

**Holographic Friedmann equation and  $\mathcal{N} = 4$  supersymmetric Yang-Mills theory**Kazuo Ghoroku<sup>1,\*</sup> and Akihiro Nakamura<sup>2,†</sup><sup>1</sup>*Fukuoka Institute of Technology, Wajiro, Higashi-ku, Fukuoka 811-0295, Japan*<sup>2</sup>*Department of Physics, Kagoshima University, Korimoto1-21-35, Kagoshima 890-0065, Japan*

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According to the AdS/CFT correspondence, the  $\mathcal{N} = 4$  supersymmetric Yang-Mills (SYM) theory has been studied by solving the dual supergravity. In solving the bulk Einstein equation, we find that it could be related to the 4D Friedmann equation, which is solved by using the cosmological constant and the energy density of the matter on the boundary, and they are dynamically decoupled from the SYM theory. We call this combination of the bulk Einstein equations and the 4D Friedmann equation as holographic Friedmann equations. Solving the holographic Friedmann equations, it is shown how the 4D decoupled matter and the cosmological constant control the dynamical properties of the SYM theory, quark confinement, chiral symmetry breaking, and baryon stability. From their effect on the SYM, the various kinds of matter are separated to two groups. Our results would give important information in studying the cosmological development of our universe.

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**I. INTRODUCTION**

Up to now, various holographic approaches to the  $\mathcal{N} = 4$  supersymmetric Yang-Mills (SYM) theory with strong coupling have been performed from the dual supergravity [1–10]. In these, the research has been extended to the SYM in the background  $dS_4(\text{AdS}_4)$  by introducing 4D cosmological constant ( $\Lambda_4$ ) [11–18]. Then, it has been found that the dynamics of the SYM theory is largely controlled by the 4D geometry, the  $dS_4$  and  $\text{AdS}_4$  [15,16].

In the case of  $dS_4$  ( $\Lambda_4 > 0$ ), a horizon<sup>1</sup> exists in the bulk geometry as in the case of  $\text{AdS}_5$ -Schwarzschild background, which is dual to the SYM theory in the high temperature deconfinement phase. As expected from this similarity of the background, the gauge theory in  $dS_4$  is in the deconfinement phase even if the theory is in the confinement phase at the limit of  $\Lambda_4 = 0$  [15]. In fact, the positive  $\Lambda_4$  plays a role similar to the temperature, then we could see the screening of the confining force above the corresponding scale of  $\Lambda_4$ .

For  $\Lambda_4 = 0$ , the 4D boundary background is represented by the Minkowski space-time, and the confinement phase is realized by introducing a nontrivial dilaton, which implies the condensate of the gauge field strength. This condensate provides the tension of the linear potential between the quark and the anti-quark [9]. In the present universe, however, very small positive  $\Lambda_4$  would be believed to exist, then the confining force responsible to dilaton would be screened. However the screening effect would appear at very large distance between quarks, so we could fortunately find stable mesons and baryons in our universe since the effect of  $\Lambda_4$  is negligible within the

hadron scale. However, at early universe, the situation would be changed.

On the other hand, in the case of  $\text{AdS}_4$  ( $\Lambda_4 < 0$ ), the horizon disappears and we could find both the quark confinement and chiral symmetry breaking even if we neglect the effect of the non-trivial dilaton, which is necessary in the Minkowski space-time for the confinement.<sup>2</sup> In  $dS_4$ , however, the screening effect of the positive  $\Lambda_4$  overwhelms the dilaton effect at large scale. In the  $\text{AdS}_4$ , we find that the negative  $\Lambda_4$  and the dilaton are cooperative to realize the confinement, and furthermore the negative  $\Lambda_4$  induces the chiral symmetry breaking [16].

In these approaches, we found that the deformation of the boundary space-time due to the  $\Lambda_4$  plays an important role in determining the dynamical properties of the SYM theory living in this curved space-time. This point is understood from the bulk metric which is also deformed from the  $\text{AdS}_5$ , and here we denote it as  $\widetilde{\text{AdS}}_5$ . An explicit example of such a  $\widetilde{\text{AdS}}_5$  is shown in the Sec. IV by the Eqs. (49)–(51),<sup>3</sup> and we should notice that it is reduced to the undeformed  $\text{AdS}_5$  in the limit of  $\lambda = 0$ . The situation is the same with the case of  $\text{AdS}_5$ -Schwarzschild background which reduces to  $\text{AdS}_5$  at the zero temperature limit. Thus the deformation of  $\widetilde{\text{AdS}}_5$  is characterized by the parameter ( $s$ ) of the boundary theory, e.g., temperature, 4D cosmological constant, etc. These parameters are essential to determine the dynamical properties of the CFT on the

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<sup>1</sup>Here we express the horizon by the zero points of the metric, which is not necessarily the time component.<sup>2</sup>It would be interesting to compare this result with the observation given in Ref. [19] many years ago. The authors in Ref. [19] has found a discrete mass spectrum for a free scalar field in  $\text{AdS}_4$ , and this spectrum coincides with our result for the mass spectrum of a scalar meson, which is considered as a bound state of quark and anti-quark in our holographic approach.<sup>3</sup>The Eqs. (22)–(25) in Sec. II also represent another kind of  $\widetilde{\text{AdS}}_5$ .

boundary. Thus, as a result of the analyses up to now, it would be able to extend the duality relation as follows,

$$\begin{aligned} \text{SUGRA in } \widetilde{\text{AdS}}_5 &\leftarrow \text{dual} \\ &\rightarrow \text{CFT in 4D curved space time,} \end{aligned} \quad (1)$$

where the bulk  $\widetilde{\text{AdS}}_5$  comes back to the  $\text{AdS}_5$  by reducing the parameter(s) of the 4D theory living on the boundary, and we find

$$\begin{aligned} \text{SUGRA in } \text{AdS}_5 &\leftarrow \text{dual} \\ &\rightarrow \text{CFT in 4D Minkowski space time.} \end{aligned} \quad (2)$$

According to the above idea, we extend our analysis to the case of time dependent  $\lambda$  by introducing various kinds of matter living in the boundary. In our universe, there are many other ingredients, which control the 4D space-time of our universe, other than  $\Lambda_4$ . It is therefore important to make clear how do they could change the properties of the SYM theory. This issue is examined here by extending our holographic analysis to the case where various kinds of matter are included. This is performed simply by replacing the  $\Lambda_4$  to a time dependent form,  $\lambda(t)$ , which is given as

$$\lambda(t) = \frac{1}{3} \left( \Lambda_4 + \kappa^2 \sum_u \frac{\rho_u}{a_0(t)^{3(1+u)}} \right) \equiv \sum_{n=3u+2} \lambda_n, \quad (3)$$

where  $\kappa^2$  and  $a_0(t)$  denote the 4D gravitational constant and the three dimensional scale factor of the Robertson-Walker metric,

$$ds_{(4)}^2 = -dt^2 + a_0(t)^2 \gamma_{ij} dx^i dx^j, \quad (4)$$

which is set as the boundary metric in our analysis. Here  $\gamma_{ij} = \delta_{ij}/(1 + k \sum_i x_i^2)^2/4$  and  $k = \pm 1$ , or 0. The energy density of the various kinds of matter are expressed by  $\rho_u$ , where  $u$  denotes the ratio of the pressure  $p$  to the energy density of the matter,

$$u = \frac{p}{\rho_u}. \quad (5)$$

The  $\lambda_n$  of the right-hand side is introduced to describe each term in the form of  $\lambda_n \propto 1/a_0(t)^{n+1}$ , where  $n = 3u + 2$ . In this notation, we could express various kinds of ordinary matter by integer  $n$ , however we extend to the case of noninteger in order to include abnormal kinds of matter.

The generalized  $\lambda(t)$  appears in the bulk Einstein equations, then the differential equation for the dilaton becomes a little complicated [18] due to the time-dependence of the  $\lambda$ . So we set the dilaton as a trivial one for the simplicity since our purpose is to make clear the role of the various kinds of matter as in the case that we have studied the effects of  $\Lambda_4$  on the SYM theory. As for the competition with the dilaton contribution, it will be postponed to study it into the future.

When  $\lambda$  depends on the time, the form of the time component metric ( $g_{00}$ ) is largely changed from the one of the time-independent case of  $\Lambda_4$ . On the other hand, other components are not changed in the form. Because of this modification of the metric, it is not so simple to see the changing of the dynamical properties of SYM from the case of the constant  $\Lambda_4$ . By picking up one component  $\lambda_n$  from  $\lambda(t)$ , its effect on the SYM theory is studied, and we find that many kinds of matter are classified to two groups, (A)  $\lambda_n < 0$  and  $n < 0$ , and (B) others. For group (A), quark confinement and chiral symmetry breaking are realized, and negative  $\Lambda_4$  is included in this group. On the other hand, positive  $\Lambda_4$  and the ordinary kinds of matter are included in the group (B), and the quark deconfinement phase is realized. As for the chiral symmetry, it is restored for most cases except for very large  $n$  case. We discuss about the possibility of this exotic matter of large  $n$  and positive  $\lambda_n$ .

Furthermore, we examined the effect of the matter on the baryon vertex which is expressed by the D5 brane in the present type IIB model. The D5 brane wraps on  $S^5$ , then its action depends only on  $g_{00}$  of (A)dS<sub>5</sub> and the metric of  $S^5$ . As a result, we can see the effect of  $g_{00}$  directly in this case, and we find the stability of the vertex in both groups when the condition,  $n\lambda_n > 0$ , is satisfied.

In the next section, the model for the bulk theory is given and the holographic Friedmann equation is explained. Then the background solution used here is obtained. In Sec. III, we examine the energy momentum tensor of holographic SYM theory in order to see the relation to the one of the matter. In Sec. IV, the role of various kinds of matter in determining the dynamical properties of the SYM is examined through the Wilson loop and the chiral condensate by introducing the probe D7 brane. In Sec. V, the stability of the baryon vertex is examined. Summary and discussions are given in the final section.

## II. SETUP OF HOLOGRAPHIC THEORY

First, we briefly review our model. We start from the 10d type IIB supergravity retaining the dilaton  $\Phi$ , axion  $\chi$  and selfdual five form field strength  $F_{(5)}$ ,

$$\begin{aligned} S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} &\left( R - \frac{1}{2} (\partial\Phi)^2 \right. \\ &\left. + \frac{1}{2} e^{2\Phi} (\partial\chi)^2 - \frac{1}{4 \cdot 5!} F_{(5)}^2 \right), \end{aligned} \quad (6)$$

where other fields are neglected since we do not need them, and  $\chi$  is Wick rotated [20]. Under the Freund-Rubin ansatz for  $F_{(5)}$ ,  $F_{\mu_1 \dots \mu_5} = -\sqrt{\Lambda}/2 \epsilon_{\mu_1 \dots \mu_5}$  [21,22], and for the 10d metric as  $M_5 \times S^5$  or

$$\begin{aligned} ds_{10}^2 &= g_{MN} dx^M dx^N + g_{ij} dx^i dx^j \\ &= g_{MN} dx^M dx^N + R^2 d\Omega_5^2, \end{aligned}$$

we consider the solution. Here, the parameter is set as  $(\mu =)1/R = \sqrt{\Lambda}/2$ .

The equations of motion of noncompact five dimensional part  $M_5$  are written as<sup>4</sup>

$$R_{MN} = \frac{1}{2}(\partial_M \Phi \partial_N \Phi - e^{2\Phi} \partial_M \chi \partial_N \chi) - \Lambda g_{MN} \quad (8)$$

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N \Phi) = -e^{2\Phi} g^{MN} \partial_M \chi \partial_N \chi, \quad (9)$$

$$\partial_M (\sqrt{-g} e^{2\Phi} g^{MN} \partial_N \chi) = 0. \quad (10)$$

These equations have a supersymmetric solutions when the following ansatz is imposed for the axion  $\chi$  [20–22],

$$\chi = -e^{-\Phi} + \chi_0. \quad (11)$$

And this ansatz is also useful in getting nonsupersymmetric solutions. The merit to use this ansatz (11) is to be able to reduce the above Eqs. (8)–(10) to the following two forms,

$$R_{MN} = -\Lambda g_{MN} \quad (12)$$

and

$$\partial_M (\sqrt{-g} g^{MN} \partial_N e^\Phi) = 0, \quad (13)$$

where we notice that the two Eqs. (9) and (10) are rewritten to the same form with (13). So  $\Phi$  and  $\chi$  are obtained by using the solution of (12), however the dilaton is not important here as mentioned in the introduction. Then we set as  $\Phi = \chi = 0$  hereafter for the simplicity.

### A. Holographic Friedmann equation

We solve the 5D Einstein equation (12) by supposing the following metric and coordinates,

$$ds_E^2 = -n^2(t, y) dt^2 + a(t, y)^2 \gamma_{i,j} dx^i dx^j + dy^2 \quad (14)$$

for the Einstein frame metric [23] since this metric is useful to study the cosmological development of the universe.

#### 1. Dark radiation (C)

In terms of this metric, the following equation is obtained from the Einstein equation of  $tt$  and  $yy$  components of (12) [23,24],

$$\left(\frac{\dot{a}}{na}\right)^2 + \frac{k}{a^2} = -\frac{\Lambda}{4} + \left(\frac{a'}{a}\right)^2 + \frac{C}{a^4}, \quad (15)$$

<sup>4</sup>The five dimensional  $M_5$  part of the solution is obtained by solving the following reduced Einstein frame 5d action,

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left( R + 3\Lambda - \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial\chi)^2 \right), \quad (7)$$

which is written in the string frame and taking  $\alpha' = g_s = 1$  and the opposite sign of the kinetic term of  $\chi$  is due to the fact that the Euclidean version is considered here [20].

where  $\dot{a} = \partial a / \partial t$  and  $a' = \partial a / \partial y$ . The parameter  $k$  is set as  $\pm 1$  or 0 according to the situation and the sign of 4D cosmological constant. The integration constant  $C$  must be a constant with respect to both  $y$  and  $t$  in order to satisfy other components of Einstein equations. The term proportional to  $C$  is called as “dark radiation” since it is proportional to  $a^{-4}$ . From holographic viewpoint, it has been cleared that this term corresponds to the energy density of the thermal Yang Mills fields with a definite temperature [17,25].

### 2. Holographic Friedmann equation

Further, by setting the following ansatz [23,24],

$$n(t, y) = \frac{\dot{a}(t, y)}{\dot{a}_0(t)}, \quad a = a_0(t)A(t, y), \quad (16)$$

the Eqs. (15) and (16) are rewritten as,

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{k}{a_0^2} = -\frac{\Lambda}{4}A^2 + (A')^2 + \frac{C}{a_0^4 A^2}, \quad (17)$$

$$ds_E^2 = A^2(y, t)(-\bar{n}^2(t, y)dt^2 + a_0(t)^2 \gamma_{i,j} dx^i dx^j) + dy^2,$$

$$\bar{n} = \frac{n}{A}. \quad (18)$$

At this stage, there are three unknown functions,  $a_0(t)$ ,  $A(y, t)$  and  $n(y, t)$ , in spite of the two equations, (16) and (17), to be solved. Then one of the three should be given by some reasonable assumption. Our strategy is to find  $A(y, t)$  and  $n(y, t)$  by solving (16) and (17) with  $a_0(t)$  which is given as a solution of the equation on the boundary.

Here the boundary of the present bulk is represented by the following metric

$$ds_{(4)}^2 = -dt^2 + a_0(t)^2 \gamma_{i,j} dx^i dx^j, \quad (19)$$

since our solution behaves as  $\bar{n} \rightarrow 1$  and  $A(y, t) \rightarrow \infty$  for  $y \rightarrow \infty$  as shown in the next section. Then the 4D Friedmann equation appeared in the standard cosmology is expressed as

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{k}{a_0^2} = \frac{\Lambda_4}{3} + \frac{\kappa_4^2}{3} \left( \frac{\rho_m}{a_0^3} + \frac{\rho_r}{a_0^4} + \frac{\rho_u}{a_0^{3(1+w)}} \right) = \lambda(t), \quad (20)$$

where  $\kappa_4$  ( $\Lambda_4$ ) denotes the 4D gravitational constant (cosmological constant).  $\rho_m$  and  $\rho_r$  denote the energy density of the nonrelativistic matter and the radiation of 4D theory respectively. The  $\lambda(t)$  in Eq. (20) represents the same one of (3) given above although some terms are written explicitly in Eq. (20). It is important to be able to solve the bulk Eq. (17) by relating its left-hand side to the Friedmann equation on the boundary at any  $y$ .

In this way,  $A(y, t)$  and  $n(y, t)$  are solved by using the time dependent function  $\lambda(t)$ . We notice here that, in solving for  $A(y, t)$  and  $n(y, t)$ , it is not necessary to know the explicit form of  $\lambda(t)$ . As for the explicit form of  $a_0(t)$ , we discuss it in the final stage by solving Eq. (20) for  $a_0(t)$ .

Through this procedure, we can see how the parameters of 4D Friedmann equation (20) control the 4D dual theory, the SYM theory. Further we notice following points; In a sense, Eq. (17) is similar to Eq. (20) given at the boundary. So we could interpret Eq. (17) as the Friedmann equation given at the slice of finite  $y$  off the boundary. In this sense, we call Eq. (17) here as the "holographic Friedman equation" by combining it with the 4D Friedmann equation (20).

Here we must be careful about the relation between the radiation density  $\rho_r$  and the dark radiation  $C$ . By comparing Eqs. (20) and (17), one might expect that the dark radiation term may approach to the radiation term of (20) in the boundary limit as

$$\frac{C}{A^2} \xrightarrow{y \rightarrow \infty} \frac{\kappa_4^2}{3} \rho_r. \quad (21)$$

However, this correspondence is misleading since  $1/A^2 \rightarrow 0$  at the boundary. Then  $\frac{C}{A^2}$  disappears there. This fact is consistent with the fact that the dark radiation belongs to the SYM theory, which is dual to the bulk gravity. So it should decouple from the gravity on the boundary, and then it doesn't appear in the 4D Friedmann equation of the boundary. This is assured from the fact that the effective gravitational coupling constant is expressed by  $1/A^2$  which vanishes at the boundary. So  $\rho_r$  in Eq. (20) has nothing to do with the dark radiation. The radiation  $\rho_r$  in 4D boundary is therefore independent of the dual SYM theory. However, we must notice that both radiations give dynamical effects on the holographic gauge theory.

### 3. Solution

Finally, in this section, we give the solution of  $A$  and  $n$  by using  $\lambda(t)$ . They are obtained by replacing the coordinate from  $y$  to  $r$  defined as  $r/R = e^{\mu y}$ . Then, from (16)–(18), we have

$$ds_{10}^2 = \frac{r^2}{R^2} (-\bar{n}^2 dt^2 + \bar{A}^2 a_0^2(t) \gamma^2(x) (dx^i)^2) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2, \quad (22)$$

$$\bar{A} = \left( \left( 1 - \frac{\lambda}{4\mu^2} \left( \frac{R}{r} \right)^2 \right)^2 + \tilde{c}_0 \left( \frac{R}{r} \right)^4 \right)^{1/2}, \quad (23)$$

$$\bar{n} = \frac{\left( 1 - \frac{\lambda}{4\mu^2} \left( \frac{R}{r} \right)^2 \right) \left( 1 - \frac{\lambda + \frac{a_0}{\lambda}}{4\mu^2} \left( \frac{R}{r} \right)^2 \right) - \tilde{c}_0 \left( \frac{R}{r} \right)^4}{\sqrt{\left( 1 - \frac{\lambda}{4\mu^2} \left( \frac{R}{r} \right)^2 \right)^2 + \tilde{c}_0 \left( \frac{R}{r} \right)^4}}, \quad (24)$$

where

$$\tilde{c}_0 = C/(4\mu^2 a_0^4). \quad (25)$$

We can see that the boundary geometry coincides with (20) since  $\bar{A}$  and  $\bar{n}$  approach to one for  $r \rightarrow \infty$  ( $y \rightarrow \infty$ ).

Then (20) can be used as a boundary condition, and it determines  $a_0(t)$ .

### III. ENERGY MOMENTUM $\langle T_{\mu\nu} \rangle$

At first, we study 4D stress tensor from holographic approach. In order to give it, we rewrite the 5d part of the metric (22). According to the Fefferman-Graham framework [26–28], it is given as

$$\begin{aligned} ds_{(5)}^2 &= \frac{r^2}{R^2} (-\bar{n}^2 dt^2 + \bar{A}^2 a_0^2(t) \gamma^2(x) (dx^i)^2) + \frac{R^2}{r^2} dr^2 \\ &= \frac{1}{\rho} \hat{g}_{\mu\nu} dx^\mu dx^\nu + \frac{d\rho^2}{4\rho^2} \\ &= \frac{1}{\rho} (-\bar{n}^2 dt^2 + \bar{A}^2 a_0^2(t) \gamma^2(x) (dx^i)^2) + \frac{d\rho^2}{4\rho^2}, \end{aligned} \quad (26)$$

where  $\rho = 1/r^2$ ,  $R = 1$  and

$$\bar{A} = \left( \left( 1 - \frac{\lambda}{4\mu^2} \left( \frac{\rho}{R^2} \right) \right)^2 + \tilde{c}_0 \left( \frac{\rho}{R^2} \right)^2 \right)^{1/2}, \quad (27)$$

$$\bar{n} = \frac{\left( 1 - \frac{\lambda}{4\mu^2} \left( \frac{\rho}{R^2} \right) \right) \left( 1 - \frac{\lambda + \lambda a_0/\dot{a}_0}{4\mu^2} \left( \frac{\rho}{R^2} \right) \right) - \tilde{c}_0 \left( \frac{\rho}{R^2} \right)^2}{\bar{A}}. \quad (28)$$

In the present case,  $\hat{g}_{\mu\nu}$  is expanded as [27]

$$\begin{aligned} g_{\mu\nu} &= g_{(0)\mu\nu} + g_{(2)\mu\nu} \rho \\ &\quad + \rho^2 (g_{(4)\mu\nu} + h_{1(4)\mu\nu} \log \rho + h_{2(4)\mu\nu} (\log \rho)^2) + \dots, \end{aligned} \quad (29)$$

where

$$g_{(0)\mu\nu} = (g_{(0)00}, g_{(0)ij}) = (-1, a_0(t)^2 \gamma_{ij}), \quad (30)$$

and

$$g_{(2)\mu\nu} = \frac{\lambda}{2} \left( 1 + \frac{a_0}{\lambda} \dot{\lambda}, -g_{(0)ij} \right), \quad (31)$$

$$g_{(4)\mu\nu} = \frac{\tilde{c}_0}{R^4} (3, g_{(0)ij}) + \frac{\lambda^2}{16} \left( -\frac{(\lambda + \frac{a_0}{\lambda} \dot{\lambda})^2}{\lambda^2}, g_{(0)ij} \right). \quad (32)$$

Then by using the following formula [26],

$$\begin{aligned} \langle T_{\mu\nu} \rangle &= \frac{4R^3}{16\pi G_N} \left( g_{(4)\mu\nu} - \frac{1}{8} g_{(0)\mu\nu} ((\text{Tr} g_{(2)})^2 - \text{Tr} g_{(2)}^2) \right. \\ &\quad \left. - \frac{1}{2} (g_{(2)}^2)_{\mu\nu} + \frac{1}{4} g_{(2)\mu\nu} \text{Tr} g_{(2)} \right), \end{aligned} \quad (33)$$

we find

$$\langle T_{\mu\nu} \rangle = \frac{4R^3}{16\pi G_N^{(5)}} \left( \frac{\tilde{c}_0}{R^4} (3, g_{(0)ij}) + \frac{3\lambda^2}{16} (1, \beta g_{(0)ij}) \right), \quad (34)$$

$$\beta = -\left(1 + \frac{2\frac{a_0}{\dot{a}_0}\lambda}{3\lambda}\right). \quad (35)$$

Then we have

$$\langle T_{\mu\nu} \rangle = \langle \tilde{T}_{\mu\nu}^{(0)} \rangle + \frac{4R^3}{16\pi G_N^{(5)}} \left\{ \frac{3\lambda^2}{16} (1, \beta g_{(0)ij}) \right\}. \quad (36)$$

$$\langle \tilde{T}_{\mu\nu}^{(0)} \rangle = \frac{4R^3}{16\pi G_N^{(5)}} \frac{\tilde{c}_0}{R^4} (3, g_{(0)ij}), \quad (37)$$

where  $\langle \tilde{T}_{\mu\nu}^{(0)} \rangle$  is the stress tensor corresponding to the thermal YM fields given in (3) for the case of  $\lambda = 0$ . The second term comes from the loop corrections of the YM fields in the curved space-time. While the first term does not contribute to the conformal anomaly, namely

$$\langle \tilde{T}_{\mu}^{(0)\mu} \rangle = 0, \quad (38)$$

the second term leads to the anomaly as follows

$$\langle T_{\mu}^{\mu} \rangle = -\frac{3\lambda^2(1 + \frac{\lambda}{2\lambda} \frac{a_0}{\dot{a}_0})}{8\pi^2} N^2, \quad (39)$$

where we used  $G_N^{(5)} = 8\pi^3 \alpha'^4 g_s / R^5$  and  $R^4 = 4\pi N \alpha'^2 g_s$ .

We can see that the anomaly (39) is the same one obtained from the loop corrections in the  $\mathcal{N} = 4$  SYM theory for a curved space-time, which is given by (19) as the boundary space-time here. In this background, the curvature squared terms, which are responsible to the anomaly, are given as

$$R^{\mu\nu\lambda\sigma} R_{\mu\nu\lambda\sigma} = 12 \left( 2\lambda^2 + \lambda\lambda \frac{a_0}{\dot{a}_0} + \left( \lambda \frac{a_0}{2\dot{a}_0} \right)^2 \right), \quad (40)$$

$$R^{\mu\nu} R_{\mu\nu} = 12 \left( 3\lambda^2 + 3\lambda\lambda \frac{a_0}{2\dot{a}_0} + \left( \lambda \frac{a_0}{2\dot{a}_0} \right)^2 \right), \quad (41)$$

$$\frac{1}{3} R^2 = 12 \left( 4\lambda^2 + 4\lambda\lambda \frac{a_0}{2\dot{a}_0} + \left( \lambda \frac{a_0}{2\dot{a}_0} \right)^2 \right). \quad (42)$$

In general, the conformal anomaly for  $n_s$  scalars,  $n_f$  Dirac fermions and  $n_v$  vector fields is given as [29,30]

$$\langle T_{\mu}^{\mu} \rangle = -\frac{n_s + 11n_f + 62n_v}{90\pi^2} E_{(4)} - \frac{n_s + 6n_f + 12n_v}{30\pi^2} I_{(4)}, \quad (43)$$

$$E_{(4)} = \frac{1}{64} (R^{\mu\nu\lambda\sigma} R_{\mu\nu\lambda\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2), \quad (44)$$

$$I_{(4)} = -\frac{1}{64} \left( R^{\mu\nu\lambda\sigma} R_{\mu\nu\lambda\sigma} - 2R^{\mu\nu} R_{\mu\nu} + \frac{1}{3} R^2 \right), \quad (45)$$

where  $\square R$  has been abbreviated since it does not contribute here. For the  $\mathcal{N} = 4$  SYM theory, the numbers of the fields are given by  $N^2 - 1$  times the number of each fields,

which are equivalent to  $n_s = 6$ ,  $n_f = 2$  and  $n_v = 1$ . Then we find for large  $N$ ,

$$\langle T_{\mu}^{\mu} \rangle = \frac{N^2}{32\pi^2} \left( R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right) = -\frac{3\lambda^2(1 + \frac{\lambda}{2\lambda} \frac{a_0}{\dot{a}_0})}{8\pi^2} N^2. \quad (46)$$

This result (46) is precisely equivalent to the above holographic one (39).

Thus we could show that the holographic analysis could provide correct results for the energy momentum tensor even if the metric is time dependent. Then we can say that any matter field, which decouples to the  $\mathcal{N} = 4$  SYM theory, could give an influence to the  $\mathcal{N} = 4$  SYM theory through the curvatures.

### A. Continuity equation of SYM fields

Before seeing the other effects, we see another important fact. The energy density of the SYM theory is composed of two kind of contents, the one of the thermal SYM fields and the one of the vacuum energy ( $\lambda^2$ -dependent terms) obtained as the quantum corrections. This density obeys the following continuity equation,

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (47)$$

where  $\rho$ ,  $p$  and  $H$  represent the energy density, pressure and the Hubble constant. Here  $H = \dot{a}_0/a_0$ . In the present case, we obtain them from (36) as

$$\begin{aligned} \rho &= 3\alpha \left( \frac{\tilde{c}_0}{R^4} + \frac{\lambda^2}{16} \right), \\ p &= \alpha \left( \frac{\tilde{c}_0}{R^4} - 3 \frac{\lambda^2}{16} \left( 1 + \frac{2\lambda}{3\lambda} \frac{a_0}{\dot{a}_0} \right) \right), \\ \alpha &= \frac{4R^3}{16\pi G_N^{(5)}}. \end{aligned} \quad (48)$$

It is easy to see that the continuity equation (47) is satisfied by the above  $\rho$  and  $p$ . This is satisfied even if  $\lambda = 0$ , therefore the  $\lambda$  dependent part also satisfies without the thermal part. This fact implies that the energy momentum is not transferred between the thermal part (SYM part) and the  $\lambda(t)$  dependent part (matter part). This is consistent with the fact that the SYM theory studied here decouples to the gravity and also to the matter in the  $\lambda(t)$ . In other words, the various kinds of matter are responsible for determining  $a(t)$  through the 4D Friedmann equation, however the SYM fields aren't. So the role of the matter is to reform the space-time background where the SYM theory lives. As a result, this reformed background changes the dynamical properties of the SYM theory as shown below.

### IV. DYNAMICAL ROLE OF DECOUPLED MATTER

Here we study the dynamical effect of the matter on the SYM theory. Many kinds of matter are introduced through

the time-dependent cosmological term  $\lambda(t)$  as in (20). In order to see its pure effect, we set  $\tilde{c}_0 = 0$ . Then the metric is written as

$$ds_{10}^2 = \frac{r^2}{R^2} (-\bar{n}^2 dt^2 + \bar{A}^2 a_0^2(t) \gamma^2(x) (dx^i)^2) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2. \quad (49)$$

$$\bar{A} = \left(1 - \frac{\lambda}{4\mu^2} \left(\frac{R}{r}\right)^2\right), \quad (50)$$

$$\bar{n} = \left(1 - \frac{\lambda + \frac{a_0}{\lambda} \dot{\lambda}}{4\mu^2} \left(\frac{R}{r}\right)^2\right). \quad (51)$$

The  $\bar{n}$  is simplified when we restrict to one kind of matter field ( $\lambda_m$ ) in  $\lambda$  whose energy density behaves as  $\epsilon \propto 1/a_0(t)^{n+1}$ . In this case,  $\bar{n}$  is written as follows

$$\bar{n} = \left(1 + n \frac{\lambda_m}{4\mu^2} \left(\frac{R}{r}\right)^2\right). \quad (52)$$

We notice here the following point that, in  $\bar{n}$  for  $n > 0$  and positive  $\lambda_m$ , the horizon in  $\bar{n}$  disappears. On the other hand, it remains in  $\bar{A}$ .

### A. Wilson-loop and quark confinement

The quarks are introduced through probe D7 branes. By presuming D7 brane embedding, we can consider the Wilson-Loop whose boundary is on the D7 brane. In order to study the potential between quark and anti-quark, we consider the U-shaped (in  $r-x$  space) string whose two end-points are on D7 brane as studied in Ref. [15]. Supposing the string action whose world volume is set in  $(t, x)$  plane,<sup>5</sup> the energy  $E$  of this state is obtained as a function of the distance ( $L$ ) between the quark and anti-quark according to Ref. [15].

Taking the gauge as  $X^0 = t = \tau$  and  $X^1 = x^1 = \sigma$  for the coordinates  $(\tau, \sigma)$  of string world-volume, the Nambu-Goto Lagrangian in the present background (22) becomes

$$L_{\text{NG}} = -\frac{1}{2\pi\alpha'} \int d\sigma \bar{n}(r) \sqrt{r'^2 + \left(\frac{r}{R}\right)^4 (\bar{A}(r) a_0(t) \gamma(x))^2}, \quad (53)$$

where we notice  $r' = \partial r / \partial x = \partial r / \partial \sigma$ . Then the energy of this configuration is rewritten to more convenient form according to Gubser [31] as

$$E = -L_{\text{NG}} = \frac{1}{2\pi\alpha'} \int d\tilde{\sigma} n_s \sqrt{1 + \left(\frac{R^2}{r^2 \bar{A}} \partial_{\tilde{\sigma}} r\right)^2}, \quad (54)$$

<sup>5</sup>Here  $x$  denotes one of the three coordinate  $x^i$ , and we take  $x^1$  in the present case.

$$\tilde{\sigma} = a_0(t) \int d\sigma \gamma(\sigma) = a_0(t) \int d\sigma \frac{1}{1 - \sigma^2/4}, \quad (55)$$

$$n_s = \left(\frac{r}{R}\right)^2 \bar{A} \bar{n}. \quad (56)$$

In the present case, we use the proper coordinate  $\tilde{\sigma}$  instead of the comoving coordinate  $\sigma$  to measure the distance between the quark and anti-quark.

It is the criterion of the confinement that  $n_s$  has a finite minimum value at some appropriate  $r (= r^*)$ . Actually, in such a case,  $E$  is approximated as [15]

$$E \sim \frac{n_s(r^*)}{2\pi\alpha'} L, \quad (57)$$

where

$$L = 2 \int_{\tilde{\sigma}_{\min}}^{\tilde{\sigma}_{\max}} d\tilde{\sigma}, \quad (58)$$

and  $\tilde{\sigma}_{\min}$  ( $\tilde{\sigma}_{\max}$ ) is the value at  $r_{\min}$  ( $r_{\max}$ ) of the string configuration [16]. The tension of the linear potential between the quark and anti-quark is therefore given as

$$\tau_{q\bar{q}} = \frac{n_s(r^*)}{2\pi\alpha'}. \quad (59)$$

### 1. Two groups of the matter

In the case of the matter  $\lambda_m$  considered above,  $n_s$  is obtained by using Eqs. (50) and (52) as follows

$$\bar{A} = 1 - \frac{r_m^2}{r^2}, \quad \bar{n} = 1 + n \frac{r_m^2}{r^2}, \quad (60)$$

$$n_s = \left(\frac{r}{R}\right)^2 \left(1 + n \frac{r_m^2}{r^2}\right) \left(1 - \frac{r_m^2}{r^2}\right), \quad (61)$$

where  $r_m^2 = \lambda_m R^4 / 4$ . When this matter is dominant, we find the necessary condition for the confinement is given by

$$(A) \quad \lambda_m < 0, \quad \text{and} \quad n < 0. \quad (62)$$

In this case, we obtain

$$r^* = \left(-n \left(\frac{\lambda_m R^4}{4}\right)^2\right)^{1/4}, \quad (63)$$

$$n_s(r^*) = \frac{n \lambda_m R^2}{4} \left(1 + \frac{1}{\sqrt{|n|}}\right)^2. \quad (64)$$

On the other hand, for the case,

$$(B) \quad \text{Other than (A)}, \quad (65)$$

there is no finite minimum of  $n_s$ . Therefore, the YM theory is in the deconfinement phase in this case. As a result, the matter is separated to two groups (A) and (B) as given above. The matter of group (A) ((B)) works to confine (deconfine) quarks.

The above form of (61) is compared to the cases of the positive ( $dS_4$ ) and negative ( $AdS_4$ ) constant  $\lambda (= \Lambda_4/3)$ , in which the corresponding factors are given as follows

$$n_s^{dS_4} = \left(\frac{r}{R}\right)^2 \left(1 - \frac{r_0^2}{r^2}\right)^2. \quad (66)$$

and

$$n_s^{AdS_4} = \left(\frac{r}{R}\right)^2 \left(1 + \frac{r_0^2}{r^2}\right)^2, \quad (67)$$

for each case. Here  $r_0^2 = \lambda_0 R^4/4$ , where  $\lambda_0 > 0$ . We know that the area law of the Wilson loop for the quark and anti-quark is obtained for the case of  $n_s^{AdS_4}$ , then we find the confinement. This belongs to the group (A) with  $n = -1$ . On the other hand, we find the deconfinement phase for  $n_s^{dS_4}$  since it has zero valued minimum at  $r = r_0$ . In fact, this belongs to the group (B) with  $n = -1$ .

## 2. A mixed example

It is possible to consider the case where several kinds of matter with different  $n$  and the sign of  $\lambda_m$  are coexisting. As a simple example, we consider here the combination of 4D negative cosmological constant ( $n = -1$ ) (group (A)) and a matter ( $n \neq -1$ ) of group (B),  $\lambda_m > 0$ . In this case, the combined  $\lambda$  is given as

$$\lambda = -\lambda_0 + \lambda_m, \quad (68)$$

where  $\lambda_0 > 0$  and  $\lambda_m \propto 1/a_0^{n+1}(t)$ . Then  $n_s$  is written as

$$n_s^{(2)} = \left(\frac{r}{R}\right)^2 \left(1 + \frac{\lambda_0 + n\lambda_m}{4\mu^2 r^2}\right) \left(1 + \frac{\lambda_0 - \lambda_m}{4\mu^2 r^2}\right). \quad (69)$$

In this case, the factor  $n_s^{(2)}$  has a finite minimum at  $r = r^*$ , which is given as

$$n_s^{(2)}(r^*) = \frac{r_c^2 + nr_m^2}{R^2} \left(1 + \sqrt{\frac{r_c^2 - r_m^2}{r_c^2 + nr_m^2}}\right)^2, \quad (70)$$

$$r^* = ((r_c^2 + nr_m^2)(r_c^2 - r_m^2))^{1/4}, \quad (71)$$

where

$$r_c^2 = \frac{\lambda_0}{4\mu^4}, \quad r_m^2 = \frac{\lambda_m}{4\mu^4}. \quad (72)$$

Here (70) is obtained for  $r_c^2 > r_m^2$ . On the other hand, for  $r_c^2 < r_m^2$ , the minimum of  $n_s^{(2)}$  is zero which is obtained at  $r^2 = r_m^2 - r_c^2$ . Then the quark deconfining phase is realized in this case.

In order to understand well the results given above, the  $E - L$  relation and the tension  $\tau_{q\bar{q}}$  are examined from numerical estimation in the followings. Since the Lagrangian in (53) does not explicitly depend on the coordinate  $\sigma = x$ , we find the following relation,

$$\frac{1}{\sqrt{(r/R)^4 \bar{A}^2(r) + (r')^2}} \left(\frac{r}{R}\right)^4 \bar{n} \bar{A}^2(r) = H, \quad (73)$$

where  $H$  denotes a constant of motion. And we notice that  $r' = \partial_{\bar{\sigma}} r$ . We can fix  $H$  at any point where we like, so we fix it at  $r = r_{\min}$ . Then, taking as  $H = \left(\frac{r}{R}\right)^2 \bar{n}(r) \bar{A}(r)|_{r_{\min}}$ , we get

$$\begin{aligned} L &= 2R^2 \int_{r_{\min}}^{r_{\max}} dr \frac{1}{r^2 \bar{A}(r) \sqrt{r^4 \bar{n}(r)^2 \bar{A}(r)^2 / (r_{\min}^4 \bar{n}(r_{\min})^2 \bar{A}(r_{\min})^2) - 1}}, \\ E &= \frac{1}{\pi \alpha'} \int_{r_{\min}}^{r_{\max}} dr \frac{\bar{n}(r)}{\sqrt{1 - r_{\min}^4 \bar{n}(r_{\min})^2 \bar{A}(r_{\min})^2 / (r^4 \bar{n}(r)^2 \bar{A}(r)^2)}}. \end{aligned} \quad (74)$$

Figure 1 shows the dependence of the energy  $E$  on the distance  $L$  for  $\lambda - \lambda_0 = -0.5 < 0$  (curve A) and  $\lambda - \lambda_0 = 1.5 > 0$  (curve B). In the former case, the matter energy density  $\lambda$  is small compared to the absolute value of the negative cosmological constant  $\lambda_0$ , then we find the linear potential at large  $L$  as expected. On the other hand, in the case of  $\lambda > \lambda_0$ , we find a typical screening behavior as seen in the finite temperature deconfinement phase.

From this fact, we could say that the matter of group (B) screens the long range force which is needed to confine quarks and it comes from group (A) matter. Then the effect of the 4D matter of group (B) on the SYM theory is similar to the one given by the thermal matter in the SYM theory.

## B. Chiral condensate

In the next section, we study the effects of the decoupled matter on the chiral condensate. We introduce D7 brane to study the chiral condensate of the quark fields. The D7-brane action is written by the Dirac-Born-Infeld (DBI) and the Chern-Simons (CS) terms as follows,

$$\begin{aligned} S_{D8} &= -T_8 \int d^8 \xi e^{-\Phi} \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})} \\ &+ T_5 \int \sum_i (\exp^{2\pi\alpha' F^{(2)}} \wedge c_{(a_1 \dots a_i)})_{0 \dots 5}, \end{aligned} \quad (75)$$

$$\begin{aligned} g_{ab} &\equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}, \\ c_{a_1 \dots a_i} &\equiv \partial_{a_1} X^{\mu_1} \dots \partial_{a_i} X^{\mu_i} C_{\mu_1 \dots \mu_i}, \end{aligned}$$

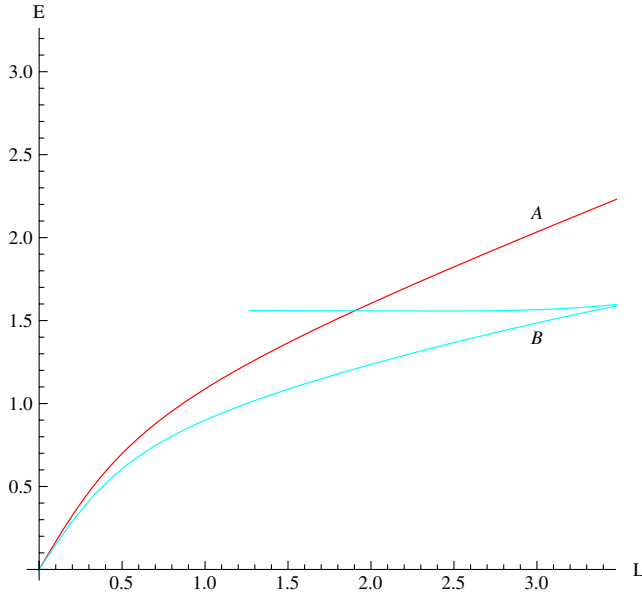


FIG. 1 (color online). Plots of  $E$  vs  $L$  for (A)  $\lambda = \lambda_0 - 0.5$  and (B)  $\lambda = \lambda_0 + 1.5$ , where  $n = 10$ ,  $\lambda_0 = 1.0$  and  $\mu = 1/R = 1$ . Further,  $\alpha' = 1$ .  $E$  increases linearly with  $L$  at large  $L$  for the case of (A),  $\lambda - \lambda_0 < 0$ . This behavior is obtained due to negative 4D cosmological constant  $-\lambda_0$ . The case of (B),  $\lambda - \lambda_0 > 0$ , shows the case of the deconfinement phase due to large matter energy density  $\lambda$ .

where  $T_5 = 1/(g_s(2\pi)^5 l_s^6)$  is the brane tension. The DBI action involves the induced metric  $g_{ab}$  and the  $U(1)$  world volume field strength  $F_{(2)} = dA_{(1)}$ .

The D7 branes are embedded in the background, which is given by (22)–(25), by rewriting the extra six dimensional of (22) as follows

$$\frac{R^2}{r^2} dr^2 + R^2 d\Omega_3^2 = \frac{R^2}{r^2} \left( d\rho^2 + \rho^2 d\Omega_3^2 + \sum_{i=8}^9 dX^i{}^2 \right), \quad (76)$$

where the new coordinate  $\rho$  is introduced instead of  $r$  with the relation

$$r^2 = \rho^2 + (X^8)^2 + (X^9)^2. \quad (77)$$

Thus, the induced metric of the D7 brane is obtained as

$$ds_8^2 = \frac{r^2}{R^2} (-\bar{n}^2 dt^2 + \bar{A}^2 a_0^2(t) \gamma^2(x) (dx^i)^2) + \frac{R^2}{r^2} ((1 + w^2) d\rho^2 + \rho^2 d\Omega_3^2), \quad (78)$$

where the profile of the D7 brane is taken as  $(X^8, X^9) = (w(\rho), 0)$  and  $w' = \partial_\rho w$ , then

$$r^2 = \rho^2 + w^2. \quad (79)$$

In the present case, there is no R-R field, so the action is given only by the one of DBI as

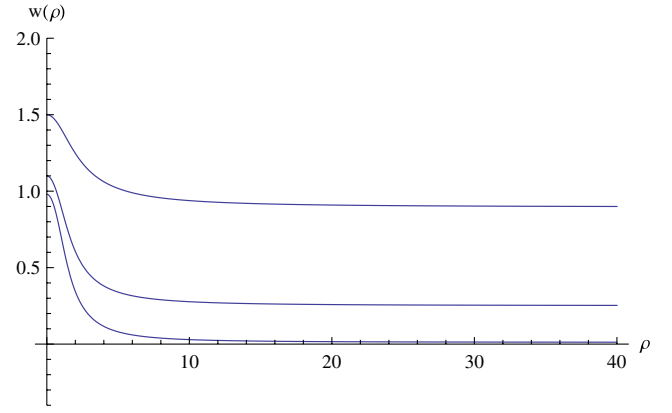


FIG. 2 (color online). An example of group (A) matter. Plots of  $w(\rho)$  vs  $\rho$  for  $\lambda = -2$ ,  $\mu = 1$ ,  $n = -0.5$ . The curves are given for  $w(0) = 1.5, 1.1, 0.98$  from the top to the bottom.

$$S_{D8} = -T_8 \Omega_3 \int d^4 x a_0^3(t) \gamma^3(x) \int d\rho \rho^3 \bar{A}^3 \bar{n} \sqrt{1 + w'^2(\rho)}, \quad (80)$$

where  $\Omega_3$  denotes the volume of  $S^3$  of the D7's world volume.

From this action, the equation of motion for  $w$  is obtained as

$$w'' + \left( \frac{3}{\rho} + \frac{\rho + ww'}{r} \partial_r (\log(\bar{A}^3 \bar{n})) \right) w' (1 + w'^2) - \frac{w}{r} (1 + w'^2)^2 \partial_r (\log(\bar{A}^3 \bar{n})) = 0. \quad (81)$$

The constant  $w$  is not the solution of this equation, so the supersymmetry is broken.

### 1. For group (A) matter

For the matter of group (A), the numerical solutions of (81) for  $w(\rho)$  are shown in Fig. 3. In general, in this case, we find finite chiral condensate  $\langle \bar{\Psi} \Psi \rangle = c$  for any  $m_q$  since the curves decrease from the above with increasing  $\rho$  according to the following asymptotic form

$$w = m_q + \frac{c}{\rho^2} + \dots, \quad (82)$$

at large  $\rho$  with  $c > 0$ . We can observe spontaneous chiral symmetry breaking from the third curve shown in Fig. 2. It shows the mass generation of a massless quark due to the chiral condensate  $\langle \bar{\Psi} \Psi \rangle$ .

As a result, we could say that the spontaneous mass generation of massless quarks is realized due to the matter of group (A) in the 4D space-time. Then the matter of group (A) contributes to both confinement and chiral symmetry breaking for the SYM theory. The situation is similar to the case of  $\text{AdS}_4$ .



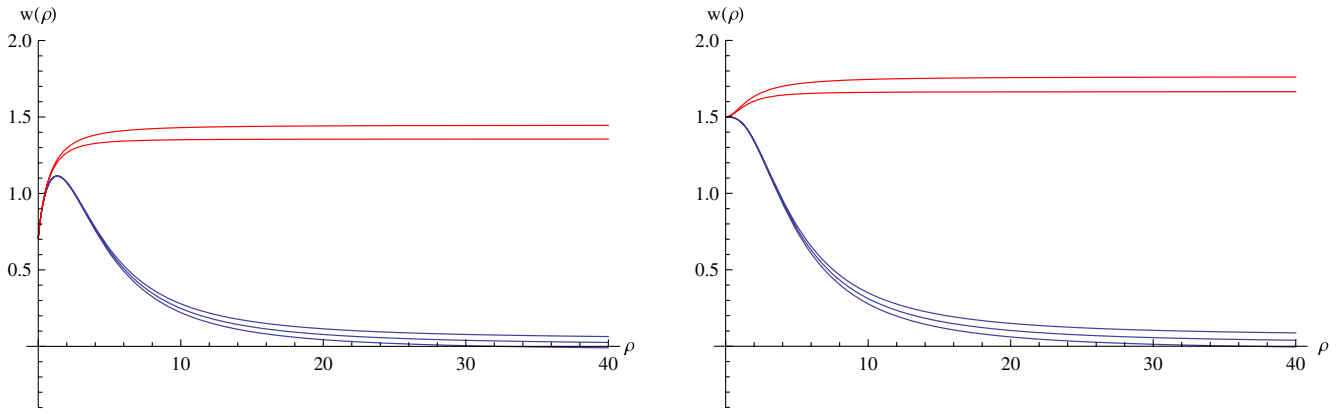


FIG. 3 (color online). For the group (B) matter. Plots of  $w(\rho)$  vs  $\rho$  for  $\lambda = 2$ ,  $\mu = 1$ . The curves are given for  $n = 2, 3, 50, 60, 70$  from the top to the bottom. The left (right) figure for  $r_m < w(0) = 0.71 < 2r_m$  ( $w(0) = 1.50 < 2r_m$ ). Each curves of the left and right figures have common behavior near  $\rho = 0$ .

### 2. For group (B) matter

For the matter of group (B), they are separated further to the two groups by the behavior near  $\rho = 0$ , which is given as

$$w = w_0 + w_2 \rho^2 + \dots, \quad (83)$$

for  $w_0 > r_m$ .<sup>6</sup> As shown below, we find two cases i) positive and ii) negative  $w_2$ , the sign of the coefficient  $w_2$ . We notice that only the case of negative  $w_2$  appears for group (A) matter.

### 3. Behavior of $w$ near $\rho = 0$

The behavior near small  $\rho$  is seen as follows. By substituting this into (81) and using (60), we find

$$w_2 = \frac{r_m^2 (4nr_m^2 + (3-n)w_0^2)}{4w_0(w_0^2 - r_m^2)(w_0^2 + nr_m^2)}. \quad (84)$$

Then we find  $w_2 > 0$  for the following two cases,

$$w_0 < 2r_m, \quad n > 0 \quad (85)$$

and

$$w_0 > 2r_m, \quad n < \frac{3w_0^2}{w_0^2 - 4r_m^2}. \quad (86)$$

On the other hand, for the following third case,

$$w_0 > 2r_m, \quad n > \frac{3w_0^2}{w_0^2 - 4r_m^2}, \quad (87)$$

we find  $w_2 < 0$ .

<sup>6</sup>In the case of  $w_0 < r_m$ , the value of  $\rho$  is restricted as  $\rho \geq \sqrt{r_m^2 - w_0^2}$  since the metric  $\bar{A}$  has zero point at  $r = r_m$ .  $\bar{n}$  also has zero point at different value of  $r$ , and it is larger than  $r_m$  for  $n < -1$  and  $\lambda_m > 0$ . However, we do not consider this case here since it does not produce new things.

In the latter case (87), we could expect finite chiral condensate as in the case of the matter of group (A). In order to see this point we need numerical analysis since it is difficult to obtain the analytic solution which describe  $w(\rho)$  for all region of  $\rho$ .

For  $n = 3$  and  $n = 4$  (non-relativistic matter and radiation).—For  $n + 1 = 3$  (for  $\rho_m$  or non-relativistic matter) and 4 (for  $\rho_r$  or radiation), it seems to be impossible to find the spontaneous mass generation even if the situation is set as  $w_2 < 0$ . This is assured from the results shown in Fig. 3, where the two cases of  $n + 1 = 3$  and 4 are shown for both  $r_m < w_0 < 2r_m$  and  $2r_m < w_0$  by the upper red curves in each case. The behavior of these solutions of  $w(\rho)$  shows the similar one of the high temperature or the negative constant  $\lambda_0$  case. In both cases, the SYM theory is in the deconfinement and chiral symmetric phase. In the previous section, we have shown that the role of the group (B) matter is similar to the one of the high temperature SYM in the deconfinement phase. In this sense, the results for the matter of  $n + 1 = 3$  and 4 is consistent with the results of the previous section.

Large  $n$  matter.—Next, we consider the matter given by  $\rho_u/a_0^{3(1+u)} = \rho_u/a_0^{n+1}$  with large  $n$  from the viewpoint of theoretical interest. Astonishingly, as shown in Fig. 3, the behavior of the solutions for very large  $n$  show the asymptotic form given by Eq. (82) with positive chiral condensate,  $c$ . So we summarize the analysis for large  $n$  as follows;

- (i) We could find the solution with positive chiral condensate,  $c$ , for any value of  $w_0$  for enough large  $u$ .
- (ii) The solution with  $m_q = 0$  and  $c > 0$  could be found at appropriate large value of  $n$  for any matter of group (B). Actually, we could show such solutions at about  $n(\sim 70)$  for both cases of  $w_2 < 0$  and  $w_2 > 0$  as shown in Fig. 3.

We give a comment on this matter of large  $u$  (or  $n$ ). Consider a model for this matter in terms of a scalar field  $\phi$

with a self-interacting potential which is given by  $V(\phi)$ . In this case, the parameter  $u$  for this scalar is given as,

$$u = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}, \quad (88)$$

where  $\phi(t)$  is assumed to be solved for an appropriate form of  $V(\phi)$ . The present purpose is not to study this model in detail but to point out the possibility of large  $u$  case. This may be found when  $V$  becomes negative at some time-interval of varying  $\phi(t)$ . It would be an interesting problem how and when this interval would appear in the universe. However we will discuss this point in other article where cosmological problem is studied at the same time.

## V. D5 BRANES AND BARYON

Here we show the stability of the baryon vertex through the analysis of the D5 brane action which corresponds to the baryon vertex, which combines the  $N_c$  quarks to make a color singlet.

First, we briefly review the model based on type IIB superstring theory [32–36]. In the type IIB model, the vertex is described by the D5 brane which wraps  $S^5$  of the 10D manifold  $M_5 \times S^5$ . In this case, in the bulk, there exists the following form of self-dual Ramond-Ramond field strength

$$G_{(5)} \equiv dC_{(4)} = \frac{4}{R}(\epsilon_{S^5} + *\epsilon_{S^5}) \quad (89)$$

$$\epsilon_{S^5} = R^5 \text{vol}(S^5) d\theta_1 \wedge \cdots \wedge d\theta_5, \quad (90)$$

where  $\text{vol}(S^5) \equiv \sin^4\theta_1 \text{vol}(S^4) \equiv \sin^4\theta_1 \sin^3\theta_2 \sin^2\theta_3 \sin\theta_4$ , and  $\epsilon_{S^5}$  denotes the volume form of  $S^5$  part. The flux from the stacked D3 branes flows into the D5 brane as  $U(1)$  field which is living in the D5 brane.

The effective action of D5 brane is given by using the Born-Infeld and Chern-Simons term as follows

$$S_{D5} = -T_5 \int d^6\xi e^{-\Phi} \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})} + T_5 \int (2\pi\alpha' F_{(2)} \wedge c_{(4)})_{0\dots 5}, \quad (91)$$

$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu},$$

$$c_{a_1\dots a_4} \equiv \partial_{a_1} X^{\mu_1} \cdots \partial_{a_4} X^{\mu_4} C_{\mu_1\dots\mu_4},$$

where  $T_5 = 1/(g_s(2\pi)^5 l_s^6)$  and  $F_{(2)} = dA_{(1)}$ , which represents the  $U(1)$  worldvolume field strength. In terms of (the pullback of) the background five-form field strength  $G_{(5)}$ , the above action can be rewritten as

$$S_{D5} = -T_5 \int d^6\xi e^{-\Phi} \sqrt{-\det(g + F)} + T_5 \int A_{(1)} \wedge G_{(5)}.$$

The embedding of the D5 brane is performed by solving the  $r(\theta)$ ,  $x(\theta)$ , and  $A_{(1)}(\theta)$  [36]. They are retained as

dynamical fields in the D5 brane action as the function of  $\theta \equiv \theta_1$  only. The equation of motion for the gauge field  $A_{(1)}$  is written as

$$\partial_\theta D = -4\sin^4\theta,$$

where the dimensionless displacement is defined as the variation of the action with respect to  $E = F_{t\theta}$ , namely  $D = \delta\tilde{S}/\delta F_{t\theta}$  and  $\tilde{S} = S/T_5\Omega_4 R^4$ . The solution to this equation is

$$D \equiv D(\nu, \theta) = \left[ \frac{3}{2}(\nu\pi - \theta) + \frac{3}{2}\sin\theta\cos\theta + \sin^3\theta\cos\theta \right]. \quad (92)$$

Here, the integration constant  $\nu$  is expressed as  $0 \leq \nu = k/N_c \leq 1$ , where  $k$  denotes the number of Born-Infeld strings emerging from one of the pole of the  $S^5$ .

Next, it is convenient to eliminate the gauge field in favor of  $D$ , then the Legendre transformation is performed for the original Lagrangian to obtain an energy functional as [34–36]:

$$U = \frac{N}{3\pi^2\alpha'} \int d\theta \bar{n} \sqrt{r^2 + r'^2 + (r/R)^4 x'^2 (\bar{A}a_0\gamma)^2} \sqrt{V_\nu(\theta)}. \quad (93)$$

$$V_\nu(\theta) = D(\nu, \theta)^2 + \sin^8\theta, \quad (94)$$

where we used  $T_5\Omega_4 R^4 = N/(3\pi^2\alpha')$ . Then, in this expression, (93),  $r(\theta)$  and  $x(\theta)$  are remained, and they are solved by minimizing  $U$ . As a result, the D5 brane configuration is determined.

For simplicity, here, we restrict to the point like configuration, namely  $r$  and  $x$  are constants. In this case, we have for the matter considered here

$$U = r\bar{n}(r)U_0 = r\left(1 + n\frac{\lambda_m}{4\mu^2}\left(\frac{R}{r}\right)^2\right)U_0, \quad (95)$$

where  $U_0$  is a constant given as

$$U_0 = \frac{N}{3\pi^2\alpha'} \int d\theta \sqrt{V_\nu(\theta)}. \quad (96)$$

From (95), we find that  $U$  has a minimum at  $r_m = \sqrt{n\lambda_m}R^2/2$ . For group (A), this assures the stability of the baryon vertex.

As for the group (B) matter, we must restrict  $r$  as  $r > \sqrt{\lambda_m}R^2/2$ . So we can see the stability for  $n > 1$ . This situation is different from the effect of the thermal Yang-Mills field which screens the long range confinement force and destabilize the baryon vertex [37]. It would be an interesting problem to study the stability for more complicated case, however, it will be postponed to the future work here.

## VI. SUMMARY AND DISCUSSIONS

Here, the holographic approach is extended to the SYM theory in a time-dependent curved space-time. Then the SYM theory in such a universe, which is represented by the RW type of metric, is examined by adding the 4D cosmological constant and the matter which is decoupled from the SYM field but control the RW metric. In the holographic approach, the SYM theory decouples also from the gravity on the boundary. Then, the form of the RW metric on the boundary is given as a solution of 4D Friedmann equation, which is obtained from the Einstein equation on the boundary with the added matter. The matter therefore determines the scale factor of the RW metric. This process to solve the 4D Friedmann equation is performed independently of the SYM theory. In spite of this fact, the dynamical properties of the SYM theory are controlled by the added matter through the holographic dual 5D Einstein equation, which leads to a deformed  $\widetilde{\text{AdS}}_5$ . This background is denoted as  $\widetilde{\text{AdS}}_5$ . Namely, the effect of the matter added is reflected to the bulk geometry  $\widetilde{\text{AdS}}_5$ , which is deeply related to the 4D Friedmann equation as a result.

The energy density of various kinds of matter is introduced in the form of  $\rho_u/a_0^{3(u+1)}(t) \propto 1/a_0^{n+1}(t)$  in the 4D Friedmann equation. For example, the non-relativistic matter and the radiation are correspond to the one of  $u = 0$  and  $u = 1/3$  respectively. The effects of these ordinary kinds of matter are similar to the case of the positive cosmological constant, namely we find the similar dynamical properties to the one found for the SYM theory

in the  $dS_4$ . In these cases, therefore, the confinement force is screened above an appropriate distance, and furthermore the chiral symmetry is restored.

On the other hand, we find confinement and chiral symmetry breaking for  $\rho_u < 0$  and  $n < 0$  (group(A)), and the negative cosmological constant belongs to this group. Then, we can separate the matter to two groups (A) and others (group (B)). As for the group (B), they are further separated to ordinary kinds of matter and exotic one. The ordinary kinds of matter are the one of  $u = 0$  and  $u = 1/3$  mentioned above, and the positive cosmological constant. They lead to deconfinement and chiral symmetric phase of the SYM theory. The one called as exotic matter has very large  $u$  (or large  $n$ ) compared to the ordinary kinds of matter. As an effect of this matter, the chiral symmetry of the theory is spontaneously broken.

Furthermore we have examined the effect of these kinds of matter on the D5 brane as the baryon vertex, which is in general unstable in the case of deconfining phase. However, in the case of the matter of group (B) with any  $n \geq 1$ , we find that the D5 brane is stable.

The results obtained here are important when we study the developing universe since the matter belonging to the SYM theory is the important ingredient of the universe, and the SYM is surrounded by other kind of matter which control the dynamical properties of the SYM theory.

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