

**Shift in weak phase  $\gamma$  due to  $CP$  asymmetries in  $D$  decays to two pseudoscalar mesons**Bhubanjyoti Bhattacharya,<sup>1</sup> Michael Gronau,<sup>2</sup> David London,<sup>1</sup> and Jonathan L. Rosner<sup>3</sup><sup>1</sup>*Physique des Particules, Université de Montréal, C.P. 6128, succursale Centre-ville, Montréal, Québec, Canada H3C 3J7*<sup>2</sup>*Physics Department, Technion-Israel Institute of Technology, Haifa 3200, Israel*<sup>3</sup>*Enrico Fermi Institute and Department of Physics, University of Chicago, 5620 S. Ellis Avenue, Chicago, Illinois 60637, USA*

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A difference of several tenths of a percent has been observed between the direct  $CP$  asymmetries of  $D^0 \rightarrow K^+K^-$  and  $D^0 \rightarrow \pi^+\pi^-$ . It has been noted recently that  $CP$  asymmetries in such singly Cabibbo-suppressed decays can affect the determination of the weak phase  $\gamma$  using the Gronau-London-Wyler method of comparing rates for  $B^+ \rightarrow DK^+$  and  $B^- \rightarrow DK^-$ , where  $D$  is a superposition of  $D^0$  and  $\bar{D}^0$  decaying to a  $CP$  eigenstate. Using an analysis of the  $CP$  asymmetries in singly Cabibbo-suppressed decays based on a  $c \rightarrow u$  penguin amplitude with a standard model weak phase but enhanced by  $CP$ -conserving strong interactions, we estimate typical shifts in  $\gamma$  of several degrees and pinpoint measurements which would reduce uncertainties to an acceptable level.

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**I. INTRODUCTION**

The precise determination of phases of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is crucial to the understanding of  $CP$  violation. At present there appears to be reasonable agreement on magnitudes and phases of CKM matrix elements [1,2]. However, discrepancies among different determinations of these quantities can signal new physics, for example, due to new heavy particles entering into loop diagrams. Consequently, it is essential to pursue the widest variety of measurements of CKM elements.

One quantity which is determined indirectly with reasonable accuracy but whose direct measurement has lagged with respect to many others is the weak phase  $\gamma$ , related to CKM elements  $V_{ij}$  by  $\gamma = \text{Arg}(-V_{ub}^*V_{ud}/V_{cb}^*V_{cd})$ . A promising method for measuring  $\gamma$  directly, proposed by Gronau-London-Wyler (GLW) [3], compared rates for  $B^+ \rightarrow DK^+$  and  $B^- \rightarrow DK^-$ , where  $D$  is a superposition of  $D^0$  and  $\bar{D}^0$  decaying to a  $CP$  eigenstate. In the initial formulation of this method,  $CP$  violation in charm decays was assumed negligible, as suggested by standard model (SM) estimates [4].

Variants of the GLW method include  $B^\pm \rightarrow DK^{*\pm}$ ,  $D^*(\rightarrow D\pi^0, D\gamma)K^\pm$  and  $B^0 \rightarrow DK^{*0}$  where  $D$  decays to  $CP$  eigenstates, and processes of this kind in which  $D^0$  and  $\bar{D}^0$  decay to a common flavor state such as  $K^-\pi^+$  [5] or to a three-body self-conjugate final state such as  $K_S\pi^+\pi^-$  [6]. Results obtained in these processes have been reported by the BABAR [7,8], Belle [9,10], CDF [11], and LHCb [12–15] collaborations.

A value of several tenths of a percent has now been seen for  $\Delta A_{CP}$ , the difference between the  $CP$  asymmetries of  $D^0 \rightarrow K^+K^-$  and  $D^0 \rightarrow \pi^+\pi^-$  [16,17]. These two asymmetries have been included in a recent experimental study by LHCb of  $B^\pm \rightarrow DK^\pm$  [13], concluding that their effect on determining  $\gamma$  is marginal at the current level of

experimental precision. References [18,19] have shown that unless  $CP$  violation in  $D$  decays is taken into account, the determination of  $\gamma$  via the GLW method can be shifted from its true value by up to several degrees.

Three of us have previously assumed that  $CP$  violation in charm decays is due to a penguin amplitude with the SM phase but enhanced by  $CP$ -conserving strong-interaction effects [20–22]. The possibility of such an enhancement was pointed out some time ago, in analogy to the likely enhancement of penguin amplitudes in  $K \rightarrow 2\pi$  decays [23,24]. A large number of authors have suggested that asymmetries at the level observed in  $\Delta A_{CP}$  cannot be excluded within the CKM framework [25]. A consistent description of singly Cabibbo-suppressed (SCS)  $CP$ -violating charm decays was found in Refs. [20–22], and predictions were made for correlations between  $CP$  asymmetries in several decays of charmed mesons to pairs of light pseudoscalar mesons  $P$ .

In the present paper we apply this description to  $B^\pm \rightarrow DK^\pm$ ,  $D \rightarrow \pi^+\pi^-$ ,  $K^+K^-$ , studying within the CKM framework shifts in  $\gamma$  due to  $CP$  violation in  $D \rightarrow PP$  decays. We find shifts of up to a few degrees as noted in Refs. [18,19], where no specific scheme has been used for  $CP$  violation and no correlations existed between  $CP$  asymmetries in  $D$  decays. We identify the most crucial measurements for reducing the uncertainty in effects of these shifts to acceptable levels, i.e., below effects of other uncertainties.

We review the GLW method in Sec. II allowing for  $CP$  violation in charm decays, using observables involving minimal systematic uncertainties. Present information on direct  $CP$  asymmetries in SCS  $D$  decays to two pseudoscalars is summarized in Sec. III. The approach of Ref. [20] is then outlined in Sec. IV, and applied to obtain predictions for shifts in  $\gamma$  in Sec. V. We summarize in Sec. VI.

## II. GLW METHOD IN PRESENCE OF CHARM $CP$ VIOLATION

We shall be concerned with a single source of information on  $\gamma$ : the decays  $B^\pm \rightarrow DK^\pm$ , with  $D$  decaying to a  $CP$ -even eigenstate such as  $f_D = \pi^+\pi^-$  or  $K^+K^-$ . Our notation will follow that of Ref. [19]. We define

$$\begin{aligned} A(B^- \rightarrow D^0 K^-) &\equiv A_B \\ A(B^- \rightarrow \bar{D}^0 K^-) &\equiv A_B r_B e^{i(\delta_B - \gamma)} \\ A(D^0 \rightarrow f_D) &\equiv A_f, \end{aligned} \quad (1)$$

where we have taken a strong phase to be zero in  $B^- \rightarrow D^0 K^-$  with no loss of generality. By  $CP$  conjugation we then have

$$\begin{aligned} A(B^+ \rightarrow \bar{D}^0 K^+) &\equiv A_B \\ A(B^+ \rightarrow D^0 K^+) &\equiv A_B r_B e^{i(\delta_B + \gamma)} \\ A(\bar{D}^0 \rightarrow f_D) &\equiv \bar{A}_f. \end{aligned} \quad (2)$$

The parameters  $r_B$  and  $\delta_B$  are measurable by combining information from  $B^\pm \rightarrow DK^\pm$ , where neutral  $D$  mesons decay to  $CP$  eigenstates, flavor-specific states or  $K_S \pi^+ \pi^-$ . Current values taken from Ref. [1] are

$$r_B = 0.099 \pm 0.008, \quad \delta_B = (110 \pm 15)^\circ. \quad (3)$$

The magnitudes of the amplitudes  $|A_f|$  and  $|\bar{A}_f|$  are measurable through the  $CP$ -averaged branching ratio for  $D^0 \rightarrow f$  and  $\bar{D}^0 \rightarrow f$  (giving  $|A_f|^2 + |\bar{A}_f|^2$ ) and the direct  $CP$  asymmetry

$$A_{CP}^{\text{dir}}(f) = \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2}. \quad (4)$$

We define the weak ( $CP$ -violating) phase  $\alpha_f \equiv \text{Arg}(A_f/\bar{A}_f)$ . Its value is, as yet, unspecified. If  $\phi$  is taken as the phase of  $A_f$  we have

$$A_f = |A_f| e^{i\phi}; \quad \bar{A}_f = |\bar{A}_f| e^{i(\phi - \alpha_f)}. \quad (5)$$

Using the above definitions we may now construct the following amplitudes:

$$\begin{aligned} A(B^- \rightarrow f_D K^-) &= A_B A_f + \bar{A}_f A_B r_B e^{i(\delta_B - \gamma)} \\ &= A_B (|A_f| + |\bar{A}_f| r_B e^{i(\delta_B - \gamma - \alpha_f)}) e^{i\phi}, \end{aligned} \quad (6)$$

$$\begin{aligned} A(B^+ \rightarrow f_D K^+) &= A_B \bar{A}_f + A_f A_B r_B e^{i(\delta_B + \gamma)} \\ &= A_B (|\bar{A}_f| + |A_f| r_B e^{i(\delta_B + \gamma + \alpha_f)}) e^{i(\phi - \alpha_f)}. \end{aligned} \quad (7)$$

The squared magnitudes of Eqs. (6) and (7) give [19]

$$\begin{aligned} |A(B^- \rightarrow f_D K^-)|^2 &= |A_B|^2 (|A_f|^2 + r_B^2 |\bar{A}_f|^2 \\ &\quad + 2r_B |A_f| |\bar{A}_f| \cos(\delta_B - \gamma - \alpha_f)), \end{aligned} \quad (8)$$

$$\begin{aligned} |A(B^+ \rightarrow f_D K^+)|^2 &= |A_B|^2 (|\bar{A}_f|^2 + r_B^2 |A_f|^2 \\ &\quad + 2r_B |A_f| |\bar{A}_f| \cos(\delta_B + \gamma + \alpha_f)). \end{aligned} \quad (9)$$

Adding and subtracting the above equations we may form quantities that are relevant in constructing the GLW observables:

$$\begin{aligned} |A(B^- \rightarrow f_D K^-)|^2 + |A(B^+ \rightarrow f_D K^+)|^2 \\ = |A_B|^2 (|A_f|^2 + |\bar{A}_f|^2) \\ \times \left( 1 + r_B^2 + 2r_B \cos \delta_B \cos(\gamma + \alpha_f) \sqrt{1 - (A_{CP}^{\text{dir}}(f))^2} \right), \end{aligned} \quad (10)$$

$$\begin{aligned} |A(B^- \rightarrow f_D K^-)|^2 - |A(B^+ \rightarrow f_D K^+)|^2 \\ = |A_B|^2 (|A_f|^2 + |\bar{A}_f|^2) \left( A_{CP}^{\text{dir}}(1 - r_B^2) \right. \\ \left. + 2r_B \sin \delta_B \sin(\gamma + \alpha_f) \sqrt{1 - (A_{CP}^{\text{dir}}(f))^2} \right). \end{aligned} \quad (11)$$

The last expression differs from a similar one in Ref. [19] by a term  $(1 - r_B^2)(|A_f|^2 - |\bar{A}_f|^2)$ , which vanishes only in the absence of direct  $CP$  violation in charm and hence cannot be neglected.

We now take the expressions for  $A_f$  and  $\bar{A}_f$  to be [19]

$$\begin{aligned} A_f &= |A_f^0| (1 + r_f e^{i(\delta_f - \gamma)}), \\ \bar{A}_f &= |\bar{A}_f^0| (1 + r_f e^{i(\delta_f + \gamma)}), \end{aligned} \quad (12)$$

where  $A_f^0$  is the amplitude in the absence of a  $CP$ -violating term, and we have assumed that the source of  $CP$  violation has the SM phase  $-\gamma$  as in Ref. [20]. The direct  $CP$  asymmetry [Eq. (4)] is then given by

$$A_{CP}^{\text{dir}}(f) = \frac{2r_f \sin \delta_f \sin \gamma}{1 + r_f^2 + 2r_f \cos \delta_f \cos \gamma}. \quad (13)$$

The fact that  $A_{CP}^{\text{dir}}(f)$  is of order a few times  $10^{-3}$  in  $D^0 \rightarrow \pi^+ \pi^-$  and  $D^0 \rightarrow K^+ K^-$  (see Tables I and II in Sec. III) suggests that  $r_f$  in these processes is also of this order as argued in Refs. [20,25]. Our subsequent analysis (in particular the discussion in Secs. IV and V) applies also to larger values of  $r_f$ , for instance of order  $10^{-2}$ , with an order of magnitude further suppression provided by  $\sin \delta_f$ . We ignore the fine-tuned solution where  $r_f$  is large, but the strong phases in these decays are of order  $10^{-3}$ . One can show that

$$\begin{aligned} \alpha_f &= -\tan^{-1} \left( \frac{2r_f \cos \delta_f \sin \gamma + r_f^2 \sin 2\gamma}{1 + 2r_f \cos \delta_f \cos \gamma + r_f^2 \cos 2\gamma} \right) \\ &\approx -A_{CP}^{\text{dir}}(f) \cot \delta_f, \end{aligned} \quad (14)$$

where the last approximation holds to leading order in  $r_f$ .

It has been suggested in Ref. [26] that when applying the GLW method one normalizes the  $CP$ -averaged rate for  $B^\pm \rightarrow f_D K^\pm$  by that for  $B^\pm \rightarrow f_D \pi^\pm$ ,

$$R_{K/\pi}^f \equiv \frac{\Gamma(B^- \rightarrow f_D K^-) + \Gamma(B^+ \rightarrow f_D K^+)}{\Gamma(B^- \rightarrow f_D \pi^-) + \Gamma(B^+ \rightarrow f_D \pi^+)}, \quad (15)$$

and one takes the ratio of this fraction and a corresponding fraction for the  $D^0$  flavor state,

$$R(K/\pi) \equiv \frac{\Gamma(B^- \rightarrow D^0 K^-)}{\Gamma(B^- \rightarrow D^0 \pi^-)}. \quad (16)$$

Significant experimental systematic uncertainties cancel in these fractions [7,9,11,13]. Defining ratios of amplitudes and strong phases in  $B^- \rightarrow D\pi^-$  in analogy with  $B^- \rightarrow DK^-$ ,

$$\frac{A(B^- \rightarrow \bar{D}^0 \pi^-)}{A(B^- \rightarrow D^0 \pi^-)} \equiv r_{B(\pi)} e^{i(\delta_{B(\pi)} - \gamma)}, \quad (17)$$

one may express the double fraction

$$R_{CP+}^f = \frac{1 + r_B^2 + 2r_B \cos \delta_B \cos(\gamma + \alpha_f) \sqrt{1 - (A_{CP}^{\text{dir}}(f))^2}}{1 + r_{B(\pi)}^2 + 2r_{B(\pi)} \cos \delta_{B(\pi)} \cos(\gamma + \alpha_f) \sqrt{1 - (A_{CP}^{\text{dir}}(f))^2}}. \quad (20)$$

Neglecting corrections in  $R_{CP+}^f$  which are quadratic in  $A_{CP}^{\text{dir}}(f)$  [ $\mathcal{O}(10^{-5})$ ] and using Eq. (14), we note that corrections linear in  $A_{CP}^{\text{dir}}(f) \sim \text{few} \times 10^{-3}$  are multiplied by  $r_B$  and are therefore negligible relative to  $R_{CP+}^f = 1 + \mathcal{O}(r_B)$ . Thus, a comparison of Eqs. (19) and (20) shows that the determination of  $\gamma$  is not affected in a significant way by including  $A_{CP}^{\text{dir}}(f)$  in  $R_{CP+}^f$ .

The other measurable quantity used in the GLW method is the  $CP$  asymmetry  $A_{CP+}^f$ ,

$$A_{CP+}^f \equiv \frac{\Gamma(B^- \rightarrow f_D K^-) - \Gamma(B^+ \rightarrow f_D K^+)}{\Gamma(B^- \rightarrow f_D K^-) + \Gamma(B^+ \rightarrow f_D K^+)}. \quad (21)$$

In the absence of  $CP$  violation in  $D^0 \rightarrow f_D$ , this asymmetry may be defined as  $[\Gamma(B^- \rightarrow D_{CP+} K^-) - \Gamma(B^+ \rightarrow D_{CP+} K^+)]/[\Gamma(B^- \rightarrow D_{CP+} K^-) + \Gamma(B^+ \rightarrow D_{CP+} K^+)]$  which is independent of  $f_D$  and is given by [26]

$$A_{CP+}^f = \frac{2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma}. \quad (22)$$

When including  $CP$  nonconservation in  $D^0 \rightarrow f_D$  the asymmetry  $A_{CP+}^f$  becomes dependent on  $A_{CP}^{\text{dir}}(f)$ . Neglecting terms quadratic in  $A_{CP}^{\text{dir}}(f)$ , one finds

$$A_{CP+}^f = \frac{A_{CP}^{\text{dir}}(f)(1 - r_B^2) + 2r_B \sin \delta_B \sin(\gamma + \alpha_f)}{1 + r_B^2 + 2r_B \cos \delta_B \cos(\gamma + \alpha_f)}. \quad (23)$$

$$R_{CP+}^f \equiv \frac{R_{K/\pi}^f}{R(K/\pi)} \quad (18)$$

in terms of  $\gamma$  and these parameters.

In the absence of  $CP$  violation in  $D^0 \rightarrow f_D$ , one has [26]

$$R_{CP+}^f = \frac{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma}{1 + r_{B(\pi)}^2 + 2r_{B(\pi)} \cos \delta_{B(\pi)} \cos \gamma}, \quad (19)$$

where the parameter  $r_{B(\pi)}$  is expected to be very small,  $r_{B(\pi)} \sim r_B \tan^2 \theta_C \sim 0.005$ . [See Eq. (3).] We note that in the approximation of neglecting  $CP$  violation in  $D^0 \rightarrow f_D$  the ratio  $R_{K/\pi}^f$  may be defined as  $R_{K/\pi}^f \equiv [\Gamma(B^- \rightarrow D_{CP+} K^-) + \Gamma(B^+ \rightarrow D_{CP+} K^+)]/[\Gamma(B^- \rightarrow D_{CP+} \pi^-) + \Gamma(B^+ \rightarrow D_{CP+} \pi^+)]$ . Consequently this ratio and the double ratio  $R_{CP+}^f$  do not depend on  $f_D$ .

Including  $CP$  violation in  $D^0 \rightarrow f_D$  one finds an expression for  $R_{CP+}^f$  which depends on  $f_D$  through the  $CP$  asymmetry  $A_{CP}^{\text{dir}}(f)$ ,

The two terms in the numerator and the last term in the denominator involve corrections linear in  $A_{CP}^{\text{dir}}(f)$  modifying the expression (22) for the case of no direct asymmetry.

We note that an expression independent of  $A_{CP}^{\text{dir}}(f)$  similar to (22), but with an opposite overall sign and an  $\mathcal{O}(r_B^2)$  correction with opposite sign, describes the asymmetry for Cabibbo-favored  $D^0$  decays to  $CP$ -odd eigenstates (such as  $K_S \phi$  and  $K_S \pi^0$ ) where  $CP$  violation is negligible in the CKM framework,

$$A_{CP-}^f = -\frac{2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 - 2r_B \cos \delta_B \cos \gamma}. \quad (24)$$

A precise measurement of this asymmetry could avoid uncertainties from  $A_{CP}^{\text{dir}}(f)$ .

Writing

$$A_{CP+}^f = \frac{2r_B \sin \delta_B \sin \gamma_{\text{eff}}}{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma_{\text{eff}}}, \quad (25)$$

we assume a measurement of  $A_{CP+}^f$  from which  $\gamma_{\text{eff}}$  is determined. Our purpose is then to study the shift  $\delta\gamma \equiv \gamma - \gamma_{\text{eff}}$  as a function of  $A_{CP}^{\text{dir}}(f)$ . Defining  $\gamma + \alpha_f = \gamma_{\text{eff}} + \delta\gamma + \alpha_f \equiv \gamma_{\text{eff}} + x$ , we expand the numerator and denominator in (23) to first order in  $x$ . Comparing this expression with (25), cross-multiplying the two ratios, canceling the leading terms and keeping terms linear in  $x$  and  $A_{CP}^{\text{dir}}(f)$ , one finds

$$\begin{aligned}
\delta\gamma &= -\alpha_f - A_{CP}^{\text{dir}}(f) \left[ \frac{1 - r_B^2}{2r_B \sin \delta_B} \frac{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma_{\text{eff}}}{(1 + r_B^2) \cos \gamma_{\text{eff}} + 2r_B \cos \delta_B} \right] \\
&\approx A_{CP}^{\text{dir}}(f) \left[ \cot \delta_f - \frac{1 - r_B^2}{2r_B \sin \delta_B} \frac{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma_{\text{eff}}}{(1 + r_B^2) \cos \gamma_{\text{eff}} + 2r_B \cos \delta_B} \right] \\
&\approx 2r_f \cos \delta_f \sin \gamma_{\text{eff}} - A_{CP}^{\text{dir}}(f) \left[ \frac{1 - r_B^2}{2r_B \sin \delta_B} \frac{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma_{\text{eff}}}{(1 + r_B^2) \cos \gamma_{\text{eff}} + 2r_B \cos \delta_B} \right]. \quad (26)
\end{aligned}$$

While both corrections are exhibited as proportional to  $A_{CP}^{\text{dir}}(f)$ , the first term appears to diverge as  $\delta_f \rightarrow 0$ . Recalling that this correction is actually  $\approx 2r_f \cos \delta_f \sin \gamma_{\text{eff}}$  [Eq. (13)], which is finite at  $\delta_f = 0$ , we will use the last expression in Sec. V. The second term in (26) illustrates an enhancement by  $1/2r_B$  of the shift in  $\gamma$  due to  $A_{CP}^{\text{dir}}(f)$  [18,19].

We mention in passing two approximations which have been applied to the  $CP$  asymmetry in (23). Neglecting  $\alpha_f$  [which is of the same order in  $r_f$  as  $A_{CP}^{\text{dir}}(f)$ , but leads to a correction linear in  $r_B$ ] and approximating the overall coefficient of  $A_{CP}^{\text{dir}}(f)$  by one, Eq. (23) reduces to an expression employed in Ref. [13],

$$A_{CP+}^f \approx \frac{2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma} + A_{CP}^{\text{dir}}(f). \quad (27)$$

A further approximation, keeping only  $\mathcal{O}(r_B)$  in the first term, leads to [19]

$$A_{CP+}^f \approx 2r_B \sin \delta_B \sin \gamma + A_{CP}^{\text{dir}}(f). \quad (28)$$

A  $CP$  asymmetry in  $B^\pm \rightarrow f_D K^\pm$  has been measured in Refs. [7,11,12] consistent with an estimate  $A_{CP+}^f(B^\pm \rightarrow f_D K^\pm) \approx 2r_B \sin \delta_B \sin \gamma \sim 0.15\text{--}0.20$ . The current error in the world-averaged value [27],  $A_{CP+} = 0.19 \pm 0.03$ , is still too large for sensitivity to  $A_{CP}^{\text{dir}}(f)$ . The corresponding error in the world-averaged measurement,  $A_{CP-} = -0.11 \pm 0.05$ , favoring an opposite sign as anticipated, is somewhat larger.

A much smaller asymmetry is expected in  $B^\pm \rightarrow D\pi^\pm$  where we estimated  $r_{B(\pi)} \sim 0.005$ . This asymmetry is given by an expression similar to (23),

$$\begin{aligned}
A_{CP+}^f(B \rightarrow f_D \pi) &= \frac{A_{CP}^{\text{dir}}(f)(1 - r_{B(\pi)}^2) + 2r_{B(\pi)} \sin \delta_{B(\pi)} \sin(\gamma + \alpha_f)}{1 + r_{B(\pi)}^2 + 2r_{B(\pi)} \cos \delta_{B(\pi)} \cos(\gamma + \alpha_f)} \\
&\approx 2r_{B(\pi)} \sin \delta_{B(\pi)} \sin \gamma + A_{CP}^{\text{dir}}(f). \quad (29)
\end{aligned}$$

The two contributions, which in principle may be disentangled by measuring also  $A_{CP-}(B \rightarrow f_D \pi) \approx -2r_{B(\pi)} \sin \delta_{B(\pi)} \sin \gamma$ , are of comparable magnitudes, each less than a percent. Thus, while the  $CP$ -averaged rate for  $B^\pm \rightarrow f_D \pi^\pm$  is suitable for normalization [see Eq. (15)], a useful measurement of the corresponding

asymmetry does not seem feasible in the foreseeable future.

### III. PRESENT INFORMATION ON $A_{CP}^{\text{dir}}$

Dedicated measurements of the difference  $\Delta A_{CP} \equiv A_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^-) - A_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+ \pi^-)$  (in which many systematic errors cancel) have been performed by the LHCb [16] and CDF [17] collaborations, while Belle has combined independent measurements of the two asymmetries [28] to obtain a value of  $\Delta A_{CP}$ . The results are shown in Table I. We shall assume that measured asymmetries are equal to direct ones, neglecting possible contributions from indirect (mixing-induced) asymmetries which would lead to slightly different averages [1,27].

The average in Table I will be used in the next section to constrain the magnitude of a SM penguin amplitude as a function of its strong phase. (Slightly different averages were used in Refs. [18,19].) In addition weak constraints on this strong phase will be seen to result from measured  $CP$  asymmetries in individual final states, quoted in Table II.

TABLE I. Experimental results on  $\Delta A_{CP} \equiv A_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^-) - A_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+ \pi^-)$ .

Reference	Value (%)
LHCb [16]	$-0.82 \pm 0.21 \pm 0.11$
CDF [17]	$-0.62 \pm 0.21 \pm 0.10$
Belle [28]	$-0.87 \pm 0.41 \pm 0.06$
Average	$-0.74 \pm 0.15$

TABLE II. Experimental results on some direct  $CP$  asymmetries in  $D$  decays.

Decay	Reference	Value (%)
$D^0 \rightarrow \pi^+ \pi^-$	CDF [29]	$0.22 \pm 0.24 \pm 0.11$
	Belle [28]	$0.55 \pm 0.36 \pm 0.09$
	Average	$0.33 \pm 0.22$
$D^0 \rightarrow K^+ K^-$	CDF [29]	$-0.24 \pm 0.22 \pm 0.09$
	Belle [28]	$-0.32 \pm 0.21 \pm 0.09$
	Average	$-0.28 \pm 0.16$
$D^+ \rightarrow K^+ \bar{K}^{0a}$	BABAR [30]	$0.46 \pm 0.36 \pm 0.25$
	Belle [28,31]	$0.08 \pm 0.28 \pm 0.14$
	Average	$0.21 \pm 0.25$

<sup>a</sup>After subtraction of  $CP$  asymmetry due to  $K^0\text{--}\bar{K}^0$  mixing.

#### IV. CHARM $CP$ VIOLATION WITH ENHANCED SM PENGUIN

In Ref. [20] three of us have calculated  $CP$  asymmetries for several  $D \rightarrow PP$  decays, where  $P = \pi, K$ , assuming that the nonzero value of  $A_{CP}^{\text{dir}}$  is due to a SM penguin amplitude with the weak phase of the standard model  $c \rightarrow b \rightarrow u$  loop diagram, but with a  $CP$ -conserving enhancement as if due to the strong interactions. In this case the magnitude and strong phase of this amplitude  $P_b$  are correlated in order to fit the observed  $CP$  asymmetry, allowing the prediction of  $CP$  asymmetries for other singly Cabibbo-suppressed modes. We refer the reader to that work for details, but outline the method briefly here.

For the decay of a charmed meson  $D$  to any final state  $f$  we are defining a direct  $CP$  asymmetry using the same convention as in (4)

$$A_{CP}^{\text{dir}}(f) \equiv \frac{\Gamma(D \rightarrow f) - \Gamma(\bar{D} \rightarrow \bar{f})}{\Gamma(D \rightarrow f) + \Gamma(\bar{D} \rightarrow \bar{f})}. \quad (30)$$

We take the  $CP$ -conserving amplitudes from a flavor-SU(3) description of charm decays presented previously [32,33]. A new ingredient [20] with respect to that work is a U-spin breaking  $c \rightarrow u$  penguin amplitude whose  $V_{cd(s)}^* V_{ud(s)}$  term reproduces satisfactorily decay rates for singly Cabibbo-suppressed (SCS) processes, while its  $V_{cb}^* V_{ub}$  term accounts for  $\Delta A_{CP}$ . This scheme allows the prediction of other  $CP$  asymmetries in SCS charmed meson decays.

Denoting by  $\delta_f$  a phase defined in Ref. [20] as  $\phi^f$ , we write the amplitude for a decay  $D \rightarrow f$  in a manner similar to (12)

$$\mathcal{A}(D \rightarrow f) \equiv A_f = |T_f| e^{i\phi_T^f} (1 + r_f e^{i(\delta_f - \gamma)}). \quad (31)$$

Here  $T_f$  represents terms with the weak phase of the tree-level terms contributing to that amplitude,  $\phi_T^f$  is its strong phase,  $r_f$  is the ratio of the magnitude of the  $CP$ -violating penguin contribution to that of  $T_f$ ,  $-\gamma$  is the weak phase of the  $CP$ -violating penguin, and  $\delta_f$  is the strong phase of the  $CP$ -violating penguin relative to  $T_f$ . Here one has

$$\delta_f = \text{Arg}(P_b) - \phi_T^f + \gamma, \quad P_b = p e^{i(\delta - \gamma)}, \quad (32)$$

leading to the relation  $\delta_f = \delta - \phi_T^f$ . The magnitudes  $T_f$  and phases  $\phi_T^f$  for some  $D \rightarrow PP$  processes [20] are summarized in Table III. For  $D^0 \rightarrow \pi^0 \pi^0$ , the amplitude must be multiplied by an additional factor of  $-1/\sqrt{2}$  [20].

The  $CP$  asymmetries are then

$$\begin{aligned} A_{CP}^{\text{dir}}(f) &= \frac{2r_f \sin \gamma \sin \delta_f}{1 + r_f^2 + 2r_f \cos \gamma \cos \delta_f} \\ &= \frac{2p|T_f| \sin \gamma \sin(\delta - \phi_T^f)}{|T_f|^2 + p^2 + 2p|T_f| \cos \gamma \cos(\delta - \phi_T^f)}. \end{aligned} \quad (33)$$

TABLE III. Magnitudes  $T_f$  and phases  $\phi_T^f$  for some  $D \rightarrow PP$  processes.

Decay mode	$ T_f $ ( $10^{-7}$ GeV)	$\phi_T^f = \text{Arg}(T_f)$ (degrees)
$D^0 \rightarrow \pi^+ \pi^-$	4.70	-158.5
$D^0 \rightarrow K^+ K^-$	8.48	32.5
$D^0 \rightarrow \pi^0 \pi^0$	3.51	60.0
$D^+ \rightarrow K^+ \bar{K}^0$	6.87	-4.2

Taking  $\gamma = (67.2_{-4.6}^{+4.4})^\circ$  from Ref. [1], the world-averaged asymmetry  $\Delta A_{CP} = (-0.74 \pm 0.15)\%$  from Table I is used to constrain the magnitude  $p$  of the penguin amplitude as a function of its strong phase  $\delta$ . The value of  $p$ , plotted in Fig. 1, is nearly constant at several tenths of a percent of the amplitudes in Table III for a wide range of  $\delta$ . This corresponds to values of  $r_f$  of this order. The value of  $p$  and corresponding values of  $r_f$  are about an order of magnitude larger for extreme values of  $\delta$  in Fig. 1. The constraint on  $p$  as a function of  $\delta$  allows one to predict asymmetries for (e.g.,)  $D^0 \rightarrow \pi^+ \pi^-$ ,  $K^+ K^-$ ,  $\pi^0 \pi^0$  and  $D^+ \rightarrow K^+ \bar{K}^0$ , as plotted in Fig. 2 for  $\gamma = 67.2^\circ$ . Very similar results (not shown) are found for  $\gamma = 71.6^\circ$  and  $62.6^\circ$ .

For much of the range of  $\delta$ ,  $A_{CP}^{\text{dir}}(\pi^+ \pi^-)$  is predicted to be positive while  $A_{CP}^{\text{dir}}(K^+ K^-)$  is predicted to be negative. (In the U-spin limit they would be equal and opposite.)

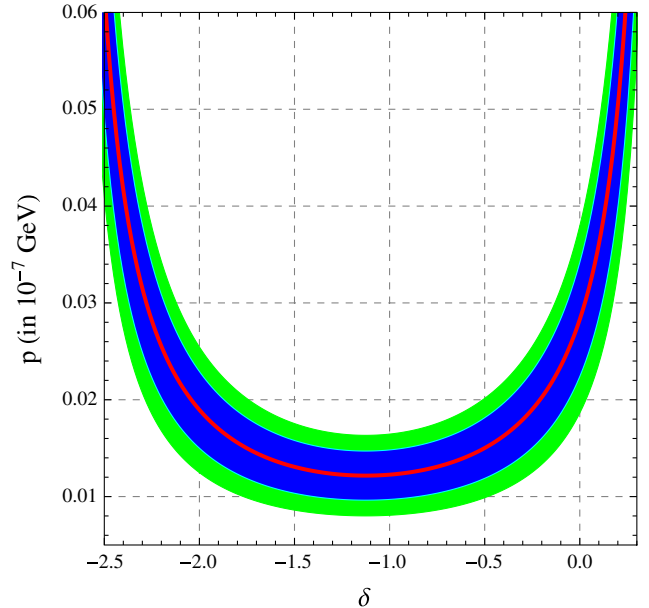


FIG. 1 (color online). Magnitude  $p$  of the  $CP$ -violating penguin amplitude as a function of its strong phase  $\delta$ . The central (red) curve was obtained using the value  $\Delta A_{CP} = -0.74\%$ , while the inner (blue) and outer (green) bands, respectively, correspond to  $\pm 1\sigma$  and  $\pm 1.64\sigma$  shifts from this value, where  $\sigma(\Delta A_{CP}) = 0.15\%$ . The plot is shown only for  $\gamma = 67.2^\circ$ , as  $p$  is not very sensitive to the exact value of  $\gamma$ .

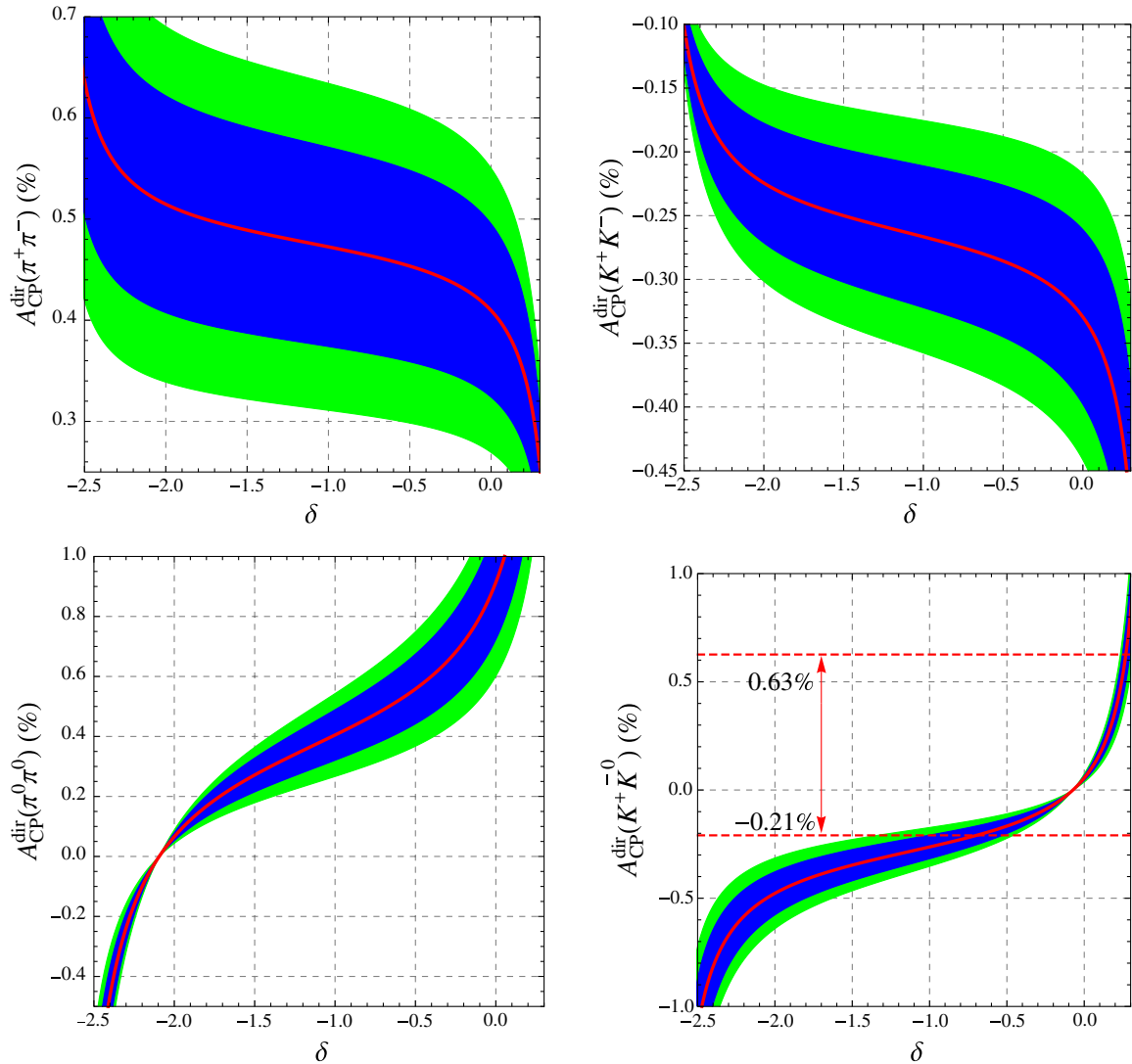


FIG. 2 (color online). Direct  $CP$  asymmetries for some SCS  $D \rightarrow PP'$  decays. Curves and bands as in Fig. 1.  $\gamma = 67.2^\circ$  is assumed. Very similar plots (not shown) are obtained for  $\gamma = 71.6^\circ$  and  $62.6^\circ$ . The dashed horizontal (red) lines in the lower right panel denote 90% C.L. limits based on the average of Belle [28,31] and BABAR [30] results.

This is consistent with the central values in Table II. However, the predicted central values for  $A_{CP}^{\text{dir}}(K^+ \bar{K}^0)$  are negative for much of the range of  $\delta$ , whereas the  $2\sigma$  lower limit  $A_{CP}^{\text{dir}}(K^+ \bar{K}^0) > -0.3$  would tend to favor values of  $\delta > -\pi/2$ . Improved measurements of all these individual  $CP$  asymmetries would of course be highly desirable.

## V. PREDICTIONS FOR SHIFTS IN WEAK PHASE $\gamma$

Taking the predicted values of  $A_{CP}^{\text{dir}}$ , Eq. (26) implies a shift in the weak phase  $\gamma$  associated with the use of each  $D$ -decay process with a  $CP$ -even final state. In Fig. 3 we present these shifts for the final states  $\pi^+ \pi^-$  and  $K^+ K^-$ . In addition to the central value of shifts  $\delta\gamma$  we also present errors in  $\delta\gamma$  due to the variation of the various measurable

parameters. In our calculations of the shifts  $\delta\gamma$  we have used the following values of  $r_B$ ,  $\delta_B$ , and  $\gamma$  taken from Ref. [1]:

$$r_B = 0.099 \pm 0.008, \quad \delta_B = (110 \pm 15)^\circ, \quad \gamma = (67.2_{-4.6}^{+4.4})^\circ. \quad (34)$$

In Fig. 3 the central black curves represent  $\delta\gamma$  as obtained from Eq. (26) using central values of the measurable parameters  $r_B$ ,  $\delta_B$ ,  $\Delta A_{CP}$  and  $\gamma$ . The errors in  $\delta\gamma$  from the  $1\sigma$  error in the measurement of the first three parameters are shown in blue using short, medium, and long dashes, respectively. In order to obtain the effect of varying  $\gamma$  on the shifts  $\delta\gamma$  we use Eq. (26) with the value of  $\gamma$  set to its  $\pm 1\sigma$  limits given in Eq. (34), while the other parameters are held fixed at their respective central values. This effect is represented by the red dots in Fig. 3.

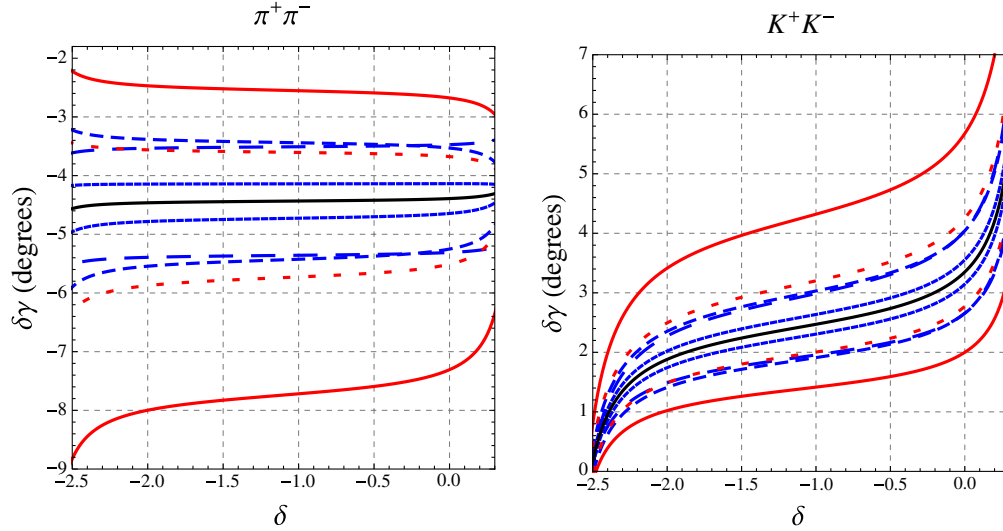


FIG. 3 (color online). Shifts  $\delta\gamma$  as calculated from Eq. (26) for  $D^0 \rightarrow \pi^+\pi^-$  (left panel) and  $D^0 \rightarrow K^+K^-$  (right panel). The solid central (black) curves denote the central value for  $\delta\gamma$ . Also shown are  $\pm 1\sigma$  errors in  $r_B$  (short dashes [blue]),  $\delta_B$  (medium dashes [blue]), and  $\Delta A_{CP}$  (long dashes [blue]). The (red) dots represent the effect of  $\pm 1\sigma$  shift in the measured value of  $\gamma$  on the central (black) curve. The solid outer (red) curves denote the effect of a  $\pm 1\sigma$  shift in the measured value of  $\gamma$  on the curve that is obtained by adding in quadrature the errors in  $r_B$ ,  $\delta_B$ , and  $\Delta A_{CP}$ . Here  $\gamma = (67.2^{+4.4}_{-4.6})^\circ$ .

In order to obtain the overall error in  $\delta\gamma$  we first add in quadrature the errors due to  $r_B$ ,  $\delta_B$ , and  $\gamma$ . We then estimate the effect of varying  $\gamma$  between its  $\pm 1\sigma$  limits on the quadrature sum. This effect is represented by the solid red curves in Fig. 3.

We see in Fig. 3 that the central value of the shift in  $\gamma$  for the  $\pi^+\pi^-$  state is nearly constant over the entire range of allowed values of the strong phase  $\delta$ . This appears to be the effect of an accidental cancellation between a variation of  $A_{CP}^{\text{dir}}$  which is modest to begin with and a compensating factor due to the variation in  $\delta_f$  for  $\gamma = 67.2^\circ$ . When the various sources of errors on  $\delta\gamma$  are included, this no longer holds, as depicted by the  $1\sigma$  boundary curves in Fig. 3. On the other hand,  $\delta\gamma$  obtained using the  $K^+K^-$  state shows appreciable variation ( $\sim 7^\circ$ ) over the allowed range of  $\delta$ . A similar exercise when performed using the  $\pi^0\pi^0$  state yields an even larger variation in  $\delta\gamma$ . However, rate asymmetries involving multiple neutral pions in the final state are not expected to be measured with adequate precision in the foreseeable future. We have therefore chosen to omit the  $\pi^0\pi^0$  decay mode from the present discussion.

In view of the variety of dependences of direct  $CP$  asymmetries on  $\delta$ , and different behavior of shifts in  $\gamma$  for  $\pi^+\pi^-$  and  $K^+K^-$  final states, it would be beneficial to further pin down  $\delta$  [e.g., with a better measurement of  $A_{CP}^{\text{dir}}(K^+\bar{K}^0)$ ], and to apply the GLW determination of  $\gamma$  to the widest assortment of  $CP$  eigenstates in neutral  $D$  meson decays. After the appropriate shifts  $\delta\gamma$  have been taken into account, inconsistencies in final values of  $\gamma$  obtained for different charm final states could point the way to effects of new physics.

## VI. DISCUSSION AND SUMMARY

The determination of the weak phase  $\gamma$  by means of the  $CP$  asymmetry in  $B^\pm \rightarrow DK^\pm$ , followed by the decay of the neutral  $D$  meson to a  $CP$  eigenstate [3], must take account of asymmetries in the decays of the neutral  $D$  mesons [13,18,19]. We have calculated the corresponding shifts  $\delta\gamma$  in an approach which imagines these  $CP$  asymmetries as due to a  $c \rightarrow u$  penguin amplitude with a weak phase of the standard model but enhanced by (presumably nonperturbative) strong interaction effects beyond those anticipated by the majority of authors. The observed value  $\Delta A_{CP} \equiv A_{CP}^{\text{dir}}(D^0 \rightarrow K^+K^-) - A_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+\pi^-) = (-0.74 \pm 0.15)\%$  has been taken as a constraint, leading to a correlation between the magnitude  $p$  of the  $CP$ -violating penguin and its strong phase  $\delta$ .

For  $\gamma = 67.2^\circ$  the central value of the shift associated with the decay  $D^0 \rightarrow \pi^+\pi^-$  is approximately  $\delta\gamma(\pi^+\pi^-) \simeq -4.4^\circ$ , with little dependence on the allowed range of the strong phase  $\delta$ . However, the factor multiplying  $A_{CP}^{\text{dir}}(f)/2r_B$  in the last expression for  $\delta\gamma$  in Eq. (26) is roughly inversely proportional to  $\cos \gamma$ , which is fairly small and fairly sensitive to  $\gamma$ . Thus, when  $\gamma$  is varied within its currently allowed range of about  $\pm 4.5^\circ$ , the value of  $\delta\gamma$  varies considerably. It is further affected by uncertainties in  $\Delta A_{CP}$ ,  $r_B$ , and  $\delta_B$ , and acquires some dependence on  $\delta$ .

The shift associated with  $D^0 \rightarrow K^+K^-$  is of the other sign (as is  $A_{CP}^{\text{dir}}$ ) and depends on both  $\Delta A_{CP}$  and  $\delta$  as well as the uncertainties in  $r_B$  and  $\delta_B$ . Using measurements of  $A_{CP}^{\text{dir}}$  for both these two decays, with the help of improved knowledge of  $A_{CP}^{\text{dir}}(D^+ \rightarrow K^+\bar{K}^0)$  to pin down  $\delta$ , the

uncertainty in  $\gamma$  due to  $CP$  violation in charm decay can be reduced to a level where it is no longer the dominant uncertainty when applying the GLW method to the decays  $B^\pm \rightarrow DK^\pm$ .

Let us be clear about the limitation of our study. The shift in  $\gamma$  we calculate using our  $c \rightarrow u$  penguin amplitude model is based on measuring  $R_{CP^+}$  and  $A_{CP^+}$  in  $B^- \rightarrow (\pi^+ \pi^-)_D K^-$ ,  $B^- \rightarrow (K^+ K^-)_D K^-$  and their  $CP$  conjugates, taking  $r_B$  and  $\delta_B$  as given. This assumes that one has measured  $r_B$  and  $\delta_B$  first and then uses only these GLW processes to determine  $\gamma$ .

An actual analysis for determining  $r_B$ ,  $\delta_B$ , and  $\gamma$  in  $B^\mp \rightarrow DK^\mp$  [8,10–13] combines information from  $D$  decays to  $CP$  eigenstates [3], flavor states [5] and three-body self-conjugate final states [6]. In this global analysis  $\gamma$  is also constrained by rates and asymmetries in  $B \rightarrow DK$  where there is no direct  $CP$  violation in  $D$  decay, for instance in decays to flavor states and  $CP$ -odd eigenstates. [See Eq. (24).] Thus any actual determination of  $\gamma$  from  $B \rightarrow DK$  will involve a considerably smaller shift than we calculate.

Furthermore, it has been noted [26,34,35] that in the self-tagged decays  $B^0 \rightarrow DK^{*0}$  the ratio  $r_B^*$  of

$B^0 \rightarrow D^0 K^{*0}$  and  $B^0 \rightarrow \bar{D}^0 K^{*0}$  amplitudes, both of which are color suppressed, is expected to be about three times larger than  $r_B$  defined in  $B^+ \rightarrow DK^+$ . We have seen an enhancement by  $1/2r_B$  of the shift in  $\gamma$  due to  $A_{CP}^{\text{dir}}(\pi^+ \pi^-, K^+ K^-)$  in  $B^+ \rightarrow DK^+$ . This implies that when applying the GLW method to  $B^0 \rightarrow DK^{*0}$  the shift in  $\gamma$  due to these direct  $CP$  asymmetries in  $D^0$  decays is expected to be about three times smaller than calculated above. First measurements of relevant observables in  $B^0 \rightarrow DK^{*0}$  and its  $CP$  conjugate have been reported very recently by the LHCb collaboration [15]. Early measurements of these processes have been performed by BABAR [36].

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