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# (Un)determined finite regularization-dependent quantum corrections: The Higgs boson decay into two photons and the two-photon scattering examples

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We investigate the appearance of arbitrary, regularization-dependent parameters introduced by divergent integrals in two *a priori* finite but superficially divergent amplitudes: the Higgs decay into two photons and the two-photon scattering. We use a general parametrization of ultraviolet divergences which makes explicit such ambiguities. Thus we separate in a consistent way using implicit regularization the divergent, finite, and regularization-dependent parts of the amplitudes which in turn are written as surface terms. We find that, although finite, these amplitudes are ambiguous before the imposition of physical conditions, namely, momentum routing invariance in the loops of Feynman diagrams. In the examples we study, momentum routing invariance turns out to be equivalent to gauge invariance. We also discuss the results obtained by different regularizations and show how they can be reproduced within our framework allowing for a clear view on the origin of regularization ambiguities.

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## I. INTRODUCTION

On the last 4th of July a new boson was announced using its decay into two photons as one of the main channels of discovery [1,2]. The immediate question that arose was whether this new boson corresponds to the one predicted by the Standard Model (Higgs boson) or not. To help answer this question theoretical predictions (loop corrections) on such decays must be set on consistent grounds.

Some time ago the $W$-loop calculation of the Higgs decay into two photons was performed in the unitary gauge and the result obtained [3] contradicted previous ones found in the literature [4–6]. The reason pointed to by the authors was the use of dimensional regularization (DReg). Soon after many authors performed calculations in the framework of dimensional regularization [7] and lattice [8] and loop regularization [9]. In all cases the old results were recovered shedding many doubts on the statements presented in Ref. [3]. Other authors used cutoff regularization [10,11] obtaining the same result of Ref. [3] thus concluding that such regularization is nonpredictive if one works on the unitary gauge. Other works were devoted to the discussion of the decoupling theorem [12,13] questioning the reliability of the predictions made in Ref. [3].

Contemporary to the work of Gastmans *et al.*, another paper questioned an old established result in the literature: the cross section of the two-photon scattering [14]. Once again, doubts were raised against the use of regularization. A work followed in which this issue was explained [15] in the framework of dimensional and Pauli-Villars regularization recovering the old results found in the literature [16,17].

The aim of the present work is to revise these two calculations with the purpose of illustrating that *a priori* undefined quantum corrections in Feynman diagram calculations, which entail regularization scheme dependence, are the common denominator of such discussion. Such arbitrarinesses must not be mistaken by finite parameters related to the freedom of defining renormalization constants to be fixed by renormalization conditions (i.e., the choice of a renormalization point). We propose a general parametrization valid at arbitrary loop order to handle such ambiguities which acts on the physical dimension of the theory, thus being particularly useful to dimensional specific models. Moreover an alternative exhibition of such arbitrariness in terms of arbitrary $n$-loop integrals is proposed. In this context such arbitrariness is expressed by differences between divergent loop integrals with the same degree of divergence and independent of external momenta with the purpose of bringing about its physical interpretation, namely, its relation to momentum routing invariance (MRI) in an arbitrary Feynman diagram. Some regularizations may break MRI, an inevitable consequence of energy-momentum conservation at the vertices of Feynman diagrams. The striking connection between momentum routing invariance and preservation of gauge symmetry was realized long ago by t' Hooft and Veltman [18] and Jackiw in Ref. [19] as well as by Elias *et al.* in Ref. [20]. In Ref. [21] some of us established the interplay between the vanishing of such arbitrary parameters expressed by surface terms and Abelian gauge invariance in the framework of implicit regularization (IReg). In this

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four-dimensional method, regularization-dependent terms (surface terms) can be extracted out in a consistent way, allowing a clear discussion of the ambiguities involved in the manipulation of divergent integrals. Such a scheme may be generalized to arbitrary (integer) dimensions and to arbitrary loop order in perturbation theory complying with Lorentz invariance and unitarity as dictated by the local Bogoliubov's $R$ operation based on the Bogoliubov, Parasiuk, Hepp, and Zimmermann (BPHZ) theorem [22–26]. Therefore, instead of just adding the result of a different method to the literature we intend to show that the discussions presented in Refs. [7–11] can all be explained using just one framework.

The paper is organized as follows: In Sec. II we discuss some regularization-dependent integrals and present our parametrizations. Section III is dedicated to the calculation of the Higgs decay into two photons in the unitary gauge. In Sec. IV we discuss the result of two-photon scattering in the framework of IReg. Finally, Sec. V is devoted to our concluding remarks.

## II. A GENERAL VIEW OF REGULARIZATION-DEPENDENT INTEGRALS

In this section we discuss on general grounds the issue of regularization-dependent integrals leaving the physical calculations of the Higgs decay as well as of the two-photon scattering to subsequent sections. Proceeding this way we hope to set the subject, both from a conceptual and technical point of view, in a consistent and self-contained way, allowing a clearer discussion of the examples just cited.

As is well known, perturbative quantum field theoretical calculations involve integration in the momentum loops which must be regularized due to ultraviolet and sometimes infrared divergences. The renormalization program consistently redefines physical degrees of freedom order by order in perturbation theory. Symmetry requirements may be either ensured by an invariant regularization or imposed as constraint equations dictated by Ward-Slavnov-Taylor identities order by order in the loops. Yet a little calculational tedious, the latter procedure is perfectly sound for both anomaly free theories and models in which the quantum symmetry breaking mechanism is well known.

A plethora of regularization schemes have been constructed to be used where gauge invariant DReg may fail, namely, in the so-called dimensional specific theories among which supersymmetric, chiral, and topological quantum field theories figure in. A natural question would be which basic properties should a method that does not resort to analytical continuation in the space-time dimension retain in order to be invariant. We start by illustrating with simple examples following [27]. Let $\Delta$ be the superficial degree of divergence of a 1-loop integral where the momentum $k$ runs. Consider the following $\Delta = 2$ integrals,

$$A = \int_k \frac{k^2}{(k^2 - m^2)^2}, \tag{1}$$

and

$$B = I_{\rm quad}(m^2) + m^2 I_{\rm log}(m^2), \tag{2}$$

where $\int_k \equiv \int d^4k/(2\pi)^4$ and we recover the standard notation of implicit regularization

$$I_{\rm log}(m^2) \equiv \int_k \frac{1}{(k^2 - m^2)^2}, \tag{3}$$

and

$$I_{\rm quad}(m^2) \equiv \int_k \frac{1}{(k^2 - m^2)}. \tag{4}$$

We expect $A = B$ guaranteed by any regularization procedure. However this is not the case. Proper-time regularization [28], for instance, introduces a cutoff $\Lambda$ after Wick rotation via the following identity at the level of propagators

$$\frac{\Gamma(n)}{(k^2 + m^2)^n} = \int_0^\infty d\tau \tau^{n-1} e^{-\tau(k^2+m^2)} \to \int_{1/\Lambda^2}^\infty d\tau \tau^{n-1} e^{-\tau(k^2+m^2)}. \tag{5}$$

Thus it is trivial to obtain within the proper-time method that $A \neq B$ since

$$A_\Lambda = \frac{-2i}{(4\pi)^2}(\Lambda^2 - m^2 \ln \Lambda^2/m^2), \tag{6}$$

whereas

$$B_\Lambda = \frac{-i}{(4\pi)^2}(\Lambda^2 - 2m^2 \ln \Lambda^2/m^2). \tag{7}$$

On the other hand it is straightforward to show that standard DReg leads to $A = B$. To circumvent such discrepancy the authors of Ref. [27] define an $n$-dimensional integral

$$I(\alpha, \beta) = \int_k^n \frac{1}{(\alpha k^2 + \beta m^2)}, \tag{8}$$

with arbitrary $\alpha$ and $\beta$, in order to write

$$A = -\frac{\partial}{\partial \alpha} I(\alpha, \beta)|_{\alpha=\beta=1, n=4}, \tag{9}$$

and

$$B = I(\alpha, \beta)\Big|_{\alpha=\beta=1,n=4} + \frac{\partial}{\partial \beta} I(\alpha, \beta)\Big|_{\alpha=\beta=1,n=4}. \tag{10}$$

Then resorting to proper-time regularization one gets

$$I(\alpha, \beta)_\Lambda = \alpha^{-n/2} \int_k^n \frac{1}{(k^2 - \beta m^2)} = \frac{-\alpha^{n/2} i}{(4\pi)^2}(\Lambda^2 - \beta m^2 \ln (\Lambda^2/m^2)), \tag{11}$$

from which is obtained

$$A_\Lambda^n = \frac{-i}{(4\pi)^2}\left(\frac{n}{2}\Lambda^2 - \frac{n}{2}m^2 \ln(\Lambda^2/m^2)\right), \quad (12)$$

and

$$B_\Lambda^n = \frac{-i}{(4\pi)^2}(\Lambda^2 - 2m^2 \ln(\Lambda^2/m^2)). \quad (13)$$

While keeping $n = 4$ violates $A = B$, the choices $n = 4$ in the term $\propto \ln \Lambda^2$ and $n = 2$ in the term $\propto \Lambda^2$ lead $A$ to coincide with $B$ at regularized level. Yet arbitrary the authors consider such a prescription, which is generalizable to other integrals in Feynman amplitudes, a concrete realization for a four-dimensional regularization. They claim that Veltman in Ref. [29] already notices that quadratic divergences are associated with $n = 2$ whereas logarithmic divergences have to be treated in $n = 4$ in DReg. Other authors have used a similar approach [30–32].

Let us now consider another related example. Consider the effect of a shift in the integration variable of a four-dimensional integral. As is well known such shifts accompany surface terms in more than logarithmically divergent integrals. Their value is highly regularization dependent. For instance take the difference between two linearly divergent integrals for $\omega = 2$:

$$\Delta_1 = \int_k^{2\omega} \frac{k_\mu}{[(k-p)^2 - m^2]^2} - \int_k^{2\omega} \frac{(k+p)_\mu}{[k^2 - m^2]^2}. \quad (14)$$

Clearly $\Delta_1 = 0$ in DReg because in this method no surface terms accompany shifts in the integration variable. In Ref. [20] the authors generalize the procedure adopted by Jauch and Rohrlich in Ref. [33] to evaluate $\Delta_1$ for $\omega$ exactly equal to 2. Their purpose was founded on the physical motivation of constructing four-dimensional regularizations with properties compatible with DReg. By defining

$$I^{2n+1,r}_{\mu_1\cdots\mu_{2n+1}} = \int_k^{2\omega} \frac{\prod_{j=1}^{2n+1} k_{\mu_j}}{[(k-p)^2 - m^2]^r}, \quad (15)$$

and

$$J^{2n+1,r}_{\mu_1\cdots\mu_{2n+1}} = \int_k^{2\omega} \frac{\prod_{j=1}^{2n+1} (k+p)_{\mu_j}}{[k^2 - m^2]^r}, \quad (16)$$

it is shown in Ref. [20] that while $I = J$ for $2\omega + 2n + 1 - 2r < 1$, if $2 > 2\omega + 2n + 1 - 2r > 1$ then

$$I^{2n+1,r}_{\mu_1\cdots\mu_{2n+1}} - J^{2n+1,r}_{\mu_1\cdots\mu_{2n+1}} = \frac{-i(2\pi)^4 \pi^\omega G_{n,2n+1}(p)}{\Gamma(\omega)} \delta_{r,\omega+n}, \quad (17)$$

with

$$G_{n,2n+1}(p) = \frac{g_{\mu_{j_1}\mu_{j_2}} \cdots g_{\mu_{j_{2n-1}}\mu_{j_{2n}}} p_{\mu_{j_{2n+1}}} \sigma^{j_1\cdots j_{2n+1}}}{\Gamma(\omega)^{-1}\Gamma(\omega+n+1)n!2^{2n}}, \quad (18)$$

and

$$\sigma^{j_1\cdots j_{2n+1}} = \epsilon^{j_1\cdots j_{2n+1}}(-)^{\mathrm{sign}(\epsilon)}. \quad (19)$$

For $n = 0$ we immediately obtain

$$\Delta_1 = \frac{-i\pi^2(2\pi)^4}{2} \delta_{\omega 2} p_\mu. \quad (20)$$

A similar expression may be obtained for more than linearly divergent variable shifted integrals. It is evident from above that the Kronecker delta signs a discontinuity in the dimensionality $\omega$. The authors use these results to back up an integer dimensional regularization called preregularization where the freedom of momentum routing in the loops is chosen to cancel out some surface terms, thus preserving Ward identities in chiral anomalies or supersymmetry [34–36]. A relevant question, given that shifts of integration variables are regularization dependent, would be to verify whether the argument could be turned the other way around, namely, to exploit the consequences of momentum routing invariance over regularization schemes. Some technicalities deserve attention. Symmetric integration in $n$ (integer) dimensions, namely, $k_\mu k_\nu \to g_{\mu\nu}k^2/n$, under integration in $k$ for divergent integrals does *not* hold in general and has been a source of disagreements in loop calculations, discussed as well in Ref. [37] in the context of $CPT$ violation in quantum field theory and used in Ref. [3] to study the Higgs decay into two photons. In particular symmetric integration was used in Ref. [33] to evaluate $\Delta_1$.

We proceed to write down a general parametrization for loop integrals which incorporates explicitly arbitrary constants which will be fixed on physical grounds. Later on we propose an alternative description of ultraviolet (and infrared) divergences in terms of basic divergent integrals. In such a description undetermined regularization-dependent constants are expressed in terms of a special set of well known surface terms, namely, integrals of total divergences in momentum space, whose contact with momentum routing invariance in the diagrams is immediate as well as is their generalization to arbitrary loop order. In order to isolate the basic loop integrals from Feynman amplitudes, an identity at the level of the integrand,

$$\frac{1}{[(k+p)^2 - m^2]} = \sum_{j=0}^{N} \frac{(-1)^j (p^2 + 2p\cdot k)^j}{(k^2 - m^2)^{j+1}} + \frac{(-1)^{N+1}(p^2 + 2p\cdot k)^{N+1}}{(k^2 - m^2)^{N+1}[(k+p)^2 - m^2]}, \quad (21)$$

can be judiciously used to extract external momentum dependence from loop integrals. Such an operation at the level of integrands somewhat resembles the renormalization procedure originally proposed by BPHZ [22–25] in which divergent Green functions are Taylor expanded up to the order needed to reach convergent integrals. We assume an implicit regulator under the integration sign which acts on the physical dimension of the underlying theory avoiding conflicts with space-time and internal algebras sensitive to dimensional continuation. Consider the derivative of $I_{\log}(m^2)$ in $d$ (integer) space-time dimensions,

$$\frac{dI_{\log}(m^2)}{dm^2} = -\frac{b_d}{m^2}, \qquad \frac{dI^{\mu\nu}_{\log}(m^2)}{dm^2} = -\frac{g^{\mu\nu}}{d}\frac{b_d}{m^2}, \tag{22}$$

where for future reference

$$b_d = \frac{i}{(4\pi)^{d/2}}\frac{(-)^{d/2}}{\Gamma(d/2)}. \tag{23}$$

A general parametrization which obeys the relations above is given by

$$\begin{aligned} I_{\log}(m^2) &= b_d \ln\left(\frac{\Lambda^2}{m^2}\right) + \alpha_1, \\ I^{\mu\nu}_{\log}(m^2) &= \frac{g^{\mu\nu}}{d}\left[b_d \ln\left(\frac{\Lambda^2}{m^2}\right) + \alpha_1'\right], \end{aligned} \tag{24}$$

where $\alpha_1$, $\alpha_1'$ are arbitrary dimensionless regularization-dependent constants, $\Lambda$ is an ultraviolet cutoff, and

$$I^{\mu\nu}_{\log}(m^2) = \int_k \frac{k^\mu k^\nu}{(k^2 - m^2)^3}. \tag{25}$$

In a similar fashion

$$\begin{aligned} \frac{dI_{\text{quad}}(m^2)}{dm^2} &= \frac{(d-2)}{2} I_{\log}(m^2), \\ \frac{dI^{\mu\nu}_{\text{quad}}(m^2)}{dm^2} &= \left(\frac{d}{2}\right) I^{\mu\nu}_{\log}(m^2), \end{aligned} \tag{26}$$

where the expression for $I^{\mu\nu}_{\text{quad}}(m^2)$ is now clear, namely, a basic quadratically divergent integral containing two-loop momenta with Lorentz indices $\mu$ and $\nu$. Again, a parametrization that complies with the relations above is

$$\begin{aligned} I_{\text{quad}}(m^2) &= \frac{(d-2)}{2}\left[\alpha_2 \Lambda^2 + b_d m^2 \ln\left(\frac{\Lambda^2}{m^2}\right) + \alpha_3 m^2\right], \\ I^{\mu\nu}_{\text{quad}}(m^2) &= \frac{g^{\mu\nu}}{2}\left[\alpha_2' \Lambda^2 + b_d m^2 \ln\left(\frac{\Lambda^2}{m^2}\right) + \alpha_3' m^2\right], \end{aligned} \tag{27}$$

in which all regularization dependence is encoded in the $\alpha$'s. Some comments are in order. It is economical and neat to write basic divergent integrals in terms of $\{I_{\log}(m^2), I_{\text{quad}}(m^2)\ldots\}$ without Lorentz indices in internal momenta, in other words expressing $I^{\mu\nu}_{\log}(m^2)$ and $I^{\mu\nu}_{\text{quad}}(m^2)$, etc., in terms of $I_{\log}(m^2)$ and $I_{\text{quad}}(m^2)$, respectively, both without resorting to symmetric integration and in a regularization-independent way through surface terms. For instance it is straightforward to show that

$$\begin{aligned} \Upsilon^{\mu\nu}_0 &\equiv \int^d_k \frac{\partial}{\partial k_\mu}\frac{k^\nu}{(k^2 - m^2)^{\frac{d}{2}}} \\ &= d\left[\frac{g^{\mu\nu}}{d} I_{\log}(m^2) - I^{\mu\nu}_{\log}(m^2)\right], \end{aligned} \tag{28}$$

and

$$\begin{aligned} \Upsilon^{\mu\nu}_2 &\equiv \int^d_k \frac{\partial}{\partial k_\mu}\frac{k^\nu}{(k^2 - m^2)^{\frac{d-2}{2}}} \\ &= (d-2)\left[\frac{g^{\mu\nu}}{(d-2)} I_{\text{quad}}(m^2) - I^{\mu\nu}_{\text{quad}}(m^2)\right]. \end{aligned} \tag{29}$$

The surface terms $\Upsilon$'s are regularization-dependent terms which however can be shown to be physically meaningful and therefore be fixed. That is because although the intrinsic (regularization-dependent) parameters in loop integrals are indeed ambiguous, the well adjusted relation between them expressed by the $\Upsilon$'s is not. In other words in the process of reducing the set of loop integrals to basic divergent integrals it can be shown that the vanishing of surface terms expressed by the $\Upsilon$'s reflects momentum routing invariance in the loops of a Feynman diagram [21,38]. Attributing spurious values to such surface terms is the root of quantum symmetry breaking by regularizations. Once we attach a physical meaning to them, as it is proposed in the implicit regularization program, we may regularize infinities in a regularization-independent fashion because the renormalization constants can be defined in terms of basic divergent integrals themselves. To see that they are regularization dependent we can use the parametrizations (24) and (27) to obtain

$$\Upsilon^{\mu\nu}_0 \propto g^{\mu\nu}(\alpha_1 - \alpha_1'), \tag{30}$$

and

$$\Upsilon^{\mu\nu}_2 \propto g^{\mu\nu}[(\alpha_2 - \alpha_2')\Lambda^2 + (\alpha_3 - \alpha_3')m^2]. \tag{31}$$

For instance in the four-dimensional case $\Upsilon^{\mu\nu}_0 = g^{\mu\nu}[i/8(4\pi)^2]$ and $\Upsilon^{\mu\nu}_2 = g^{\mu\nu}\Lambda^2[i/4(4\pi)^2]$ in sharp cutoff regularization [39] while they are both zero in DReg. As for the examples we presented earlier, it is immediate that $A = B$ within our approach because summing and subtracting $m^2$ in the numerator of $A$ leads to $B$. Whenever even powers of internal momenta appear in the numerator, one can always make use of such artifice to avoid ambiguous symmetric integration [40]. As for $\Delta_1$ in Eq. (14) one obtains within implicit regularization

$$\Delta_1^{\text{IR}} = \Upsilon^{\mu\nu}_0 p_\nu. \tag{32}$$

In Ref. [21] we demonstrate that momentum routing invariance in Feynman diagrams (enforced by setting all surface terms $\Upsilon$'s to zero) leads automatically to Abelian gauge invariance at arbitrary loop order.

For the sake of completeness we draw a few remarks regarding renormalization and generalization to arbitrary loop order in four space-time dimensions within this approach. To define a mass independent scheme we use the regularization-independent relation

$$I_{\log}(m^2) = I_{\log}(\lambda^2) + b \ln\left(\frac{\lambda^2}{m^2}\right), \tag{33}$$

where $\lambda \neq 0$ plays the role of renormalization group scale (see Ref. [21] and references therein). After subtraction of subdivergences according to BPHZ formalism we may define the divergence of $n$th-loop order in terms of basic divergent integrals for both massive and massless theories [26] in the form

$$I_{\log}^{(n)}(m^2) \equiv \int_k \frac{1}{(k^2-m^2)^2} \ln^{n-1}\left(-\frac{(k^2-m^2)}{\lambda^2}\right), \tag{34}$$

which obeys

$$I_{\log}^{(n+1)}(m^2) = I_{\log}^{(n+1)}(\lambda^2) - b \sum_{i=1}^{n+1} \frac{n!}{i!} \ln^i\left(\frac{m^2}{\lambda^2}\right). \tag{35}$$

Likewise

$$\begin{aligned} \frac{dI_{\log}^{(n)}(\lambda^2)}{d\lambda^2} &= -\frac{(n-1)}{\lambda^2} I_{\log}^{(n-1)}(\lambda^2) + \frac{b_d}{\lambda^2} A^{(n)}, \\ \frac{dI_{\log}^{(n)\mu\nu}(\lambda^2)}{d\lambda^2} &= -\frac{(n-1)}{\lambda^2} I_{\log}^{(n-1)\mu\nu}(\lambda^2) + \frac{g^{\mu\nu}}{2}\frac{b_d}{\lambda^2} B^{(n)}. \end{aligned} \tag{36}$$

After some algebra one can demonstrate that the parametrization below respects (36),

$$\begin{aligned} I_{\log}^{(n)}(\lambda^2) &= \sum_{i=1}^{n} \frac{(n-1)!}{(i-1)!}\left[\frac{(-b_d)A^{(i)}}{(n-i+1)!}\ln^{n-i+1}\left(\frac{\Lambda^2}{\lambda^2}\right) + \sum_{j=0}^{n-i}\frac{a_{n-j-i+1}}{j!(n-j-i)!}\ln^j\left(\frac{\Lambda^2}{\lambda^2}\right)\right], \\ I_{\log}^{(n)\mu\nu}(\lambda^2) &= \frac{g^{\mu\nu}}{2}\sum_{i=1}^{n} \frac{(n-1)!}{(i-1)!}\left[\frac{(-b_d)B^{(i)}}{(n-i+1)!}\ln^{n-i+1}\left(\frac{\Lambda^2}{\lambda^2}\right) + \sum_{j=0}^{n-i}\frac{a'_{n-j-i+1}}{j!(n-j-i)!}\ln^j\left(\frac{\Lambda^2}{\lambda^2}\right)\right], \end{aligned} \tag{37}$$

where

$$\begin{aligned} A^{(i)} &\equiv \Gamma(d/2)\lim_{\delta\to 0}\left[-(n-1)\sum_{l=0}^{n-2}\binom{n-2}{l}\frac{(-1)^{1+l}}{\delta^{n-2}} \times \frac{\Gamma(1-\delta(n-2-l))}{\Gamma(d/2+1-\delta(n-2-l))} + \left(\frac{d}{2}\right)\sum_{l=0}^{n-1}\binom{n-1}{l} \times \frac{(-1)^{1+l}}{\delta^{n-1}}\frac{\Gamma(1-\delta(n-1-l))}{\Gamma(d/2+1-\delta(n-1-l))}\right], \\ B^{(i)} &\equiv \Gamma(d/2)\lim_{\delta\to 0}\left[-(n-1)\sum_{l=0}^{n-2}\binom{n-2}{l}\frac{(-1)^{1+l}}{\delta^{n-2}} \times \frac{\Gamma(1-\delta(n-2-l))}{\Gamma(d/2+2-\delta(n-2-l))} + \left(\frac{d+2}{2}\right)\sum_{l=0}^{n-1}\binom{n-1}{l} \times \frac{(-1)^{1+l}}{\delta^{n-1}}\frac{\Gamma(1-\delta(n-1-l))}{\Gamma(d/2+2-\delta(n-1-l))}\right], \end{aligned} \tag{38}$$

and $a_i$, $a'_i$ are arbitrary constants. The surface terms read

$$\frac{1}{2}\sum_{j=1}^{n}\left(\frac{2}{d}\right)^j \frac{(n-1)!}{(n-j)!}\Upsilon_0^{(n)\mu\nu} = -I_{\log}^{(n)\mu\nu}(\lambda^2) + \frac{g^{\mu\nu}}{2}\sum_{j=1}^{n}\left(\frac{2}{d}\right)^j \times \frac{(n-1)!}{(n-j)!} I_{\log}^{(l-j+1)}(\lambda^2). \tag{39}$$

Generalization to an arbitrary number of Lorentz indices is equally straightforward.

## III. HIGGS DECAY INTO TWO PHOTONS

In this section we will study the $W$-loop contributions to the Higgs decay into two photons. Using the unitary gauge we have only three Feynman diagrams to evaluate (Fig. 1). Notice that we are not choosing a specific routing for the diagrams since we want to study how the final amplitude depends on it.

The sum of the three diagrams can be simplified to the expression (Feynman rules as well as the basic steps to arrive at the equations below are presented in the Appendix)[1]

$$M = ie^2 g M_w [M_{\mu\nu}^{(a)} + M_{\mu\nu}^{(b)} + M_{\mu\nu}^{(c)}](\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^* + (p_1 \leftrightarrow p_2, \mu \leftrightarrow \nu), \tag{40}$$

$$\begin{aligned} M_{\mu\nu}^{(a)} = &-\frac{4}{M_w^2}[g_{\mu\nu}(p_1)^\alpha(p_2)^\beta I_{\alpha\beta}^{(3)} + (p_1\cdot p_2)I_{\mu\nu}^{(3)} - (p_1)_\nu(p_2)^\alpha I_{\mu\alpha}^{(3)} - (p_2)_\mu(p_1)^\alpha I_{\nu\alpha}^{(3)}] \\ &+ \frac{2}{M_w^2}[g_{\mu\nu}(p_1\cdot p_2) - (p_2)_\mu(p_1)_\nu]I_2^{(3)}, \end{aligned} \tag{41}$$

$$M_{\mu\nu}^{(b)} = \int_k \frac{3(g_{\mu\nu}k^2 - 4k_\mu k_\nu)}{(q_1^2 - M_w^2)(q_2^2 - M_w^2)(q_3^2 - M_w^2)}, \tag{42}$$

$$\begin{aligned} M_{\mu\nu}^{(c)} = &\,6g_{\mu\nu}\left[(p_1\cdot p_2)I_0^{(3)} - (p_1)^\alpha I_\alpha^{(3)} - \frac{M_w^2}{2}I_0^{(3)}\right] \\ &+ 6[2(p_1)_\nu I_\mu^{(3)} - (p_2)_\mu(p_1)_\nu I_0^{(3)}], \end{aligned} \tag{43}$$

$$I_{0,2,\mu,\mu\nu}^{(3)} = \int_k \frac{1, k^2, k_\mu, k_\mu k_\nu}{(q_1^2 - M_w^2)(q_2^2 - M_w^2)(q_3^2 - M_w^2)}. \tag{44}$$

As one may notice only $M_{\mu\nu}^{(a)}$ and $M_{\mu\nu}^{(b)}$ contain divergent terms. At this point we must choose a regularization in order to deal properly with such terms. We employ IReg, which allows us to express divergent integrals in terms of loop momenta only (for a review see Ref. [21] and references therein). A characteristic of IReg is that all regularization-dependent objects (surface terms) can be consistently treated, allowing a clear discussion about ambiguities as will be seen below. Explicitly they are given by

$$\begin{aligned} \Upsilon_0^{\mu\nu} = g^{\mu\nu}\Gamma_0 &= \int_k \frac{\partial}{\partial k_\mu}\frac{k^\nu}{(k^2 - M_w^2)^2} \\ &= \int_k \frac{g^{\mu\nu}}{(k^2 - M_w^2)^2} - 4\int_k \frac{k^\mu k^\nu}{(k^2 - M_w^2)^3}. \end{aligned} \tag{45}$$

Therefore the first term can be expressed as

[1] We define $q_i = k + \chi_i$, $\bar{q}_i = k + \bar{\chi}_i$, $\int_k = \int \frac{d^4k}{(2\pi)^4}$ and use relations $p_i^2 = 0$ and $(p_1 + p_2)^2 = M_h^2$.

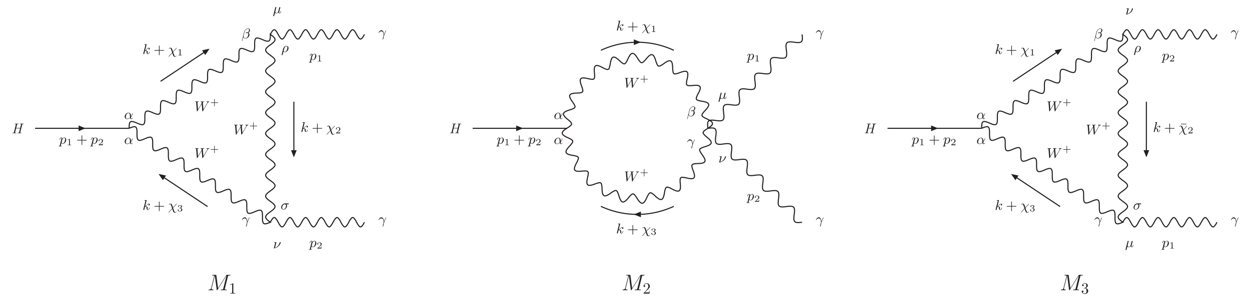


FIG. 1. Diagrams with arbitrary momentum routing $\chi$.

$$M^{(a)}_{\mu\nu} = \frac{[(p_2)_\mu (p_1)_\nu - g_{\mu\nu}(p_1 \cdot p_2)]}{M_w^2}\left[\frac{i}{16\pi^2} - 2\Gamma_0\right]. \quad (46)$$

The first point to be noticed is that this term is gauge invariant and, in general, ambiguous since it depends on a surface term. Another feature is that it does not depend on[2] $\tau$ which gives us a clue that it may be the term missing on Ref. [3]. In fact, if one performs a symmetric regularization in four dimensions (by replacing $k_\mu k_\nu \to g_{\mu\nu}k^2/4$), it will be null. In other words a four-dimensional regularization that resorts to such substitution evaluates the surface term to a precise value, in this case $i/32\pi^2$. On the other hand, if one uses DReg the surface term will vanish which furnishes a *non-null* amplitude in the limit $\tau^{-1} \to 0$. In the framework of IReg there is no reason *a priori* to favor one of these two values since we are dealing with ambiguous objects in nature. From our perspective, physical conditions, other than the regularization method, should constrain the value the surface term should assume. In general, one such condition is to impose gauge invariance; however, since this term is *already* gauge invariant, this consideration will not fix it. Therefore, we should leave it arbitrary and proceed with the calculation of the amplitude for now. The sum of the two last terms is[3]

$$M^{(b)}_{\mu\nu} + M^{(c)}_{\mu\nu} = \frac{i}{16\pi^2 M_w^2}[(p_2)_\mu (p_1)_\nu - g_{\mu\nu}(p_1 \cdot p_2)] \times \left[\frac{3\tau^{-1}}{2} + \frac{3(2\tau^{-1} - \tau^{-2})f(\tau)}{2}\right] + g_{\mu\nu}(p_1 \cdot p_2)\left(\frac{3\tau^{-1}}{2M_w^2}\Gamma_0\right). \quad (47)$$

Readily one may notice the appearance of another surface term due to $M^{(b)}_{\mu\nu}$ which explicitly breaks gauge invariance. Since there are no other terms to consider, one should impose gauge invariance as a physical condition that the whole amplitude should fulfill. Thus the otherwise arbitrary surface term must assume a precise value which in our case is null. This choice also fixes the surface term appearing in (46) since in the framework of IReg there is no distinction between surface terms coming from integrals with the same degree of divergence and the same Lorentz structure. This approach is different from the one found in Ref. [10] where a cutoff scheme is used and the ambiguities are parametrized by different boundary conditions for the integrals appearing in (46) and (47). Since the authors of Ref. [10] consider that each integral is arbitrary and integrals are unrelated to each other, they conclude that the imposition of gauge invariance is not enough to give an unambiguous result.

After all these considerations we obtain the amplitude for the Higgs decay into two photons in the framework of IReg,

$$M = -\frac{e^2 g}{16\pi^2 M_w}[(p_2)_\mu (p_1)_\nu - g_{\mu\nu}(p_1 \cdot p_2)] \times [2 + 3\tau^{-1} + 3(2\tau^{-1} - \tau^{-2})f(\tau)](\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*, \quad (48)$$

which agrees with previous ones found in the literature [4–6].

In the time this work was written another paper devoted to this decay appeared [41]. Its authors have a point of view similar to ours in the sense that ambiguities should be fixed on physical grounds.[4] They use the equivalence theorem as well as the conservation of charge as inputs that their amplitude must fulfill. Since these are consequences of gauge invariance there is no surprise that just the imposition of such a requirement gives us an unambiguous result.

Another interesting point discussed there is the role played by momentum routing freedom. From their point of view the loop momentum of the three diagrams must be

[2] We define $\tau = \frac{M_h^2}{4M_w^2}$.

[3] Where we define

$$f(\tau) = \begin{cases} \arcsin^2(\sqrt{\tau}) & \text{for } \tau \le 1, \\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right]^2 & \text{for } \tau > 1. \end{cases}$$

[4] It should be emphasized that their definition of the ambiguity is more closely related to the one found in Ref. [42]. We, on the other hand, define it by (45), which is more closely related to the preservation of Abelian gauge invariance [21].

chosen in a particular way as to reduce the superficial degree of divergence of the amplitude to a logarithmic one.[5] However, from our point of view MRI is a symmetry that must be respected since it is connected with Abelian gauge invariance as well as supersymmetry preservation [21]. The importance of this statement is particularly clear if, instead of considering the calculation of the whole amplitude, one evaluates each diagram *individually*. Following the reasoning of Ref. [21] one finds out that momentum routing dependent terms will arise always multiplied by arbitrary-valued objects (surface terms). Therefore, since individual diagrams are not supposed to be gauge invariant, the only symmetry left in order to fix the ambiguities is demanding momentum routing invariance. As can be seen, we could have adopted this approach since the beginning of our work, avoiding completely the discussion of gauge invariance (since the two symmetries are connected it is not a surprise that the surface terms must be null in both cases). However, in order to make contact with the literature we performed the calculation of the whole amplitude with the same routing for all three diagrams, which evidently is not the more general situation. Therefore, it is not a surprise that our result is independent of the momentum routing $\chi_1$ even though we still have an ambiguity expressed by $\Gamma_0$.

## IV. TWO-PHOTON SCATTERING

In this brief section we would like to comment on the result found in Ref. [14]. As in the case just analyzed, the problem lies on divergent integrals which appear as intermediate steps of the calculation. Explicitly we have [15]

$$A^{\mu\nu\rho\sigma} = \int_k \frac{m^4 S_1^{\mu\nu\rho\sigma} + 2m^2(2S_2^{\mu\nu\rho\sigma} - k^2 S_1^{\mu\nu\rho\sigma})}{(k^2 - m^2)^4} + \int_k \frac{24k^\mu k^\nu k^\rho k^\sigma + (k^2)^2 S_1^{\mu\nu\rho\sigma} - 4k^2 S_2^{\mu\nu\rho\sigma}}{(k^2 - m^2)^4}, \tag{49}$$

where

$$S_1^{\mu\nu\rho\sigma} = g^{\mu\nu}g^{\rho\sigma} + g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\rho\nu},$$
$$S_2^{\mu\nu\rho\sigma} = g^{\mu\nu}k^\rho k^\sigma + g^{\mu\rho}k^\nu k^\sigma + g^{\mu\sigma}k^\rho k^\nu + g^{\rho\nu}k^\mu k^\sigma + g^{\sigma\nu}k^\rho k^\mu + g^{\rho\sigma}k^\mu k^\nu. \tag{50}$$

As can be readily seen, the integral above is divergent, thus ambiguous. Such a statement is particularly clear in the framework of IReg since it is evaluated to $(\Gamma_0^{(2)} - 4\Gamma_0^{(4)})S_1^{\mu\nu\rho\sigma}$ where $\Gamma_0^{(i)}$ is a surface term coming from an integral with Lorentz structure $k^{\nu_1} \dots k^{\nu_i}$. Therefore, there is no preferred value this integral should assume—it should be left arbitrary being fixed by the imposition of physical conditions. As discussed in Ref. [15], a non-null value for $A^{\mu\nu\rho\sigma}$ implies the breaking of gauge invariance which means the surface terms must obey $\Gamma_0^{(2)} = 4\Gamma_0^{(4)}$ in order to respect such symmetry. Thus, there is no ambiguity left on the final amplitude which as expected agrees with previous results found in the literature [15]. In summary, as in the case of Ref. [3], the authors of Ref. [14] performed a symmetric regularization on the integral above which in turn gave a precise value to the surface terms ($A^{\mu\nu\rho\sigma} = (i/96\pi^2)S_1^{\mu\nu\rho\sigma}$). Such a choice resulted in a different cross section for the two-photon scattering than the one found previously in the literature [16,17]. However, since the integral is ambiguous in nature there is no reason to assume a precise value for the surface terms which must be fixed on physical grounds.

## V. CONCLUDING REMARKS

In this work we studied the decay of the Higgs boson into two photons as well as the two-photon scattering amplitude. Both processes must have only finite corrections since the photon does not couple with the Higgs boson and neither with itself. However, in the intermediate steps of the calculation one may encounter divergent integrals and the issue of regularization is particularly important in order to give a meaningful result. To discuss the ambiguities inherent in such processes we used the framework of implicit regularization which can consistently separate the divergent, finite, and ambiguous part of any integral. We found out that although the divergent parts cancel as expected, there are some ambiguities left (parametrized as surface terms). These should not be fixed by the regularization scheme *a priori*, but should be left arbitrary as has been determined by physical conditions. In the cases studied here, the condition used was the gauge invariance of the final result which univocally fixed the surface terms thus recovering the amplitude for the Higgs decay as well as the cross section of the two-photon scattering found previously in the literature.

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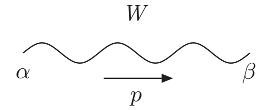
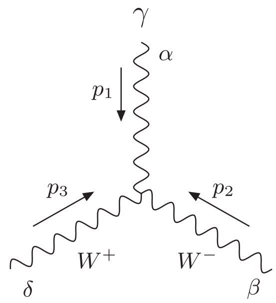
## APPENDIX A: SOME EXPLICIT CALCULATIONS OF THE AMPLITUDE $H \to \gamma\gamma$

In this Appendix we will show how the terms presented in the calculation of the Higgs decay into two photons can be simplified. We will use the Feynman rules defined in Fig. 2.

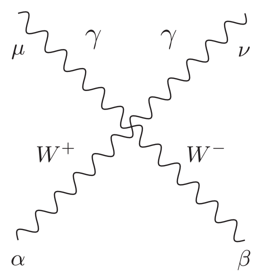
[5]They find that all three diagrams must contain the same momentum routing. Therefore it is no surprise that our result before regularization (44) contains at most logarithmic divergent integrals since we also adopted the same momentum routing for all three diagrams ($\chi_1$).



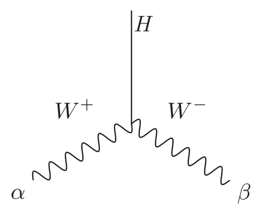

$$\frac{i}{p^2-M_w^2}\Big(-g^{\alpha\beta}+\frac{p^\alpha p^\beta}{M_w^2}\Big)$$




$$ieV_{\alpha\beta\delta}(p_1,p_2,p_3)\equiv ie[(p_2-p_3)_\alpha g_{\beta\delta}+(p_3-p_1)_\beta g_{\delta\alpha}+(p_1-p_2)_\delta g_{\alpha\beta}]$$




$$-ie^2U_{\alpha\beta\mu\nu}\equiv -ie^2[2g_{\alpha\beta}g_{\mu\nu}-g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}]$$




$$igM_wg_{\alpha\beta}$$

FIG. 2. Feynman rules.

Defining $q_i = k + x_i$, $\bar{q}_i = k + \bar{x}_i$, and $\int_k = \int \frac{d^4k}{(2\pi)^4}$, the diagrams of Fig. 1 are expressed as

$$M_1 = ie^2gM_w\int_k \frac{1}{q_1^2-M_w^2}\left(-g^{\alpha\beta}+\frac{q_1^\alpha q_1^\beta}{M_w^2}\right)V_{\mu\rho\beta}(-p_1,-q_2,q_1)\frac{1}{q_2^2-M_w^2}\left(-g^{\rho\sigma}+\frac{q_2^\rho q_2^\sigma}{M_w^2}\right)$$
$$\times V_{\nu\gamma\sigma}(-p_2,-q_3,q_2)\frac{1}{q_3^2-M_w^2}\left(-g^{\gamma\alpha}+\frac{q_3^\alpha q_3^\gamma}{M_w^2}\right)(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*, \tag{A1}$$

$$M_2 = ie^2gM_w\int_k \frac{1}{q_1^2-M_w^2}\left(-g^{\alpha\beta}+\frac{q_1^\alpha q_1^\beta}{M_w^2}\right)U_{\beta\gamma\mu\nu}\frac{1}{q_3^2-M_w^2}\left(-g^{\gamma\alpha}+\frac{q_3^\alpha q_3^\gamma}{M_w^2}\right)(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*, \tag{A2}$$

$$M_3 = ie^2gM_w\int_k \frac{1}{q_1^2-M_w^2}\left(-g^{\alpha\beta}+\frac{q_1^\alpha q_1^\beta}{M_w^2}\right)V_{\nu\rho\beta}(-p_2,-\bar{q}_2,q_1)\frac{1}{\bar{q}_2^2-M_w^2}\left(-g^{\rho\sigma}+\frac{\bar{q}_2^\rho \bar{q}_2^\sigma}{M_w^2}\right)\times V_{\mu\gamma\sigma}(-p_1,-q_3,\bar{q}_2)$$
$$\times\frac{1}{q_3^2-M_w^2}\left(-g^{\gamma\alpha}+\frac{q_3^\alpha q_3^\gamma}{M_w^2}\right)(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*. \tag{A3}$$

The strategy now is to classify the terms of the integrand according to their dependence on $M_w^{-n}$.

### 1. Terms $M_w^{-6}$

The term coming from $M_1$, which we call $M_1^{(-6)}$, is[6]

$$M_1^{(-6)} = \left(\frac{q_1^\alpha q_1^\beta}{M_w^2}\right) V_{\mu\rho\beta}(-p_1, -q_2, q_1)\left(\frac{q_2^\rho q_2^\sigma}{M_w^2}\right) \times V_{\nu\gamma\sigma}(-p_2, -q_3, q_2)\left(\frac{q_3^\alpha q_3^\gamma}{M_w^2}\right)\frac{(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}, \tag{A4}$$

where

$$\frac{1}{D} \equiv \frac{1}{(q_1^2 - M_w^2)}\frac{1}{(q_1^3 - M_w^2)}. \tag{A5}$$

Using that $q_1^\beta q_2^\rho V_{\mu\rho\beta}(-p_1, -q_2, q_1)(\epsilon_1{}^\mu)^* = 0$ we obtain a null contribution. A similar reasoning can be applied to the term coming from $M_3$.

### 2. Terms $M_w^{-4}$

The diagram $M_1$ has two contributions. However, one of them is null [due to identity $q_1^\beta q_2^\rho V_{\mu\rho\beta}(-p_1, -q_2, q_1) \times (\epsilon_1{}^\mu)^* = 0$] leaving us with the following term:

$$M_1^{(-4)} = \frac{q_1^\alpha q_1^\beta}{M_w^2} V_{\mu\rho\beta}(-p_1, -q_2, q_1)(-g^{\rho\sigma}) \times V_{\nu\gamma\sigma}(-p_2, -q_3, q_2)\frac{q_3^\alpha q_3^\gamma}{M_w^2}\frac{(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}. \tag{A6}$$

With the help of identity

$$q_1^\beta V_{\mu\rho\beta}(-p_1, -q_2, q_1)(\epsilon_1{}^\mu)^* = \{-(q_2)_\mu(q_2)_\rho + M_w^2 g_{\mu\rho} + [(q_2)^2 - M_w^2]g_{\mu\rho}\}(\epsilon_1{}^\mu)^*, \tag{A7}$$

we can separate $M_1^{(-4)}$ into three terms. The first one is null due to identity

$$q_3^\gamma q_2^\sigma V_{\nu\gamma\sigma}(-p_2, -q_3, q_2)(\epsilon_2{}^\nu)^* = 0. \tag{A8}$$

The second is proportional to $M_w^{-2}$ and will be treated in the next section (we call it $M_1^{(-2;-4)}$). In the third one ($M_1^{(-4;2d)}$) we cancel one of the denominators to obtain

$$M_1^{(-4;2d)} = \frac{q_1^\alpha}{M_w^2} g_{\mu\rho}(-g^{\rho\sigma})V_{\nu\gamma\sigma}(-p_2, -q_3, q_2) \times \frac{q_3^\alpha q_3^\gamma}{M_w^2}\frac{(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*}{D}. \tag{A9}$$

Performing the exchange $p_1 \leftrightarrow p_2$ and $\mu \leftrightarrow \nu$ we obtain the contributions from diagram $M_3$. For diagram $M_2$ we have

$$M_2^{(-4)} = \frac{q_1^\alpha q_1^\beta}{M_w^2} U_{\beta\gamma\mu\nu}\frac{q_3^\alpha q_3^\gamma}{M_w^2}\frac{(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*}{D}. \tag{A10}$$

Summing $M_1^{(-4;2d)} + M_2^{(-4)} + M_3^{(-4;2d)}$ we obtain a null result.

### 3. Terms $M_w^{-2}$

The contributions coming from the diagram $M_1$ are

$$M_1^{(-2)} = \frac{q_1^\alpha q_1^\beta}{M_w^2} V_{\mu\rho\beta}(-p_1, -q_2, q_1)(-g^{\rho\sigma})V_{\nu\gamma\sigma}(-p_2, -q_3, q_2)(-g^{\gamma\alpha})\frac{(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)} + (-g^{\alpha\beta})V_{\mu\rho\beta}(-p_1, -q_2, q_1) \times \frac{q_2^\rho q_2^\sigma}{M_w^2} V_{\nu\gamma\sigma}(-p_2, -q_3, q_2)(-g^{\gamma\alpha})\frac{(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)} + (-g^{\alpha\beta})V_{\mu\rho\beta}(-p_1, -q_2, q_1)(-g^{\rho\sigma})V_{\nu\gamma\sigma}(-p_2, -q_3, q_2) \times \frac{q_3^\alpha q_3^\gamma}{M_w^2}\frac{(\epsilon_1{}^\mu)^*(\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}. \tag{A11}$$

We use identities

$$q_1^\beta V_{\mu\rho\beta}(-p_1, -q_2, q_1)(\epsilon_1{}^\mu)^* = \{-(q_2)_\mu(q_2)_\rho + M_w^2 g_{\mu\rho} + [(q_2)^2 - M_w^2]g_{\mu\rho}\}(\epsilon_1{}^\mu)^*, \tag{A12}$$

$$q_3^\gamma V_{\nu\gamma\sigma}(-p_2, -q_3, q_2)(\epsilon_2{}^\nu)^* = \{-(q_2)_\nu(q_2)_\sigma + M_w^2 g_{\nu\sigma} + [(q_2)^2 - M_w^2]g_{\nu\sigma}\}(\epsilon_2{}^\nu)^*, \tag{A13}$$

in order to separate $M_1^{(-2)}$ into three terms: $M_1^{(-2;2d)}$, which has only two denominators, $M_1^{(0;-2)}$, which is proportional to $M_w^0$, and $M_1^{(-2;3d)}$.

As before, the diagram $M_3$ furnishes similar contributions. The diagram $M_2$ gives

[6] In the following we will omit the common factor $ie^2 g M_w$ as well the integral in $k$.

$$M_2^{(-2)} = \left[\frac{q_1^\alpha q_1^\beta}{M_w^2} U_{\beta\gamma\mu\nu}(-g^{\gamma\alpha}) + (-g^{\alpha\beta}) U_{\beta\gamma\mu\nu} \frac{q_3^\alpha q_3^\gamma}{M_w^2}\right] \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D}. \quad \text{(A14)}$$

Adding $M_1^{(-2;2d)}$, $M_2^{(-2)}$, and $M_3^{(-2;2d)}$, we obtain a null result. Therefore, the remaining terms proportional to $M_w^{-2}$ are $M_1^{(-2;3d)}$, $M_1^{(-2;-4)}$, and similar contributions from $M_3$. Using now the definitions of $q_i$ and identities such as $(q_2)^2 = [(q_2)^2 - M_w^2] + M_w^2$, these terms can be simplified to

$$M_1^{(-2;3d)} + M_1^{(-2;-4)} = M_1^{(-2;\mathrm{div})} + M_1^{(0;\mathrm{fin})},$$

$$M_1^{(-2;\mathrm{div})} = \frac{2k^2}{M_w^2}[g_{\mu\nu}(p_1 \cdot p_2) - (p_2)_\mu (p_1)_\nu] \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)} + \frac{4}{M_w^2}[-g_{\mu\nu}(p_1)^\alpha (p_2)^\beta k_\alpha k_\beta + (p_1)_\nu (p_2)^\alpha k_\mu k_\alpha + (p_2)_\mu (p_1)^\alpha k_\nu k_\alpha - (p_1 \cdot p_2) k_\mu k_\nu] \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}, \quad \text{(A15)}$$

$$M_1^{(0;\mathrm{fin})} = \{[g_{\mu\nu}[-(p_2)^\alpha k_\alpha + (p_1)^\alpha k_\alpha] - 2[-(p_2)^\mu k_\nu + (p_1)^\nu k_\mu]\} \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}. \quad \text{(A16)}$$

### 4. Terms $M_w^0$

As before, the terms of order $M_w^0$ coming from diagram $M_i$ will be called $M_i^{(0)}$. Therefore, all the terms we have to deal with are summarized below:

$$M_1^{(0;-2)} = [(q_1)^\gamma V_{\nu\gamma\mu}(-p_2, -q_3, q_2) + (q_3)^\beta V_{\mu\nu\beta}(-p_1, -q_2, q_1)] \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}, \quad \text{(A17)}$$

$$M_1^{(0;\mathrm{fin})} = \{[g_{\mu\nu}[-(p_2)^\alpha k_\alpha + (p_1)^\alpha k_\alpha] - 2[-(p_2)^\mu k_\nu + (p_1)^\nu k_\mu]\} \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}, \quad \text{(A18)}$$

$$M_1^{(0)} = (-g^{\alpha\beta}) V_{\mu\rho\beta}(-p_1, -q_2, q_1)(-g^{\rho\sigma}) \times V_{\nu\gamma\sigma}(-p_2, -q_3, q_2)(-g^{\gamma\alpha}) \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}, \quad \text{(A19)}$$

$$M_3^{(0;-2)} = M_1^{(0;-2)}(p_1 \leftrightarrow p_2, \mu \leftrightarrow \nu), \quad \text{(A20)}$$

$$M_3^{(0;\mathrm{fin})} = M_1^{(0;\mathrm{fin})}(p_1 \leftrightarrow p_2, \mu \leftrightarrow \nu), \quad \text{(A21)}$$

$$M_3^{(0)} = M_1^{(0)}(p_1 \leftrightarrow p_2, \mu \leftrightarrow \nu), \quad \text{(A22)}$$

$$M_2^{(0)} = (-g^{\alpha\beta}) U_{\beta\gamma\mu\nu}(-g^{\gamma\alpha}) \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D}. \quad \text{(A23)}$$

The last term can be expressed as

$$M_2^{(0)} = M_2^{(0;1)} + M_2^{(0;3)}, \quad \text{(A24)}$$

$$M_2^{(0;1)} = \frac{1}{2}(-g^{\alpha\beta}) U_{\beta\gamma\mu\nu}(-g^{\gamma\alpha}) \frac{(q_2^2 - M_w^2)}{D(q_2^2 - M_w^2)} (\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*, \quad \text{(A25)}$$

$$M_2^{(0;3)} = M_2^{(0;1)}(p_1 \leftrightarrow p_2, \mu \leftrightarrow \nu). \quad \text{(A26)}$$

Adding $M_1^{(0;-2)}$, $M_1^{(0;\mathrm{fin})}$, $M_1^{(0)}$, and $M_2^{(0;1)}$ together we obtain

$$M_1^{(0;-2)} + M_1^{(0;\mathrm{fin})} + M_1^{(0)} + M_2^{(0;1)} = M_t^{(0,\mathrm{div})} + M_t^{(0,\mathrm{fin})}, \quad \text{(A27)}$$

$$M_t^{(0,\mathrm{div})} = [3(g_{\mu\nu} k^2 - 4k_\mu k_\nu)] \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}, \quad \text{(A28)}$$

$$M_t^{(0,\mathrm{fin})} = 3g_{\mu\nu}[2(p_1 \cdot p_2) - 2(p_1)^\alpha k_\alpha - M_w^2] + 6[2(p_1)_\nu k_\mu - (p_2)_\mu (p_1)_\nu] \frac{(\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*}{D(q_2^2 - M_w^2)}. \quad \text{(A29)}$$

Therefore, it can be easily seen that the terms $M_{\mu\nu}^{(a)}$, $M_{\mu\nu}^{(b)}$, and $M_{\mu\nu}^{(c)}$ are given by

$$M_1^{(-2;\mathrm{div})} = M_{\mu\nu}^{(a)} (\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*, \quad \text{(A30)}$$

$$M_t^{(0,\mathrm{div})} = M_{\mu\nu}^{(b)} (\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*, \quad \text{(A31)}$$

$$M_t^{(0,\mathrm{fin})} = M_{\mu\nu}^{(c)} (\epsilon_1{}^\mu)^* (\epsilon_2{}^\nu)^*. \quad \text{(A32)}$$

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