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Possibility of measuring the *CP* Majorana phases in $0\nu\beta\beta$ decay

F. Šimkovic,[1,2] S. M. Bilenky,[2,3] Amand Faessler,[4] and Th. Gutsche[4]

[1]*Comenius University, Mlynská dolina F1, SK-842 48 Bratislava, Slovakia and IEAP CTU, 128-00 Prague, Czech Republic*
[2]*Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow region, Russia*
[3]*TRIUMF 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3*
[4]*Institute of Theoretical Physics, University of Tuebingen, 72076 Tuebingen, Germany*
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In view of recent measurements of the mixing angle θ_{13}, we investigate the possibility to determine the difference of two *CP* Majorana phases of the neutrino mixing matrix from the study of neutrinoless double-beta decay. We show that if cosmological measurements will reach the sensitivity of 0.1 eV for the sum of neutrino masses, i.e., the mass value of the lightest neutrino will be strongly constrained, the long-baseline neutrino oscillation experiments will determine inverted hierarchy of neutrino masses and if neutrinoless double-beta decay will be observed, this determination might be possible. The required experimental accuracies and the uncertainties in the calculated nuclear matrix elements of the process are discussed in this context.

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I. INTRODUCTION

The discovery of neutrino oscillations in the SuperKamiokande (atmospheric neutrinos) [1], SNO (solar neutrinos) [2], KamLAND (reactor neutrinos) [3], MINOS [4] (accelerator neutrinos), and other neutrino experiments gives us compelling evidence that neutrinos possess small masses and flavor neutrino fields are mixed.

Neutrino flavor states $|\nu_\alpha\rangle$ $(\alpha = e, \mu, \tau)$ are connected to the states of neutrinos with masses m_j ($|\nu_j\rangle$) by the following standard mixing relation:

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U_{\alpha j}^{*}|\nu_j\rangle \qquad (\alpha = e, \mu, \tau), \tag{1}$$

where U is the 3×3 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) unitary mixing matrix. If the massive neutrinos are Dirac (Majorana) particles, the PMNS matrix contains one (three) *CP* phases.

The problem to determine the *CP* phases is one of the major challenges in today's neutrino physics. Some information about lepton phases could help to solve the problem of the baryon asymmetry of the Universe [5]. The discovery in the Daya Bay [6], RENO [7], T2K [8], and Double Chooz [9] experiments of a relatively large mixing angle θ_{13} opened a possibility of measuring the Dirac phase δ in long baseline accelerator experiments. The Majorana *CP* phases can only be determined through the observation of neutrinoless double β decay ($0\nu\beta\beta$ decay) [10–12].

The *CP* phases enter into the effective Majorana mass [13–17] defined as

$$m_{\beta\beta} = \sum_j U_{ej}^2 m_j. \tag{2}$$

This quantity depends also on the neutrino oscillation parameters θ_{12}, θ_{13}, $\Delta m^2_{\mathrm{SUN}}$ $\Delta m^2_{\mathrm{ATM}}$, the lightest neutrino mass, and the type of the neutrino mass spectrum (normal or inverted).

The effective Majorana mass $m_{\beta\beta}$ can be determined in the neutrinoless double beta decay of even-even nuclei [18–20]

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- \tag{3}$$

by relating the $0\nu\beta\beta$-decay half-life to $|m_{\beta\beta}|$ using calculated nuclear matrix elements (NMEs).

The goal of this paper is to discuss a possibility for determining the Majorana *CP* phases from data of $0\nu\beta\beta$ experiments of the next generation assuming that the $0\nu\beta\beta$ decay will be observed. The problem of the experimental accuracies and the theoretical uncertainties of the calculated NMEs will also be addressed.

II. NEUTRINO OSCILLATIONS AND EFFECTIVE MAJORANA MASS

It was proved in experiments with atmospheric, solar, reactor, and accelerator neutrinos that flavor neutrinos oscillate from one flavor (electron-, muon-, and tau-) to another due to neutrino mixing and nonzero neutrino mass-squared differences. All existing neutrino oscillation data (with the exception of the LSND [21], MiniBooNE [22], short baseline reactor [23], and gallium [24]) anomalies) are perfectly described by the minimal scheme of three-neutrino mixing.

In the case of Dirac neutrinos, the unitary 3×3 PMNS neutrino mixing matrix can be parametrized as follows:

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix}, \tag{4}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$. θ_{12}, θ_{13}, and θ_{23} are mixing angles and δ is the CP phase. If neutrinos are Majorana particles, the matrix U in Eq. (4) is multiplied by a diagonal phase matrix $P = \mathrm{diag}(e^{i(\alpha_1/2-\delta)}, e^{i(\alpha_2/2-\delta)}, 1)$, which contains two additional CP phases α_1 and α_2.

With the discovery of neutrino oscillations we know:

(i) The values of the large mixing angles θ_{12} and θ_{23}. The value of the relatively small angle θ_{13} recently measured in the Double Chooz [9], the Daya Bay [6], and RENO [7] reactor neutrino experiments.

(ii) The solar and atmospheric mass-squared differences[1] $\Delta m^2_{\mathrm{SUN}} = \Delta m^2_{12}$ and $\Delta m^2_{\mathrm{ATM}} = \Delta m^2_{23}$ (normal spectrum), $\Delta m^2_{\mathrm{ATM}} = -\Delta m^2_{13}$ (inverted spectrum).

We do not know the value of the lightest neutrino mass, the CP phases, and the character of the neutrino mass spectrum (normal or inverted).

From the data of the MINOS experiment [4] it was found that $\Delta m^2_{\mathrm{ATM}} = (2.43 \pm 0.13) \times 10^{-3}\ \mathrm{eV}^2$. From the analysis of the KamLAND and solar data it was obtained that $\tan^2\theta_{12} = 0.452^{+0.035}_{-0.033}$ [3]. From the global fit to all data it was inferred that [25] $\Delta m^2_{\mathrm{SUN}} = (7.65^{+0.13}_{-0.20}) \times 10^{-5}\ \mathrm{eV}^2$ and $\sin^2\theta_{23} = 0.50^{+0.07}_{-0.06}$. Finally, from the analysis of the Daya Bay [6] and RENO data [7], one obtains $\sin^2 2\theta_{13} = 0.092 \pm 0.016(\mathrm{stat}) \pm 0.005(\mathrm{syst})$ and $\sin^2 2\theta_{13} = 0.103 \pm 0.013(\mathrm{stat}) \pm 0.011(\mathrm{syst})$, respectively.

The effective Majorana mass is given by

$$|m_{\beta\beta}| = |c^2_{12}c^2_{13}e^{i\alpha_1}m_1 + s^2_{12}c^2_{13}e^{i\alpha_2}m_2 + s^2_{13}m_3| \tag{5}$$

or by the full expression

$$\begin{aligned} |m_{\beta\beta}|^2 = {} & c^4_{12}c^4_{13}m^2_1 + s^4_{12}c^4_{13}m^2_2 + s^4_{13}m^2_3 \\ & + 2c^2_{12}s^2_{12}c^4_{13}m_1m_2\cos(\alpha_1 - \alpha_2) \\ & + 2c^2_{12}c^2_{13}s^2_{13}m_1m_3\cos\alpha_1 \\ & + 2s^2_{12}c^2_{13}s^2_{13}m_2m_3\cos\alpha_2. \end{aligned} \tag{6}$$

From this equation it simply follows that the effective Majorana mass depends on the character of the neutrino mass spectrum and three unknown parameters: the lightest neutrino mass and the two CP phases. We note that for two sets of phases α_1, α_2 and $(2\pi - \alpha_1)$, $(2\pi - \alpha_2)$, the same value of $|m_{\beta\beta}|$ is reproduced.

In the three-neutrino case, two mass spectra are currently possible:

(i) Normal spectrum (NS): $m_1 < m_2 < m_3$: $\Delta m^2_{12} \ll \Delta m^2_{23}$. In this case

$$m_2 = \sqrt{\Delta m^2_{\mathrm{SUN}} + m^2_0},$$
$$m_3 = \sqrt{\Delta m^2_{\mathrm{ATM}} + \Delta m^2_{\mathrm{SUN}} + m^2_0}$$

with $m_0 = m_1$.

(ii) Inverted spectrum (IS), $m_3 < m_1 < m_2$: $\Delta m^2_{12} \ll |\Delta m^2_{13}|$. We have

$$m_1 = \sqrt{\Delta m^2_{\mathrm{ATM}} + m^2_0},$$
$$m_2 = \sqrt{\Delta m^2_{\mathrm{ATM}} + \Delta m^2_{\mathrm{SUN}} + m^2_0}$$

with $m_0 = m_3$.

For both cases $m_0 = m_1(m_3)$ is the lightest neutrino mass.

For the two neutrino mass hierarchies we can set constraints on the effective Majorana mass:

(1) Normal hierarchy (NH): $m_1 \ll m_2 \ll m_3$: In this case for the neutrino masses we have

$$m_1 \ll \sqrt{\Delta m^2_{\mathrm{SUN}}}, \qquad m_2 \simeq \sqrt{\Delta m^2_{\mathrm{SUN}}},$$
$$m_3 \simeq \sqrt{\Delta m^2_{\mathrm{ATM}}}.$$

Neglecting the negligibly small contribution of m_1 we find

$$\cos\alpha_2 \simeq \frac{|m_{\beta\beta}|^2 - s^4_{12}c^4_{13}\Delta m^2_{\mathrm{SUN}} - s^4_{13}\Delta m^2_{\mathrm{ATM}}}{2s^2_{12}c^2_{13}s^2_{13}\sqrt{\Delta m^2_{\mathrm{SUN}}\Delta m^2_{\mathrm{ATM}}}}. \tag{7}$$

For the effective Majorana mass we then have the following range of values:

$$\begin{aligned} & |s^2_{12}c^2_{13}\sqrt{\Delta m^2_{\mathrm{SUN}}} - s^2_{13}\sqrt{\Delta m^2_{\mathrm{ATM}}}| \\ & \quad \le |m_{\beta\beta}| \le s^2_{12}c^2_{13}\sqrt{\Delta m^2_{\mathrm{SUN}}} + s^2_{13}\sqrt{\Delta m^2_{\mathrm{ATM}}}. \end{aligned} \tag{8}$$

Using the best-fit values of the mass squared differences and the mixing angles we find

$$1.5\ \mathrm{meV} \le |m_{\beta\beta}| \le 3.8\ \mathrm{meV}.$$

(2) Inverted hierarchy (IH): $m_3 \ll m_1 < m_2$: In the IH scenario $m_3 \ll \sqrt{\Delta m^2_{\mathrm{ATM}}}$ and $m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_{\mathrm{ATM}}}$. We find

$$\cos\alpha_{12} = \frac{|m_{\beta\beta}|^2 - c^4_{13}(1 - 2s^2_{12}c^2_{12})\Delta m^2_{\mathrm{ATM}}}{2c^2_{12}s^2_{12}c^4_{13}\Delta m^2_{\mathrm{ATM}}}, \tag{9}$$

where $\alpha_{12} = \alpha_1 - \alpha_2$. For the absolute value of the effective Majorana mass we have

[1] We use the following definition $\Delta m^2_{ij} = m^2_j - m^2_i$.

$$|\cos 2\theta_{12}| c_{13}^2 \sqrt{\Delta m_{\rm ATM}^2} \leq |m_{\beta\beta}| \leq c_{13}^2 \sqrt{\Delta m_{\rm ATM}^2}. \tag{10}$$

Using the best-fit values of the parameters we find the following range for $|m_{\beta\beta}|$ in the case of the IH:

$$18 \text{ meV} \leq |m_{\beta\beta}| \leq 48 \text{ meV}.$$

The absolute value of the neutrino mass can be determined from a precise measurement of the end-point part of the β spectrum of the tritium [26] and other β-decay measurements [27]. Cosmological observations allow to infer the sum of the neutrino masses

$$m_{\rm cosmo} = \sum_k^3 m_k. \tag{11}$$

For inverted and normal hierarchy of neutrino masses there is a minimal value of $m_{\rm cosmo}$ allowed by the oscillation data as follows:

$$\begin{aligned} m_{\rm cosmo} &\simeq 2\sqrt{\Delta m_{\rm ATM}^2} \simeq 105 \text{ meV} \quad \text{(IH)} \\ &\simeq \sqrt{\Delta m_{\rm ATM}^2} \simeq 62 \text{ meV} \quad \text{(NH)}. \end{aligned} \tag{12}$$

The current limits on $m_{\rm cosmo}$ depend on the type of observations included in the fit [28]. The CMB primordial gives ≤ 1.3 eV, CMB + distance ≤ 0.58 eV, galaxy distribution and lensing of galaxies ≤ 0.6 eV. On the other hand, the largest photometric redshift survey yields ≤ 0.28 eV [29]. It is expected that future cosmological observables will provide precise constraints on the sum of neutrino masses $m_{\rm cosmo}$ [30]. These constraints will be such that they are even sensitive to the minimal values of 0.105 and 0.062 eV allowed by the oscillation data for the IH and NH, respectively (see, e.g., the recent summary [28]). In the case of the IH and for the lowest value of $m_{\rm cosmo}$, the value of the lightest neutrino mass m_0 can be restricted to values below a value of about 10 meV depending on the accuracy of the cosmological measurement. We note that the neutrino mass hierarchy can be probed with accelerator based neutrino oscillation experiments through earth matter effects [31,32]. However, from neutrino oscillation experiments alone one cannot determine the absolute neutrino mass scale or even constrain the mass of the lightest neutrino unlike for the case of cosmological measurements.

In Fig. 1, by exploiting Eq. (6), the Majorana *CP* phase α_2 (or difference of phases $\alpha_{21} = \alpha_2 - \alpha_1$) is plotted as a function of the absolute value of the effective Majorana mass for chosen values of m_0 and by assuming the NH (IH) of neutrino masses. The second phase α_1 is considered to be arbitrary. The results strongly depend on the value of m_0 and the type of neutrino mass hierarchy, normal or inverted. We find that when m_0 lies within a range of 0 to 10 meV and when the IH is considered, the phase difference α_{21} depends only weakly on $|m_{\beta\beta}|$. This is due to the fact that the third term on the right-hand side of Eq. (5) is small in comparison with the first two terms. A different situation occurs in the case of the NH. The results depend strongly on m_0 in the considered range of (0–10) meV. There is practically no chance to determine a value for m_0 by any laboratory or cosmological measurement, if it is lower than a few meV. Thus, by measuring $0\nu\beta\beta$ decay it will not be possible to obtain model independent information on the value of at least one of the three *CP* Majorana phases when there is a normal hierarchy of neutrino

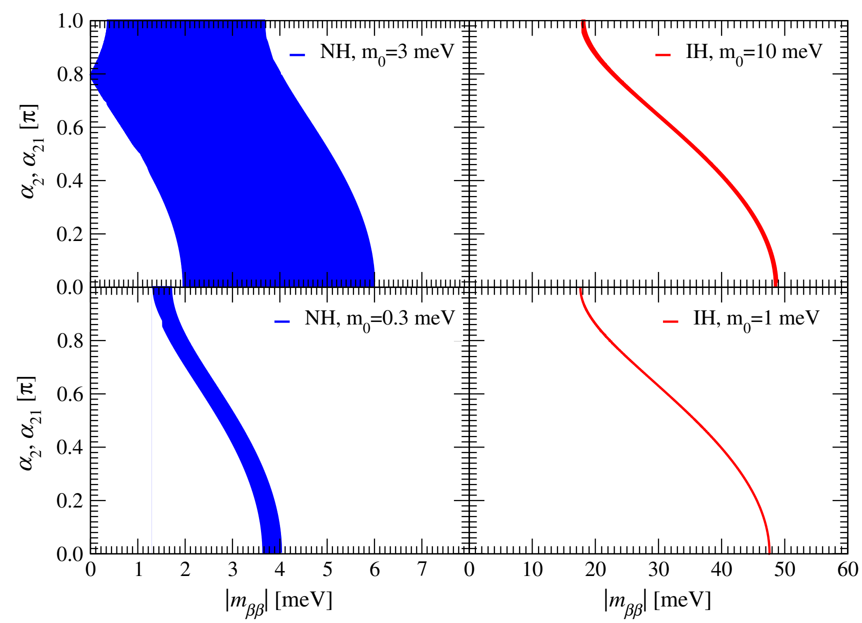

FIG. 1 (color online). The *CP* phase α_2 (left panels) and difference of phases α_{21} (right panels) are plotted, respectively, as function of the absolute value of the effective Majorana mass for the IH and NH of neutrino masses and the chosen value of m_0.

masses. One can rely only on those particle physics models which allow one to predict all three neutrino masses. In these cases values for one of the *CP* Majorana phases could be obtained when considering the second phase to be arbitrary and by observing the $0\nu\beta\beta$ decay. From Fig. 1 it follows that if m_0 is about 3 meV this possibility is also very much limited. It is interesting to note that for this value of m_0 the minimal value of $|m_{\beta\beta}|$ does not appear for the case of *CP* conservation (see the left upper panel of Fig. 1) but for $\alpha_2 \simeq 0.79\pi$.

III. EFFECTIVE MAJORANA MASS AND THE $0\nu\beta\beta$ DECAY

Assuming that the $0\nu\beta\beta$ decay is driven by the Majorana neutrino mass mechanism, we have for the effective Majorana mass $m_{\beta\beta}$,

$$|m_{\beta\beta}| = \frac{m_e}{\sqrt{T^{0\nu}_{1/2} G^{0\nu}(Q_{\beta\beta}, Z) g_A^2 |M'^{0\nu}|}}. \tag{13}$$

Here, $T^{0\nu}_{1/2}$, $G_{0\nu}(Q_{\beta\beta}, Z)$, g_A, and $M'^{0\nu}$ are, respectively, the half-life of the $0\nu\beta\beta$ decay, the known phase-space factor, the unquenched axial-vector coupling constant, and the nuclear matrix element, which depends on nuclear structure. Recently, a complete and improved calculation of phase-space factors for $0\nu\beta\beta$ decay was presented in Ref. [33]. The exact Dirac wave functions with finite nuclear size and electron screening were considered. It is believed that the calculated phase factors are not a source of uncertainty in the determination of the effective Majorana mass from the measured half-life.

The future experiments, CUORE (^{130}Te), EXO, KamLAND-Zen (^{136}Xe), MAJORANA/GERDA (^{76}Ge), SuperNEMO (^{82}Se), SNO + (^{150}Nd), and others [18], with a sensitivity

$$|m_{\beta\beta}| \simeq \text{a few} 10^{-2}\ \text{eV} \tag{14}$$

will probe the IH of neutrino masses. In the case of the normal mass hierarchy $|m_{\beta\beta}|$ is much too small in order that $0\nu\beta\beta$ decay will be detected in experiments of the next generation.

If the $0\nu\beta\beta$ decay will be observed, the measured half-life $T^{0\nu-\exp}_{1/2}$ with experimental error $\sigma_{\exp}$ can be converted into an "observed effective Majorana mass" $|m_{\beta\beta}|^{\text{obs}}$ and its error $\sigma_{\beta\beta}$ as

$$\frac{\sigma_{\beta\beta}}{|m_{\beta\beta}|^{\text{obs}}} = \sqrt{\frac{1}{4}\left(\frac{\sigma_{\exp}}{T^{0\nu-\text{obs}}_{1/2}}\right)^2 + \left(\frac{\sigma_{\text{th}}}{|M'^{0\nu}|}\right)^2}. \tag{15}$$

Here, σ_{th} is the "theoretical error" of the nuclear matrix element $|M'^{0\nu}|$.

In Fig. 2 we plot the difference α_{21} of the *CP* Majorana phases as a function of $|m_{\beta\beta}|^{\text{obs}}$ for the inverted hierarchy of neutrino masses and by assuming 0% (blue region), 15% (red region), and 25% uncertainty (orange region) in $|m_{\beta\beta}|^{\text{obs}}$. The current experimental errors of neutrino mixing parameters and mass squared differences are taken into account. The lightest neutrino mass m_0 is assumed to be within a range from 0 to 10 meV and one of the *CP* violating phases α_1 (or α_2) is taken to be arbitrary. We see that if the considered accuracies are achieved it can be possible to determine the value of the *CP* phase difference α_{12}. However, for

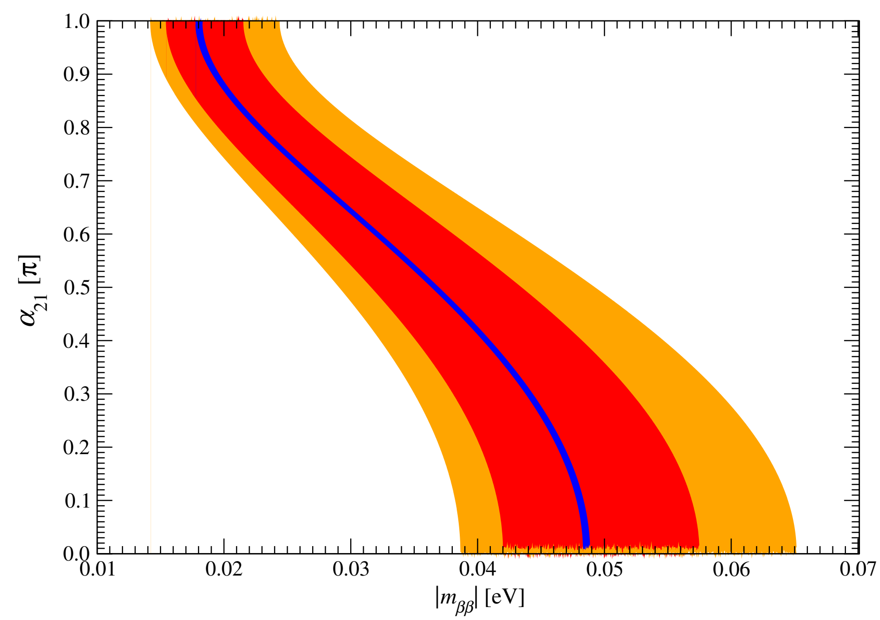

FIG. 2 (color online). The difference of *CP* phases $\alpha_{21} = \alpha_2 - \alpha_1$ plotted as a function of $|m_{\beta\beta}|^{\text{obs}}$ for the inverted hierarchy of neutrino masses. The current experimental errors of neutrino mixing parameters and mass squared differences are taken into account; m_0 is taken to be in the range of 0 to 10 meV. The blue, red, and orange regions correspond to $|m_{\beta\beta}|^{\text{obs}}$ with errors ($\sigma_{\beta\beta}/|m_{\beta\beta}|^{\text{obs}}$) of 0%, 15%, and 25%, respectively.

$\sigma_{\beta\beta}/|m_{\beta\beta}|^{\rm obs} > 50\%$ it will be difficult, or even impossible, to gain reliable information on the value of α_{12}.

The uncertainty $\sigma_{\rm th}$ in the calculated $0\nu\beta\beta$ decay NME is a complicated and more involved problem. $M'^{0\nu}$ consists of the Fermi (F), Gamow-Teller (GT), and tensor (T) parts [34–36]:

$$M'^{0\nu} = \left(\frac{g_A^{\rm eff}}{g_A}\right)^2 \left(-\frac{M_{\rm F}^{0\nu}}{(g_A^{\rm eff})^2} + M_{\rm GT}^{0\nu} - M_{\rm T}^{0\nu}\right). \qquad (16)$$

Here, $g_A^{\rm eff}$ is the quenched axial-vector coupling constant. $M'^{0\nu}$ is a function of $(g_A^{\rm eff})^2$, which appears in the Fermi matrix element and also enters in the calculation of the Gamow-Teller and tensor constituents due to a consideration of the nucleon weak-magnetism terms [37]. This definition of $M'^{0\nu}$ allows one to display the effects of the uncertainties in $g_A^{\rm eff}$ and to use the same phase factor $G^{0\nu}$ when calculating the $0\nu\beta\beta$-decay rate.

The treatment of quenching $g_A^{\rm eff}$ is an important source of difference between the calculated $0\nu\beta\beta$-decay NMEs [35,38]. Quenching of the axial-vector coupling was introduced to account for the fact that the calculated strengths of the Gamow-Teller β-decay transitions to individual final states are significantly larger than the experimental ones. Formally this is accomplished by replacing the true vacuum value of the coupling constant $g_A = 1.269$ by a quenched value $g_A^{\rm eff} = 1.0$. It is not clear whether a similar phenomenon exists for other multipoles besides $J = 1^+$.

Different nuclear structure methods have been used for the calculation of $M'^{0\nu}$, in particular the interacting shell model (ISM) [39,40], the quasiparticle random phase approximation (QRPA) [34–36], the projected Hartree-Fock Bogoliubov approach (PHFB, PQQ2 parametrization) [41], the energy density functional method (EDF) [42], and the interacting boson model (IBM) [43]. By assuming an unquenched g_A the ISM values of the NMEs are about a factor 2–3 smaller than the NMEs of other approaches (see Table 3 of Ref. [19]). The results of the QRPA, IBM, EDF, and projected Hartree-Fock-Bogoliubov approaches differ by a factor less than 2. Their results agree rather well with each other in the case of the $0\nu\beta\beta$ decay of ^{130}Te.

A detailed study of uncertainties in the calculated $0\nu\beta\beta$-decay NMEs was performed within the QRPA approach [34–36]. The average matrix element $\langle M'^{0\nu}\rangle$ (averaged over different nucleon-nucleon (NN) potentials, choices for the single particle space, variants of the QRPA approach) was evaluated as well as its variance σ:

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(M'^{0\nu}{}_i - \langle M'^{0\nu}\rangle)^2. \qquad (17)$$

Further progress was achieved by performing a self-consistent calculation of the NMEs in which the pairing and residual interactions as well as the two-nucleon short-range correlations were derived from the same modern nucleon-nucleon potentials, namely, from the charge-dependent Bonn potential (CD-Bonn) and the Argonne V18 potential [36]. The particle-particle strength of neutron-proton interaction was adjusted to the $2\nu\beta\beta$-decay half-life eliminating one of the main reasons for variability of the calculated $M'^{0\nu}$ within the QRPA-like methods [34]. We note that this procedure of fixing the particle-particle strength was also used in some earlier works [44], however, without pointing out this important consequence.

Recently, a further refinement of the QRPA method has been achieved by introducing a partial restoration of the isospin symmetry [45]. The particle-particle neutron-proton interaction was separated into its isovector and isoscalar parts and each were renormalized separately. The isoscalar channel of the NN interaction was fitted from the requirement that the calculated $2\nu\beta\beta$-decay half-life reproduces the experimental value. The strength of the isovector NN interaction was found to be close to the strength of the pairing interaction following the requirement of isospin symmetry of the particle-particle force, i.e., essentially no new parameter was introduced.

Here, we update the calculation of the average $\langle M'^{0\nu}\rangle$ and its variance σ for the $0\nu\beta\beta$ decay of ^{76}Ge, ^{130}Te, and ^{136}Xe. The recommended half-life value of $2\nu\beta\beta$ decay of ^{76}Ge [46] and the recently measured half-life of the $2\nu\beta\beta$ decay of ^{136}Xe [47,48] were considered. The calculations were performed for the CD-Bonn and Argonne potentials, three different sizes of the model space [34], the unquenched or quenched value of the axial–vector coupling constant.

The calculated sets of $N = 12$ NMEs for each of the three considered isotopes are presented in Table I. The results do not depend much on the size of the model space and on the type of the NN interaction. For a quenched weak coupling constant, the NMEs are significantly smaller than those for an unquenched g_A mostly due to the factor $(1.00/1.269) = 0.62$ entering in the definition of $M'^{0\nu}$ in Eq. (16). The largest value of the average matrix element $\langle M'^{0\nu}\rangle$ is for ^{76}Ge (4.62) followed by those for ^{130}Te (3.73) and ^{136}Xe (2.17), which is about half the value. The variance σ is about 15% of the full NME $\langle M'^{0\nu}\rangle$. Of course, these results are only valid for the QRPA approach and the considered averaging scheme.

It goes without saying that further progress in the calculation of the $0\nu\beta\beta$-decay NMEs is required. Thanks to the theoretical efforts made over the past years, the disagreement among the different NMEs is now much less severe than it was some years ago. Currently, the main issue of interest is that there exists significant disagreement of the ISM results with those of other approaches and the problem of quenching the axial-vector coupling constant. The uncertainty associated with the calculation of the $0\nu\beta\beta$-decay NMEs can be reduced by suitably chosen nuclear probes. Complementary experimental information from related processes like the $2\nu\beta\beta$ decay [46], charge-exchange [49], and particle transfer reactions [50] is also very important. The differences between the results of various nuclear structure approaches could be

TABLE I. Nuclear matrix element $M'^{0\nu}$ for ^{76}Ge, ^{130}Te, and ^{136}Xe calculated in QRPA with partial restoration of isospin symmetry [45]. Three different sizes of the single-particle space, two different types of NN interaction (CD-Bonn and Argonne), and quenched ($g_A = 1.00$) or unquenched ($g_A = 1.269$) values of the axial-vector coupling constant are considered, i.e., 12 values are presented for each isotope. The corresponding average matrix element $\langle M'^{0\nu}\rangle$ was evaluated as well as its variance σ (in parentheses) following Eq. (17). $G^{0\nu}(Q_{\beta\beta}, Z)$ is the phase-space factor, whose values are taken from Ref. [33]. The nuclear radius $R = r_0 A^{1/3}$ with $r_0 = 1.2$ fm is used.

			$M'^{0\nu}$				
Nucleus	Nucleon-nucleon potential	g_A	Minimal single-particle model space	Intermediate single-particle model space	Largest single-particle model space	$\langle M'^{0\nu}\rangle$ (σ)	$G^{0\nu}(Q_{\beta\beta}, Z)$ [y^{-1}]
^{76}Ge	Argonne	1.00	3.875	3.701	3.886	4.62(0.70)	2.36×10^{-15}
		1.269	5.134	4.847	5.157		
	CD-Bonn	1.00	4.161	4.034	4.211		
		1.269	5.514	5.290	5.571		
^{130}Te	Argonne	1.00	2.992	3.161	2.945	3.73(0.61)	14.22×10^{-15}
		1.269	3.989	4.229	3.888		
	CD-Bonn	1.00	3.317	3.492	3.297		
		1.269	4.438	4.683	4.373		
^{136}Xe	Argonne	1.00	1.761	1.867	1.643	2.17(0.37)	14.58×10^{-15}
		1.269	2.360	2.509	2.177		
	CD-Bonn	1.00	1.963	2.069	1.847		
		1.269	2.639	2.787	2.460		

understood by performing an anatomy of the $0\nu\beta\beta$-decay NME [35,38,51]. The recent development in the field is very encouraging. There is reason to believe that the uncertainty in the $0\nu\beta\beta$ decay will be further reduced.

IV. SUMMARY

The possible establishment of *CP* violation in the lepton sector is one of the most challenging problems of neutrino and astrophysics. Studies of $0\nu\beta\beta$ decay driven by Majorana neutrinos can lead to insights into *CP* violation in this sector. In view of recent measurement of the smallest neutrino mixing angle θ_{13} in the Double Chooz, Daya Bay, and RENO experiments, we revisited the problem of determining the Majorana *CP* phases by assuming the additional observation of the $0\nu\beta\beta$ decay.

Both cases of the normal and inverted hierarchy of neutrino masses were discussed. It was shown that in the case of the NH the determination of one of the Majorana *CP* phases could be possible only by knowledge of both the absolute value of the effective Majorana mass $|m_{\beta\beta}|$ and of the lightest neutrino mass m_1. This task cannot be solved by any of the planned or prepared neutrino experiments, only within some particle physics models which allow for a prediction of neutrino masses. It was also found that for some values of m_1 the minimal value of $|m_{\beta\beta}|$ is realized in the case of *CP* violation.

The case of the IH of neutrino masses offers different possibilities. Future cosmological measurements have the potential to constrain the lightest neutrino mass m_0 to values below 10 meV. The difference α_{21} of Majorana phases depends very weakly on m_0 for these low values and can be determined by an accurate value for $|m_{\beta\beta}|$. For this purpose, the $0\nu\beta\beta$-decay NME needs to be evaluated with an uncertainty of less than 30%. This is a formidable task, which might be achieved at some point in time due to further developments in the fields of nuclear structure and many-body physics also linked to a further increase of computer power.

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