

Quantum-mechanical description of spin-1 particles with electric dipole moments

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The Proca-Corben-Schwinger equations for a spin-1 particle with an anomalous magnetic moment are added by a term describing an electric dipole moment, then they are reduced to a Hamiltonian form, and finally they are brought to the Foldy-Wouthuysen representation. Relativistic equations of motion are derived. The needed agreement between quantum-mechanical and classical relativistic equations of motion is proved. The scalar and tensor electric and magnetic polarizabilities of pointlike spin-1 particles (W bosons) are calculated for the first time.

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I. INTRODUCTION

The discovery of electric dipole moments (EDMs) is one of the main goals of contemporary physics. Such a discovery would go beyond the Standard Model and open a window to new physics. To disclose the EDM, one needs to find an anomaly in spin dynamics. The classical spin dynamics of a particle with an anomalous magnetic moment (AMM) and an EDM was determined many years ago [1], but the corresponding quantum-mechanical equations have been derived only for spin-1/2 particles [2,3].

One of the experimental priorities is a search for the EDM of the deuteron [4–6] whose spin is 1. While the classical and quantum theories of spin motion should agree, a proper quantum-mechanical consideration of spin-1 particles/nuclei is also necessary. We perform such a consideration based on the Proca equations [7] with an additional term included by Corben and Schwinger [8]. We generalize the above Proca-Corben-Schwinger (PCS) equations to take also into account the EDM, then bring the generalized equations to a Hamiltonian form and perform the relativistic Foldy-Wouthuysen (FW) transformation. Unlike the original FW approach [9], we use the method [10,11] that enables transition to the FW representation for relativistic particles in external fields. This allows us to find a relativistic operator equation of spin motion and easily determine its classical limit. This result provides a deficient quantum-mechanical basis for the deuteron EDM experiment.

Finally, we calculate for the first time scalar and tensor electric and magnetic polarizabilities of pointlike (structureless) spin-1 particles. We use the system of units $\hbar = 1$, $c = 1$.

II. BASIC EQUATIONS

Proca equations [7] for spin-1 particles with the Corben-Schwinger term [8] have the form

$$U_{\mu\nu} = D_\mu U_\nu - D_\nu U_\mu, \quad \mu, \nu = 0, 1, 2, 3, \quad (1)$$

$$D^\mu U_{\mu\nu} - m^2 U_\nu + ie\kappa U^\mu F_{\mu\nu} = 0, \quad (2)$$

where $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative, A_μ is the four-potential, $F_{\mu\nu}$ is the electromagnetic field tensor, and $U_{\mu\nu} = -U_{\nu\mu}$. The Corben-Schwinger term is proportional to $\kappa = g - 1$, where $g = 2m\mu/(es) = 2m\mu/e$ for spin-1 particles. Since the Proca equations correspond to $g = 1$, this term describes not only the AMM but also a part of the normal ($g = 2$) magnetic moment [12]. Spin-1 particles can be also described by the Duffin-Kemmer-Petiaux equation [13], Stueckelberg equation [14], multispinor Bargmann-Wigner equations [15], and other equations.

Since the spin of Proca particles has three components, six components of the wave function are independent. Spatial components of Eq. (1) and a time component of Eq. (2) can be expressed in terms of the others. As a result, the equations for the ten-component wave function can be reduced to the equation for the six-component one (Sakata-Taketani transformation [16]). The distinctive feature of this transformation is that it obtains expressions for U_0 and U_{ij} ($i, j = 1, 2, 3$) which do not contain the time derivative, and then it substitutes them into equations for the remaining components. From Eq. (2) we have

$$U_0 = \frac{1}{m^2} (D^i U_{i0} + ie\kappa U^i F_{i0}).$$

Next we introduce two vector functions, ϕ and U , whose components are given by iU_{i0}/m and U^i and form the six-component Sakata-Taketani wave function

$$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi + U \\ \phi - U \end{pmatrix}.$$

As the generalized Sakata-Taketani equation can be expressed in terms of spin-1 matrices [12], the wave function of this equation is similar to a Dirac bispinor. The general form of the Hamiltonian in the Sakata-Taketani representation obtained by Young and Bludman [12] is given by

$$\begin{aligned}
\mathcal{H} = & e\Phi + \rho_3 m + i\rho_2 \frac{1}{m} (\mathbf{S} \cdot \mathbf{D})^2 - (\rho_3 + i\rho_2) \frac{1}{2m} (\mathbf{D}^2 + e\mathbf{S} \cdot \mathbf{B}) - (\rho_3 - i\rho_2) \frac{e\kappa}{2m} (\mathbf{S} \cdot \mathbf{B}) \\
& - \frac{e\kappa}{2m^2} (1 + \rho_1) [(\mathbf{S} \cdot \mathbf{E})(\mathbf{S} \cdot \mathbf{D}) - i\mathbf{S} \cdot [\mathbf{E} \times \mathbf{D}] - \mathbf{E} \cdot \mathbf{D}] + \frac{e\kappa}{2m^2} (1 - \rho_1) [(\mathbf{S} \cdot \mathbf{D})(\mathbf{S} \cdot \mathbf{E}) - i\mathbf{S} \cdot [\mathbf{D} \times \mathbf{E}] - \mathbf{D} \cdot \mathbf{E}] \\
& - \frac{e^2 \kappa^2}{2m^3} (\rho_3 - i\rho_2) [(\mathbf{S} \cdot \mathbf{E})^2 - \mathbf{E}^2], \tag{3}
\end{aligned}$$

where \mathbf{S} is the 3×3 spin matrix, ρ_i ($i = 1, 2, 3$) are the 2×2 Pauli matrices, $\kappa = \text{const}$, \mathbf{E} is the electric field strength, and \mathbf{B} is the magnetic field induction. We do not consider a nonintrinsic quadrupole moment included in Ref. [12]. Denotation $\rho_i S_j$ means the direct product of two matrices. For spin-1 particles, the polarization operator is equal to $\mathbf{\Pi} = \rho_3 \mathbf{S}$. It is analogous to the corresponding Dirac operator which can be written in a similar form (see Ref. [17]): $\mathbf{\Pi} = \rho_3 \boldsymbol{\sigma}$.

In Refs. [18,19], Hamiltonian (3) has been transformed to the FW representation for relativistic particles in electric and magnetic fields with allowance for derivatives of the electric field strength. The terms proportional to the derivatives of the magnetic field induction have not been calculated.

III. INCLUSION OF ELECTRIC DIPOLE MOMENTS

To describe the EDMs of spin-1/2 particles, the terms proportional to the γ^5 matrix can be added to the Lagrangian and the Dirac equation [20]. It has been shown in Ref. [3] that there exists another way to include the EDMs with the tensor $G_{\mu\nu} = (\mathbf{B}, -\mathbf{E})$ dual to the electromagnetic field one, $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$. In this case, the Lagrangians describing the AMM and EDM become very similar and are given by [3]

$$\mathcal{L}_{\text{AMM}} = \frac{\mu'}{2} \sigma^{\mu\nu} F_{\mu\nu}, \quad \mathcal{L}_{\text{EDM}} = -\frac{d}{2} \sigma^{\mu\nu} G_{\mu\nu} \tag{4}$$

where d is the EDM of the particle. The generalized Dirac-Pauli equation assumes the form [3]

$$\left[\gamma^\mu \pi_\mu - m + \frac{\mu'}{2} \sigma^{\mu\nu} F_{\mu\nu} - \frac{d}{2} \sigma^{\mu\nu} G_{\mu\nu} \right] \Psi = 0. \tag{5}$$

The terms describing the contributions of the AMM and EDM to the Hamiltonian are transformed into each other using the substitutions $\mathbf{B} \rightarrow \mathbf{E}$, $\mathbf{E} \rightarrow -\mathbf{B}$, $\mu' \rightarrow d$. The corresponding relativistic FW Hamiltonian and equations of motion have been derived in Ref. [3] by the method developed in Ref. [10].

Similarly, we can supplement the Lagrangian of spin-1 particles [12] containing the AMM term, $\mathcal{L}_{\text{AMM}} = (ie\kappa/2)(U_\mu^\dagger U_\nu - U_\nu^\dagger U_\mu)F^{\mu\nu}$, with the EDM one:

$$\mathcal{L}_{\text{EDM}} = -(ie\eta/2)(U_\mu^\dagger U_\nu - U_\nu^\dagger U_\mu)G^{\mu\nu}, \tag{6}$$

where $\eta = 2dm/(es) = 2dm/e$. The corresponding generalized PCS equations read

$$\begin{aligned}
U_{\mu\nu} &= D_\mu U_\nu - D_\nu U_\mu, \tag{7} \\
D^\mu U_{\mu\nu} - m^2 U_\nu + ie\kappa U^\mu F_{\mu\nu} - ie\eta U^\mu G_{\mu\nu} &= 0.
\end{aligned}$$

This generalization brings Eq. (3) to the form

$$\begin{aligned}
\mathcal{H} = & e\Phi + \rho_3 m + i\rho_2 \frac{1}{m} (\mathbf{S} \cdot \mathbf{D})^2 - (\rho_3 + i\rho_2) \frac{1}{2m} (\mathbf{D}^2 + e\mathbf{S} \cdot \mathbf{B}) - (\rho_3 - i\rho_2) \frac{e\kappa}{2m} (\mathbf{S} \cdot \mathbf{B}) \\
& - \frac{e\kappa}{2m^2} (1 + \rho_1) [(\mathbf{S} \cdot \mathbf{E})(\mathbf{S} \cdot \mathbf{D}) - i\mathbf{S} \cdot [\mathbf{E} \times \mathbf{D}] - \mathbf{E} \cdot \mathbf{D}] + \frac{e\kappa}{2m^2} (1 - \rho_1) [(\mathbf{S} \cdot \mathbf{D})(\mathbf{S} \cdot \mathbf{E}) - i\mathbf{S} \cdot [\mathbf{D} \times \mathbf{E}] - \mathbf{D} \cdot \mathbf{E}] \\
& - \frac{e^2 \kappa^2}{2m^2} (\rho_3 - i\rho_2) [(\mathbf{S} \cdot \mathbf{E})^2 - \mathbf{E}^2] - (\rho_3 - i\rho_2) \frac{e\eta}{2m} (\mathbf{S} \cdot \mathbf{E}) + \frac{e\eta}{2m^2} (1 + \rho_1) [(\mathbf{S} \cdot \mathbf{B})(\mathbf{S} \cdot \mathbf{D}) - i\mathbf{S} \cdot [\mathbf{B} \times \mathbf{D}] - \mathbf{B} \cdot \mathbf{D}] \\
& - \frac{e\eta}{2m^2} (1 - \rho_1) [(\mathbf{S} \cdot \mathbf{D})(\mathbf{S} \cdot \mathbf{B}) - i\mathbf{S} \cdot [\mathbf{D} \times \mathbf{B}] - \mathbf{D} \cdot \mathbf{B}], \tag{8}
\end{aligned}$$

where only first-order terms in η are taken into account.

The use of the appropriate method of the FW transformation for relativistic particles in external fields [10,11] leads to the following FW Hamiltonian:

$$\begin{aligned}
 \mathcal{H}_{\text{FW}} = & \rho_3 \epsilon' + e\Phi + \frac{e}{4m} \left[\left\{ \left(\frac{g-2}{2} + \frac{m}{\epsilon' + m} \right) \frac{1}{\epsilon'}, (S \cdot [\boldsymbol{\pi} \times \mathbf{E}] - S \cdot [\mathbf{E} \times \boldsymbol{\pi}]) \right\} - \rho_3 \left\{ \left(g - 2 + \frac{2m}{\epsilon'} \right), S \cdot \mathbf{B} \right\} \right. \\
 & + \rho_3 \frac{g-2}{4} \left\{ \frac{1}{\epsilon'(\epsilon' + m)}, \{S \cdot \boldsymbol{\pi}, (\boldsymbol{\pi} \cdot \mathbf{B} + \mathbf{B} \cdot \boldsymbol{\pi})\} \right\} \left. \right] + \frac{e\eta}{8m} \left[\left\{ \frac{1}{\epsilon'}, (S \cdot [\mathbf{B} \times \boldsymbol{\pi}] - S \cdot [\boldsymbol{\pi} \times \mathbf{B}]) \right\} \right. \\
 & \left. - 4\rho_3 S \cdot \mathbf{E} + \frac{\rho_3}{2} \left\{ \frac{1}{\epsilon'(\epsilon' + m)}, \{S \cdot \boldsymbol{\pi}, (\boldsymbol{\pi} \cdot \mathbf{E} + \mathbf{E} \cdot \boldsymbol{\pi})\} \right\} \right], \quad (9)
 \end{aligned}$$

where $\epsilon' = \sqrt{m^2 + \boldsymbol{\pi}^2}$. In this Hamiltonian, terms neither bilinear in the field strengths nor those containing derivatives of these strengths are taken into account. Commutators and anticommutators are defined by $[\dots, \dots]$ and $\{\dots, \dots\}$, respectively. Equation (9) generalizes the corresponding one derived in Ref. [18] without allowance for the EDM terms.

IV. EQUATIONS OF MOTION AND THEIR CLASSICAL LIMIT

Equation (9) allows us to derive quantum-mechanical equations of motion and then obtain their classical limit.

Such equations are defined by the commutators of the FW Hamiltonian with appropriate operators:

$$\begin{aligned}
 \frac{d\boldsymbol{\pi}}{dt} &= \frac{i}{\hbar} [\mathcal{H}_{\text{FW}}, \boldsymbol{\pi}] + \frac{\partial \boldsymbol{\pi}}{\partial t} = \frac{i}{\hbar} [\mathcal{H}_{\text{FW}}, \boldsymbol{\pi}] - e \frac{\partial \mathbf{A}}{\partial t}, \\
 \frac{d\boldsymbol{\Pi}}{dt} &= \frac{i}{\hbar} [\mathcal{H}_{\text{FW}}, \boldsymbol{\Pi}] = \frac{1}{2} (\boldsymbol{\Omega} \times \boldsymbol{\Pi} - \boldsymbol{\Pi} \times \boldsymbol{\Omega}), \quad (10)
 \end{aligned}$$

where $\boldsymbol{\Omega}$ is the operator of angular velocity of spin motion. The operator equation of spin motion has the form

$$\begin{aligned}
 \frac{d\boldsymbol{\Pi}}{dt} = & \frac{1}{4} \left\{ \left(\frac{\mu_0 m}{\epsilon' + m} + \mu' \right) \frac{1}{\epsilon'}, (\boldsymbol{\Pi} \times [\mathbf{E} \times \boldsymbol{\pi}] - \boldsymbol{\Pi} \times [\boldsymbol{\pi} \times \mathbf{E}]) \right\} + \frac{1}{2} \left\{ \left(\frac{\mu_0 m}{\epsilon'} + \mu' \right), [\boldsymbol{\Sigma} \times \mathbf{B}] \right\} \\
 & - \frac{\mu'}{4} \left\{ \frac{1}{\epsilon'(\epsilon' + m)}, ([\boldsymbol{\Sigma} \times \boldsymbol{\pi}](\boldsymbol{\pi} \cdot \mathbf{B}) + (\mathbf{B} \cdot \boldsymbol{\pi})[\boldsymbol{\Sigma} \times \boldsymbol{\pi}]) \right\} - \frac{d}{dt} \left\{ \frac{1}{\epsilon'}, (\boldsymbol{\Pi} \times [\mathbf{B} \times \boldsymbol{\pi}] - \boldsymbol{\Pi} \times [\boldsymbol{\pi} \times \mathbf{B}]) \right\} \\
 & + d[\boldsymbol{\Sigma} \times \mathbf{E}] - \frac{d}{dt} \left\{ \frac{1}{\epsilon'(\epsilon' + m)}, ([\boldsymbol{\Sigma} \times \boldsymbol{\pi}](\boldsymbol{\pi} \cdot \mathbf{E}) + (\mathbf{E} \cdot \boldsymbol{\pi})[\boldsymbol{\Sigma} \times \boldsymbol{\pi}]) \right\}, \quad (11)
 \end{aligned}$$

where $\mu_0 = e/m$, $\boldsymbol{\Sigma} = I\mathbf{S}$ (I is the unit 2×2 matrix). In this equation, second-order terms in spin are not taken into account. Their contribution into the Hamiltonian is considered below. Equation (11) is fully consistent with the corresponding one for spin-1/2 particles with the EDM [3]. Finding its classical limit reduces to the replacement of operators by respective classical quantities [21] and results in

$$\begin{aligned}
 \frac{ds}{dt} = & \frac{e}{m} \left\{ \frac{1}{\epsilon'} \left(\frac{g-2}{2} + \frac{m}{\epsilon' + m} \right) [s \times [\mathbf{E} \times \boldsymbol{\pi}]] \right. \\
 & + \left(\frac{g-2}{2} + \frac{m}{\epsilon'} \right) [s \times \mathbf{B}] - \frac{g-2}{2\epsilon'(\epsilon' + m)} [s \times \boldsymbol{\pi}](\boldsymbol{\pi} \cdot \mathbf{B}) \\
 & - \frac{\eta}{2\epsilon'} [s \times [\mathbf{B} \times \boldsymbol{\pi}]] + \frac{\eta}{2} [s \times \mathbf{E}] \\
 & \left. - \frac{\eta}{2\epsilon'(\epsilon' + m)} [s \times \boldsymbol{\pi}](\boldsymbol{\pi} \cdot \mathbf{E}) \right\}. \quad (12)
 \end{aligned}$$

Equation (12) coincides with the respective classical one [1] which generalizes the Thomas-Bargmann-Michel-Telegdi equation [22,23].

This result perfectly proves self-consistency of the relativistic wave equations for spin-1 particles which

was called in question for a long time (see Refs. [24,25] and references therein). Their self-consistency is also confirmed by the form of the quantum-mechanical equation of particle motion:

$$\begin{aligned}
 \frac{d\boldsymbol{\pi}}{dt} = & e\mathbf{E} + \rho_3 \frac{e}{4} \left\{ \frac{1}{\epsilon'}, ([\boldsymbol{\pi} \times \mathbf{B}] - [\mathbf{B} \times \boldsymbol{\pi}]) \right\} \\
 & + \frac{1}{4} \left\{ \left(\frac{\mu_0 m}{\epsilon' + m} + \mu' \right) \frac{1}{\epsilon'}, \nabla(\boldsymbol{\Sigma} \cdot [\mathbf{E} \times \boldsymbol{\pi}]) \right. \\
 & \left. - \boldsymbol{\Sigma} \cdot [\boldsymbol{\pi} \times \mathbf{E}] \right\} + \frac{1}{2} \left\{ \left(\frac{\mu_0 m}{\epsilon'} + \mu' \right), \nabla(\boldsymbol{\Pi} \cdot \mathbf{B}) \right\} \\
 & - \frac{\mu'}{8} \left\{ \frac{1}{\epsilon'(\epsilon' + m)}, \{(\boldsymbol{\Pi} \cdot \boldsymbol{\pi}), \nabla(\boldsymbol{\pi} \cdot \mathbf{B} + \mathbf{B} \cdot \boldsymbol{\pi})\} \right\}. \quad (13)
 \end{aligned}$$

In Eq. (13), the terms dependent on the EDM are omitted. This equation agrees with the corresponding one describing spin-1/2 particles [10], and it is also in accord with classical theory. Spin-dependent terms define a relativistic Stern-Gerlach force. For spin-1 particles in a uniform magnetic field, the quantum-mechanical equations of motion have been derived in Ref. [26].

V. SCALAR AND TENSOR POLARIZABILITIES OF POINTLIKE PARTICLES

The W boson being a charged pointlike (structureless) spin-1 particle can be described by the PCS equations. Such a particle possesses some electric and magnetic moments. The W boson may have an AMM (see Refs. [27–29] and references therein). This AMM is defined by radiative corrections. The quantum mechanics allows us to derive other moments of the pointlike particle with the definite g factor. Of course, moments of pointlike particles can be affected by radiative corrections, and moments of pointlike and composed particles can significantly differ.

The quadrupole and contact interactions of a charged structureless spin-1 particle possessing the AMM were first determined by Young and Bludman [12] in the nonrelativistic approximation. In Ref. [18], a relativistic description of these interactions has been made. Pomeransky and Khriplovich [30] have obtained relativistic expressions for the quadrupole interaction of arbitrary-spin particles (without analysis of the contact interaction). The nonrelativistic formula for the quadrupole and contact interactions of a charged structureless spin-1 particle is

$$W = \frac{e(g-1)}{4m^2} (S_i S_j + S_j S_i) \frac{\partial E_i}{\partial x_j} - \frac{e(g-1)}{2m^2} \nabla \cdot \mathbf{E}. \quad (14)$$

The corresponding quadrupole moment is equal to $Q = -e(g-1)/m^2$ [12,18,30].

The nonrelativistic FW transformation of the initial Young-Bludman Hamiltonian (9) makes it possible to determine the polarizabilities defined as follows:

$$\Delta \mathcal{H}_{\text{FW}} = -\frac{1}{2} \alpha_S E^2 - \frac{1}{2} \beta_S B^2 - \alpha_T (\mathbf{S} \cdot \mathbf{E})^2 - \beta_T (\mathbf{S} \cdot \mathbf{B})^2. \quad (15)$$

Here α_S and β_S are the scalar electric and magnetic polarizabilities, and α_T and β_T are the tensor electric and magnetic ones. The related terms in the nonrelativistic FW Hamiltonian calculated for the first time read

$$\Delta \mathcal{H}_{\text{FW}} = \rho_3 \frac{e^2 \hbar^2 (g-1)^2}{2m^3} \mathbf{E}^2 - \rho_3 \frac{e^2 \hbar^2 (g-1)^2}{2m^3} (\mathbf{S} \cdot \mathbf{E})^2 - \rho_3 \frac{e^2 \hbar^2}{8m^3} [(g-1)^2 + 3] (\mathbf{S} \cdot \mathbf{B})^2. \quad (16)$$

For positive-energy states, the polarizabilities are given by

$$\alpha_S = -\frac{e^2 \hbar^2 (g-1)^2}{m^3}, \quad \alpha_T = \frac{e^2 \hbar^2 (g-1)^2}{2m^3}, \quad (17)$$

$$\beta_T = \frac{e^2 \hbar^2}{8m^3} [(g-1)^2 + 3],$$

and the scalar magnetic polarizability is zero. The tensor electric and magnetic polarizabilities of spin-1 particles without the AMM are equal to each other. The nonzero AMM brings a difference between them.

VI. DISCUSSION AND SUMMARY

The present work shows that the PCS equations can be added by the EDM-dependent term. Next, the transformations allow us to obtain the self-consistent Hamiltonians in the Sakata-Taketani and FW representations. The classical limit of derived quantum-mechanical equations of motion coincides with corresponding classical ones. These results demonstrate self-consistency of quantum mechanics of spin-1 particles and create the sufficient quantum-mechanical basis for the planned deuteron EDM experiment [4–6].

The deuteron EDM experiment in storage rings is very important. This experiment is not only complementary to other EDM searches, but for some potential sources of EDMs, it is superior (see Ref. [5] and references therein).

The calculation of the scalar and tensor electric and magnetic polarizabilities of pointlike spin-1 particles has been fulfilled for the first time. The scalar magnetic polarizability happens to be zero. The tensor electric and magnetic polarizabilities of a spin-1 particle with the zero AMM are equal to each other. Equation (17) defines parameters of W boson being a charged structureless spin-1 particles. In particular, $\alpha_S = -1.1 \times 10^{-10} \text{ fm}^3$, $\alpha_T = \beta_T = 5.4 \times 10^{-11} \text{ fm}^3$ for $g = 2$ and $\alpha_S = -9.9 \times 10^{-11} \text{ fm}^3$, $\alpha_T = 5.0 \times 10^{-11} \text{ fm}^3$, $\beta_T = 5.3 \times 10^{-11} \text{ fm}^3$ for Ref. [29] $g - 2 = -4.05 \times 10^{-2}$. The polarizabilities of composed spin-1 particles are much greater. For example, the tensor electric and magnetic polarizabilities of the deuteron are of the order of $10^{-2}/10^{-1} \text{ fm}^3$ [31], while the corresponding values for a pointlike particle of the same mass calculated by using Eq. (17) are of the order of 10^{-6} fm^3 . We can note an importance of allowance for the tensor polarizabilities in the EDM experiment (see Ref. [32] and references therein).

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