the seesaw mechanism for neutrino masses. This outcome is achieved by the introduction of three new chiral superfields, S, which is a gauge singlet and a pair of $SU(3) \otimes SU(2) \otimes U(1)_Y$ singlet fields Φ, $\bar{\Phi}$ which are oppositely charged under $U(1)'$. We then exhibit characteristic sparticle spectra for the model; the calculations involved to obtain these are essentially as described in Refs. [10,11], but allowing for a larger gravitino mass. We also discuss the fine-tuning issue raised by

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this, and compare the results of our model with results from the most popular (but, we will argue, less elegant) version of AMSB, generally called mAMSB. We will see that sAMSB generally keeps sleptons lighter than in mAMSB, which means that the contribution to the muon anomalous magnetic moment $a_\mu \equiv (g_\mu - 2)/2$ is typically higher for a given Higgs mass. In a separate paper [12], we consider different patterns of supersymmetry breaking, and also explore the cosmological consequences of the model in more detail.

The theory incorporates a natural mechanism for supersymmetric F-term inflation, with the scalar component of S as the inflaton. Previously, we concentrated on a region of parameter space such that inflation ended with a transition to a state with only the $U(1)'$ broken. There is, however, an interesting alternative that inflation ends with the development of vacuum expectation values (vevs) for the Higgs multiplets, $h_{1,2}$, breaking the electroweak symmetry. A combination of the Higgs fields $h_1 \cdot h_2$ and the scalar components of the singlet fields $\phi\bar{\phi}$ is a flat direction, lifted by soft supersymmetry-breaking terms, and the normal low energy electroweak vacuum is achieved after a later period of thermal inflation. Approximately 15 e-foldings of thermal inflation reduce the number of e-foldings of high-scale inflation, and therefore reduce the spectral index of scalar cosmic microwave background (CMB) fluctuations to about 0.975. This is within about 1σ of the WMAP7 value.

The reheat temperature after this period of thermal inflation is around 10^9 GeV, which means that there is no gravitino problem: gravitinos are very massive, more than 40 TeV, and so decay early enough not to be in conflict with nucleosynthesis. Indeed, the gravitino problem can turn into the gravitino solution for the typical AMSB feature of too low a dark matter density generated at freeze-out: the lightest supersymmetric particle (LSP) is mostly wino and has a relatively high annihilation cross section. In our model a critical density of LSPs can be generated by gravitino decays, if the gravitino (and hence the LSP) is heavy enough.

The model also has the possibility of baryogenesis via leptogenesis following thermal inflation, with *CP* violation supplied by the neutrino sector. The field giving mass to right-handed neutrinos has an inflation scale (10^{16} GeV) vev, but if the lightest right-handed neutrino is sufficiently light to be generated at reheating after thermal inflation, a lepton asymmetry can be generated by its out-of-equilibrium decay.

There is a broken U(1) symmetry in the model, and cosmic strings with a 10^{16} GeV mass scale are formed,

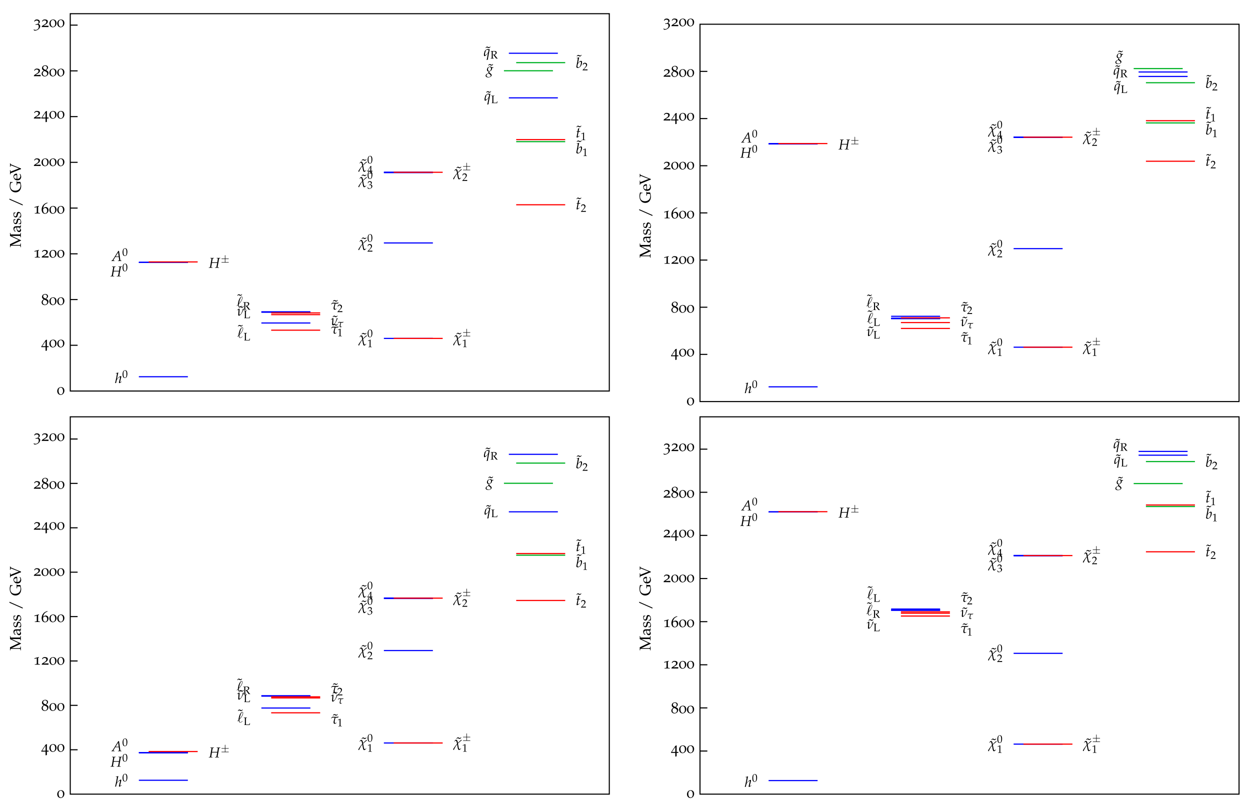

FIG. 1 (color online). Comparison between sAMSB (left) and mAMSB (right) mass spectra, drawn from columns 4 and 7 of $\tan\beta = 16$ in Table II(c) and (d) (left), and columns 5 and 6 of Table III(b) (right). The gravitino masses are 140 TeV, and $\tan\beta = 16$ in all cases. The resulting Higgs masses are between 125 and 126 GeV. Note how the increase in the magnitude of the sAMSB D-term contribution to the soft masses decreases the masses of the non-SM Higgs particles.

although not until the end of thermal inflation. There is a large Higgs condensate in the core of the string which spreads the string out to a width of order the supersymmetry-breaking scale, and reduces its mass per unit length by well over an order of magnitude. The strings satisfy CMB constraints on the mass per unit length from combined WMAP7 and small-scale observations. Their decays are constrained by pulsar timing observations in the case of gravitational waves and the diffuse γ-ray background in the case of particle production: the latter means that less than about 0.1% of the energy of the strings should end up as particles, and the former puts constraints on the average size of the loops at formation.

The model has the same field content as the F_D hybrid inflation model [13,14], but different charge assignments and couplings. F_D hybrid inflation also has a singlet which is a natural inflaton candidate, but differs in other ways: for example, right-handed neutrinos have electroweak-scale masses, and the gravitino problem is countered by entropy generation.

The model also has the same field content as the B-L model of Refs. [15], although as explained in Sec. III, the $U(1)'$ symmetry cannot be $U(1)_{B-L}$ in AMSB. It is also closely related to the model of Ref. [16], in which the fields Φ, $\bar{\Phi}$ are $SU(2)_R$ triplets. This also has a flat direction involving the Higgs, although the authors did not pursue its consequences.

To summarize our results: at $\tan\beta = 10$, sAMSB can accommodate a Higgs mass above 120 GeV for gravitino masses over 80 TeV, while accounting for the discrepancy in a_μ between the SM theory and experiment to within 2σ would have favored 80 TeV or lower. Larger values of $\tan\beta$ allow a more massive Higgs: for $\tan\beta = 16$ we find a Higgs mass of 125 GeV for a gravitino mass of 140 TeV.

sAMSB also allows for an observationally consistent dark matter density, if the gravitino mass is over about 100 TeV, with the dark matter deriving from the decay of gravitinos produced from reheating after thermal inflation. The spectral index of scalar cosmological perturbations is within 1σ of the WMAP7 value, and the observational bounds on cosmic strings can be satisfied if the strings decay into gravitational radiation. The model has also has a natural mechanism for baryogenesis via leptogenesis through the decays of right-handed neutrinos.

II. THE AMSB SOFT TERMS

We will assume that supersymmetry breaking arises via the renormalization group invariant form characteristic of anomaly mediation, so that the soft parameters for the gaugino mass M, the ϕ^3 interaction h and the $\phi^*\phi$ and ϕ^2 mass terms m^2 and m_3^2 in the MSSM take the generic RG invariant form

$$M_i = m_{\frac{3}{2}}\beta_{g_i}/g_i, \tag{1}$$

$$h_{U,D,E,N} = -m_{\frac{3}{2}}\beta_{Y_{U,D,E,N}}, \tag{2}$$

$$(m^2)^i{}_j = \frac{1}{2}m_{\frac{3}{2}}^2\mu\frac{d}{d\mu}\gamma^i{}_j, \tag{3}$$

$$m_3^2 = \kappa m_{\frac{3}{2}}\mu_h - m_{\frac{3}{2}}\beta_{\mu_h}. \tag{4}$$

Here μ is the renormalization scale, and $m_{\frac{3}{2}}$ is the gravitino mass; β_{g_i} are the gauge β functions and γ is the chiral supermultiplet anomalous dimension matrix. $Y_{U,D,E,N}$ are the 3×3 Yukawa matrices, and μ_h is the superpotential Higgs μ-term. We will see that in our low energy theory, Eq. (3) is replaced, in fact, by

$$(m^2)^i{}_j = \frac{1}{2}m_{\frac{3}{2}}^2\mu\frac{d}{d\mu}\gamma^i{}_j + kY_i\delta^i{}_j, \tag{5}$$

where the Y_i are charges corresponding to a U(1) symmetry. This kY term corresponds in form to the contribution of a FI D-term, and can be employed to obviate the tachyonic scalar problem characteristic of AMSB. How such a term can be generated (with AMSB) was first discussed in Ref. [5], and first applied to the MSSM in Ref. [6]. The basic idea was pursed in a number of papers [17–21]. For example, Ref. [17] demonstrated explicitly the UV insensitivity of the result, and Ref. [18] emphasized that the tachyonic problem could be solved using a single U(1) rather than a linear combination of two, the approach followed in Ref. [6]. An extension of the MSSM such that the spontaneous breaking of a gauged $U(1)'$ with a FI term gave rise to the kY term was written down in Ref. [10]. In Ref. [7] we developed an improved version of this model, retaining the possibility of a *primordial* FI term for $U(1)'$; here we will dispense with the FI term, and emphasize that we can nevertheless generate the kY term naturally with k of $O(m_{\frac{3}{2}}^2)$, by breaking a $U(1)'$ symmetry at a large scale, *without introducing an explicit FI term*.

At first sight Eq. (5) resembles the formula for the scalar masses employed in the so-called mAMSB model, where the kY_i term is replaced by a universal scalar mass contribution m_0^2. The differences are as follows:

(i) The mAMSB involves the introduction of an additional source of supersymmetry breaking independent of the gravitino mass, while, as we shall see, Eq. (5) does not.

(ii) The parameter k in Eq. (5) turns out to be more constrained than m_0^2. This is associated with the fact that inevitably all the Y_i cannot have the same sign.

(iii) The elegant RG invariance of Eqs. (1)–(4) is preserved by Eq. (5).

It is these observations that prompts us to refer to our model as sAMSB. Note that, of course, we cannot "promote" the mAMSB into the sAMSB by the addition of additional heavier fields which cancel the associated $U(1)'$ anomaly; with an unbroken $U(1)'$, any massive chiral multiplets will obviously make no contribution to this anomaly.

Equation (4) is the most general form for m_3^2 that is consistent with RG invariance, as first remarked explicitly in Ref. [10]; the parameter κ is an arbitrary constant. For discussion of possible origins of m_3^2 from the underlying superconformal calculus formulation of supergravity see Refs. [5,20,21]. We will simply assume that the model can be generalized to produce such a term; the procedure which has, in fact, been generally followed. The presence of κ means that in sparticle spectrum calculations one is free to calculate m_3^2 (and the value of the Higgs μ-term, μ_h) by minimizing the Higgs potential at the electroweak scale in the usual way. (For $\kappa = 1$, which is the value suggested by a straightforward use of the conformal compensator field [5], one might have hoped to use the minimization conditions to determine $\tan\beta$, but it turns out this leads to a very small value of $\tan\beta$ incompatible with gauge unification, because of the correspondingly large top Yukawa coupling [19]) We will see, however, that in our model the result for μ_h has implications for other parameters in the *underlying* theory which are constrained by cosmological considerations.

III. THE U(1)′ SYMMETRY

The MSSM (including right-handed neutrinos) admits two independent generation-blind anomaly-free U(1) symmetries. The possible charge assignments are shown in Table I.

The SM gauged $U(1)_Y$ is $q_L = -1$, $q_E = 2$; this U(1) is of course anomaly-free even in the absence of N. $U(1)_{\text{B-L}}$ is $q_E = -q_L = 1$; in the absence of N this would have $U(1)^3$- and U(1)-gravitational anomalies, but no mixed anomalies with the SM gauge group.

Our model will have, in addition, a pair of MSSM singlet fields Φ, $\bar{\Phi}$ with $U(1)'$ charges $q_{\Phi,\bar{\Phi}} = \pm(4q_L + 2q_E)$ and a gauge singlet S. In order to solve the tachyon slepton problem we will need that, for our new gauge symmetry $U(1)'$, the charges q_L, q_E have the same sign at low energies. As explained in Ref. [11], however, it is in fact more appropriate to input parameters at high energies, when in fact although necessarily $q_E > 0$, the range of acceptable values of q_L includes negative ones, not negative enough, however, to allow $U(1)'$ to be $U(1)_{\text{B-L}}$.

Thus sAMSB has three input parameters $m_{\frac{3}{2}}$, kq_L, kq_E, associated with the supersymmetry-breaking sector, while mAMSB only has two: $m_{\frac{3}{2}}$, m_0. However, it turns out that because the allowed (q_L, q_E) region is so restricted, sAMSB is the more predictive of the two. We will see this explicitly in Sec. VI.

IV. THE SUPERPOTENTIAL AND SPONTANEOUS U(1)′ BREAKING

The complete superpotential for our model is

$$W = W_A + W_B, \tag{6}$$

where W_A is the MSSM superpotential, omitting the Higgs μ-term, and augmented by Yukawa couplings for the right-handed neutrinos, N:

$$W_A = H_2 Q Y_U U + H_1 Q Y_D D + H_1 L Y_E E + H_2 L Y_N N \tag{7}$$

and

$$W_B = \lambda_1 \Phi\bar{\Phi} S + \frac{1}{2}\lambda_2 N N \Phi - \lambda_3 S H_1 H_2 - M^2 S, \tag{8}$$

where M, λ_1, λ_3 are real and positive and λ_2 is a symmetric 3×3 matrix. The sign of the λ_3 term above is chosen because with our conventions, in the electroweak vacuum where

$$H_1 = \left(\frac{v_1}{\sqrt{2}}, 0\right)^T \quad \text{and} \quad H_2 = \left(0, \frac{v_2}{\sqrt{2}}\right)^T, \tag{9}$$

we have $H_1 H_2 \to -\frac{1}{2}v_1 v_2$.

The $U(1)'$ symmetry forbids the renormalizable B and L violating superpotential interaction terms of the form QLD, UDD, LLE, H_1H_2N, NS^2, N^2S and N^3, as well as the mass terms NS, N^2 and LH_2 and the linear term N. Moreover W_B contains the only cubic term involving Φ, $\bar{\Phi}$ that is allowed. Our superpotential Eq. (6) is completely natural, in the sense that it is invariant under a global R symmetry, with superfield charges

$$\begin{aligned} S = 2, \qquad & L = E = N = U = D = Q = 1, \\ & H_1 = H_2 = \Phi = \bar{\Phi} = 0, \end{aligned} \tag{10}$$

which forbids the remaining gauge invariant renormalizable terms (S^2, S^3, $\Phi\bar{\Phi}$ and H_1H_2). This R symmetry also forbids the quartic superpotential terms $QQQL$ and $UUDE$, which are allowed by the $U(1)'$ symmetry and give rise to dimension 5 operators capable of causing proton decay [22–24]. It is easy to see, in fact, that the charges in Eq. (10) disallow B-violating operators in the superpotential of arbitrary dimension. Of course this R symmetry is broken by the soft supersymmetry breaking.

TABLE I. Anomaly-free U(1) symmetry for arbitrary lepton doublet and singlet charges q_L and q_E respectively.

	Q	U	D	H_1	H_2	N
q	$-\frac{1}{3}q_L$	$-q_E - \frac{2}{3}q_L$	$q_E + \frac{4}{3}q_L$	$-q_E - q_L$	$q_E + q_L$	$-2q_L - q_E$

V. THE HIGGS POTENTIAL

In this section we discuss the spontaneous breaking of the U(1)$'$ symmetry and its consequences. We shall assume M is much larger than the scale of supersymmetry breaking. (Such a large tadpole term has been disfavored in the past; moreover it has been argued that it would generally be expected to lead to a large vev $\langle s\rangle$, but as we shall see this does not happen in our model.) It is then clear from the form of the superpotential W_B as given in Eq. (8) that for an extremum that is supersymmetric (when we neglect supersymmetry breaking) we will require nonzero vevs for ϕ, $\bar{\phi}$ and/or $h_{1,2}$ (in order to obtain $F_S = 0$). The existence of competing vacua of this nature was noted in by Dvali *et al.* in Ref. [16]; their model differs from ours in choice of gauge group [they have $SU(3)\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$] and supersymmetry-breaking mechanism.

Let us consider these two possibilities in turn.

A. The ϕ, $\bar{\phi}$, s extremum

Retaining for the moment only the scalar fields ϕ, $\bar{\phi}$, s (the scalar component of their upper case counterpart superfields) we write the scalar potential:

$$\begin{aligned} V &= \lambda_1^2(|\phi s|^2 + |\bar{\phi} s|^2) + |\lambda_1\phi\bar{\phi} - M^2|^2 \\ &\quad + \frac{1}{2}q_\Phi^2 g'^2(|\phi|^2 - |\bar{\phi}|^2)^2 + m_\phi^2|\phi|^2 + m_{\bar{\phi}}^2|\bar{\phi}|^2 \\ &\quad + m_s^2|s|^2 + \rho M^2 m_{\frac{3}{2}}(s + s^*) + h_{\lambda_1}\phi\bar{\phi}s + \text{c.c.} \qquad (11) \end{aligned}$$

Here, as well as soft terms dictated by Eqs. (2) and (3), we also introduce a soft breaking term linear in s. The status of this term in anomaly mediation is somewhat ambiguous. According to Ref. [25], without a quadratic term in the superpotential, the *RG invariant* solution is $\rho = 0$. On the other hand, a naive application of the superconformal calculus to the superpotential term $M^2 S$ would conclude that $\rho = 2$. Pending a definitive resolution of this puzzle, we will presently assume the ρ term is small, while also considering its possible impact.

The potential depends on two explicit mass parameters, the gravitino mass $m_{\frac{3}{2}}$ and M. Let us establish its minimum. Writing $\langle\phi\rangle = v_\phi/\sqrt{2}$, $\langle\bar{\phi}\rangle = v_{\bar{\phi}}/\sqrt{2}$ and $\langle s\rangle = v_s/\sqrt{2}$, we find

$$v_\phi\left[m_\phi^2 + \frac{1}{2}\lambda_1^2 v_s^2 + \frac{1}{2}g^2 q_\Phi^2(v_\phi^2 - v_{\bar{\phi}}^2)\right] + v_{\bar{\phi}}\left[\lambda_1\left(\frac{1}{2}\lambda_1 v_\phi v_{\bar{\phi}} - M^2\right) + \frac{h_{\lambda_1}}{\sqrt{2}}v_s\right] = 0, \qquad (12)$$

$$v_{\bar{\phi}}\left[m_{\bar{\phi}}^2 + \frac{1}{2}\lambda_1^2 v_s^2 - \frac{1}{2}g^2 q_\Phi^2(v_\phi^2 - v_{\bar{\phi}}^2)\right] + v_\phi\left[\lambda_1\left(\frac{1}{2}\lambda_1 v_\phi v_{\bar{\phi}} - M^2\right) + \frac{h_{\lambda_1}}{\sqrt{2}}v_s\right] = 0, \qquad (13)$$

$$v_s\left[m_s^2 + \frac{1}{2}\lambda_1^2(v_\phi^2 + v_{\bar{\phi}}^2)\right] + \frac{h_{\lambda_1}}{\sqrt{2}}v_\phi v_{\bar{\phi}} + \sqrt{2}\rho M^2 m_{\frac{3}{2}} = 0. \qquad (14)$$

It follows easily from Eqs. (12) and (13) that

$$\lambda_1\left(\frac{1}{2}\lambda_1 v_\phi v_{\bar{\phi}} - M^2\right) = -\frac{v_\phi v_{\bar{\phi}}}{v_\phi^2 + v_{\bar{\phi}}^2}\left[m_\phi^2 + m_{\bar{\phi}}^2 + \lambda_1^2 v_s^2\right] - \frac{h_{\lambda_1}}{\sqrt{2}}v_s, \qquad (15)$$

$$\frac{1}{2}g'^2 q_\Phi^2(v_\phi^2 - v_{\bar{\phi}}^2) = \frac{v_{\bar{\phi}}^2 m_{\bar{\phi}}^2 - v_\phi^2 m_\phi^2 + (v_{\bar{\phi}}^2 - v_\phi^2)\frac{1}{2}\lambda_1^2 v_s^2}{v_\phi^2 + v_{\bar{\phi}}^2}. \qquad (16)$$

We now assume that $M \gg m_{\frac{3}{2}}$. It is immediately clear from Eqs. (15) and (16) that

$$v_\phi^2 \simeq v_{\bar{\phi}}^2 \simeq \frac{2}{\lambda_1}M^2, \qquad (17)$$

and then from Eq. (14) that v_s is $O(m_{\frac{3}{2}})$. We thus obtain from Eq. (16) that

$$v_\phi^2 - v_{\bar{\phi}}^2 = \frac{m_{\bar{\phi}}^2 - m_\phi^2}{g'^2 q_\Phi^2} + O(m_{\frac{3}{2}}^4/M^2), \qquad (18)$$

and from Eq. (14) that

$$v_s = -\frac{h_{\lambda_1}}{\sqrt{2}\lambda_1^2} - \frac{m_{\frac{3}{2}}\rho}{\sqrt{2}\lambda_1} + O(m_{\frac{3}{2}}^2/M). \qquad (19)$$

Now the h_{λ_1} term is determined in accordance with Eq. (2):

$$h_{\lambda_1} = -m_{\frac{3}{2}}\frac{\lambda_1}{16\pi^2}\left(3\lambda_1^2 + \frac{1}{2}\,\mathrm{Tr}\lambda_2^2 + 2\lambda_3^2 - 4q_\phi^2 g'^2\right), \qquad (20)$$

denoting the U(1)$'$ charge by g'.

If we assume that $|q_\Phi g'| \gg |\lambda_{1,2,3}|$ then we find

$$v_s \simeq -\frac{\sqrt{2}\lambda_1 m_{\frac{3}{2}}}{16\pi^2}\left(\frac{2q_\phi^2 g'^2}{\lambda_1^2}\right) - \frac{m_{\frac{3}{2}}\rho}{\sqrt{2}\lambda_1}. \qquad (21)$$

For simplicity we shall assume that $|\rho| \ll q_\phi^2 g'^2/(16\pi^2)$, so that the ρ contributions to Eqs. (19) and (21) are negligible.

Substituting back from Eqs. (17) and (19) into Eq. (11), we obtain to leading order

$$V_\phi = \frac{1}{\lambda_1}M^2\left(m_\phi^2 + m_{\bar{\phi}}^2 - \frac{h_{\lambda_1}^2}{2\lambda_1^2}\right). \qquad (22)$$

Presently we shall compare this result with the analogous one associated with the $h_{1,2}$, s extremum.

Supposing, however, that the ϕ, $\bar{\phi}$, s extremum is indeed the relevant one, we obtain the Higgs μ-term

$$\mu_h = \frac{\lambda_1 \lambda_3 m_{\frac{3}{2}}}{16\pi^2}\left(\frac{2q_\phi^2 g'^2}{\lambda_1^2}\right). \quad (23)$$

One might think that since v_s is naturally determined above to be associated with the supersymmetry-breaking scale [rather than the U(1)$'$ breaking scale] it would be necessary to minimize the whole Higgs potential (including $\langle h_{1,2}\rangle$) in order to determine it. But if we retain, for example, the m_s^2 term in Eq. (14), the resulting correction to Eq. (19) is easily seen to be $O(m_{\frac{3}{2}}^4/M^2)$. Similarly, the Higgs vevs responsible for electroweak symmetry breaking do not affect Eqs. (19) and (23) to an appreciable extent.

In Ref. [7], we naively estimated $\mu_h \sim \lambda_1\lambda_3 m_{\frac{3}{2}}$, concluding that μ_h would be at most O(GeV) rather than O(100 GeV). The improved formula, Eq. (23), changes this conclusion.

If we neglect terms of $O(m_{\frac{3}{2}})$, it is easy to see from Eqs. (15) and (16) that the breaking of U(1)$'$ preserves supersymmetry [since in this limit the two equations correspond to vanishing of the S F-term and the U(1)$'$ D-term respectively]; thus the U(1)$'$ gauge boson, its gaugino (with one combination of $\psi_{\phi,\bar{\phi}}$) and the Higgs boson form a massive supermultiplet with mass $m_V = g'\sqrt{v_\phi^2 + v_{\bar{\phi}}^2}$, while the remaining combination of ϕ and $\bar{\phi}$ and the other combination of $\psi_{\phi,\bar{\phi}}$, form a massive chiral supermultiplet (corresponding to the X field of Ref. [5]). Both this multiplet and S have supersymmetric mass $m_S = \lambda_1\sqrt{v_\phi^2 + v_{\bar{\phi}}^2}$.

For large $M \gg m_{\frac{3}{2}}$ and $g'^2 \gg \lambda_1$ a hierarchy develops between the $U(1)'$ breaking scale m_V, the S mass scale m_s, and the supersymmetry-breaking scale $m_{\frac{3}{2}}$. Above m_V, the soft terms obey Eqs. (1)–(3) with all states contributing. Between m_V and m_S the theory is the MSSM with two singlets, and with Eq. (5) replacing Eq. (3). The extra contributions to the masses of the matter fields arise from the U(1)$'$ D-term as explained in Ref. [5]. Below m_S the singlets can be integrated out, which does not affect the D-term contribution. Hence the only traces of the U(1)$'$ symmetry in the effective low energy Lagrangian are contributions to the masses of the matter fields, arising from the U(1)$'$ D-term, which are naturally of the same order as the AMSB ones.

Evidently the N triplet also gains a large supersymmetric mass thus naturally implementing the seesaw mechanism. The generation of an appropriate μ-term via the vev of a singlet is reminiscent of the next-to-minimal supersymmetric standard model (NMSSM) (for a review of and references for the NMSSM see Ref. [26]). We stress, however, that our model differs in a crucial way from the NMSSM, in that the spectrum at $m_{3/2}$ is precisely that of the MSSM.

It is easy to show by substituting Eq. (18) back into the potential, Eq. (11), that the contribution to the slepton masses arising from the U(1)$'$ term which resolves the tachyonic slepton problem is given by

$$\delta m_{l,e}^2 \sim \frac{q_{L,E}}{2q_\Phi}(m_{\bar{\phi}}^2 - m_\phi^2), \quad (24)$$

with corresponding contributions for the other scalar MSSM fields proportional to their U(1)$'$ charges. Now

$$m_{\bar{\phi}}^2 - m_\phi^2 = \frac{1}{2} m_{\frac{3}{2}}^2 \mu \frac{d}{d\mu}(\gamma_{\bar{\phi}} - \gamma_\phi) = -\frac{1}{2}\frac{m_{\frac{3}{2}}^2}{16\pi^2}\mathrm{Tr}\lambda_2\beta_{\lambda_2}, \quad (25)$$

where (at one loop)

$$16\pi^2\beta_{\lambda_2} = \lambda_2\Big[\lambda_1^2 + 2\lambda_2^2 + \frac{1}{2}\mathrm{Tr}\lambda_2^2 + 2Y_N^\dagger Y_N - (2q_\phi^2 + 4q_N^2)g'^2\Big], \quad (26)$$

and we have for simplicity taken λ_2 to be diagonal.

Let us consider what sort of values of $\delta m_{l,e}^2$ we require. In this context it is interesting to compare Fig. 1 of Ref. [10] with Fig. 1 of Ref. [11]. In both references, (L, e) correspond to our $(\delta m_l^2, \delta m_e^2)$ respectively. In the former case the scalar masses are calculated at low energies, whereas in the latter they are calculated at gauge unification and then run down to the electroweak scale. This is why the allowed (L, e) regions are different in the two cases. Since we are assuming M is large, it is clear that the latter are more relevant to our situation. From Fig. 1 of Ref. [11] we see that suitable values would be

$$\delta m_l^2 \simeq 0, \qquad 0.16\left(\frac{m_{\frac{3}{2}}}{40}\right)^2 \lesssim \delta m_e^2 \lesssim 0.35\left(\frac{m_{\frac{3}{2}}}{40}\right)^2. \quad (27)$$

Notice that δm_e^2 must necessarily be positive.

So, if we assume that the one-loop β_{λ_2} is dominated by its gauge contribution, consistent with our previous assumption that $|q_\Phi g'| \gg |\lambda_{1,2,3}|$, we obtain

$$\delta m_{l,e}^2 \simeq \frac{q_{L,E}(q_\Phi^2 + 2q_N^2)}{2q_\Phi}\frac{g'^2 m_{\frac{3}{2}}^2 \mathrm{Tr}\lambda_2^2}{(16\pi^2)^2}. \quad (28)$$

Now $q_\Phi = 4q_L + 2q_E$, so we see that it is easy to obtain the correct sign for δm_e^2.

For $q_L = 0$, we find

$$\delta m_e^2 \simeq 3q_E^2 \frac{g'^2 m_{\frac{3}{2}}^2 \mathrm{Tr}\lambda_2^2}{2(16\pi^2)^2} \quad (29)$$

or

$$1.6 \lesssim q_E^2 g'^2 \mathrm{Tr}\lambda_2^2 \lesssim 3.6. \quad (30)$$

Of course with $q_L = 0$, we have $\delta m_l^2 = 0$; but as described earlier, it was shown in Ref. [11] that acceptable slepton masses nevertheless result when we run down to low energies. Clearly there are similar contributions to the masses of the other matter fields similar to Eq. (28), thus for example

$$\delta m^2_{h_1,h_2} \simeq \frac{q_{H_1,H_2}(q_\Phi^2 + 2q_N^2)}{2q_\Phi} \frac{g'^2 m^2_{\frac{3}{2}} \mathrm{Tr}\lambda_2^2}{(16\pi^2)^2}. \tag{31}$$

In the notation of Ref. [11], Eq. (28), for example, is simply replaced by $\delta m_l^2 = Lk'$ and $\delta m_e^2 = ek'$ with (L, e) replacing $q_{L,E}$, and all results presented for $k' = 1(\mathrm{TeV})^2$.

We emphasize once again the contrast between our model and conventional versions of the NMSSM, which does not, in basic form, contain an extra gauged U(1), but where a vev (of the scale of supersymmetry breaking) for the gauge singlet s generates a Higgs μ-term in much the same way, as is done here. However, while in the NMSSM case the s fields are very much part of the Higgs spectrum, here, in spite of the comparatively small s vev, the s quanta obtain large supersymmetric masses and are decoupled from the low energy physics, which becomes simply that of the MSSM. Another nice feature is the natural emergence of the seesaw mechanism via the spontaneous breaking of the U(1)$'$. Evidently it will be feasible to associate the U(1)$'$ breaking scale given by Eq. (17) with the scale of gauge unification.

Although, as indicated above, we will be regarding M as source of significant physics, it is worth briefly considering the limit $M \to \infty$. In that limit, the theory becomes simply the MSSM (including the Higgs μ_h-term) with the soft breaking terms given in Eqs. (1)–(3) including the additional kY term, which resolves the tachyon problem. The explicit form of the terms proportional to the gravitino mass in these equations is easily derived using the conformal compensator field as described in Ref. [5]. Of course, although the resulting kY term in Eq. (3) has the *form* of a FI term, in the effective theory (for $M \to \infty$) $U(1)'$ is not gauged and so we do not fall foul of the strictures of Ref. [9]. The conformal compensator field does not provide us with a straightforward derivation of Eq. (4); as described earlier, we will, like most previous authors, rely on the electroweak minimization process to determine the Higgs B term.

B. The $h_{1,2}$, s extremum

We now consider the scalar potential

$$\begin{aligned} V = {} & \lambda_3^2(|h_1 s|^2 + |h_2 s|^2) + |\lambda_3 h_1 h_2 - M^2|^2 \\ & + \frac{1}{2} g'^2 q_{H_1}^2 (|h_1|^2 - |h_2|^2)^2 + \frac{1}{8} g_1^2 (h_1^\dagger h_1 - h_2^\dagger h_2)^2 \\ & + \frac{1}{8} g_2^2 (h_1^\dagger \sigma^a h_1 + h_2^\dagger \sigma^a h_2)^2 + m_{h_1}^2 |h_1|^2 + m_{h_2}^2 |h_2|^2 \\ & + m_s^2 |s|^2 + \rho M^2 m_{\frac{3}{2}} (s + s^*) + h_{\lambda_3} h_1 h_2 s + \text{c.c.} \end{aligned} \tag{32}$$

In Eq. (32) we have written the $U(1)_Y$ gauge coupling as g_1, although its normalization corresponds to the usual SM convention, not that appropriate for SU(5) unification. This is to avoid confusion with the U(1)$'$ coupling, g'.

We see that the potential is very similar to Eq. (11), the main difference being the presence of SU(2) and $U(1)_Y$ D-terms. To leading order in M, only the SU(2) D-term depends on the relative direction in SU(2) space of the two doublets; it follows that we can choose without loss of generality to set $h_1 = (v_1/\sqrt{2}, 0)$ and $h_2 = (0, v_2/\sqrt{2})$, as in electroweak breaking, in order to obtain zero for the SU(2) D-term for $v_1 = v_2$. Minimization of the potential then proceeds in a similar way to the previous section (with the replacement $\lambda_1 \to \lambda_3$) leading to

$$V_h = \frac{M^2}{\lambda_3}\left(m_{h_1}^2 + m_{h_2}^2 - \frac{h_{\lambda_3}^2}{2\lambda_3^2}\right) \tag{33}$$

at the extremum. Here

$$\begin{aligned} h_{\lambda_3} = {} & -m_{\frac{3}{2}} \frac{\lambda_3}{16\pi^2} (\mathrm{Tr} Y_E Y_E^\dagger + 3\,\mathrm{Tr} Y_D Y_D^\dagger + 3\,\mathrm{Tr} Y_U Y_U^\dagger \\ & + \lambda_1^2 + 4\,\mathrm{Tr}\lambda_3^2 - 3g_2^2 - g_1^2 - 4q_{H_1}^2 g'^2). \end{aligned} \tag{34}$$

Let us compare the result for V_h with that obtained for V_ϕ, in the previous section, Eq. (22). If we assume that the g' terms dominate throughout we obtain simply

$$V_\phi = -\frac{M^2}{\lambda_1}\left(\frac{m_{\frac{3}{2}} g'^2}{16\pi^2}\right)^2 [4Qq_\Phi^2 + 8q_\Phi^4] \tag{35}$$

and

$$V_h = -\frac{M^2}{\lambda_3}\left(\frac{m_{\frac{3}{2}} g'^2}{16\pi^2}\right)^2 [4Qq_{H_1}^2 + 8q_{H_1}^4], \tag{36}$$

where we have written the one-loop g' β function as

$$\beta_{g'} = Q\frac{g'^3}{16\pi^2} \tag{37}$$

and

$$\begin{aligned} Q &= n_G\left(\frac{40}{3}q_L^2 + 8q_E^2 + 16q_E q_L\right) + 36q_L^2 + 40q_E q_L + 12q_E^2 \\ &= 76q_L^2 + 36q_E^2 + 88q_E q_L \quad (\text{for } n_G = 3). \end{aligned} \tag{38}$$

The coefficient Q is in general large, and larger than both q_Φ^2 and $q_{H_1}^2$, so the condition for the ϕ, $\bar{\phi}$, s extremum to have a lower energy than the h_1, h_2, s one may be written

$$\lambda_1\left(\frac{q_{H_1}}{q_\Phi}\right)^2 \lesssim \lambda_3. \tag{39}$$

Alternatively, for the specific choice $q_L = 0$, which we will see in the next section leads to an acceptable electroweak vacuum, we find that the same condition becomes

$$\frac{19}{88}\lambda_1 \lesssim \lambda_3. \tag{40}$$

VI. THE SPARTICLE SPECTRUM

In this section we calculate sparticle spectra for the sAMSB model, and compare the results with typical mAMSB spectra. We shall be interested in seeking regions of parameter space with a "high" Higgs mass—that is, close to about 125 GeV as suggested by recent LHC data [1,2]—and a supersymmetric contribution to the muon anomalous magnetic moment δa_μ compatible with the experimental deviation from the Standard Model prediction, $\delta a_\mu^{\rm exp} = 29.5(8.8) \times 10^{-10}$ [27]. We will also wish to remain consistent with the negative results of recent LHC supersymmetry searches, see for example Refs. [28,29].

We use the methodology of Ref. [11], which, as explained in Sec. II, can also be applied to mAMSB by replacing the characteristic (L, e) FI-type terms of sAMSB by a universal mass term m_0^2.

We begin by choosing input values for $m_{\frac{3}{2}}$, $\tan\beta$, L, e and signμ_h at the gauge unification scale M_X. Then we calculate the appropriate dimensionless coupling input values at the scale M_Z by an iterative procedure involving the sparticle spectrum, and the loop corrections to $\alpha_{1\ldots3}$, m_t, m_b and m_τ, as described in Ref. [30]. We define gauge unification by the meeting point of α_1 and α_2; this scale, of around 10^{16} GeV, we assume to be equal or close to the scale of U(1)$'$ breaking. For the top quark pole mass we use $m_t = 172.9$ GeV. All calculations are done in the approximation that we retain only third generation Yukawa couplings, $\lambda_{t,b,\tau}$; thus the squarks and sleptons of the second generation are degenerate with the corresponding ones of the first generation.

We then determine a given sparticle pole mass by running the dimensionless couplings up to a certain scale chosen (by iteration) to be equal to the pole mass itself, and then implementing full one-loop corrections from Ref. [30], and two-loop corrections to the top quark mass [31]. We use two-loop anomalous dimensions and β functions throughout.

A. Mass spectra in sAMSB

We display some examples of spectra in Table II(a)–(d). In each table, the columns are for different gravitino masses, all with $L = 0$ with e increasing with increasing gravitino mass so as to remain within the allowed (L, e) region; obviously e scales like $m_{\frac{3}{2}}^2$ from Eq. (28). [As already indicated, we input (L, e) at M_X, so the allowed (L, e) region corresponds to that in Ref. [11] rather than that in Ref. [10]]. In Table II(a) and (c) the (L, e) values are in the center of the allowed (L, e) region (at least for smaller values of $m_{\frac{3}{2}}$), whereas in Table II(b) and (d) e is smaller so that lighter sleptons result. We see that $\mu_h/m_{\frac{3}{2}}$ varies little with $m_{\frac{3}{2}}$; for example in Table II(a) changing from 0.014 at $m_{\frac{3}{2}} = 40$ TeV to 0.012 at $m_{\frac{3}{2}} = 140$ TeV. We thus find from Eq. (23) that

$$\lambda_1\lambda_3\frac{2q_\phi^2 g'^2}{\lambda_1^2} \simeq 2.2 \qquad (41)$$

in order for the electroweak vacuum to exist. We shall return to this formula when we have discussed the cosmological constraints.

In Table II(a) and (b) we have $\tan\beta = 10$, whereas in Table II(c) and (d) we have $\tan\beta = 16$. Increasing $\tan\beta$ generally leads to a slight increase in the light Higgs mass m_h, and a *decrease* in the heavy Higgs masses; this decrease is a signal of the fact that (for given $m_{\frac{3}{2}}$, L, e) there is an upper limit on $\tan\beta$; above that limit, the electroweak vacuum fails.

Increasing the scale of supersymmetry breaking (by increasing $m_{\frac{3}{2}}$) will, generally speaking, allow us to remain compatible with the more stringent limits on beyond the Standard Model physics emerging from LHC searches and B decay. Recent LHC publications on supersymmetry searches (see for example Refs. [28,29]) tend to focus on sparticle spectra which are not compatible with AMSB; but it seems clear that for $m_{\frac{3}{2}} \gtrsim 60$ TeV or so, our model is not (yet) ruled out. One search result that explicitly targets anomaly mediation is that of Ref. [32]; this sets a lower limit on the wino mass of 92 GeV, which in sAMSB would correspond to $m_{\frac{3}{2}} \simeq 28$ TeV.

Increasing $m_{\frac{3}{2}}$ so as to reduce squark or gluino production will, however, reduce the supersymmetric contribution to the muon anomalous magnetic moment a_μ, and hence the opportunity to account for the existing discrepancy between theory and experiment. But it is a feature of AMSB, and in particular sAMSB, that the sleptons are comparatively light compared to the gluino and squarks. Therefore it turns out to be possible to combine heavier colored states with sleptons and electroweak gauginos still light enough to contribute appreciably to a_μ. We demonstrate this by including in the tables the result for the supersymmetric contribution to a_μ. For $m_{\frac{3}{2}} = 60$ TeV, the result is manifestly compatible with the aforementioned discrepancy [33].

Notice that increasing $m_{\frac{3}{2}}$ so as to increase m_h to bring it closer to the recent announcement of evidence [1,2] for a SM-like Higgs in the region of 125 GeV can be done, but at the cost of reducing δa_μ; see columns 4 and 7 of $\tan\beta = 16$ in Table II(c) and (d). It also increases the degree of fine-tuning, as we shall discuss presently.

Generally speaking, in all the regions of parameter space which we have explored, there is one light neutral Higgs scalar, with all other Higgs scalars being much heavier, and roughly equal in mass. One might then define an effective low energy Higgs theory, by integrating out these heavy Higgses. It is then easy to check that that $\cos(\beta - \alpha) = 0$ up to small corrections [where α is the (h, H) mixing angle]. Consequently all the light Higgs couplings to SM particles closely approximate their SM values, and our model is broadly consistent with the recent LHC analyses

TABLE II. sAMSB mass spectra (in GeV) and δa_μ for $m_t = 172.9$ GeV and different values of (L, e) and $\tan\beta$.

$m_{\frac{3}{2}}$	40 TeV	80 TeV	140 TeV	40 TeV	80 TeV	140 TeV
		(a) $\tan\beta = 10$			(b) $\tan\beta = 10$	
(L, e)	(0, 0.25)	(0, 1)	(0, 3.0625)	(0, 0.16)	(0, 0.64)	(0, 1.96)
$\tilde{g}$	900	1684	2802	900	1684	2802
$\tilde{t}_1$	757	1346	2120	770	1369	2237
$\tilde{t}_2$	507	925	1473	548	1023	1668
$\tilde{u}_L$	819	1531	2542	825	1545	2568
$\tilde{u}_R$	766	1408	2304	795	1474	2428
$\tilde{b}_1$	714	1322	2181	723	1342	2891
$\tilde{b}_2$	946	1798	3031	909	1721	2237
$\tilde{d}_L$	822	1533	2544	829	1547	2922
$\tilde{d}_R$	955	1816	3062	919	1740	2569
$\tilde{\tau}_1$	199	419	758	119	270	502
$\tilde{\tau}_2$	266	512	882	198	366	623
$\tilde{e}_L$	212	433	776	145	295	532
$\tilde{e}_R$	261	512	887	187	363	627
$\tilde{\nu}_e$	249	506	883	170	354	622
$\tilde{\nu}_\tau$	247	502	876	167	349	612
χ_1	131	265	461	131	265	460
χ_2	362	734	1294	363	736	1296
χ_3	588	1084	1773	635	1186	1964
χ_4	599	1091	1778	645	1192	1968
$\chi_1^\pm$	131	265	461	131	265	460
$\chi_2^\pm$	597	1089	1777	643	1190	1967
h	115	120	124	115	120	124
H, A	366	595	802	499	907	1448
$H^\pm$	374	601	806	506	911	1451
$\delta\chi_1^{\pm,0}$	0.236	0.214	0.194	0.223	0.212	0.197
μ_h	571	1041	1675	618	1142	1867
δa_μ	32×10^{-10}	8.4×10^{-10}	2.9×10^{-10}	40×10^{-10}	10×10^{-10}	3.5×10^{-10}
		(c) $\tan\beta = 16$			(d) $\tan\beta = 16$	
(L, e)	(0, 0.25)	(0, 1)	(0, 3.0625)	(0, 0.18)	(0, 0.72)	(0, 1.96)
$\tilde{g}$	899	1683	2801	899	1684	2801
$\tilde{t}_1$	750	1328	2168	761	1348	2199
$\tilde{t}_2$	504	924	1745	536	1001	1629
$\tilde{u}_L$	819	1532	2543	928	1543	2563
$\tilde{u}_R$	766	1409	2305	824	1460	2402
$\tilde{b}_1$	703	1301	2153	710	1348	2181
$\tilde{b}_2$	929	1768	2983	901	1708	2872
$\tilde{d}_L$	823	1534	2544	828	1545	2564
$\tilde{d}_R$	955	1812	3062	928	1757	2954
$\tilde{\tau}_1$	182	400	733	111	280	532
$\tilde{\tau}_2$	271	512	877	223	405	683
$\tilde{e}_L$	212	433	776	163	331	595
$\tilde{e}_R$	262	512	887	206	401	693
$\tilde{\nu}_e$	249	506	883	190	393	689
$\tilde{\nu}_\tau$	244	497	867	184	381	668
χ_1	132	199	461	132	265	460
χ_2	362	734	1294	363	735	1295
χ_3	585	1077	1763	621	1156	1910
χ_4	594	1083	1767	630	1161	1914
$\chi_1^\pm$	132	265	461	132	265	460
$\chi_2^\pm$	592	1082	1766	628	1160	1913
h	116	121	125	116	121	125
H, A	284	417	374	410	729	1125
$H^\pm$	285	425	384	419	734	1129
$\delta\chi_1^{\pm,0}$	0.229	0.213	0.194	0.218	0.211	0.195
μ_h	566	1032	1662	603	1111	1852
δa_μ	51×10^{-10}	14×10^{-10}	4.7×10^{-10}	60×10^{-10}	16×10^{-10}	5.4×10^{-10}

demonstrating compatibility with SM predictions. In particular we do not expect important contributions to the $h \to \gamma\gamma$ rate beyond those due to the top quark and the W boson, compatible with recent analyses (see for example Ref. [36]).

We can increase δa_μ by choosing (L, e) closer to one of the boundaries of the allowed region corresponding to either the charged slepton doublets or singlets becoming too light; but the effect of doing this is limited in that the gaugino masses are not sensitive to (L, e). The bottom line is that with $\tan\beta = 10$, to account for the whole of $\delta a_\mu^{\rm exp}$ we need a light Higgs mass of around 115–120 GeV. Increasing $\tan\beta$ also leads to larger δa_μ, but also a *smaller* charged Higgs mass, and a potentially over-large contribution to the branching ratio $B \to X_s\gamma$. This effect is particularly noticeable in Table II(c), where the heavy Higgs masses actually *decrease* as $m_{\frac{3}{2}}$ is increased from 80 to 140 TeV. We will return to this issue in Sec. VI C.

As in most versions of AMSB, the LSP is mostly neutral wino, with the charged wino a few hundred MeV heavier. In all the tables we denote this mass difference as $\delta\chi_1^{\pm,0}$.

B. Comparison with mAMSB

It is interesting to compare the sAMSB spectra presented in Table II(a)–(d) with some mAMSB spectra.

In Table III(a) we present results for $m_{\frac{3}{2}} = 60$ TeV, for different values of m_0. The second column of this table corresponds to the benchmark point mAMSB1.3 of Ref. [37]; our results for the masses agree reasonably well with those presented there: for example, the gluino masses differ by 2%, and the lightest third generation squarks by 1%. They are also not inconsistent with those of Ref. [38], who quote an upper limit for m_h of 120.4 GeV; note that there the parameter scan is restricted to $m_0 <$ 2 TeV. For a detailed comparison of mAMSB results with recent LHC data see Ref. [39]. We see that by increasing m_0, we can eventually make all the squarks heavier than the the gluino; this is not possible in sAMSB, because increasing L and e soon leads to loss of the electroweak vacuum. We will discuss this fact in more detail in Sec. VI C.

In Table III(b) we present the corresponding results for $m_{\frac{3}{2}} = 140$ TeV. Note the (comparatively) light sleptons when $m_0 = 900$ GeV; these occur because for this value

TABLE III. Mass spectra (in GeV) and δa_μ for $m_t = 172.9$ GeV and different values of $m_{\frac{3}{2}}$, m_0 and $\tan\beta$.

	(a) $m_{\frac{3}{2}} = 60$ TeV, $\tan\beta = 10$				(b) $m_{\frac{3}{2}} = 140$ TeV, $\tan\beta = 16$		
m_0	450	900	1800	2700	900	1800	2700
$\tilde g$	1310	1342	1398	1438	2824	2881	2939
$\tilde t_1$	1156	1303	1783	2398	2382	2682	3114
$\tilde t_2$	940	1052	1384	1804	2038	2248	2548
$\tilde u_L$	1295	1499	2135	2912	2776	3162	3720
$\tilde u_R$	1285	1489	2126	2903	2756	3143	3703
$\tilde b_1$	1120	1278	1773	2392	2362	2668	3105
$\tilde b_2$	1288	1489	2121	2892	2704	3085	3636
$\tilde d_L$	1287	1491	2128	2904	2757	3144	3707
$\tilde d_R$	1303	1506	2141	2917	2795	3179	3735
$\tilde\tau_1$	355	851	1764	2664	620	1652	2573
$\tilde\tau_2$	399	870	1774	2671	710	1691	2605
$\tilde e_L$	381	865	1778	2680	707	1717	2634
$\tilde e_R$	390	871	1784	2687	723	1707	2643
$\tilde\nu_e$	372	861	1776	2679	703	1705	2632
$\tilde\nu_\tau$	367	856	1768	2668	670	1678	2560
χ_1	199	200	201	202	461	464	465
χ_2	550	555	558	559	1297	1306	1311
χ_3	1031	1027	1004	950	2240	2211	2162
χ_4	1037	1032	1009	956	2243	2214	2164
$\chi_1^\pm$	200	201	201	202	461	464	465
$\chi_2^\pm$	1036	1031	1009	955	2242	2213	2164
h	118	119	120	122	125	126	126
H, A	1076	1314	2006	2802	2186	2618	3214
$H^\pm$	1079	1317	2008	2804	2188	2620	3216
$\delta\chi_1^{\pm,0}$	0.209	0.209	0.208	0.209	0.191	0.175	0.161
μ_h	1000	989	956	889	2136	2095	2032
δa_μ	13×10^{-10}	5.1×10^{-10}	1.7×10^{-10}	0.82×10^{-10}	4.7×10^{-10}	2.0×10^{-10}	1.1×10^{-10}

the m_0^2 contribution to the slepton (masses)^2 almost cancels the (negative) $m_{\frac{3}{2}}^2$ one. (We do not give results in Table III(b) for $m_0 = 450$ GeV, because in that case there are still tachyonic sleptons.) This is analogous to being close to a boundary in the allowed (L, e) space in the sAMSB case, and, as there, does not in itself result in a large δa_μ, because the wino masses are unaffected. Moreover, away from the (L, e) boundary (in sAMSB) the slepton masses remain relatively small, whereas for fixed $m_{\frac{3}{2}}$, increasing m_0 (in mAMSB) leads rapidly to larger slepton masses.

It is interesting that in mAMSB, increasing m_0 (for fixed $m_{\frac{3}{2}}$) leads to a slight decrease in μ_h, and a consequent slight decrease in the masses of the heavy neutralinos and chargino. Note also that the supersymmetric contribution to a_μ is compatible with $\delta a_\mu^{\rm exp}$ for $m_0 = 450$ GeV, in Table III(a), but decreases rapidly as m_0 increases. If we increase $m_{\frac{3}{2}}$ to 140 TeV as in Table III(b), we are able to obtain $m_h = 125$ GeV, but, as in sAMSB at the price of a small contribution to δa_μ. In order to aid comparison, Fig. 1 shows a graphical representation of the features of sAMSB and mAMSB mass spectra with $m_h = 125$ GeV. The diagrams demonstrate that the non-SM Higgs particles are generically lighter in sAMSB.

C. Fine-tuning

Noting that as $m_{\frac{3}{2}}$ is increased we find that μ_h increases, we should comment on the issue of the fine-tuning required to produce the electroweak scale. From the well-known tree level relation

$$\frac{m_{h_1}^2 - m_{h_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu_h^2 = \frac{1}{2} M_Z^2, \tag{42}$$

we see that unless $|m_{h_1}^2| \gg |m_{h_2}^2|$ then, for typical values of $\tan\beta$, we have

$$\mu_h^2 \simeq -m_{h_2}^2 - \frac{1}{2} M_Z^2, \tag{43}$$

which since generically $|m_{h_2}| \gg M_Z$ represents a fine-tuning, sometimes called the "little hierarchy" problem.

One might have hoped, since $q_{H_2} = q_L + q_E > 0$, to reduce $|m_{h_2}^2|$, and hence μ_h^2, by increasing $q_L + q_E$; see Eq. (31). But from Fig. 1 of Ref. [11] we see $L + e$ is severely constrained by the requirement of a stable electroweak vacuum; the failure of this is manifested by a tachyonic m_A. The tree formula for m_A is

$$m_A^2 = 2\mu_h^2 + m_{h_1}^2 + m_{h_2}^2 \simeq m_{h_1}^2 - m_{h_2}^2 - M_Z^2, \tag{44}$$

and it is apparent from Eq. (31) that the overall effect of increasing $q_L + q_E$ actually decreases m_A^2.

For example, if we use $m_{\frac{3}{2}} = 80$ TeV and $(L, e) = (0, 1.2)$, then we find that m_A is sharply reduced to 295 GeV while μ_h changes only to 980 GeV. A small further increase in e takes m_A rapidly to zero. A similar outcome is the result of increasing $\tan\beta$. For example, with $m_{\frac{3}{2}} = 80$ TeV and $(L, e) = (0, 1)$, as in the third columns of Table II(a) and (c), μ_h decreases with increasing $\tan\beta$ but m_A decreases more sharply. For $\tan\beta = 19$, we find $\mu_h = 1025$ GeV, but $m_A = 207$ GeV, and for $\tan\beta = 20$, $m_A^2 < 0$.

If we increase $m_{\frac{3}{2}}$ then the upper limit on $\tan\beta$ decreases; for example with $m_{\frac{3}{2}} = 120$ TeV and $(L, e) = (0, 2.25)$, we find that the maximum value of $\tan\beta$ is $\tan\beta = 17$, with $m_h = 124$ GeV and $m_A = 309$ GeV, and $\delta a_\mu = 6.6 \times 10^{-10}$. Note that δa_μ increases as $\tan\beta$ increases; however, in Table II(c), the concomitant decrease in the Higgs masses (in particular the charged Higgs mass) leads to an increased supersymmetric contribution to the branching ratio for $B \to X_s \gamma$, and potential conflict with experiment. See Fig. 4 of Ref. [40]. This problem is avoided in Table II(d); but with $m_{\frac{3}{2}}$ large enough to produce $m_h = 125$ GeV, there is no region in (L, e) space permitting a $\tan\beta$ large enough to generate $\delta a_\mu \approx 20 - 30 \times 10^{-10}$.

Within the context of our model we see no clean way to avoid the fine-tuning problem. It is interesting to note that with the alternative grand unified theory compatible assignment considered in Sec. 5 of Ref. [11], $L + e$ can be increased if desired (see Fig. 2 of Ref. [11]). However in that case we have $q_{H_1} = -e - L$ and $q_{H_2} = -2e$, so increasing $L + e$ does not reduce $|m_2^2|$ or m_A^2.

VII. COSMOLOGICAL HISTORY

A. F-term inflation

As detailed in a previous paper [7], the theory naturally produces F-term inflation [41–43], with the singlet scalar s as the inflaton. In this paper we are assuming that the FI-term vanishes, which considerably simplifies the radiative corrections driving the evolution of s during inflation. We also assume that the quartic term in s in the Kähler potential is negligible.

The relevant terms in the tree potential are

$$\begin{aligned} V_{\rm tree} = &|\lambda_1 \phi\bar{\phi} - \lambda_3 h_1 h_2 - M^2|^2 + [\lambda_1^2(|\phi|^2 + |\bar{\phi}|^2) \\ &+ \lambda_3^2(|h_1|^2 + |h_2|^2)]|s|^2 + \frac{1}{2} g'^2 (q_\Phi(\phi^*\phi - \bar{\phi}^*\bar{\phi}) \\ &+ q_{H_1}(h_1^\dagger h_1 - h_2^\dagger h_2))^2 + \frac{1}{8} g_2^2 \sum_a (h_1^\dagger \sigma^a h_1 + h_2^\dagger \sigma^a h_2)^2 \\ &+ \frac{1}{8} g_1^2 (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + V_{\rm soft}, \end{aligned} \tag{45}$$

where we have used $q_{\bar{\Phi}} = -q_\Phi$, $q_{H_2} = -q_{H_1}$ arising from the anomaly cancellation and gauge invariance conditions. The AMSB soft terms $V_{\rm soft}$ are the sum of those appearing in Eqs. (11) and (32), and are all suppressed by at least one power of $m_{\frac{3}{2}}$, which we are assuming to be much less than

M. The most important soft term is the linear one, which we are assuming is absent or at least small (see the discussion following Eq. (11)).

At large s and vanishing ϕ, $\bar{\phi}$, h_1 and h_2, and neglecting soft terms, we have

$$V_{\rm tree} = M^4 + \Delta V_1, \tag{46}$$

where ΔV_1 represents the one-loop corrections, given as usual by

$$\Delta V_1 = \frac{1}{64\pi^2} {\rm Str}[(M^2(s))^2 \ln (M^2(s)/\mu^2)]. \tag{47}$$

Here

$$\mathrm{Str} \equiv \sum_{\rm scalars} - 2 \sum_{\rm fermions} + 3 \sum_{\rm vectors}. \tag{48}$$

In the absence of the FI-term, ΔV is in fact dominated by the Φ, $\bar{\Phi}$ and $H_{1,2}$ subsystems, and the contribution to the one-loop scalar potential is [7]

$$\begin{aligned}\Delta V_1 = \frac{1}{32\pi^2}\Big[&(\lambda_1^2 s^2 + \lambda_1 M^2)^2 \ln\left(\frac{\lambda_1^2 s^2 + \lambda_1 M^2}{\mu^2}\right)\\ &+ (\lambda_1^2 s^2 - \lambda_1 M^2)^2 \ln\left(\frac{\lambda_1^2 s^2 - \lambda_1 M^2}{\mu^2}\right)\\ &+ 2(\lambda_3^2 s^2 + \lambda_3 M^2)^2 \ln\left(\frac{\lambda_3^2 s^2 + \lambda_3 M^2}{\mu^2}\right)\\ &+ 2(\lambda_3^2 s^2 - \lambda_3 M^2)^2 \ln\left(\frac{\lambda_3^2 s^2 - \lambda_3 M^2}{\mu^2}\right)\\ &- 2\lambda_1^4 s^4 \ln\left(\frac{\lambda_1^2 s^2}{\mu^2}\right) - 4\lambda_3^4 s^4 \ln\left(\frac{\lambda_3^2 s^2}{\mu^2}\right)\Big].\end{aligned} \tag{49}$$

For values of s for which $\lambda_{1,3}s^2 \gg M^2$ it is easy to show that, after removing a finite local counterterm, this reduces to

$$V(s) \simeq M^4\left[1 + \alpha \ln\frac{2s^2}{s_c^2}\right], \tag{50}$$

where

$$\alpha = \frac{\lambda^2}{16\pi^2}, \qquad \lambda = \sqrt{\lambda_1^2 + 2\lambda_3^2}, \qquad s_c^2 = M^2/\lambda. \tag{51}$$

Note that neglecting the linear soft term $\rho M^2 m_{\frac{3}{2}} s$ + c.c. is equivalent to assuming

$$\rho \ll \frac{\lambda^3}{16\pi^2}\frac{s_c}{m_{\frac{3}{2}}}. \tag{52}$$

With the parametrization (50), the scalar and tensor power spectra $\mathcal{P}_s$, $\mathcal{P}_t$ and the scalar spectral index n_s generated N e-foldings before the end of inflation are

$$\mathcal{P}_s(k) \simeq \frac{1}{24\pi^2}\frac{2N_k}{\alpha}\left(\frac{M}{m_p}\right)^4 = \frac{4N_k}{3}\left(\frac{s_c}{m_p}\right)^4, \tag{53}$$

$$\mathcal{P}_t(k) \simeq \frac{1}{6\pi^2}\left(\frac{M}{m_p}\right)^4 = \frac{8}{3}\left(\frac{s_c}{m_p}\right)^4, \tag{54}$$

$$n_s \simeq \left(1 - \frac{1}{N_k}\right). \tag{55}$$

The WMAP7 best-fit values for $\mathcal{P}_s(k_0)$ and n_s at $k = k_0 = 0.002h\ {\rm Mpc}^{-1}$ in the standard Lambda cold dark matter model are [44]

$$\begin{aligned}\mathcal{P}_s(k_0) &= (2.43 \pm 0.11) \times 10^{-9},\\ n_s &= 0.963 \pm 0.012 (68\%\ {\rm CL}),\end{aligned} \tag{56}$$

which correspond to

$$\frac{s_c}{m_p} \simeq 2.9 \times 10^{-3}\left(\frac{27}{N_{k_0}}\right)^{\frac{1}{4}}, \qquad N_{k_0} = 27^{+13}_{-7}. \tag{57}$$

There is an approximately 2σ discrepancy with the standard hot big bang result $N_{k_0} \simeq 58 + \ln(T_{\rm rh}/10^{15}\ {\rm GeV})$. We will see later how this is ameliorated by $N_\theta \simeq 15$ e-foldings of thermal inflation, leading to $N_{k_0} \simeq 40$ (1SF), and $n_s \simeq 0.975$. Hence the the discrepancy in the spectral index is reduced to approximately 1σ [45].

If $\lambda_3 > \lambda_1$, inflation ends at the critical value $s_{c1}^2 = M^2/\lambda_1$, followed by transition to the U(1)$'$-broken phase described by Eqs. (12)–(14). On the other hand, if $\lambda_3 < \lambda_1$, we find that $s_{c3}^2 = M^2/\lambda_3$, and the Higgses develop vevs of order the unification scale rather than ϕ, $\bar{\phi}$.

At first sight this rules out this latter possibility and in Ref. [7] we did not explore it. However, we saw in Sec. V that the condition for the correct (small Higgs vev) electroweak vacuum to have the lowest energy density (39) is slightly less restrictive than the condition for inflation to exit to the $\phi - \bar{\phi}$ direction, and that there is a range of parameters

$$\lambda_1\left(\frac{q_{H_1}}{q_\Phi}\right)^2 \lesssim \lambda_3 < \lambda_1 \tag{58}$$

for which the Universe exits to the false high Higgs vev h vacuum. It then should evolve to the true ground state: in this section we will see that this evolution leads to a very interesting cosmological history, with some distinctive features.

B. Reheating

If inflation exits to the h vacuum the symmetry breaking is

$${\rm SU}(2) \times {\rm U}(1)_Y \times {\rm U}(1)' \to {\rm U}(1)_{\rm em} \times {\rm U}(1)'', \tag{59}$$

where the U(1)$''$ is generated by the linear combination of hypercharge and U(1)$'$ generators which leaves the Higgses invariant:

$$Y'' = Y' - (q_L + q_E)Y. \tag{60}$$

Topologically, the symmetry breaking is the same as in the Standard Model, and hence cosmic strings are not formed at this transition.

Reheating after hybrid inflation [46] is expected in our model to be very rapid, as the nonperturbative field interactions of the scalars with fermions [47] and with gauge fields [48] are very efficient at transferring energy out of the zero-momentum modes of the fields s, h_1 and h_2. Higgs modes decay rapidly into b quarks, leading to the universe regaining a relativistic equation of state in much less than a Hubble time. Hence the Universe thermalizes at a temperature $T_{\rm rh} \simeq M$.

One notices that before thermal effects and soft terms are taken into account, the minimum of the scalar potential is determined by the requirement that both the F- and D-terms vanish. The vanishing of the D-terms ensures that $|\phi| = |\bar{\phi}|$, $|h_1| = |h_2|$ and $h_1^\dagger h_2 = 0$, while the vanishing of the F-term is assured by $\lambda_1 \phi\bar{\phi} - \lambda_3 h_1 h_2 = M^2$. The minimum can therefore be parametrized by an SU(2) gauge transformation and angles χ, φ defined by

$$\begin{aligned} \langle h_1\rangle &\simeq -i\sigma_2\langle h_2\rangle^* \simeq \left(\frac{M}{\sqrt{\lambda 3}}\cos\chi, 0\right), \\ \langle\phi\rangle &\simeq \langle\bar{\phi}^*\rangle \simeq \frac{M}{\sqrt{\lambda_1}}\sin\chi e^{i\varphi}. \end{aligned} \tag{61}$$

The φ angle can always be removed by a U(1)$''$ gauge transformation, so the physical flat direction just maps out the interval $0 \leq \chi \leq \pi/2$. At the special point $\chi = 0$ the U(1)$''$ symmetry is restored, and at $\chi = \pi/2$ the SU(2) $\otimes$ U(1)$_Y$ is restored. Away from these special points only U(1)$_{\rm em}$ is unbroken.

With this parametrization, it is straightforward to show that the leading $O(M^2 m_{\frac{3}{2}}^2)$ terms in the effective potential for χ are, after solving for s,

$$\begin{aligned} V(\chi) \simeq &-\frac{M^2}{2}\frac{(\tilde{h}_{\lambda_1}\sin^2\chi + \tilde{h}_{\lambda_3}\cos^2\chi)^2}{\lambda_1\sin^2\chi + \lambda_3\cos^2\chi} \\ &+ M^2\left(\frac{\bar{m}_\phi^2}{\lambda_1}\sin^2\chi + \frac{\bar{m}_h^2}{\lambda_3}\cos^2\chi\right), \end{aligned} \tag{62}$$

where we have defined $\tilde{h}_{\lambda_1} = \frac{h_{\lambda_1}}{\lambda_1}$, $\tilde{h}_{\lambda_3} = \frac{h_{\lambda_3}}{\lambda_3}$, $\bar{m}_\phi^2 = m_\phi^2 + m_{\bar{\phi}}^2$ and $\bar{m}_h^2 = m_{h_1}^2 + m_{h_2}^2$. A little more algebra demonstrates that

$$V''(0) \simeq \frac{2M^2}{\lambda_3}\left[-\frac{\tilde{h}_{\lambda_3}^2}{2}\left(2\frac{\tilde{h}_{\lambda_1}}{\tilde{h}_{\lambda_3}} - \frac{\lambda_1}{\lambda_3} - 1\right) + \bar{m}_\phi^2\frac{\lambda_3}{\lambda_1} - \bar{m}_h^2\right], \tag{63}$$

while the expansion around the true vacuum (the ϕ vacuum) at $\chi = \pi/2$ is easily obtained by the replacements $1 \leftrightarrow 3$ and $\bar{m}_\phi^2 \leftrightarrow \bar{m}_h^2$.

In sAMSB we have, under our assumption that the U(1)$'$ couplings dominate the β functions,

$$\tilde{h}_{\lambda_1} \simeq \left(\frac{m_{\frac{3}{2}}g'^2}{16\pi^2}\right)4q_\phi^2, \qquad \tilde{h}_{\lambda_3} \simeq \left(\frac{m_{\frac{3}{2}}g'^2}{16\pi^2}\right)4q_{H_1}^2, \tag{64}$$

and

$$\bar{m}_h^2 \simeq -\left(\frac{m_{\frac{3}{2}}g'^2}{16\pi^2}\right)^2 4Qq_{H_1}^2, \qquad \bar{m}_\phi^2 \simeq -\left(\frac{m_{\frac{3}{2}}g'^2}{16\pi^2}\right)^2 4Qq_\phi^2. \tag{65}$$

As pointed out in Sec. V, Q is in general much larger than both q_ϕ^2 and $q_{H_1}^2$, so we see that the h vacuum is unstable only if

$$\bar{m}_\phi^2\frac{\lambda_3}{\lambda_1} - \bar{m}_h^2 \lesssim 0, \tag{66}$$

or

$$\frac{\lambda_3}{\lambda_1} \gtrsim \frac{q_{H_1}^2}{q_\phi^2}. \tag{67}$$

This coincides with the condition (39) that the h vacuum has higher energy than the ϕ vacuum, and that the ϕ vacuum is stable.

Note that we can define a canonically normalized U(1)$''$-charged complex scalar modulus field X, related to χ and φ in the neighborhood of the h-vacuum by

$$X \simeq \sqrt{\frac{2M^2}{\lambda_1}}\chi e^{i\varphi}, \tag{68}$$

and whose mass m_X is given by

$$m_X^2 \simeq \bar{m}_\phi^2\frac{\lambda_3}{\lambda_1} - \bar{m}_h^2. \tag{69}$$

C. High temperature ground state

As we outlined in the previous section, reheating is expected to take place in much less than a Hubble time $H \sim M^2/m_{\rm P}$, while the relaxation rate to the true ground state, the ϕ vacuum, is from Eq. (69) m_X. Given that we expect $m_X \sim 1$ TeV and $M \sim 10^{14}$ GeV, reheating happens much faster than the relaxation, and the Universe is trapped in the U(1)$''$-symmetric vacuum with the large Higgs vev.

The high temperature effective potential, or free energy density, can be written

$$f(X, T) = -\frac{\pi^2}{90}g_{\rm eff}(X, T)T^4, \tag{70}$$

where $g_{\rm eff}(X, T)$ is the effective number of relativistic degrees of freedom at temperature T. At weak coupling, $g_{\rm eff}(X, T)$ can be calculated in the high-temperature expansion for all particles of mass $m_i \ll T$ [49],

$$g_{\rm eff}(X, T) \simeq g_{\rm eff}^0 - \frac{90}{\pi^2}\sum_i c_{1,i}\frac{m_i^2(X)}{T^2}, \tag{71}$$

where $g_{\rm eff}^0$ is the effective number of degrees of freedom at $X = 0$, and $c_1 = \frac{1}{24}, \frac{1}{48}$ for bosons and fermions respectively. For particles with $m > T$, $g_{\rm eff}$ is exponentially suppressed.

We can see that $X = 0$ is a local minimum for temperatures $m_X \lesssim T \lesssim M$, because away from that point the $U(1)''$ gauge boson develops a mass $q_\phi g' |X|$, and so $g_{\rm eff}$ decreases. For similar reasons the ϕ vacuum at $X_\phi \sim \sqrt{M^2/\lambda_1}$ is also a local minimum: away from that point the MSSM particles develop masses and again reduce $g_{\rm eff}$.

In fact, by counting relativistic degrees of freedom at temperatures $m_{\frac{3}{2}} \ll T \lesssim M$ one finds that X_ϕ is the global minimum. In the h vacuum the relativistic species are the Φ, $\bar\Phi$ chiral multiplets and the $\mathrm{U}(1)''$ gauge multiplet. In the ϕ vacuum, the particles of the MSSM are all light relative to T. Hence

$$f(0,T) \simeq -\frac{15}{2}\frac{\pi^2}{90}T^4, \tag{72}$$

$$f(X_\phi,T) \simeq -\frac{915}{4}\frac{\pi^2}{90}T^4. \tag{73}$$

The minima of the free energy density are separated by a free energy barrier of height $\sim T^4$. The transition rate can be calculated in the standard way [50] by calculating the free energy of the critical bubble E_c, and it is not hard to show that the transition rate is suppressed by a factor $\exp(-X_\phi/T)$. Hence we expect that the Universe is trapped in the h vacuum at temperatures $T \gtrsim m_X/q_\phi g'$.

D. Gravitinos and dark matter

Gravitinos are an inevitable consequence of supersymmetry and general relativity, and there are strict constraints on their mass in the cosmological models with a standard thermal history and an R symmetry guaranteeing the existence of a lightest supersymmetry particle (LSP) [51]. Even when unstable, they cause trouble either by decaying after nucleosynthesis and photodissociating light elements, or by decaying into the LSP. The result is a constraint on the reheat temperature $T_{\rm rh}$ in order to suppress the production of gravitinos. The relic abundance of thermally produced gravitinos is approximately

$$Y_{\frac{3}{2}} \simeq 2.4\times 10^{-12}\omega_{\tilde G}\left(\frac{T_{\rm rh}}{10^{10}\ \mathrm{GeV}}\right), \tag{74}$$

where gravitinos are taken much more massive than the other superparticles, and $\omega_{\tilde G}$ is an O(1) factor taking into account the theoretical uncertainty [15,52,53]. The LSP density parameter arising from a particular relic abundance in the MSSM is

$$\Omega_{\rm LSP}h^2 \simeq 2.8\times 10^{10}\frac{m_{\rm LSP}}{100\ \mathrm{GeV}}Y_{\frac{3}{2}}. \tag{75}$$

The LSP density parameter from thermally produced gravitinos is therefore

$$\Omega_{\rm LSP}h^2 \simeq 6\times 10^{-2}\omega_{\tilde G}\frac{m_{\rm LSP}}{100\ \mathrm{GeV}}\left(\frac{T_{\rm rh}}{10^{10}\ \mathrm{GeV}}\right). \tag{76}$$

In our model, we will see that the gravitinos generated by the first stage of reheating, or by nonthermal production from decaying long-lived scalars [54], are diluted by a period of thermal inflation. The constraint therefore applies to reheating after thermal inflation.

E. Thermal inflation in the *h* vacuum

In this section we continue with the assumption that the Universe exits inflation into the h vacuum. As the temperature falls, eventually soft terms in the potential become comparable to thermal energy density, and the Universe can seek its true ground state, which we established in Sec. V was $\chi = \pi/2$, the ϕ vacuum. This leads to a second period of inflation, akin to the complementary modular inflation model of Ref. [55]. Unlike this model, we will see that the reheating temperature is high enough to regenerate an interesting density of gravitinos, and also to allow baryogenesis by leptogenesis.

At zero temperature the difference in energy density between the h vacuum and the ϕ vacuum is [see Eqs. (35) and (36)]

$$\Delta V^0_{\rm eff} \simeq s_c^2\left(1+2\frac{\lambda_3^2}{\lambda_1^2}\right)^{\frac{1}{2}}\left(\frac{m_{\frac{3}{2}}g'^2}{16\pi^2}\right)^2 4Qq_\phi^2\left(1-\frac{q_{H_1}^2}{q_\phi^2}\frac{\lambda_1}{\lambda_3}\right). \tag{77}$$

Defining an effective supersymmetry-breaking scale

$$m_{\rm sb} = \left(\frac{m_{\frac{3}{2}}g'^2}{16\pi^2}\right)q_\phi\sqrt{Q}, \tag{78}$$

we see that a period of thermal inflation [56] starts at

$$T_{\rm i} \simeq \left(\frac{30}{g_{\rm eff}\pi^2}s_c^2 m_{\rm sb}^2\right)^{\frac{1}{4}}. \tag{79}$$

Using the CMB normalization for N e-foldings of standard hybrid inflation, $(s_c/m_{\rm P}) \simeq 3\times 10^{-3}$ (dropping the unimportant dependence on N), and the MSSM value for the degrees of freedom $g_{\rm eff} = 915/4$, we have

$$T_{\rm i} \simeq 1.0\times 10^9\left(\frac{m_{\rm sb}}{1\ \mathrm{TeV}}\right)^{\frac{1}{2}}\ \mathrm{GeV}. \tag{80}$$

Thermal inflation continues until the quadratic term in the thermal potential $q_\phi^2 g'^2 T^2|X|^2$ becomes the same size as the negative soft mass terms $m_X^2|X|^2$. Hence the transition which ends thermal inflation takes place at $T_{\rm e} \sim m_{\rm sb}$, and the number of e-foldings of thermal inflation is

$$N_\theta \simeq \frac{1}{2}\ln\left(\frac{s_c}{m_{\rm sb}}\right) \simeq 15, \tag{81}$$

taking $m_{\rm sb} \sim 1$ TeV. Thus any gravitinos will be diluted to unobservably low densities, as will any baryon number generated prior to thermal inflation.

There is another period of reheating as the energy of the modulus X is converted to particles. Around the true vacuum, the X is mostly Higgs, and so its large amplitude oscillations will be quickly converted into the particles of the MSSM in much less than an expansion time, and the vacuum energy will be efficiently converted into thermal energy. With the assumption of complete conversion of vacuum energy into thermal energy, the reheat temperature following thermal inflation will be

$$T_{\rm rh3} = \left(\frac{30}{g_{\rm eff}\pi^2}\Delta V^0_{\rm eff}\right)^{\frac{1}{4}} \simeq T_{\rm i}. \tag{82}$$

This reheating regenerates the gravitinos, and we may again apply the gravitino constraint Eq. (76), finding

$$\Omega_{\rm LSP}h^2 \simeq 6\times 10^{-3}\omega_{\tilde{G}}\frac{m_{\rm LSP}}{100~{\rm GeV}}\left(\frac{m_{\rm sb}}{1~{\rm TeV}}\right)^{\frac{1}{2}}. \tag{83}$$

We can convert the relic density into a constraint on the gravitino mass, requiring that the LSP density is less than or equal to the observed dark matter abundance, $\Omega_{\rm dm}h^2 \simeq 0.1$, obtaining

$$m_{\frac{3}{2}} \lesssim \frac{5\times 10^4}{g'^2 q_\phi\sqrt{Q}}\left(\omega_{\tilde{G}}\frac{m_{\rm LSP}}{100~{\rm GeV}}\right)^{-2}~{\rm TeV}. \tag{84}$$

Hence this class of models requires a high gravitino mass in order to saturate the bound and generate the dark matter.

We can be a bit more precise if we use use the phenomenological relations derived in Sec. VI. First, in order to fit μ_h we have from Eq. (41)

$$q_\phi^2 g'^2 \simeq 1.1\frac{\lambda_1}{\lambda_3}, \tag{85}$$

while we can derive a phenomenological formula for the LSP mass from Table II(a)

$$m_{\rm LSP} \simeq 3.3\times 10^{-3} m_{\frac{3}{2}}. \tag{86}$$

Hence

$$m_{\frac{3}{2}} \lesssim 350\left(\frac{1}{\omega^2_{\tilde{G}}}\frac{q_\phi}{\sqrt{Q}}\frac{\lambda_3}{\lambda_1}\right)^{\frac{1}{3}}~{\rm TeV}, \tag{87}$$

with the inequality saturated if the gravitino decays supply all the dark matter.

In the case where the dark matter consists of LSPs derived from gravitino decay, we can derive a range of acceptable values for the gravitino mass, as we have a constraint (58) on $\frac{\lambda_3}{\lambda_1}$ from requiring the exit to a false h vacuum. Hence, in order for gravitino-derived LSPs in this model to comprise all the dark matter, we have

$$\left(\frac{1}{\omega^2_{\tilde{G}}}\frac{q_\phi}{\sqrt{Q}}\frac{q^2_{H_1}}{q^2_\phi}\right)^{\frac{1}{3}} \lesssim \frac{m_{\frac{3}{2}}}{350~{\rm TeV}} \lesssim \left(\frac{1}{\omega^2_{\tilde{G}}}\frac{q_\phi}{\sqrt{Q}}\right)^{\frac{1}{3}}. \tag{88}$$

For example, taking $q_L = 0$ as in Sec. VI, we find that $m_{\frac{3}{2}}$ is independent of q_E and in the range

$$150\omega_{\tilde{G}}^{-\frac{2}{3}}~{\rm TeV} \lesssim m_{\frac{3}{2}} \lesssim 240\omega_{\tilde{G}}^{-\frac{2}{3}}~{\rm TeV}. \tag{89}$$

Interestingly, a Higgs with mass near 125 GeV also demands a high gravitino mass, consistent with this range and the uncertainties in $\omega_{\tilde{G}}$. In order to fit the central value of δa_μ we require a gravitino mass of 60 TeV, which would require $\omega_{\tilde{G}} \simeq 4$, or another source of dark matter.

F. Cosmic string formation and constraints

The breaking of the U(1)$''$ gauge symmetry at the end of thermal inflation results in the formation of cosmic strings [57–59]. The string tension in models with flat directions is much less than the naive calculation, as the potential energy density in core the string is of order $\Delta V \sim s_c^2 m_{\rm sb}^2$ rather than M^4. The vacuum expectation of the modulus field defined in Sec. VII B is still $X_0 \sim s_c$, so as a rough approximation we can therefore take the potential as

$$V_{\rm string} \sim \frac{m^2_{\rm sb}}{s_c^2}(X^2 - X_0^2)^2, \tag{90}$$

showing that there is an effective scalar coupling of order $(m^2_{\rm sb}/s_c^2)$. The string tension is approximately

$$\mu \simeq 2\pi B\left(\frac{m^2_{\rm sb}}{q_\phi^2 g'^2 s_c^2}\right)\frac{2M^2}{\lambda_1}, \tag{91}$$

where B is a slowly varying function of its argument, with [60]

$$B(\beta) \simeq 2.4/\ln(2/\beta), \qquad (\beta < 10^{-2}). \tag{92}$$

Hence, for $q_\phi^2 g'^2 = 2$, $s_c = 3\times 10^{-3} m_{\rm P}$, and $m_{\rm sb} = 1$ TeV as above,

$$B \simeq 0.04, \tag{93}$$

demonstrating that the string tension is more than an order of magnitude below its naive value $4\pi s_c^2$, which reduces the CMB constraint on this model. Hence the string tension in this model is

$$G\mu = \frac{B}{4}\frac{s_c^2}{m_{\rm P}^2} \sim 10^{-7}, \tag{94}$$

well below the 95% confidence limit for CMB fluctuations from strings [61,62].

There are also other bounds on strings depending on uncertain details about their primary decay channel. Pulsar timing provides a strong bound if the long strings lose a significant proportion of energy into loops with sizes above a light year or so (smaller loops radiate at frequencies to which pulsar timing is not very sensitive). In this case recent European Pulsar Timing Array data [63] can be used to place a conservative upper bound of $G\mu < 5.3\times 10^{-7}$ [64] for strings with a reconnection probability of close to unity (as is the case in field theory), and loops formed with a typical size of about 10^{-5} of the horizon size. Future experiments will place tighter (but still model-dependent) bounds [64,65]. For example, the Large European Array for Pulsars will be two orders of magnitude more sensitive than European Pulsar Timing Array [66] and will be able to detect the gravitational radiation from the loops in this model if they are large enough to radiate into the Large European Array for Pulsars sensitivity window. Current string modeling [67] indicates this is likely if loop production is significant.

Strings may also produce high energy particles, whose decays can produce cosmic rays over a very wide spectrum of energies. If $f_{\rm cr}$ is the fraction of the energy density going into cosmic rays, then the diffuse γ-ray background provides a limit [58] $G\mu \lesssim 10^{-10} f_{\rm cr}^{-1}$. Given that the strings in our model contain a large Higgs condensate, we would expect that all particles produced by the strings would end up as Standard Model particles or neutralinos. Thus we require that the decays are primarily gravitational in order to avoid the cosmic ray bound.

G. Baryogenesis

Baryon asymmetry requires baryon number (B) violation, C violation, and CP violation [68]. In common with the standard model, our model has C violation and sphaleron-induced B violation. It can also support CP-violating phases in the neutrino Yukawa couplings. In Ref. [7], it was pointed out that leptogenesis [69] was natural in the model, provided that the reheat temperature is greater than about 10^9 GeV.

As we saw in Sec. VII E, this is the approximate value of the reheat temperature after thermal inflation, and so we require at least one right-handed neutrino which is sufficiently light to be generated in the reheating process, i.e., with a mass less than around 10^9 GeV. The baryogenesis in our model should therefore be similar to that of Ref. [70].

In sAMSB, the light scalars are weakly coupled to the Higgs (the stops are both at the TeV scale), and so the electroweak phase transition is a crossover [71]. This means that there is no conventional electroweak baryogenesis (see e.g., Ref. [72] for a recent review).

VIII. CONCLUSIONS

The sAMSB model, as described here, is in our opinion the most attractive way of resolving the tachyonic slepton problem of anomaly mediated supersymmetry breaking. The low energy spectrum is similar to that of regions of constrained minimal supersymmetric standard model or MSUGRA parameter space, but with characteristic features, most notably a wino LSP. We have seen that, while it is possible to obtain a light SM-like Higgs with a mass of 125 GeV, this requires fine-tuning and also results in a suppression of the supersymmetric contribution to a_μ, so that the current prediction for a_μ in our model is about 3σ below the experimental value [73].

Moreover, to produce a Higgs of over 120 GeV, we must increase the gravitino mass to over 80 TeV. If the gravitino mass is over 100 TeV we can use wino LSPs derived from gravitino decays to account for all dark matter.

Assuming that the $U(1)'$ introduced to solve the tachyonic slepton problem is broken at a high scale, M, we have seen that sAMSB naturally realizes F-term hybrid inflation. The Universe may exit the inflationary era into a vacuum dominated by large vevs for the MSSM Higgs fields, $h_{1,2}$, with the true vacuum with unbroken $SU(3) \otimes SU(2) \otimes U(1)_Y$ (above the electroweak scale) attained only after a later period of approximately 15 e-foldings of thermal inflation.

The thermal inflation reduces the number of e-foldings of high-scale inflation to about 40, and hence the spectral index of scalar CMB fluctuations is reduced to about 0.975, within about 1σ of the WMAP7 value. Cosmic strings are formed at the end of thermal inflation, with a low mass per unit length, satisfying observational bounds provided their main decay channel is gravitational, and the typical size of string loops at formation is about 10^{-5} of the horizon size, or so small that they radiate at a frequency below 1 yr^{-1}, to which pulsar timing is not sensitive. The Large European Array for Pulsars will be two orders of magnitude more sensitive, and be capable of closing the window in the loop size at 10^{-5} of the horizon, or detecting the gravitational radiation.

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Note added.—Planck has recently reported $n_s = 0.9624 \pm 0.0075$ (at pivot scale $k_0 = 0.002$ Mpc^{-1}) [75] while the bounds on the string tension remain above $G\mu \simeq 10^{-7}$ [76]. Hence the model remains consistent with the string data, and suffers a mild increase in the tension with the scalar spectral index to 2σ.

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