Broken S_3 neutrinos

H. B. Benaoum*

Prince Mohammad Bin Fahd University, Al-Khobar 31952, Saudi Arabia

(Received 6 February 2013; published 23 April 2013)

Motivated by recent measurements which strongly support a nonzero reactor mixing angle θ_{13}, we study a deviation from S_3 neutrino discrete symmetry by explicitly breaking the neutrino mass matrix with a general retrocirculant matrix. We show that nonzero θ_{13} and nonzero CP violation parameter J_{CP} arise due to the difference between y_2 and y_3. We demonstrate that it is possible to obtain the experimentally favored results for neutrino masses and mixing angles from this mass matrix. Furthermore, we estimate the effective masses m_β and $m_{\beta\beta}$ and total neutrino mass $\sum |m_i|$ predicted by this mass matrix.

DOI: 10.1103/PhysRevD.87.073010 PACS numbers: 14.60.Pq, 12.15.Ff

Experiments using solar, atmospheric, and reactor neutrinos have made considerable progress in establishing two different mass squared differences (Δm^2_{21} and Δm^2_{31}) and two large mixing angles (θ_{12} and θ_{23}) in the lepton sector. Recently, MINOS [1,2], T2K [3], Double CHOOZ [4], Daya Bay [5], and RENO [6] have revealed that the reactor mixing angle θ_{13} is not only nonzero but relatively large.

The phenomenon of neutrino mixing can be simply described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix $V_{\rm PMNS}$ [7], which links the neutrino flavor eigenstates ν_e, ν_μ, ν_τ to the mass eigenstates ν_1, ν_2, ν_2:

$$V = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix}. \tag{1}$$

In the standard parametrization used by the particle data group, the PMNS matrix is expressed by three mixing angles θ_{12}, θ_{23}, and θ_{13} and one intrinsic CP violating phase δ for Dirac neutrinos,

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} .P_{Maj}, \tag{2}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and P_{Maj} is a diagonal matrix with Majorana CP violating phases.

The three mixing angles are related to the moduli of the elements of the PMNS mixing matrix as

$$\sin^2\theta_{13} = |V_{e3}|^2 \qquad \sin^2\theta_{12} = \frac{|V_{e2}|^2}{1-|V_{e3}|^2}$$
$$\sin^2\theta_{23} = \frac{|V_{\mu 3}|^2}{1-|V_{e3}|^2}. \tag{3}$$

The well-known tri-bimaximal (TBM) mixing pattern, which corresponds to $\theta_{13} = 0$, $\theta_{23} = \pm\frac{\pi}{4}$ and $\theta_{12} = \sin^{-1}(\frac{1}{\sqrt{3}})$, has attracted a degree of attention in the literature because it suggests some underlying flavor symmetry among lepton's generations. This flavor symmetry is expected to explain the mass spectrum and neutrino mixing pattern.

The TBM form is

$$V_0 = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4}$$

Models based on discrete symmetries were successful in reproducing this matrix. Among a number of interesting discrete flavor symmetries discussed in the literature, the S_3 symmetry which is the permutation group of three objects, is the simplest [8]. S_3 is the smallest non-Abelian discrete group. The three-dimensional reducible representations of all S_3 group elements are

$$S^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \qquad S^{(12)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$
$$S^{(13)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \qquad S^{(23)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \tag{5}$$
$$S^{(123)} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}; \qquad S^{(132)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

The most general neutrino mass matrix M^0_ν invariant under S_3 is

*hbenaoum@pmu.edu.sa

TABLE I. Global oscillation analysis with best fit for Δm^2_{21}, Δm^2_{31}, $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, and δ the upper and/or lower corresponds to normal and/or inverted neutrino mass hierarchy.

Parameter	Best fit	2σ	3σ
$\Delta m^2_{21}[10^{-5}\ \mathrm{eV}^2]$	7.62	7.27–8.01	7.12–8.20
$\lvert\Delta m^2_{31}\rvert[10^{-3}\ \mathrm{eV}^2]$	2.55	2.38–2.68	2.31–2.74
	2.43	2.29–2.58	2.21–2.64
$\sin^2\theta_{12}$	0.320	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	0.613	0.38–0.66	0.36–0.68
	0.600	0.39–0.65	0.37–0.67
$\sin^2\theta_{13}$	0.0246	0.019–0.030	0.017–0.033
	0.0250	0.020–0.030	0.017–0.033
δ	0.80π	0–2π	0–2π
	-0.03π	0–2π	0–2π

$$M^0_\nu = \alpha S^{(1)} + \beta(S^{(12)} + S^{(13)} + S^{(23)}), \tag{6}$$

where α and β are, in general, complex numbers.

In the basis where the charged lepton mass matrix is diagonal, the TBM mixing matrix diagonalizes the neutrino matrix M^0_ν:

$$V_0^T M^0_\nu V_0 = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha+\beta & 0 \\ 0 & 0 & \alpha \end{pmatrix}. \tag{7}$$

This matrix leads to two degenerate masses, namely, m_1 and m_3. However, this is not correct experimentally. To overcome this problem, it was suggested in Ref. [9] that in fact the three masses are degenerate by letting the complex number α lies in the third quadrant,

$$\alpha = -i|\alpha|e^{-i\frac{\psi}{2}} \quad \text{for } 0 \le \psi < \pi, \tag{8}$$

and taking β as real number such that

$$\beta = \frac{2}{3}|\alpha|\sin\frac{\psi}{2}. \tag{9}$$

In this work, we consider the neutrino matrix M^0_ν, which is invariant under S_3, as zeroth order with degenerate masses and TBM mixing angles. Nondegenerate mass spectrum and nonzero θ_{13} were realized in Refs. [9,10] by introducing small perturbations that violate S_3 symmetry.

Here, we investigate the phenomenological consequences of the deviation from an exact S_3 symmetry by explicitly breaking the neutrino mass matrix M^0_ν with a general retrocirculant matrix:

$$\Delta M_\nu = -\alpha\begin{pmatrix} y_1 & y_2 & y_3 \\ y_2 & y_3 & y_1 \\ y_3 & y_1 & y_2 \end{pmatrix}, \tag{10}$$

where the dimensionless parameters $y_i = |y_i|e^{i\varphi_i/2}$ are complex numbers with magnitude less than one and $0 \le \varphi_i \le \pi$. We also consider the charged lepton to be diagonal so the leptonic mixing solely comes from the neutrino sector. It is easy to see that ΔM_ν can be written as a linear combination of $S^{(23)}$, $S^{(12)}$ and $S^{(13)}$ as

$$\Delta M_\nu = -\alpha(y_1 S^{(23)} + y_2 S^{(12)} + y_3 S^{(13)}). \tag{11}$$

As a result the broken neutrino matrix M_ν becomes

$$M_\nu = M^0_\nu + \Delta M_\nu = \begin{pmatrix} \alpha+\beta-\alpha y_1 & \beta-\alpha y_2 & \beta-\alpha y_3 \\ \beta-\alpha y_2 & \alpha+\beta-\alpha y_3 & \beta-\alpha y_1 \\ \beta-\alpha y_3 & \beta-\alpha y_1 & \alpha+\beta-\alpha y_2 \end{pmatrix}. \tag{12}$$

The eigenvalues of the above matrix are

$$\begin{aligned} m_1 &= \alpha - \alpha\sqrt{y_1^2 + y_2^2 + y_3^2 - y_1y_2 - y_1y_3 - y_2y_3}, \\ m_2 &= \alpha + 3\beta - \alpha(y_1 + y_2 + y_3), \\ m_3 &= \alpha + \alpha\sqrt{y_1^2 + y_2^2 + y_3^2 - y_1y_2 - y_1y_3 - y_2y_3}. \end{aligned} \tag{13}$$

The matrix M_ν is called magic mass matrix since every row and column add up to the same value which is m_2 for this case. It implies that this mass matrix has a trimaximal eigenvector $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})^T$ [11]. Moreover, the μ-τ symmetry corresponding to $\theta_{13} = 0$ and $\theta_{23} = \pm\frac{\pi}{4}$ is broken for M_ν,

$$[M_\nu, S^{(23)}] = -\alpha(y_2 - y_3)(S^{(123)} - S^{(132)}) \tag{14}$$

due to the difference between y_2 and y_3.

Interestingly, by rotating the above matrix by the TBM mixing matrix V_0, we get

$$V_0^T M_\nu V_0 = \begin{pmatrix} \alpha - \frac{\alpha}{2}(2y_1 - y_2 - y_3) & 0 & \frac{\alpha\sqrt{3}}{2}(y_2 - y_3) \\ 0 & \alpha + 3\beta - \alpha(y_1 + y_2 + y_3) & 0 \\ \frac{\alpha\sqrt{3}}{2}(y_2 - y_3) & 0 & \alpha + \frac{\alpha}{2}(2y_1 - y_2 - y_3) \end{pmatrix}. \tag{15}$$

As a consequence, the neutrino matrix M_ν is diagonalized by the total unitary matrix $V = V_0 U P_{Maj}$. The mixing matrix U is given by

$$U = \begin{pmatrix} \cos\theta & 0 & e^{-i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}. \tag{16}$$

A straightforward calculation yields that the angle θ and the CP-phase δ are

$$\tan 2\theta = \frac{\sqrt{X^2+Y^2}}{Z}, \qquad \tan\delta = \frac{Y}{X}, \tag{17}$$

where

$$\begin{aligned} X &= \sqrt{3}\Big(|y_2|\cos\frac{\varphi_2}{2} - |y_3|\cos\frac{\varphi_3}{2}\Big),\\ Y &= \sqrt{3}\Big(|y_1||y_2|\sin\Big(\frac{\varphi_1-\varphi_2}{2}\Big) - |y_1||y_3|\sin\Big(\frac{\varphi_1-\varphi_3}{2}\Big)\\ &\quad + |y_2||y_3|\sin\Big(\frac{\varphi_2-\varphi_3}{2}\Big)\Big),\\ Z &= 2|y_1|\cos\frac{\varphi_1}{2} - |y_2|\cos\frac{\varphi_2}{2} - |y_3|\cos\frac{\varphi_3}{2}. \end{aligned} \tag{18}$$

The explicit expression of V is

$$V = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}e^{-i\delta}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{e^{i\delta}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\theta}{\sqrt{2}} - \frac{e^{-i\delta}\sin\theta}{\sqrt{6}} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{e^{i\delta}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{2}} - \frac{e^{i\delta}\sin\theta}{\sqrt{6}} \end{pmatrix} . P_{Maj}. \tag{19}$$

We immediately obtain the mixing angles:

$$\begin{aligned} \sin^2\theta_{13} &= \frac{2}{3}\sin^2\theta, \qquad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta},\\ \sin^2\theta_{23} &= \frac{1}{2}\Big(1 + \frac{\sqrt{3}\sin 2\theta\cos\delta}{2+\cos 2\theta}\Big), \end{aligned} \tag{20}$$

which for $|y_3| = |y_2| = 0$ (i.e., $\theta = 0$) give the TBM mixing angles.

The first and second equation in (20) show that the solar and reactor neutrino mixing angles are related by

$$\sin^2\theta_{12} = \frac{1}{3\cos^2\theta_{13}}. \tag{21}$$

Next, by considering the third equation in (20), a simple relation between the CP-phase δ and the mixing angles can be derived. The result for $\cos\delta$ reads

$$\cos\delta = -\frac{1}{\sqrt{3}}\frac{\cos 2\theta_{23}}{\cos\theta_{12}\sqrt{3\sin^2\theta_{12}-1}}. \tag{22}$$

This, in turn, would imply that the solar mixing angle θ_{12} has to be

$$\sin^2\theta_{12} > \frac{1}{3}, \tag{23}$$

which is right on the edge from the global fits to the neutrino oscillation data. Such a constraint could be confirmed or ruled out with a little better data.

The strength of CP violation in neutrino oscillations is described by the Jarlskog rephasing invariant parameter. It is given by

$$\begin{aligned} J_{CP} &= \mathrm{Im}(V_{e2}V_{\mu 3}V^\star_{e3}V^\star_{\mu 2}) = -\frac{1}{6\sqrt{3}}\sin 2\theta\sin\delta,\\ &= -\frac{1}{6\sqrt{3}}\frac{Y}{\sqrt{X^2+Y^2+Z^2}}. \end{aligned} \tag{24}$$

Similarly, the Jarlskog parameter J_{CP} can be written in terms of the solar θ_{12} and the atmospheric θ_{23} mixing angles.

It proves convenient to use the ratio $R_\nu = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$, to write the mass squared differences as

$$\begin{aligned} \Delta m^2_{21} &= 2|\alpha|^2 R_\nu\sqrt{X^2+Y^2+Z^2},\\ \Delta m^2_{31} &= 2|\alpha|^2\sqrt{X^2+Y^2+Z^2},\\ \Delta m^2_{32} &= 2|\alpha|^2(1-R_\nu)\sqrt{X^2+Y^2+Z^2}. \end{aligned} \tag{25}$$

In order to confront the above neutrino matrix with the experimental observations (Table I), we use the constraints on neutrino parameters at 2σ and 3σ [12].

For numerical analysis, we use the 3σ ranges of neutrino oscillation parameters. The neutrino mass matrix M_ν depends on $|\alpha|$, $|y_1|$, $|y_2|$, $|y_3|$ and the phases ψ, φ_1, φ_2, and φ_3.

Since there are many unknown parameters, we consider a particular set of those parameters and show how the measured values of neutrino experiments can be accommodated in our neutrino mass matrix M_ν. For simplicity, we take $\varphi_2 = \varphi_3 = 0$ as inputs.

To see how the nonzero value of the phase φ_1 can lift the degeneracy between m_1 and m_2, we notice that for $y_3 = y_2 = 0$, the mass squared differences become

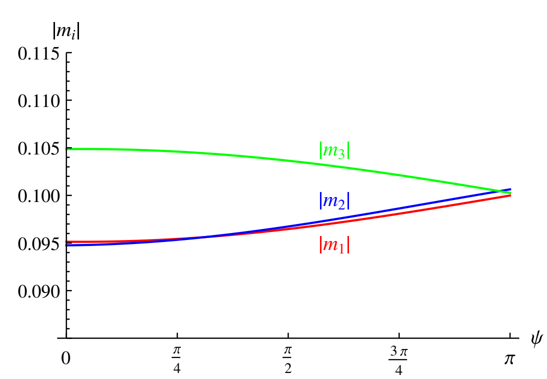

FIG. 1 (color online). Neutrino's masses versus ψ.

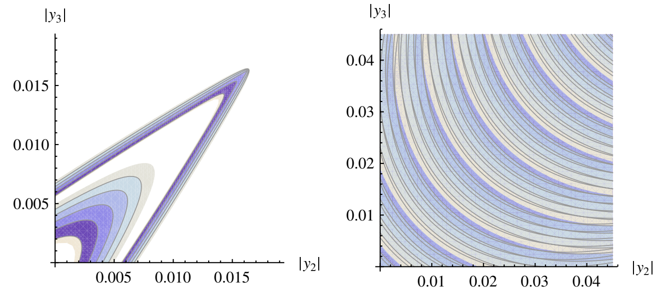

FIG. 2 (color online). Contour plot of the mass squared differences Δm_{21}^2 (left panel) and Δm_{31}^2 (right panel) in the parameter space $(|y_1|, |y_2|)$.

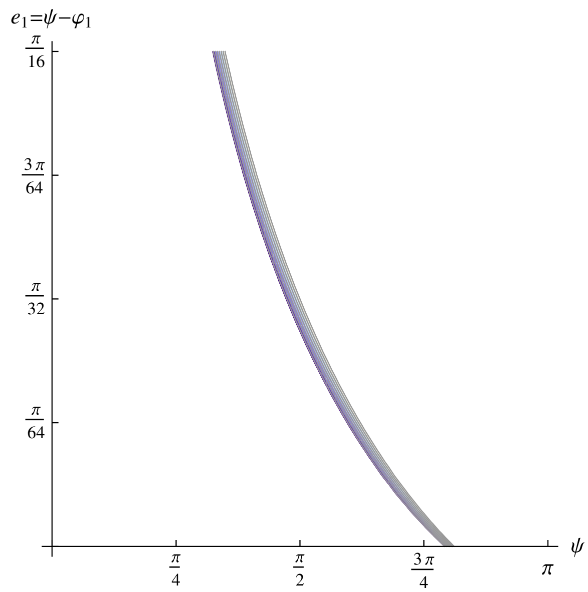

FIG. 3 (color online). Contour plot of the ratio R_ν in the parameter space (ψ, ϵ_1).

$$\Delta m_{21}^2 = 4|\alpha|^2|y_1| \sin\frac{\psi}{2} \sin\frac{\psi - \varphi_1}{2},$$
$$\Delta m_{31}^2 = 4|\alpha|^2|y_1| \cos\frac{\varphi_1}{2}, \quad (26)$$
$$\Delta m_{32}^2 = 4|\alpha|^2|y_1| \cos\frac{\psi}{2} \cos\frac{\psi - \varphi_1}{2}.$$

It is worthwhile to remark that for $\varphi_1 = \psi$, the masses m_1 and m_2 are degenerate and $|m_3| > |m_2|$. Now to separately obtain a nonzero mass squared difference Δm_{21}^2, which is smaller than Δm_{31}^2, we introduce a small phase difference $\epsilon_1 = \psi - \varphi_1$ between the phases ψ and φ_1. Such a small phase difference will be responsible for lifting the mass degeneracy between the first and second generation.

To see the behavior of the mass eigenvalues with respect to ψ and other observables, we take $\epsilon_1 = 6°$ as a typical value with $|\alpha| = 0.1$ eV, $|y_1| = 5 \times 10^{-2}$, $|y_2| = 1.3 \times 10^{-3}$, and $|y_3| = 1.2 \times 10^{-3}$. Figure 1 shows the variation of the masses $|m_i|$ as a function of the phase ψ. One clearly observes that Fig. 1 suggests a normal hierarchical ordering pattern for $\psi > 70°$.

Based on the expression of the neutrino mass squared differences, we numerically scan over a broader range of $|y_1|$ $(0.05 \leq |y_1| \leq 0.4)$ to obtain the restriction on the parameter space of $|y_2|$ and $|y_3|$. We have plotted in Fig. 2 the contour plots of the mass squared differences in the two-dimensional parameter spaces $(|y_2|, |y_3|)$ where we take $\psi = 120°$, $\epsilon_1 = 6°$, and $|\alpha| = 0.1$ eV as inputs.

From Fig. 2, we obtain the allowed ranges of $|y_2|$ and $|y_3|$ for the normal mass hierarchy,

$$|y_i| \leq 0.02 \quad \text{for } i = 2, 3. \quad (27)$$

The ratio R_ν for normal mass hierarchy ordering in the 3σ allowed range is

$$R_\nu = (2.99^{+0.32}_{-0.34}) \times 10^{-2}. \quad (28)$$

Figure 3 shows contour plot of the ratio R_ν in the two parameter space $(\psi, \epsilon_1 = \psi - \varphi_1)$, where $|y_1| = 0.05$,

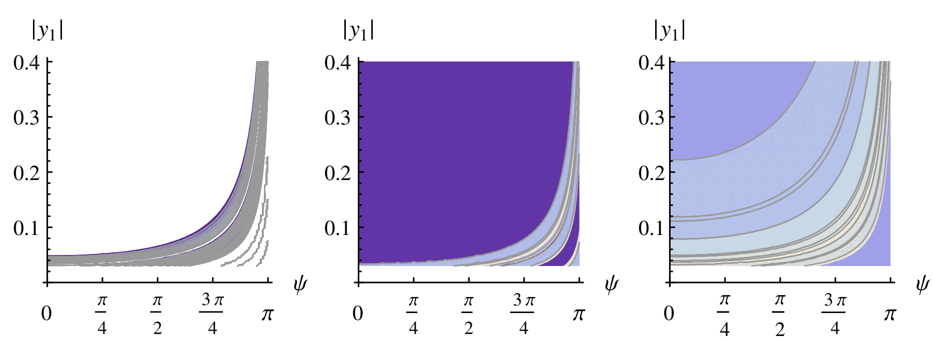

FIG. 4 (color online). Contour plots of reactor angle θ_{13} (left panel), solar angle θ_{12} (middle panel), and atmospheric angle θ_{23} (right panel) in the parameter space $|y_1|$ and ψ.

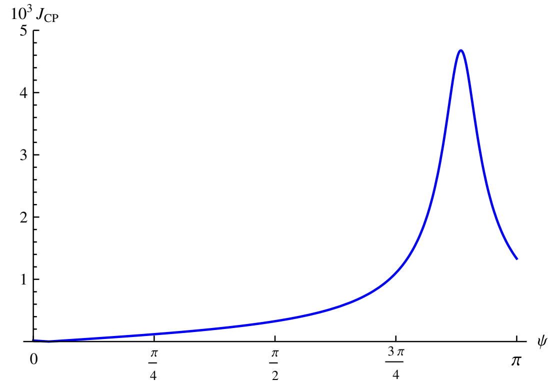

FIG. 5 (color online). CP-violating parameter J_{CP} versus ψ.

$|y_2| = 1.3 \times 10^{-3}$, and $|y_3| = 1.2 \times 10^{-3}$. As expected, the ratio R_ν at 3σ indicates that ψ has to be greater than 70°.

The departure of the mixing angles from TBM mixing angles depend on the phase ϵ_1 and the two parameters $|y_2|$ and $|y_3|$. Scanning over $|y_2|$ and $|y_3|$ within their allowed ranges ($|y2|$, $|y_3| \leq 0.02$), we investigate how a nonzero θ_{13} can be obtained for normal mass hierarchy. As a result of numerical analysis, contour plots in the $(\psi, |y_1|)$ parameter plane of the mixing angles θ_{13}, θ_{12} and θ_{23} are shown in Fig. 4 using the experimental constraints on the measured angles.

The CP violation parameter J_{CP}, which is directly related to the Dirac phase δ, arises due to nonzero value of the difference $|y_3| - |y_2|$ in the neutrino matrix M_ν. We have plotted in Fig. 5, J_{CP} with respect to ψ using the allowed region of $|y_i$. It leads to values of $|J_{CP}|$ around 10^{-3}. Nonvanishing $|J_{CP}|$ will be explored by the next generation high performance long-baseline neutrino experiments.

The absolute neutrino mass scale can be probed by nonoscillatory neutrino experiments. Cosmology is sensitive to the sum of neutrino masses $\sum |m_i|$. The beta decay endpoint measurements probe the so-called effective electron neutrino mass m_β. The rate of the neutrinoless double beta decay depends on the effective Majorana mass of the electron neutrino $m_{\beta\beta}$.

Both $m_{\beta\beta}$ and m_β and the sum of neutrino masses are given by

$$m_{\beta\beta} = \left|\sum_i m_i V_{ei}^2\right|, \qquad m_\beta = \sqrt{\sum_i m_i^2 |V_{ei}|^2}, \qquad \sum |m_i| = |m_1| + |m_2| + |m_3|. \tag{29}$$

From the neutrino mass M_ν, the effective Majorana masses $m_{\beta\beta}$ and m_β can be written as

$$m_{\beta\beta} = \frac{|\alpha|}{3}\sqrt{5 + 4\cos\psi - 6|y_1|\left(\sin\frac{\psi - \varphi_1}{2} + 2\sin\frac{\varphi_1}{2}\right) + 9|y_1|^2}, \tag{30}$$

$$m_\beta = |\alpha|\sqrt{1 - \frac{2}{3}y_1\left(2\cos\frac{\varphi_1}{2} + \cos\left(\psi - \frac{\varphi_1}{2}\right)\right) + \frac{4}{3}(y_2 + y_3)\cos^2\frac{\psi}{2} + y_1^2 + y_2^2 + y_3^2}.$$

Present cosmological constraints on the sum of neutrino masses $\sum |m_i|$ are in the range 0.44–0.76 eV [13]. The Mainz [14] and Troitsk [15] experiments on the high precision measurement of the end-point part of the β spectrum of ^3H decay found the 95% C.L. upper bounds $m_\beta \leq 2.3$ eV (Mainz) and $m_\beta \leq 2.1$ eV (Troitsk). Experimental bound on $m_{\beta\beta}$ is below 0.36 eV [16].

Figure 6 gives the effective electron neutrino mass m_β, the effective Majorana mass $m_{\beta\beta}$ and the sum of neutrino masses $\sum |m_i|$ with respect to ψ for $\epsilon_1 = 6°$. It shows the predicted m_β, $m_{\beta\beta}$ and $\sum |m_i|$ are well below the experimental bounds. The magnitude of $m_{\beta\beta}$ increases with larger $|y_1|$ values.

I would like to thank S. Nasri and J. Schechter for reading the manuscript and useful comments.

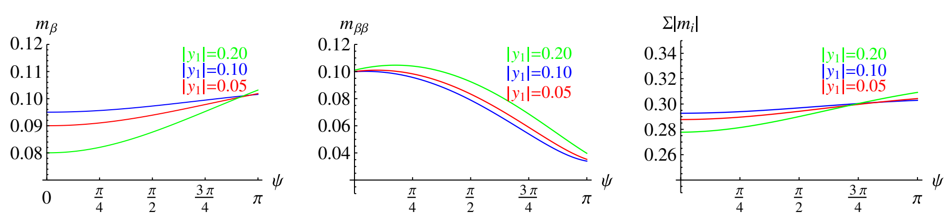

FIG. 6 (color online). m_β, $m_{\beta\beta}$ and $\sum |m_i|$ versus ψ.

[1] L. Whitehead (MINOS Collaboration), recent results from MINOS, *Joint Experimental—Theoretical Seminar, June 24, 2011, Fermilab, USA* (unpublished), http://theory.fnal.gov/jetp, http://www-numi.fnal.gov/pr_plots.

[2] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **107**, 181802 (2011).

[3] K. Abe *et al.* (T2K Collaboration), Phys. Rev. Lett. **107**, 041801 (2011).

[4] H. De Kerrect, *Low Nu 2011, Seoul, South Korea* (unpublished), http://workshop.kias.re.kr/lownu11/.

[5] F. P. An *et al.* (The Daya Bay Collaboration), Phys. Rev. Lett. **108**, 171803 (2012).

[6] J. K. Ahn *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012).

[7] B. Pontecorvo, J. Exp. Theor. Phys. **33**, 549 (1957) [Sov. Phys. JETP **6**, 429 (1958)]; J. Exp. Theor. Phys. **34**, 247 (1958) [Sov. Phys. JETP **7**, 172 (1958)]; Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[8] L. Wolfenstein, Phys. Rev. D **18**, 958 (1978); S. Pakvasa and H. Sugawara, Phys. Lett. B **73**, 61 (1978); **82**, 105 (1979); E. Derman and H. S. Tsao, Phys. Rev. D **20**, 1207 (1979); S.-L. Chen, M. Frigerio, and E. Ma, Phys. Rev. D **70**, 073008 (2004); **70**, 079905(E) (2004); M. Fukugita, M. Tanimoto, and T. Yanagida, Phys. Rev. D **57**, 4429 (1998); E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001); Z.-z. Xing, D. Yang, and S. Zhou, Phys. Lett. B **690**, 304 (2010); D. A. Dicus, S.-F. Ge, and W. W. Repko, Phys. Rev. D **82**, 033005 (2010); H. Fritzsch and Z.-Z. Xing, Phys. Lett. B **440**, 313 (1998); P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002); Z.-Z. Xing, Phys. Lett. B **533**, 85 (2002); X. G. He and A. Zee, Phys. Lett. B **560**, 87 (2003); P. F. Harrison and W. G. Scott, Phys. Lett. B **557**, 76 (2003); C. I. Low and R. R. Volkas, Phys. Rev. D **68**, 033007 (2003); A. Zee, Phys. Rev. D **68**, 093002 (2003); J. D. Bjorken, P. F. Harrison, and W. G. Scott, Phys. Rev. D **74**, 073012 (2006); R. N. Mohapatra, S. Nasri, and H. B. Yu, Phys. Lett. B **639**, 318 (2006); S. F. King, Nucl. Phys. **B576**, 85 (2000); S. F. King and N. N. Singh, Nucl. Phys. **B591**, 3 (2000); E. Ma, Phys. Rev. D **70**, 031901 (2004); M. Hirsch, A. Velanova del Morel, J. W. F. Valle, and E. Ma, Phys. Rev. D **72**, 119904 (2005); A. Montdragon, M. Montdragon, and E. Peinado, J. Phys. A **41**, 304035 (2008); T. Fukuyama and H. Nishiura, arXiv:hep-ph/9702253; R. N. Mohapatra and S. Nussinov, Phys. Rev. D **60**, 013002 (1999); E. Ma and M. Raidal, Phys. Rev. Lett. **87**, 011802 (2001); C. S. Lam, Phys. Lett. B **507**, 214 (2001); T. Kitabayashi and M. Yasue, Phys. Rev. D **67**, 015006 (2003); W. Grimus and L. Lavoura, Phys. Lett. B **572**, 189 (2003); Y. Koide, Phys. Rev. D **69**, 093001 (2004); Y. H. Ahn, Sin Kyu Kang, C. S. Kim, and Jake Lee, Phys. Rev. D **73**, 093005 (2006); A. Ghosal, arXiv:hep-ph/0304090; W. Grimus and L. Lavoura, Phys. Lett. B **572**, 189 (2003); J. Phys. G **30**, 73 (2004); W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka, and M. Tanimoto, Nucl. Phys. **B713**, 151 (2005); R. N. Mohapatra, J. High Energy Phys. 10 (2004) 027; A. de Gouvea, Phys. Rev. D **69**, 093007 (2004); R. N. Mohapatra and W. Rodejohann, Phys. Rev. D **72**, 053001 (2005); T. Kitabayashi and M. Yasue, Phys. Lett. B **621**, 133 (2005); R. N. Mohapatra and S. Nasri, Phys. Rev. D **71**, 033001 (2005); R. N. Mohapatra, S. Nasri, and H. B. Yu, Phys. Lett. B **615**, 231 (2005); K. Matsuda and H. Nishiura, Phys. Rev. D **73**, 013008 (2006); A. Joshipura, Eur. Phys. J. C **53**, 77 (2007); R. N. Mohapatra, S. Nasri, and H. B. Yu, Phys. Lett. B **636**, 114 (2006); T. Kaneko and H. Sugawara, Phys. Lett. B **697**, 329 (2011).

[9] R. Jora, S. Nasri, and J. Schechter, Int. J. Mod. Phys. A **21**, 5875 (2006); R. Jora, J. Schechter, and M. Naeem Shahid, Phys. Rev. D **80**, 093007 (2009); **82**, 079902(E) (2010); **82**, 053006 (2010); arXiv:1210.6755 [Phys. Rev. D (to be published)].

[10] S. Dev, S. Gupta, and R. R. Gautam, Phys. Lett. B **702**, 28 (2011); **708**, 284 (2012).

[11] P. F. Harrison and W. G. Scott, Phys. Lett. B **557**, 76 (2003); C. S. Lam, Phys. Lett. B **640**, 260 (2006); W. Grimus and L. Lavoura, J. High Energy Phys. 09 (2008) 106; C. H. Albright and W. Rodejohann, Eur. Phys. J. C **62**, 599 (2009); S. Kumar, Phys. Rev. D **82**, 013010 (2010).

[12] D. V. Forero, M. Tortola, and J. W. F. Valle, Phys. Rev. D **86**, 073012 (2012).

[13] S. Hannestad, A. Mirizzi, G. G. Raffelt, and Y. Wong, J. Cosmol. Astropart. Phys. 08 (2010) 001; M. C. Gonzalez-Garcia, M. Maltoni, and J. Salvado, J. High Energy Phys. 08 (2010) 117.

[14] C. Kraus *et al.*, Eur. Phys. J. C **40**, 447 (2005).

[15] V. N. Aseev *et al.*, Phys. Rev. D **84**, 112003 (2011).

[16] J. J. Gomez-Cadenas, J. Martin-Albo, M. Mezzetto, F. Monrabal, and M. Sorel, Riv. Nuovo Cimento **35**, 29 (2012).