PHYSICAL REVIEW D **87,** 066003 (2013)

Leading finite-size effects on some three-point correlators in $AdS_5 \times S^5$

Plamen Bozhilov*

Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria
(Received 15 January 2013; published 12 March 2013)

In the framework of the semiclassical approach, we find the leading finite-size effects on the normalized structure constants in some three-point correlation functions in $AdS_5 \times S^5$, expressed in terms of the conserved string angular momenta J_1, J_2, and the world-sheet momentum p_w, identified with the momentum p of the magnon excitations in the dual spin chain arising in $\mathcal{N} = 4$ super Yang-Mills theory in four dimensions.

DOI: 10.1103/PhysRevD.87.066003 PACS numbers: 11.25.Tq

I. INTRODUCTION

The correspondence between type IIB string theory on $AdS_5 \times S^5$ target space and the $\mathcal{N} = 4$ super Yang-Mills theory (SYM) in four space-time dimensions, in the planar limit, is the most studied example of the AdS/CFT duality [1]. A lot of impressive progress has been made in this field of research based on the integrability structures discovered on both sides of the correspondence (for a recent overview on AdS/CFT integrability, see Ref. [2]).

Various classical string solutions play an important role in testing and understanding the AdS/CFT correspondence. To establish relations with the dual gauge theory, we have to take the semiclassical limit of *large* conserved charges like string energy E and spins $S_{1,2}$ on AdS_5 and angular momenta $J_{1,2,3}$ on S^5 [3].

An example of such a string solution is the so-called "giant magnon," for which the energy E and the angular momentum J_1 go to infinity, but the difference $E - J_1$ is finite, while $S_{1,2} = 0$, $J_{2,3} = 0$ [4]. It lives on $R_t \times S^2$ subspace of $AdS_5 \times S^5$, and gave a strong support for the conjectured all-loop $SU(2)$ spin chain, arising in the dual $\mathcal{N} = 4$ SYM, and made it possible to get a deep insight into the AdS/CFT duality. This was extended to the giant magnon bound state ($J_2 \neq 0$), or *dyonic* giant magnon, corresponding to a string moving on $R_t \times S^3$ and related to the complex sine-Gordon model [5]. Further extension to $R_t \times S^5$ has also been worked out in Ref. [6], where it was also shown that such type of string solutions can be obtained by reduction of the string dynamics to the Neumann-Rosochatius integrable system. It can be used also for studding the *finite-size effects*, related to the wrapping interactions in the dual field theory [7]. From the string theory viewpoint, the leading, and even subleading finite size, effect on the giant magnon dispersion relation was first found and described in Ref. [8]. The case of the leading finite-size effect on the *dyonic* giant magnon dispersion relation was considered in Ref. [9]. There, the string theory result was compared with the result coming from the μ-term Lüscher correction, based on the S-matrix description. Both results coincide.

During the years, many important achievements concerning correlation functions in the AdS/CFT context have been obtained. Recently, interesting developments have been accomplished by considering general heavy string states [10–70].[1]

In Refs. [37,39], the three-point correlation functions of finite-size (dyonic) giant magnons [4,5] and three different "light" states have been obtained. They are given in terms of hypergeometric functions and several parameters. However, it is important to know their dependence on the conserved string charges J_1, J_2 and the world-sheet momentum p, because, namely, these quantities are related to the corresponding operators in the dual gauge theory, and the momentum of the magnon excitations in the dual spin chain. That is why we are going to find this dependence here. Unfortunately, this cannot be done exactly for the finite-size case due to the complicated dependence between the above mentioned parameters and J_1, J_2, p. Because of that, we will consider only the leading order finite-size effects on the three-point correlators. In this paper, we will restrict ourselves to the case of $AdS_5 \times S^5/\mathcal{N} = 4$ SYM duality.

The paper is organized as follows. In Sec. II, we first give a short review of the giant magnon solution. Then, we explain the limitations under which the three-point correlation functions considered here are computed and give the exact results in the semiclassical limit. Section III is devoted to the computation of the leading order finite-size effects on the three-point correlators given in Sec. II in terms of the conserved string angular momenta and the world-sheet momentum p. In Sec. IV we conclude with some final remarks.

II. FINITE-SIZE GIANT MAGNONS AND THREE-POINT CORRELATORS

A. Review of the giant magnon solutions

We denote with Y, X the coordinates in AdS_5 and S^5 parts of the background $AdS_5 \times S^5$:

*plbozhilov@gmail.com

[1] Some papers devoted to the field theory side of the problem are also included here.

$$Y_1 + iY_2 = \sinh\rho \sin\eta e^{i\varphi_1}, \quad Y_3 + iY_4 = \sinh\rho\cos\eta e^{i\varphi_2},$$
$$Y_5 + iY_0 = \cosh\rho e^{it}.$$

The coordinates Y are related to the Poincare coordinates by

$$Y_m = \frac{x_m}{z}, \qquad Y_4 = \frac{1}{2z}(x^m x_m + z^2 - 1),$$
$$Y_5 = \frac{1}{2z}(x^m x_m + z^2 + 1),$$

where $x^m x_m = -x_0^2 + x_i x_i$, with $m = 0, 1, 2, 3$ and $i = 1, 2, 3$. We parametrize S^5 as in Ref. [26].

Euclidean continuation of the timelike directions to $t_e = it$, $Y_{0e} = iY_0$, $x_{0e} = ix_0$ will allow the classical trajectories to approach the AdS_5 boundary $z = 0$ when $\tau_e \to \pm\infty$, and to compute the corresponding correlation functions.

The dyonic finite-size giant magnon solution, where (τ, σ) are the world-sheet coordinates, can be written as $(t = \sqrt{W}\tau, i\tau = \tau_e)$

$$x_{0e} = \tanh(\sqrt{W}\tau_e), \qquad x_i = 0, \qquad z = \frac{1}{\cosh(\sqrt{W}\tau_e)},$$

$$\cos\theta = \sqrt{\chi_p}\, dn\left(\frac{\sqrt{1-u^2}}{1-v^2}\sqrt{\chi_p}(\sigma - v\tau)\Big|1-\epsilon\right),$$
$$\phi_1 = \frac{\tau - v\sigma}{1-v^2} + \frac{vW}{\sqrt{1-u^2}\sqrt{\chi_p}(1-\chi_p)}$$
$$\times\, \Pi\left(-\frac{\chi_p}{1-\chi_p}(1-\epsilon),\right.$$
$$\left. am\left(\frac{\sqrt{1-u^2}}{1-v^2}\sqrt{\chi_p}(\sigma - v\tau)\right)\Big|1-\epsilon\right)$$
$$\phi_2 = u\frac{\tau - v\sigma}{1-v^2}, \tag{2.1}$$

where θ is the angle on which the metric on $S^3 \subset S^5$ depends, while $\phi_{1,2}$ are the isometric angles on it. $dn(\alpha|1-\epsilon)$ is one of the Jacobi elliptic functions, $\Pi(\alpha, \beta|1-\epsilon)$ is the incomplete elliptic integral of the third kind, and $am(x)$ is the Jacobi amplitude. Let us also mention that χ_p, χ_m are related to u, v, W parameters according to

$$\chi_p + \chi_m = \frac{2 - (1+v^2)W - u^2}{1-u^2},$$
$$\chi_p\chi_m = \frac{1 - (1+v^2)W + (vW)^2}{1-u^2}, \tag{2.2}$$

and

$$\epsilon \equiv \frac{\chi_m}{\chi_p}. \tag{2.3}$$

For the finite-size dyonic giant magnon string solution, the explicit expressions for the conserved quantities and the world-sheet momentum p can be written as [30]

$$\mathcal{E} = \frac{2\sqrt{W}(1-v^2)}{\sqrt{1-u^2}\sqrt{\chi_p}}\mathbf{K}(1-\epsilon),$$
$$\mathcal{J}_1 = \frac{2\sqrt{\chi_p}}{\sqrt{1-u^2}}\left[\frac{1-v^2W}{\chi_p}\mathbf{K}(1-\epsilon) - \mathbf{E}(1-\epsilon)\right], \tag{2.4}$$

$$\mathcal{J}_2 = \frac{2u\sqrt{\chi_p}}{\sqrt{1-u^2}}\mathbf{E}(1-\epsilon),$$
$$p = \frac{2v}{\sqrt{1-u^2}\sqrt{\chi_p}}\left[\frac{W}{1-\chi_p}\Pi\left(-\frac{\chi_p}{1-\chi_p}(1-\epsilon)\Big|1-\epsilon\right)\right.$$
$$\left. - \mathbf{K}(1-\epsilon)\right], \tag{2.5}$$

where[2]

$$\mathcal{E} = \frac{2\pi E}{\sqrt{\lambda}}, \qquad \mathcal{J}_{1,2} = \frac{2\pi J_{1,2}}{\sqrt{\lambda}}$$

are the string energy and the two angular momenta. $\mathbf{K}(1-\epsilon)$, $\mathbf{E}(1-\epsilon)$, and $\Pi(-\frac{\chi_p}{1-\chi_p}(1-\epsilon)|1-\epsilon)$ are the complete elliptic integrals of first, second, and third kind. As explained in Ref. [8],[3] (2.5) should be identified with the momentum of the magnon excitations in the spin chain arising in the dual $\mathcal{N} = 4$ SYM theory.

The dyonic giant magnon dispersion relation, including the leading finite-size correction, can be written as

$$\mathcal{E} - \mathcal{J}_1 = \frac{\sqrt{\lambda}}{2\pi}\left[\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)} - \frac{\sin^4(p/2)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}}\epsilon\right], \tag{2.6}$$

where

$$\epsilon = 16\exp\left[-\frac{2(\mathcal{J}_1 + \sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)})\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}\sin^2(p/2)}{\mathcal{J}_2^2 + 4\sin^4(p/2)}\right]. \tag{2.7}$$

[2]The relation between the string tension T and the 't Hooft coupling λ in the dual $\mathcal{N} = 4$ SYM is $TR^2 = \sqrt{\lambda}/2\pi$, where R is the common radius of AdS_5 and S^5 subspaces. Here R is set to 1.

[3]See also Ref. [9] for the dyonic case.

The second term in (2.6) represents the leading finite-size effect on the energy-charge relation, which disappears for $\epsilon \to 0$, or equivalently $\mathcal{J}_1 \to \infty$. It is nonzero only for $\mathcal{J}_1$ finite.

The above two equalities are found under the following conditions on the parameters:

$$0 < u < 1, \qquad 0 < v < 1, \qquad 0 < W < 1,$$
$$0 < \chi_m < \chi_p < 1.$$

The case of finite-size giant magnons with one angular momentum can be obtained by setting $u = 0$, or $\mathcal{J}_2 = 0$, as can be seen from (2.4).

B. Three-point correlation functions

It is known that the correlation functions of any conformal field theory can be determined in principle in terms of the basic conformal data $\{\Delta_i, C_{ijk}\}$, where Δ_i are the conformal dimensions defined by the two-point correlation functions

$$\langle \mathcal{O}_i^\dagger(x_1)\mathcal{O}_j(x_2)\rangle = \frac{C_{12}\delta_{ij}}{|x_1 - x_2|^{2\Delta_i}},$$

and C_{ijk} are the structure constants in the operator product expansion

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1 - x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1}}.$$

Therefore, the determination of the initial conformal data for a given conformal field theory is the most important step in the conformal bootstrap approach.

The three-point functions of two "heavy" operators and a light operator can be approximated by a supergravity vertex operator evaluated at the heavy classical string configuration [14,26]:

$$\langle V_H(x_1)V_H(x_2)V_L(x_3)\rangle = V_L(x_3)_{\mathrm{classical}}.$$

For $|x_1| = |x_2| = 1$, $x_3 = 0$, the correlation function reduces to

$$\langle V_H(x_1)V_H(x_2)V_L(0)\rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_H}}.$$

Then, the normalized structure constants

$$\mathcal{C} = \frac{C_{123}}{C_{12}}$$

can be found from

$$\mathcal{C} = c_\Delta V_L(0)_{\mathrm{classical}}, \tag{2.8}$$

where c_Δ is the normalized constant of the corresponding light vertex operator.

Recently, first results describing *finite-size* effects on the three-point correlators appeared [30,31,35,37,39]. This was done for the cases when the heavy string states are *finite-size* giant magnons, carrying one or two angular momenta, and for three different choices of the light state:

(1) Primary scalar operators: $V_L = V_j^{\mathrm{pr}}$.

(2) Dilaton operator: $V_L = V_j^d$.

(3) Singlet scalar operators on higher string levels: $V_L = V^q$.

The corresponding (unintegrated) vertices are given by [14]

$$V_j^{\mathrm{pr}} = (Y_4 + Y_5)^{-\Delta_{\mathrm{pr}}}(X_1 + iX_2)^j \times [z^{-2}(\partial x_m \bar\partial x^m - \partial z \bar\partial z) - \partial X_k \bar\partial X_k], \tag{2.9}$$

where the scaling dimension is $\Delta_{\mathrm{pr}} = j$. The corresponding operator in the dual gauge theory is $\mathrm{Tr}(Z^j)$:[4]

$$V_j^d = (Y_4 + Y_5)^{-\Delta_d}(X_1 + iX_2)^j[z^{-2}(\partial x_m \bar\partial x^m + \partial z \bar\partial z) + \partial X_k \bar\partial X_k], \tag{2.10}$$

where now the scaling dimension $\Delta_d = 4 + j$ to the leading order in the large $\sqrt{\lambda}$ expansion. The corresponding operator in the dual gauge theory is proportional to $\mathrm{Tr}(F_{\mu\nu}^2 Z^j + \cdots)$, or for $j = 0$, just to the SYM Lagrangian:

$$V^q = (Y_4 + Y_5)^{-\Delta_q}(\partial X_k \bar\partial X_k)^q. \tag{2.11}$$

This operator corresponds to a scalar *string* state at level $n = q - 1$, and to leading order in $\frac{1}{\sqrt{\lambda}}$ expansion

$$\Delta_q = 2\left(\sqrt{(q-1)\sqrt{\lambda} + 1 - \frac{1}{2}q(q-1)} + 1\right). \tag{2.12}$$

The value $n = 1(q = 2)$ corresponds to a massive string state on the first exited level and the corresponding operator in the dual gauge theory is an operator contained within the Konishi multiplet. Higher values of n label higher string levels.

The results obtained for the normalized structure constants (2.8), for the case of finite-size giant magnons in $AdS_5 \times S^5$, and the above three vertices, are as follows [37,39]:

[4] Z is one of the three complex scalars contained in $\mathcal{N} = 4$ SYM.

$$\mathcal{C}_j^{\mathrm{pr}} = \pi^{3/2} c_j^{\mathrm{pr}} \frac{\Gamma(\frac{j}{2})}{\Gamma(\frac{3+j}{2})} \frac{\chi_p^{\frac{j-1}{2}}}{\sqrt{(1-u^2)W}} \bigg[(1 - W + j(1 - v^2 W))\,{}_2F_1\bigg(\frac{1}{2}, \frac{1}{2} - \frac{j}{2}; 1; 1 - \epsilon\bigg) - (1+j)(1-u^2)\chi_p\,{}_2F_1\bigg(\frac{1}{2}, -\frac{1}{2} - \frac{j}{2}; 1; 1 - \epsilon\bigg)\bigg], \tag{2.13}$$

$$\mathcal{C}_j^{d} = 2\pi^{3/2} c_{4+j}^{d} \frac{\Gamma(\frac{4+j}{2})}{\Gamma(\frac{5+j}{2})} \frac{\chi_p^{\frac{j-1}{2}}}{\sqrt{(1-u^2)W}} \bigg[(1-u^2)\chi_p\,{}_2F_1\bigg(\frac{1}{2}, -\frac{1}{2} - \frac{j}{2}; 1; 1 - \epsilon\bigg) - (1-W)\,{}_2F_1\bigg(\frac{1}{2}, \frac{1}{2} - \frac{j}{2}; 1; 1 - \epsilon\bigg)\bigg], \tag{2.14}$$

$$\mathcal{C}^{q} = c_{\Delta_q} \pi^{3/2} \frac{\Gamma(\frac{\Delta_q}{2})}{\Gamma(\frac{\Delta_q+1}{2})} \frac{(-1)^q [2 - (1+v^2)W]^q}{(1-v^2)^{q-1}\sqrt{(1-u^2)W\chi_p}}, \qquad \sum_{k=0}^{q} \frac{q!}{k!(q-k)!} \bigg[-\frac{1-u^2}{1 - \frac{1}{2}(1+v^2)W} \bigg]^k \chi_p^k\,{}_2F_1\bigg(\frac{1}{2}, \frac{1}{2} - k; 1; 1 - \epsilon\bigg), \tag{2.15}$$

where ${}_2F_1(a, b; c; z)$ is Gauss's hypergeometric function.

III. LEADING ORDER FINITE-SIZE EFFECTS

As we already point out in the beginning, (2.4) and (2.5) cannot be solved *exactly* with respect to the parameters involved, in order to express the relevant three-point correlation functions in terms of the conserved charges and p. That is why we will consider here only the leading order finite-size effects on the three-point correlators. This means that we will consider the limit $\mathcal{J}_1$ large, i.e., $J_1 \gg \sqrt{\lambda}$, where the finite-size corrections to both conformal dimensions and energies of string states have been computed also from the Lüscher corrections. Practically, the problem reduces to consider the limit $\epsilon \to 0$, since $\epsilon = 0$ corresponds to the infinite-size case, i.e., $\mathcal{J}_1 = \infty$. The relevant expansions of the parameters are [30]

$$\begin{aligned} &\chi_p = \chi_{p0} + (\chi_{p1} + \chi_{p2}\log(\epsilon))\epsilon, \qquad \chi_m = \chi_{m1}\epsilon, \\ &W = 1 + W_1\epsilon, \qquad v = v_0 + (v_1 + v_2\log(\epsilon))\epsilon, \\ &u = u_0 + (u_1 + u_2\log(\epsilon))\epsilon. \end{aligned} \tag{3.1}$$

The coefficients on the first line in (3.1) can be obtained by using the equalities (2.2) and the definition of ϵ (2.3) to be

$$\begin{aligned} \chi_{p0} &= 1 - \frac{v_0^2}{1-u_0^2}, \\ \chi_{p1} &= \frac{v_0}{(1-v_0^2)(1-u_0^2)^2}\{v_0[(1-v_0^2)^2 - 3(1-v_0^2)u_0^2 \\ &\quad + 2u_0^4 - 2(1-v_0^2)u_0u_1] - 2(1-v_0^2)(1-u_0^2)v_1\}, \\ \chi_{p2} &= -2v_0\frac{v_2 + (v_0u_2 - u_0v_2)u_0}{(1-u_0^2)^2}, \\ \chi_{m1} &= 1 - \frac{v_0^2}{1-u_0^2}, \quad W_1 = -\frac{(1-u_0^2-v_0^2)^2}{(1-u_0^2)(1-v_0^2)}. \end{aligned} \tag{3.2}$$

The coefficients in the expansions of v and u we take from Ref. [71], where for the case under consideration we have to set $K_1 = \chi_{n1} = 0$, or equivalently $\Phi = 0$. This leads to

$$\begin{aligned} v_0 &= \frac{\sin(p)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}}, \\ u_0 &= \frac{\mathcal{J}_2}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)d}}, \\ v_1 &= \frac{v_0(1-v_0^2-u_0^2)}{4(1-u_0^2)(1-v_0^2)}[(1-v_0^2)(1-\log(16)) \\ &\quad - u_0^2(5 - v_0^2(1+\log(16)) - \log(4096))], \\ v_2 &= \frac{v_0(1-v_0^2-u_0^2)}{4(1-u_0^2)(1-v_0^2)}[1 - v_0^2 - u_0^2(3+v_0^2)], \\ u_1 &= \frac{u_0(1-v_0^2-u_0^2)}{4(1-v_0^2)}[1 - \log(16) - v_0^2(1+\log(16))], \\ u_2 &= \frac{u_0(1-v_0^2-u_0^2)}{4(1-v_0^2)}(1+v_0^2). \end{aligned} \tag{3.3}$$

We need also the expression for ϵ. It can be found from the expansion of $\mathcal{J}_1$, and to the leading order is given by (2.7).

A. Giant magnons and primary scalar operators

Let us first point out that (2.13) simplifies a lot when j is odd ($j = 2m+1$, $m = 0, 1, 2, \ldots$). In that case, Gauss's hypergeometric functions in (2.13) reduce to polynomials. This results in

$$\begin{aligned} \mathcal{C}_{2m+1}^{\mathrm{pr}} &= \pi^{3/2} c_{2m+1}^{\mathrm{pr}} \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+2)} \frac{\epsilon^{m/2}\chi_p^m}{\sqrt{(1-u^2)W}} \\ &\quad \times \bigg[-2(m+1)(1-u^2)\sqrt{\epsilon}\chi_p P_{m+1}\bigg(\frac{1+\epsilon}{2\sqrt{\epsilon}}\bigg) \\ &\quad + (1 - W + (2m+1)(1-v^2W))P_m\bigg(\frac{1+\epsilon}{2\sqrt{\epsilon}}\bigg)\bigg], \end{aligned} \tag{3.4}$$

where $P_n(z)$ are Legendre's polynomials.

Since the corresponding operators in the dual gauge theory are of the type $\mathrm{Tr}(Z^j)$, we will restrict ourselves to integer-valued j.

Let us start with the simpler case when $J_2 = 0$, or equivalently $u = 0$. Expanding (2.13) in ϵ and using (3.1), (3.2), and (3.3), one finds that[5]

$$\mathcal{C}_{10}^{\mathrm{pr}} \approx 0, \qquad \mathcal{C}_{20}^{\mathrm{pr}} \approx \frac{4}{3} c_2^{\mathrm{pr}} \mathcal{J}_1 \sin^2(p/2)\epsilon, \qquad \mathcal{C}_{j0}^{\mathrm{pr}} \approx c_j^{\mathrm{pr}} a_j \sin(p/2)^{j+1}\epsilon, \qquad j = 3, \ldots, 10, \tag{3.5}$$

where

$$\epsilon = 16 \exp[-2 - \mathcal{J}_1 \csc(p/2)], \tag{3.6}$$

for the case under consideration.[6] The numerical coefficients a_j are given by

$$a_j = \left(\frac{1}{4}\pi^2, \frac{2^4}{3.5}, \frac{1}{16}\pi^2, \frac{2^7}{3^2.5.7}, \frac{3.5}{2^9}\pi^2, \frac{2^{10}}{3^3.5^2.7}, \frac{5.7}{2^{11}}\pi^2, \frac{2^{14}}{3^2.5^2.7^2.11}\right).$$

A few comments are in order. From (3.5) one can conclude that the $\mathcal{C}_{10}^{\mathrm{pr}}$ and $\mathcal{C}_{20}^{\mathrm{pr}}$ cases are exceptional, while $\mathcal{C}_{j0}^{\mathrm{pr}}$ have the same structure for $j \geq 3$. $\mathcal{C}_{10}^{\mathrm{pr}} \approx 0$ means that the small ϵ contribution to the three-point correlator is zero to the leading order in ϵ. $\mathcal{C}_{20}^{\mathrm{pr}}$ is the only one normalized structure constant of this type proportional to $\mathcal{J}_1$. It is still exponentially suppressed by ϵ. The common feature of $\mathcal{C}_{j0}^{\mathrm{pr}}$ in (3.5) is that they all vanish in the *infinite-size* case, i.e., for $\epsilon = 0$. This property was established in Ref. [26], and confirmed even for the γ-deformed case in Ref. [37]. Here, we obtained the leading finite-size corrections to it.

Now, let us turn to the dyonic case, i.e., $J_2 \neq 0$. Working in the same way, but with $u \neq 0$, we derive

$j = 1$:

$$\begin{aligned}
\mathcal{C}_1^{\mathrm{pr}} \approx c_1^{\mathrm{pr}} \frac{\pi^2}{16} & \frac{\mathcal{J}_2^2 \csc(p/2)}{[\mathcal{J}_2^2 + 4\sin^2(p/2)]^{3/2}[\mathcal{J}_2^2 + 4\sin^4(p/2)]} \\
& \times \Big\{ 8[\mathcal{J}_2^2 + 4\sin^2(p/2)][\mathcal{J}_2^2 + 4\sin^4(p/2)] + \sin^2(p/2)[40 + 17\mathcal{J}_2^2 + 2\mathcal{J}_2^4 - 20(3 + \mathcal{J}_2^2)\cos(p) \\
& + 3(8 + \mathcal{J}_2^2)\cos(2p) - 4\cos(3p) - 4\frac{\mathcal{J}_2^2 + 8\sin^2(p/2)}{\mathcal{J}_2^2 + 4\sin^4(p/2)} \times \Big(\mathcal{J}_1\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)} + \mathcal{J}_2^2 + 4\sin^2(p/2)\Big) \\
& \times (\mathcal{J}_2^2 + 4\sin^4(p/2) + 2\sin^2(p))\sin^2(p/2)]\epsilon \Big\};
\end{aligned} \tag{3.7}$$

$j = 2$:

$$\begin{aligned}
\mathcal{C}_2^{\mathrm{pr}} \approx \frac{4}{3} c_2^{\mathrm{pr}} & \frac{1}{[\mathcal{J}_2^2 + 4\sin^2(p/2)]^{3/2}[\mathcal{J}_2^2 + 4\sin^4(p/2)]} \\
& \times \Big\{ 2\mathcal{J}_2^2[\mathcal{J}_2^2 + 4\sin^2(p/2)][\mathcal{J}_2^2 + 4\sin^4(p/2)] - \sin^4(p/2) \times [20 + 3\mathcal{J}_2^2 - 2\mathcal{J}_2^4 - 2(15 \\
& + 2\mathcal{J}_2^2)\cos(p) + (12 + \mathcal{J}_2^2)\cos(2p) - 2\cos(3p) + \frac{8}{\mathcal{J}_2^2 + 4\sin^4(p/2)}\Big(\mathcal{J}_1\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)} + \mathcal{J}_2^2 + 4\sin^2(p/2)\Big) \\
& \times (-3 + 2(2 + \mathcal{J}_2^2)\cos(p) - \cos(2p))\sin^4(p/2)]\epsilon \Big\};
\end{aligned} \tag{3.8}$$

[5]We use the notation $\mathcal{C}_{j0}^{\mathrm{pr}}$ in order to say that $\mathcal{C}_j^{\mathrm{pr}}$ are computed for the case $J_2 = 0$.

[6]This expression for ϵ comes from (2.7) after setting $\mathcal{J}_2 = 0$.

$j = 3$:

$$\begin{aligned}
\mathcal{C}_3^{\rm pr} \approx{}& c_3^{\rm pr}\frac{\pi^2}{256}\csc(p/2)\frac{[\mathcal{J}_2^2+4\sin^2(p/2)]^{5/2}}{\mathcal{J}_2^2+4\sin^4(p/2)}\\
&\times\Bigg\{48\mathcal{J}_2^2\sin^2(p/2)\frac{\mathcal{J}_2^2+4\sin^4(p/2)}{[\mathcal{J}_2^2+4\sin^2(p/2)]^3}-\Bigg[\frac{25\mathcal{J}_2^4}{[\mathcal{J}_2^2+4\sin^2(p/2)]^2}-\mathcal{J}_2^2\frac{\mathcal{J}_2^2+4\sin^4(p/2)}{[\mathcal{J}_2^2+4\sin^2(p/2)]^3}(21-16\cos(p)\\
&-5\cos(2p)+8\mathcal{J}_2^2)-\frac{3}{2}\mathcal{J}_2^6\frac{11-12\cos(p)+\cos(2p)+6\mathcal{J}_2^2}{[\mathcal{J}_2^2+4\sin^2(p/2)]^4}+(3\mathcal{J}_2^2(\mathcal{J}_2^2+4\sin^2(p/2)\\
&+\mathcal{J}_1\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)})\times(80+42\mathcal{J}_2^2+12\mathcal{J}_2^4-(120+47\mathcal{J}_2^2-4\mathcal{J}_2^4)\cos(p)+(8+\mathcal{J}_2^2)(6\cos(2p)\\
&-\cos(3p)))\sin^4(p/2))\frac{1}{[\mathcal{J}_2^2+4\sin^2(p/2)]^4[\mathcal{J}_2^2+4\sin^4(p/2)]}-\frac{20\mathcal{J}_2^4\sin^2(p)}{[\mathcal{J}_2^2+4\sin^2(p/2)]^3}\\
&+\frac{3\mathcal{J}_2^4\sin^4(p)}{[\mathcal{J}_2^2+4\sin^2(p/2)]^4}-8\left(\frac{\mathcal{J}_2^2+4\sin^4(p/2)}{\mathcal{J}_2^2+4\sin^2(p/2)}\right)^2\Bigg]\epsilon\Bigg\}; \qquad (3.9)
\end{aligned}$$

$j = 4$:

$$\begin{aligned}
\mathcal{C}_4^{\rm pr} \approx{}& \frac{2}{45}c_4^{\rm pr}\frac{[\mathcal{J}_2^2+4\sin^2(p/2)]^{5/2}}{\mathcal{J}_2^2+4\sin^4(p/2)}\Bigg\{\frac{32\mathcal{J}_2^2[\mathcal{J}_2^2+4\sin^4(p/2)]\sin^2(p/2)}{[\mathcal{J}_2^2+4\sin^2(p/2)]^3}\\
&-\Bigg[\frac{17\mathcal{J}_2^4}{[\mathcal{J}_2^2+4\sin^2(p/2)]^2}-\frac{1}{2}\mathcal{J}_2^2\frac{\mathcal{J}_2^2+4\sin^4(p/2)}{\mathcal{J}_2^2+4\sin^2(p/2)]^3}(39-32\cos(p)-7\cos(2p)+16\mathcal{J}_2^2)\\
&-\mathcal{J}_2^6\frac{11-12\cos(p)+\cos(2p)+6\mathcal{J}_2^2}{[\mathcal{J}_2^2+4\sin^2(p/2)]^4}+(2\mathcal{J}_2^2(\mathcal{J}_2^2+4\sin^2(p/2)+\mathcal{J}_1\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)})\\
&\times(75+44\mathcal{J}_2^2+16\mathcal{J}_2^4-2(58+23\mathcal{J}_2^2-4\mathcal{J}_2^4)\cos(p)+4(13+\mathcal{J}_2^2)\cos(2p)-2(6+\mathcal{J}_2^2)\cos(3p)\\
&+\cos(4p))\sin^4(p/2))\times\frac{1}{[\mathcal{J}_2^2+4\sin^2(p/2)]^4[\mathcal{J}_2^2+4\sin^4(p/2)]}-\frac{13\mathcal{J}_2^4\sin^2(p)}{[\mathcal{J}_2^2+4\sin^2(p/2)]^3}\\
&+\frac{2\mathcal{J}_2^4\sin^4(p)}{[\mathcal{J}_2^2+4\sin^2(p/2)]^4}-3\left(\frac{\mathcal{J}_2^2+4\sin^4(p/2)}{\mathcal{J}_2^2+4\sin^2(p/2)}\right)^2\Bigg]\epsilon\Bigg\}. \qquad (3.10)
\end{aligned}$$

In the four formulas above ϵ is given by (2.7).

B. Giant magnons and dilaton operator

The leading finite-size effect on the normalized structure constant in the three-point correlator of two finite-size giant magnon's states and zero-momentum dilaton operator ($j = 0$), in the limit $J_1 \gg \sqrt{\lambda}$, has been considered in Ref. [30]. Here, we will deal with the $j > 0$ cases. Since the corresponding operators in the dual gauge theory are proportional to $\mathrm{Tr}(F_{\mu\nu}^2 Z^j + \cdots)$, we will restrict ourselves to integer-valued j.

When j is odd ($j = 2m + 1$, $m = 0, 1, 2, \ldots$), the normalized structure constants (2.14) simplify to

$$\mathcal{C}_{2m+1}^d = 2\pi^{3/2}c_{2m+5}^d\frac{\Gamma(m+\frac{5}{2})}{\Gamma(m+3)}\frac{\epsilon^{m/2}\chi_p^m}{\sqrt{(1-u^2)W}}\left[(1-u^2)\sqrt{\epsilon}\chi_p P_{m+1}\left(\frac{1+\epsilon}{2\sqrt{\epsilon}}\right)-(1-W)P_m\left(\frac{1+\epsilon}{2\sqrt{\epsilon}}\right)\right]. \qquad (3.11)$$

Expanding (2.14) in ϵ and using (3.1), (3.2), and (3.3), one finds

$j = 1$:

$$\mathcal{C}_1^d \approx \frac{3}{4}\pi^2 c_5^d \sin^3(p/2)\left\{\frac{1}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}} - \frac{1}{128(\mathcal{J}_2^2 + 4\sin^2(p/2))^{3/2}(\mathcal{J}_2^2 + 4\sin^4(p/2))^2}\right.$$
$$\times [(840 + 826\mathcal{J}_2^2 + 258\mathcal{J}_2^4 - 24\mathcal{J}_2^6 - 2(744 + 707\mathcal{J}_2^2 + 244\mathcal{J}_2^4 + 72\mathcal{J}_2^6)\cos(p)$$
$$+ 4(255 + 218\mathcal{J}_2^2 + 62\mathcal{J}_2^4 - 6\mathcal{J}_2^6)\cos(2p) - (520 + 367\mathcal{J}_2^2 + 24\mathcal{J}_2^4)\cos(3p) + 2(92$$
$$+ 47\mathcal{J}_2^2 + 3\mathcal{J}_2^4)\cos(4p) - (40 + 11\mathcal{J}_2^2)\cos(5p) + 4\cos(6p))$$
$$+ 8\mathcal{J}_1\sin^2(p/2)\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}((8 + 19\mathcal{J}_2^2 + 12\mathcal{J}_2^4)\cos(p)$$
$$\left. + (8 - 16\mathcal{J}_2^2)\cos(2p) - (8 + 3\mathcal{J}_2^2)\cos(3p) - 2(5 + 5\mathcal{J}_2^2 - 2\mathcal{J}_2^4 - \cos(4p)))]\epsilon\right\};$$

$j = 2$:

$$\mathcal{C}_2^d \approx \frac{2^8}{3^2 5} c_6^d \sin^4(p/2)\left\{\frac{1}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}} - \frac{1}{128(\mathcal{J}_2^2 + 4\sin^2(p/2))^{3/2}(\mathcal{J}_2^2 + 4\sin^4(p/2))^2}\right.$$
$$\times [(210 + 8\mathcal{J}_2^2(6 - \mathcal{J}_2^2)(7 + 4\mathcal{J}_2^2) - 8(63 + 84\mathcal{J}_2^2 + 38\mathcal{J}_2^4 + 16\mathcal{J}_2^6)\cos(p) + (585 + 576\mathcal{J}_2^2$$
$$+ 176\mathcal{J}_2^4 - 32\mathcal{J}_2^6)\cos(2p) - 4(115 + 84\mathcal{J}_2^2 + 4\mathcal{J}_2^4)\cos(3p) + 2(111 + 56\mathcal{J}_2^2 + 4\mathcal{J}_2^4)\cos(4p)$$
$$- 4(15 + 4\mathcal{J}_2^2)\cos(5p) + 7\cos(6p)) - 8\mathcal{J}_1\sin^2(p/2)\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}(15 + 8\mathcal{J}_2^2 - 8\mathcal{J}_2^4$$
$$\left. - 4(3 + 5\mathcal{J}_2^2 + 4\mathcal{J}_2^4)\cos(p) - (12 - 8\mathcal{J}_2^2)\cos(2p) + 4(3 + \mathcal{J}_2^2)\cos(3p) - 3\cos(4p))]\epsilon\right\};$$

$j = 3$:

$$\mathcal{C}_3^d \approx \frac{3.5}{2^5}\pi^2 c_7^d \sin^5(p/2)\left\{\frac{1}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}} + \frac{1}{960(\mathcal{J}_2^2 + 4\sin^2(p/2))^{3/2}(\mathcal{J}_2^2 + 4\sin^4(p/2))^2}\right.$$
$$\times [20(256(13 + 15\cos(p))\sin^{10}(p/2) + 288\mathcal{J}_2^2(5 + 7\cos(p))\sin^8(p/2) + \mathcal{J}_2^4(54 + 241\cos(p)$$
$$+ 10\cos(2p) + 15\cos(3p))\sin^2(p/2) + 10\mathcal{J}_2^6\cos(p)(5 + 3\cos(p)))$$
$$+ 60\mathcal{J}_1\sin^2(p/2)\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}(20 + 6\mathcal{J}_2^2 - 12\mathcal{J}_2^4 - (16 + 21\mathcal{J}_2^2 + 20\mathcal{J}_2^4)\cos(p)$$
$$\left. - 2(8 - 5\mathcal{J}_2^2)\cos(2p) + (16 + 5\mathcal{J}_2^2)\cos(3p) - 4\cos(4p))]\epsilon\right\};$$

$j = 4$:

$$\mathcal{C}_4^d \approx \frac{2^{11}}{3.5^2.7} c_8^d \sin^6(p/2)\left\{\frac{1}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}} + \frac{1}{8192(\mathcal{J}_2^2 + 4\sin^2(p/2))^{3/2}(\mathcal{J}_2^2 + 4\sin^4(p/2))^2}\right.$$
$$\times [64(294 + 14\mathcal{J}_2^2 - 60\mathcal{J}_2^4 + 48\mathcal{J}_2^6 - 4(51 - 49\mathcal{J}_2^2 - 53\mathcal{J}_2^4 - 36\mathcal{J}_2^6)\cos(p)$$
$$- (435 + 8\mathcal{J}_2^2(61 + 19\mathcal{J}_2^2 - 6\mathcal{J}_2^4))\cos(2p) + 2(305 + 209\mathcal{J}_2^2 + 6\mathcal{J}_2^4)\cos(3p)$$
$$- 2(179 + 83\mathcal{J}_2^2 + 6\mathcal{J}_2^4)\cos(4p) + 2(53 + 13\mathcal{J}_2^2)\cos(5p) - 13\cos(6p))$$
$$+ 512\mathcal{J}_1\sin^2(p/2)\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}(25 + 4\mathcal{J}_2^2 - 16\mathcal{J}_2^4 - (20 + 22\mathcal{J}_2^2 + 24\mathcal{J}_2^4)\cos(p)$$
$$\left. - 4(5 - 3\mathcal{J}_2^2)\cos(2p) + 2(10 + 3\mathcal{J}_2^2)\cos(3p) - 5\cos(4p))]\epsilon\right\}.$$

In the four formulas above ϵ is given by (2.7).

Actually, we computed the normalized coefficients in the three-point correlators up to $j = 10$. However, since the expressions for them are too complicated, we give here only the results for the first two odd and two even values of j. Knowing these expressions, the conclusion is that they have the same structure for any j in the small ϵ limit.[7] Namely,

$$\mathcal{C}^d_j \approx A_j c^d_{j+4} \sin^{j+2}\left(\frac{p}{2}\right)\left\{\frac{1}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(\frac{p}{2})}} + \frac{a_j}{(\mathcal{J}_2^2 + 4\sin^2(\frac{p}{2}))^{3/2}(\mathcal{J}_2^2 + 4\sin^4(\frac{p}{2}))^2}\right.$$
$$\left.\times\left[P^3_j(\mathcal{J}_2^2) + \mathcal{J}_1 \sin^2\left(\frac{p}{2}\right)\sqrt{\mathcal{J}_2^2 + 4\sin^2\left(\frac{p}{2}\right)}Q^2_j(\mathcal{J}_2^2)\right]\epsilon\right\}, \tag{3.12}$$

where ϵ is given in (2.7), A_j and a_j are numerical coefficients, while $P^3_j(\mathcal{J}_2^2)$ and $Q^2_j(\mathcal{J}_2^2)$ are polynomials of third and second order, respectively, with coefficients depending on p in a trigonometric way.

Now, let us restrict ourselves to the simpler case when $\mathcal{J}_2 = 0$, i.e., giant magnon string states with one (large) angular momentum $\mathcal{J}_1 \neq 0$. Knowing the above results for $1 \leq j \leq 10$, one can conclude that the normalized structure constants in the three-point correlators for any $j \geq 1$ in the small ϵ limit look like[8]

$$\mathcal{C}^d_{j0} \approx \frac{A_j}{2} c^d_{j+4} \sin^j\left(\frac{p}{2}\right)\left[\sin\left(\frac{p}{2}\right) + \left(B_{j0}\sin\left(\frac{p}{2}\right) + C_{j0}\sin\left(\frac{3p}{2}\right) + D_{j0}(1+\cos(p))\mathcal{J}_1\right)e^{-2-\frac{\mathcal{J}_1}{\sin\frac{p}{2}}}\right], \tag{3.13}$$

where

$$B_{j0} = \left(-2^2, 3, \frac{2.11}{3}, 11, \frac{2^3 3^2}{5}, \frac{53}{3}, \frac{2.73}{7}, 2^3 3, \ldots\right) \quad \text{for } j = (1, \ldots, 8, \ldots), \qquad C_{j0} = 1 + 3j, \qquad D_{j0} = 2(j+1).$$

C. Giant magnons and singlet scalar operators on higher string levels

For that case, the expressions for the normalized structure constants in the three-point correlation functions for *dyonic* giant magnons are too long and complicated. That is why we will write down here the results for *finite-size* giant magnon states only, i.e., for $\mathcal{J}_2 = 0$. Then, after small ϵ expansion, one can find that (2.15) reduces to[9]

$$\mathcal{C}^q_0 \approx c_{\Delta_q}\frac{\sqrt{\pi}}{Aq_0}\frac{\Gamma(\frac{\Delta q}{2})}{\Gamma(\frac{1+\Delta q}{2})}\{Aq_1\sin(p/2) + Aq_2\mathcal{J}_1 + [(Aq_3 + Aq_4\cos(p))\sin(p/2) + (Aq_5 + Aq_6\cos(p))\mathcal{J}_1$$
$$+ Aq_7\csc(p/2)(1+\cos(p))\mathcal{J}_1^2]\epsilon\}, \tag{3.14}$$

where Aq_i $(i = 0, 1, \ldots, 7)$ are numerical coefficients, and for the case at hand ϵ is given by (3.6).

This is the general structure of $\mathcal{C}^q_0$. The values of Aq_i we found are as follows $(q = 1, \ldots, 10)$:

$$Aq_0 = (8, 24, 60, 420, 2520, 27720, 180180, 180180, 3063060, 116396280),$$
$$Aq_1 = (16, -16, 152, -632, 7216, -55216, 559304, -420312, 10089896, -301915216),$$
$$Aq_2 = (-8, 24, -60, 420, -2520, 27720, -180180, 180180, -3063060, 116396280),$$
$$Aq_3 = (2, -66, 147, -2575, 13446, -272694, 1555993, -2484923, 37469109, -2088496586),$$
$$Aq_4 = (2, -10, 171, -1027, 15334, -144942, 1747825, -1523631, 41620821, -1396357874),$$
$$Aq_5 = \left(-5, 31, -\frac{187}{2}, \frac{1837}{2}, -6343, 86653, -\frac{1256569}{2}, \frac{3}{2}490499, -\frac{27342361}{2}, 587890603\right),$$
$$Aq_6 = \left(1, 13, -\frac{97}{2}, \frac{1207}{2}, -4453, 65863, -\frac{986299}{2}, \frac{3}{2}400409, -\frac{22747771}{2}, 500593393\right),$$
$$Aq_7 = \left(-1, 3, -\frac{15}{2}, \frac{105}{2}, -315, 3465, -\frac{45045}{2}, \frac{3}{2}15015, -\frac{765765}{2}, 14549535\right). \tag{3.15}$$

[7]The only difference in that sense is that for j odd an additional overall factor of π^2 appears, as can be seen from the formulas above.

[8]$\mathcal{C}^d_{j0}$ is used for $\mathcal{C}^d_j$ computed for the $\mathcal{J}_2 = 0$ case.

[9]$\mathcal{C}^q_0 \equiv \mathcal{C}^q$ computed for $\mathcal{J}_2 = 0$.

IV. CONCLUDING REMARKS

In this paper, in the framework of the semiclassical approach, we computed the leading finite-size effects on the normalized structure constants in some three-point correlation functions in $AdS_5 \times S^5$, expressed in terms of the conserved string angular momenta J_1, J_2, and the world-sheet momentum p_w, identified with the momentum p of the magnon excitations in the dual spin chain arising in $\mathcal{N} = 4$ SYM in four dimensions. Namely, we found the leading finite-size effects on the structure constants in three-point correlators of two heavy (dyonic) giant magnon's string states and the following three light states:

(1) Primary scalar operators;
(2) Dilaton operator with nonzero momentum ($j \geq 1$);
(3) Singlet scalar operators on higher string levels.

A natural generalization of the above results would be to consider the case of γ-deformed (or TsT-transformed) $AdS_5 \times S^5$ type IIB string theory background. Another possible issue to investigate is the case of $AdS_4 \times CP^3$ type IIA string theory background, dual to $\mathcal{N} = 6$ super Chern-Simons-matter theory in three space-time dimensions (Aharony-Bergman-Jafferis-Maldacena model) and its TsT deformations. We hope to report on these soon.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[2] N. Beisert *et al.*, Lett. Math. Phys. **99**, 3 (2012).
[3] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Nucl. Phys. **B636**, 99 (2002).
[4] D. M. Hofman and J. Maldacena, J. Phys. A **39**, 13095 (2006).
[5] H.-Y. Chen, N. Dorey, and K. Okamura, J. High Energy Phys. 09 (2006) 024.
[6] M. Kruczenski, J. Russo, and A. A. Tseytlin, J. High Energy Phys. 10 (2006) 002.
[7] J. Ambjorn, R. A. Janik, and C. Kristjansen, Nucl. Phys. **B736**, 288 (2006); R. A. Janik and T. Lukowski, Phys. Rev. D **76**, 126008 (2007).
[8] G. Arutyunov, S. Frolov, and M. Zamaklar, Nucl. Phys. **B778**, 1 (2007).
[9] Y. Hatsuda and R. Suzuki, Nucl. Phys. **B800**, 349 (2008).
[10] R. A. Janik, P. Surowka, and A. Wereszczynski, J. High Energy Phys. 05 (2010) 030.
[11] E. I. Buchbinder and A. A. Tseytlin, J. High Energy Phys. 08 (2010) 057.
[12] K. Zarembo, J. High Energy Phys. 09 (2010) 030.
[13] M. S. Costa, R. Monteiro, J. E. Santos, and D. Zoakos, J. High Energy Phys. 11 (2010) 141.
[14] R. Roiban and A. A. Tseytlin, Phys. Rev. D **82**, 106011 (2010).
[15] R. Hernández, J. Phys. A **44**, 085403 (2011).
[16] S. Ryang, J. High Energy Phys. 01 (2011) 092.
[17] G. Georgiou, J. High Energy Phys. 02 (2011) 046.
[18] J. Escobedo, N. Gromov, A. Sever, and P. Vieira, J. High Energy Phys. 09 (2011) 028.
[19] D. Arnaudov and R. C. Rashkov, Phys. Rev. D **83**, 066011 (2011).
[20] J. G. Russo and A. A. Tseytlin, J. High Energy Phys. 02 (2011) 029.
[21] C. Park and B. Lee, Phys. Rev. D **83**, 126004 (2011).
[22] E. I. Buchbinder and A. A. Tseytlin, J. High Energy Phys. 02 (2011) 072.
[23] D. Bak, B. Chen, and J. Wu, J. High Energy Phys. 06 (2011) 014.
[24] A. Bissi, C. Kristjansen, D. Young, and K. Zoubos, J. High Energy Phys. 06 (2011) 085.
[25] D. Arnaudov, R. C. Rashkov, and T. Vetsov, Int. J. Mod. Phys. A **26**, 3403 (2011).
[26] R. Hernández, J. High Energy Phys. 05 (2011) 123.
[27] X. Bai, B. Lee, and C. Park, Phys. Rev. D **84**, 026009 (2011).
[28] J. Escobedo, N. Gromov, A. Sever, and P. Vieira, J. High Energy Phys. 09 (2011) 029.
[29] L. F. Alday and A. A. Tseytlin, J. Phys. A **44**, 395401 (2011).
[30] C. Ahn and P. Bozhilov, Phys. Lett. B **702**, 286 (2011).
[31] B. Lee and C. Park, Phys. Rev. D **84**, 086005 (2011).
[32] T. Klose and T. McLoughlin, J. High Energy Phys. 04 (2012) 080.
[33] D. Arnaudov and R. C. Rashkov, Fortschr. Phys. **60**, 217 (2012).
[34] D. Arnaudov and R. C. Rashkov, Phys. Rev. D **84**, 086009 (2011).
[35] C. Ahn and P. Bozhilov, Phys. Rev. D **84**, 126011 (2011).
[36] G. Georgiou, J. High Energy Phys. 09 (2011) 132.
[37] P. Bozhilov, J. High Energy Phys. 08 (2011) 121.
[38] M. Michalcik, R. C. Rashkov, and M. Schimpf, Mod. Phys. Lett. A **27**, 1250091 (2012).
[39] P. Bozhilov, Nucl. Phys. **B855**, 268 (2012).
[40] S. Ryang, J. High Energy Phys. 11 (2011) 026.
[41] R. Janik and A. Wereszczynski, J. High Energy Phys. 12 (2011) 095.
[42] Y. Kazama and S. Komatsu, J. High Energy Phys. 01 (2012) 110; 06 (2012) 150(E).
[43] E. I. Buchbinder and A. A. Tseytlin, Phys. Rev. D **85**, 026001 (2012).
[44] N. Gromov, A. Sever, and P. Vieira, J. High Energy Phys. 07 (2012) 044.
[45] Omar Foda, J. High Energy Phys. 03 (2012) 096.
[46] A. Bissi, T. Harmark, and M. Orselli, J. High Energy Phys. 02 (2012) 133.
[47] G. Georgiou, V. Gili, A. Grossardt, and J. Plefka, J. High Energy Phys. 04 (2012) 038.
[48] C. A. Cardona, Nucl. Phys. **B867**, 165 (2013).

[49] N. Gromov and P. Vieira, arXiv:1202.4103v1.
[50] R. Hernandez, Nucl. Phys. **B862**, 751 (2012).
[51] C. Ahn, O. Foda, and R. I. Nepomechie, J. High Energy Phys. 06 (2012) 168.
[52] D. Serban, J. High Energy Phys. 01 (2013) 012.
[53] I. Kostov, Phys. Rev. Lett. **108**, 261604 (2012).
[54] P. Bozhilov, P. Furlan, V. B. Petkova, and M. Stanishkov, Phys. Rev. D **86**, 066005 (2012).
[55] G. Grignani and A. V. Zayakin, J. High Energy Phys. 06 (2012) 142.
[56] S. Ryang, Phys. Lett. B **713**, 122 (2012).
[57] P. Caputa, R. de Mello Koch, and K. Zoubos, J. High Energy Phys. 08 (2012) 143.
[58] S. Hirano, C. Kristjansen, and D. Young, J. High Energy Phys. 07 (2012) 006.
[59] I. Kostov, J. Phys. A **45**, 494018 (2012).
[60] G. Grignani and A. V. Zayakin, J. High Energy Phys. 9 (2012) 087.
[61] N. Gromov and P. Vieira, arXiv:1205.5288v1.
[62] Y. Kazama and S. Komatsu, J. High Energy Phys. 09 (2012) 022.
[63] D. Arnaudov and R. C. Rashkov, arXiv:1206.2613v1.
[64] J. A. Minahan, J. High Energy Phys. 07 (2012) 187.
[65] A. Bissi, G. Grignani, and A. V. Zayakin, arXiv:1208.0100v1.
[66] J. Caetano and J. Toledo, arXiv:1208.4548v2.
[67] H. Lin, J. High Energy Phys. 12 (2012) 011.
[68] P. Caputa and B. A. E. Mohammed, J. High Energy Phys. 1 (2013) 055.
[69] A. Bissi, C. Kristjansen, A. Martirosyan, and M. Orselli, J. High Energy Phys. 01 (2013) 137.
[70] B. Gwak, B.-H. Lee, and C. Park, arXiv:1211.5838.
[71] P. Bozhilov, arXiv:1010.5465v1.